Bargain for Exit

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We examine an alternate move bargaining model of an exit process in a declining industry. We show that when firms negotiate with each other, the less profitable firm will be merged to be closed down. We find that the size of the firm will affect the division of the surplus created by the merger through cost effects. The smaller firm extracts more surplus than its value under continued operation because it has cost advantage. (*JEL* Classification: C78, D43, L13)

I. Introduction

One of the important policy concerns in structural adjustment of the dynamic economy is an exit process in a declining industry. A pioneering study on this subject was undertaken by Ghemawat and Nalebuff (1985). They showed that in a declining industry where larger firms incur more capacity cost, smaller firms can endure longer and will be the last to exit. Whinston (1988) reexamined their results when firms have multi—plants with different sizes, and showed that Ghemawat-Nalebuff equilibrium may not be the unique equilibrium. The small firm may be the first to exit. In an extension of their earlier model, Ghemawat and Nalebuff (1990) allowed continuous reduction of capacities and showed that the bigger firms reduce their capacity first to the size of their rivals. Fudenberg and Tirole (1986) examined an exit game in the framework of war of attrition and showed that the more efficient firm outlasts the less efficient one.

But recent empirical studies on exit from declining industries[1] do not always confirm theoretical implications of exit decision. The unconsistency arises partly from the lack of adequate theory that explains facors

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[1]See for example Lieberman (1990) among others.

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governing the various forms of exit, whether it be merger, voluntary liquidation or bankrupcy.

In this paper we introduce a bargaining process to investigate an exit process through merger and rationalization. For this purpose we keep all the other aspects to the simplest setting; duopoly with single plant firms and complete information. We assume that firms are not allowed to merge before they suffer from losses.[2] The main result we obtain is that the profitability determines which plant-firm is to be closed down first. However, the division of the surplus created by the merger is affected by the size of the firms; the larger the firm is, the smaller its share. In case where the small firm is to exit, its exit value exceeds the value of the firm under continued operations. This implies that even if there are substantial scale economies in rationalization of existing production facilities and management system, the larger firm may incur more than its investment costs to remain in the market through rationalization strategy.

II. The Model

Consider a declining industry. The inverse demand curve is denoted by $p(q, t)$, where q is the industry output level and t represents time. Following Ghemawat and Nalebuff (1985) we assume that for all $q > 0$

$$\frac{\partial p(q,t)}{\partial t} < 0, \quad \frac{\partial p(q,t)}{\partial q} < 0, \quad \lim_{t \to \infty} p(q,t) = 0, \quad \text{and} \quad \frac{\partial p(q,t)q}{\partial q} > 0.$$

There are two firms denoted by $i = 1, 2$ with capacities K_1 and K_2, where $K_1 > K_2$. Firms have identical cost flows per unit of capacity denoted by $c > 0$. There are no other costs, hence the assumption of positive marginal revenue implies that the capacity is always fully used.

We introduce a bargaining process into the model. During the bargaining process merger can take place by agreement. We model the bargaining process à la Rubinstein (1982). Buy-out offers are made periodically by one of the firms. The time interval between the offers is denoted by Δ. Hence, the offers are made at $t = 0, \Delta, 2\Delta, \ldots$. If firm 1 makes the offer at $n\Delta$, then firm 2 makes the offer at $(n + 1)\Delta$, and

[2]In some countries governments are less strict in enforcing the anti-trust laws. See for example Itoh, kiyono, Okuno, and Suzumura (1988) for the Japanese cartel policy in declining industries.

vice versa. Each time when an offer is rejected, firms decide whether to exit or not. If a firm exits, its profit becomes zero in that particular period. If both firms stay, then the bargaining process continues until one firm exits or is bought out. As soon as the merger takes place, the less profitable firm should be shut down. Otherwise the bargaining process produces an inefficient outcome. We are interested in the bargaining outcome when Δ is small but fixed.

We assume that the government allows merger only when the industry begins to generate losses. Assume that, when both firms are in business, the market clearing price is equal to the unit cost at $t = 0$; $p(K_1 + K_2, 0) = c$. Hence bargaining starts at $t = 0$. At $t > 0$, if merger does not take place and no firm exits, then firm i will suffer (discounted) loss $l_i(t) \equiv [c - p^t]K_i e^{-rt}$, where $p^t = p(K_1 + K_2, t)$ and r is the discount rate. We assume that the market demand shrinks fast enough to insure that $l_i(t)$ increases over time. Alternatively, we could assume that the discount rate is close to 0.

