Optimal Current Account and the External Debt for Korea

Won-Am Park*

The paper investigates the optimal path of Korea’s current account and external debt by building a dynamic model of consumption and investment. It determines the investment and growth on the basis of Tobin’s q theory for a small open economy with foreign debt and examines the relationship among the market value of capital, the marginal product of capital, and the interest rate in Korea. It also discusses the problems in determining the optimal consumption under dynamic solvency constraints. (JEL Classification: F41)

I. Introduction

Korea’s debt problem has been viewed from quite different perspectives between the first and the second halves of the 1