Denote by $\pi_i(t)$ frim i's (discounted) *monopoly* profit at $t \geq 0$; $\pi_i(t) \equiv [p(K_i, t) - c]K_i e^{-rt}$. Since $p(K_i, t)$ is decreasing to 0, there is t such that $\pi_i(t) = 0$. Denote such t's by τ_i, $i = 1, 2$. It is clear that $\tau_1 < \tau_2$. Notice that each firm i will exit not later than τ_i, the bargaining process must be concluded before τ_2. Therefore, the bargaining we analyze has essentially a finite number of periods. Let us define $R_i(t)$ to be the (discounted) sum of firm i's lifetime monopoly profits starting from t; $R_i(t) \equiv \int_t^{\tau_i} \pi_i(s)\, ds$. When $R_1(0) > R_2(0)$, it is efficient from the industry point of view that the smaller firm is closed down. However, under free competition between two firms, it is usually the smaller one that will survive longer. (See Ghemawat and Nalebuff 1985.) There are therefore gains to be made (for the firms) by coordinating the exit process. One can conjecture that allowing mergers in declining industries would restore the efficient outcome. Below we will show this formally and investigate how the surplus is divided between the merging partners. On the other hand, when $R_1(0) \leq R_2(0)$, there is no need for the firms to coordinate. In this case, the bargaining outcome will be exactly the same as in the competitive model. Thus, we assume that $R_1(0) > R_2(0)$. In fact we will use a slightly stronger assumption to simplify the analysis. We assume that $\pi_1(0) > \pi_2(0)$ and that $\pi_1(t) - \pi_2(t)$ is decreasing in t until τ_1. When $\pi_1(0) > \pi_2(0)$, the second condition is satisfied by any demand functions which are time-separable; $p(q, t) = a(t)p(q)$. Under this assumption, there is a unique time $\tau_0 \in (0, \tau_1)$ such that $\pi_1(\tau_0) = \pi_2(\tau_0)$. Since $R_1(t) - R_2(t)$ is decreasing in t until τ_0, and since $R_1(0) > R_2(0)$ and $R_1(\tau_0) < R_2(\tau_0)$,

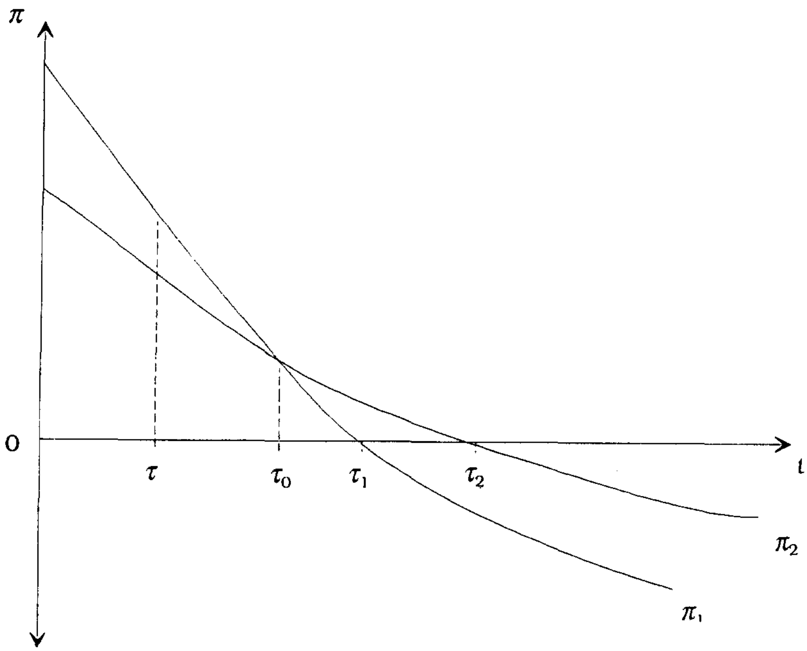

FIGURE 1

there is a unique time $\tau \in (0, \tau_0)$ such that $R_1(\tau) < R_2(\tau)$. Since there is no gains from coordination after τ, bargaining must end before τ. In Figure 1, typical profit functions are drawn, and τ, τ_0, τ_1, and τ_2 are marked.

III. The Bargaining Outcome

We investigate the subgame perfect equilibrium (SPE) bargaining outcomes. Hence, we will solve the game backwards. Our first proposition summarizes what will happen when the smaller firms is more profitable. In this case, since there are no gains from merger, the bargaining outcome is the same as the competitive outcome. The proof is rather standard and placed in the Appendix.

***Proposition* 1**

Suppose the bargaining starts at t at which the (discounted) sum of the monopoly profit of the smaller firm is larger (i.e., when $t > \tau$). Then, the unique SPE bargaining outcome coincides with the competitive outcome: the larger firm exits immediately at t, and the smaller firm stays till the last moment as a monopolist.

This confirms our conjecture. In the current setup bargaining generates an efficient outcome, which is to shut down the less profitable firm, the larger one. However, we expect a different outcome when the larger firms is more profitable over its lifetime.

At this point we introduce some notation to facilitate the exposition of the paper. Recall that $l_i(t) = [c - p^t]K_i e^{-rt}$ is firm i's instantaneous (discounted) loss at time t when both firms are in operation. Define $L_i(t)$ to be i's one period loss starting from t; $L_i(t) \equiv \int_t^{t+\Delta} l_i(s)\, ds$. Similarly, let us define $\Pi_i(t)$ to be firm i's one period monopoly profit starting from t; $\Pi_i(t) \equiv \int_t^{t+\Delta} \pi_i(s)\, ds$.

In backward induction it is crucial to know what will happen next. For example, an offer made by a proposer will be determined by what the responder can get after rejecting the current offer. A responder who rejects an offer can either continue bargaining or exit. It turns out that firm 2 will never exit before firm 1 does for two reasons. First, when both are in operation firm 2's loss is smaller than firm 1's. This gives firm 2 more bargaining power and a larger share of the surplus. Secondly, 2's advantage increases toward the end of the bargaining. As shown in Proposition 1, firm 2 can earn non-trivial sum of profit after τ. This prospect gives firm 2 strong motivation to stay in towards the end, which discourages firm 1 even further. On the other hand, firm 1 will always exit rather than stay and incur losses whenever its share in the following period becomes too small. However, as we move farther away from the last bargaining period in the backward induction, firm 1's payoff increases, eventually exceeding its one period loss, and at this point it too stays even after an offer has been rejected. The time at which this occurs, which is endogenously determined (and will be denoted by τ^* later), will be an important parameter in describing the outcome.

In the following we distinguish two cases; the case where it is always optimal for firm 1 to continue bargaining rather than exit and the case where it is optimal for firm 1 to exit rather than stay after a certain period. Let us first consider the case where it is always optimal for firm 1 to continue bargaining. In this case, responder's payoff will be its next period payoff minus one period loss. On the other hand, proposer gets firm 1's one period monopoly profit plus responder's one period loss in addition to the next period payoff. Formally, let $V_i(t)$ denote firm

i's payoff in the subgame beginning at t. The, firm i's payoff is determined according to the following rules;

$$V_i(t) = V_i(t+\Delta) + \Pi_1(t) + L_j(t), \text{ if } i \text{ is a proposer}$$

$$V_i(t) = V_i(t+\Delta) - L_i(t), \text{ if } i \text{ is a responder.}$$

Since firms alternate in making offers, we get

$$V_i(t) = V_i(t+2\Delta) - L_i(t+\Delta) + \Pi_1(t) + L_j(t), \text{ if } i \text{ is a proposer}$$

$$V_i(t) = V_i(t+2\Delta) + \Pi_1(t+\Delta) + L_j(t+\Delta) - L_i(t), \text{ if } i \text{ is a responder.}$$

As long as the payoffs are non-negative, we can generalize this to obtain

$$V_i(t) = V_i(t+2m\Delta) + \sum_{k=0}^{m-1}\{\Pi_1(t+2k\Delta) + L_j(t+2k\Delta) - L_i(t+2k\Delta+\Delta)\},$$
$$\text{if } i \text{ is a proposer,}$$

$$V_i(t) = V_i(t+2m\Delta) + \sum_{k=0}^{m-1}\{\Pi_1(t+2k\Delta+\Delta) + L_j(t+2k\Delta+\Delta) - L_i(t+2k\Delta)\},$$
$$\text{if } i \text{ is a responder.}$$

As $\Delta \rightarrow 0$, we obtain in the limit

$$V_i(t) = V_i(t') + \frac{1}{2}\int_t^{t'}\{\pi_1(s) + l_j(s) - l_i(s)\}ds,$$

provided that the integrand is positive for $t \leq s \leq t'$, whether i is a proposer or a responder. Since $l_1(s) > l_2(s)$, $V_2(t)$ will be positive for small Δ, and hence firm 2 will never exit. Whether firm 1 would exit or not is determined by whether or not $V_1(t)$ is positive. Notice that $\pi_1(s) + l_2(s) - l_1(s)$ is decreasing in s. Let us define τ_c by $\pi_1(\tau_c) + l_2(\tau_c) - l_1(\tau_c) = 0$. Suppose that $\tau < \tau_c$. This will be the case when $K_1 - K_2$ is small enough, since as $K_1 - K_2 \rightarrow 0$, $\tau \rightarrow 0$ and $\tau_c \rightarrow \tau_i$. Then, $\int_t^{\tau}\{\pi_1(s) + l_2(s) - l_1(s)\}\,ds > 0$ for all $t < \tau$. Since the bargaining must be concluded which includes τ, $V_1(t) > 0$ for small Δ, and firm 1 will not exit even after an offer is rejected. If we define $\bar{t}$ to be the period before τ, then $V_1(\bar{t}) = R_1(\bar{t}) - R_2(\bar{t})$ or 0 depending on whether 1 is a proposer or a responder in that period. Define $T_i(a, b)$ to be the set of time periods between a and b when firm i is the proposer. When there is no danger of confusion, we will just write T_i.

Proposition 2

When $\tau \leq \tau_c$, firms' payoffs at $t \leq \tau$ are

$$V_1(t) = V_1(\bar{t}) + \sum_{s \in T_1} [\Pi_1(s) + L_2(s)] - \sum_{s \in T_2} L_1(s),$$

$$V_2(t) = R_1(\bar{t}) - V_1(\bar{t}) + \sum_{s \in T_2} [\Pi_1(s) + L_1(s)] - \sum_{s \in T_1} L_2(s).$$

Next suppose that it may be the case that a responder can do better by exiting rather than staying. This will be the case if $V_i(t + \Delta) < L_i(t)$ when i is a responder. This is possible when $\tau_c < t < \tau$. In this case a responder's payoff will be determined according to the following rule;

$$V_i(t) = \max\{0,\ V_i(t + \Delta) - L_i(t)\}.$$

This makes the inductive calculation of the equilibrium payoffs complicated. There is another factor which makes it further complicated. If one firm exits, then the remaining firm's payoff is not determined by the inductive calculation which presupposes continuation of bargaining. The remaining firm's payoff will simply be the sum of its monopoly profit to be earned during its lifetime, $R_i(t)$. Hence, when it is the case that the proposer would *optimally* exit after its offer has been declined, the proposer (let us denote it by j) could extract all the surplus above $R_i(t)$, which is what the responder i would get after rejecting j's offer.

Now let us examine this case. Since firms alternate in making offers, we get

$$V_i(t) = \max\{\Pi_1(t) + L_j(t),\ V_i(t + 2\Delta) - L_i(t + \Delta) + \Pi_1(t) + L_j(t)\},$$
if i is a proposer,

$$V_i(t) = \max\{0,\ V_i(t + 2\Delta) + \Pi_1(t + \Delta) + L_j(t + \Delta) - L_i(t)\},$$
if i is a responder.

Again for Δ small enough, $L_1(t + \Delta) - L_2(t)$ will be positive, and hence firm 2 will never exit. We will now show that there exists the smallest t between τ_c and τ at which $V_1(t) = 0$. Consider firm 1's payoff when it is a responder at $t > \tau_c$. If $V_1(t + 2\Delta) > 0$ so that firm 1 stays till $t + 2\Delta$, then $V_1(t) < V_1(t + 2\Delta)$ for small enough Δ, since $\Pi_1(t + \Delta) + L_2(t + \Delta) - L_1(t) \simeq [\pi_1(t) + l_2(t) - l_1(t)]\Delta < 0$. Define

$$F(t,t') = \sum_{s \in T_1(t,t')} [\Pi_1(s) + L_2(s)] - \sum_{s \in T_1(t,t')} L_1(s).$$

For $t > \tau_c$ (and for Δ small), $F(\cdot\ ,\ t')$ is increasing in every 2 periods. Hence, as we move backwards away from τ, 1's payoff decreases, possibly to 0. (See Figure 2.)

Suppose $V_1(\hat{t}) = 0$ for some $\hat{t}$ between τ_c and τ. This implies that firm 1 is a responder at $\hat{t}$. Then, 1's payoff at $\hat{t} - \Delta$ will be $R_1(\hat{t} - \Delta) - R_2(\hat{t} - \Delta)$,

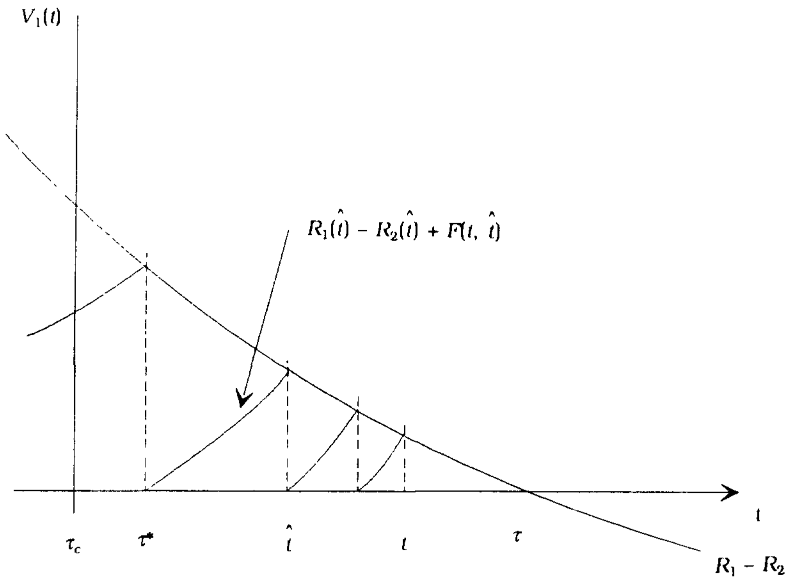

FIGURE 2

and for $t < \hat{t} - \Delta$

$$V_1(t) = V_1(\hat{t} - \Delta) + F(t, \hat{t} - \Delta),$$

as long as $V_1(s) > 0$ for all $s \in (t, \hat{t} - \Delta)$. In this way we can trace 1's payoff back to τ_c. Figure 1 shows graphically how this is done. Let τ^* be the smallest t between τ_c and τ such that $V_1(t) = 0$. Then, we get

Proposition 3

When $\tau_c < \tau$, firms' payoffs at $t \leq \tau$ are

$$V_1(t) = R_1(\tau^* - \Delta) - R_2(\tau^* - \Delta) + F(t, \tau^* - \Delta),$$

$$V_2(t) = R_2(\tau^* - \Delta) + \sum_{s \in T_2}[\Pi_1(s) + L_1(s)] - \sum_{s \in T_1} L_2(s).$$

Notice that, since the SPE payoff after rejection of an offer is unique, a proposer can always have its offer accepted by offering the responder slightly more than its SPE share after rejection. Since the firms get more in sum by agreeing now rather than delaying, the agreement will be reached immediately in equilibrium. Hence, from Propositions 2 and 3, we obtain the following.

Theorem

Suppose that the government allows merger at the time when the industry begins to generate losses. If the firms follow the bargaining procedure described in section II, then merger will take place immediately, and the less profitable firm (firm 2) will be closed. The firms' shares in SPE are

$$V_1(0) = V_1(t^*) + \sum_{s\in T_1}[\Pi_1(s) + L_2(s)] - \sum_{s\in T_2} L_1(s), \text{ and}$$

$$V_2(0) = R_1(t^*) - V_1(t^*) + \sum_{s\in T_2}[\Pi_1(s) + L_1(s)] - \sum_{s\in T_1} L_2(s),$$

where $t^* = \bar{t}$ or τ^*, depending on whether τ_c is larger or smaller than τ.

We also have the following corollaries.

Corollary 1

Suppose two firms do not differ much in capacity size ($K_1 - K_2$ is small enough) so that $\tau < \tau_c$. Then, as $\Delta \to 0$ the payoffs converge to

$$V_1(0) = \frac{1}{2}\int_0^{\tau}\{\pi_1(s) + l_2(s) - l_1(s)\}ds, \text{ and}$$

$$V_2(0) = R_1(\tau) + \frac{1}{2}\int_0^{\tau}\{\pi_1(s) + l_1(s) - l_2(s)\}ds.$$

Proof: Since $\lim_{\Delta\to 0} V_1(\bar{t}) = V_1(\tau) = 0$, the corollary follows from Proposition 2.

Q.E.D.

Corollary 2

Under the stated assumptions in the theorem, $V_2(0) > R_2(0)$ in the SPE.

In other words, the smaller firm always gains more from bargaining than under competition. Notice that under the same cost condition for two types of firms $V_1(0)$ decreases as $(K_1 - K_2)$ decreases. When K_2 converges to K_1, $V_1(0)$ eventually diminishes, and $V_2(0)$ converges to $R_1(0)$. Since $R_1(0)$ approaches to $R_2(0)$ as K_2 converges to K_1, our bargaining outcome resembles to the Ghemawat and Nalebuff competitive outcome.

IV. Concluding Comments

Mergers often take place in declining industries. The questions of practical interest, then, are what determines the buy-out price (the exit value of the merged firm) and how it is compared to the value of the firm as a going concern. For this purpose, we have examined a bargaining model of an exit process in a declining industry. We have shown that when firms negotiate with each other, the less profitable firm will be merged to be closed down. The size of the firm will affect the division of the surplus created by the merger in such a way that the smaller firm with cost advantage extracts more surplus than its value under continued operation.

To make this model more appealing, one may have to examine the following question of general interest: why horizontal mergers do not lead to monopoly everywhere, because as long as bargaining game (with complete information) starts, the merger will immediately occur independently of any market shrinkage or need for exit. We give only a partial answer: the legal restraints on mergers are often lifted in a declining industry as a part of the rationalization policy toward business whenever the decline in demand is caused by unanticipated structural factors such as oil shocks and sudden decline in export demands. Although this does not constitute a clear answer, this justification adds some realism to our model of an exit process. To enhance practicability, one may also introduce uncertainty about the other firm's cost. In this case, the bargaining process may last long, and itself constitutes an exit barrier. To facilitate an efficient exit, the government may run an information sharing mechanism which is compatible with an incentive. But this will be an area for future research.

Appendix

***Proof of Proposition* 1**

First, suppose that the game reaches $t \geq \tau_2$ with both firms in business. Obviously, further bargaining does not make sense and both firms will exit. Suppose the bargaining reaches the last round before τ_2. If no agreement is reached, firm 1 will exit since it expects losses regardless of whether firm 2 stays or exits. Firm 2 will stay and earn its monopoly profit until τ_2. Hence, whoever makes the offer in the last round before τ_2, firm 1 gets 0 and firm 2 gets its monopoly profit. Working backwards to the first round after τ_1, one concludes that firm 1 exits immediately and gets 0, whereas firm 2 stays and earns monopoly profit until τ_2.

Suppose that the game reaches the last round before τ_1 and no agreement is reached. Firm 2 will stay regardless of whether firm 1 stays because the losses to be incurred this period is smaller than the monopoly profit to be earned in the coming periods (when Δ is short enough). Thus, firm 1 will exit immediately. Working backwards, one can conclude that firm 1 earns nothing out of the bargaining hence exits immediately as long as $R_1(t) < R_2(t)$. Firm 2 on the other hand earns $R_2(t)$.

Q.E.D.

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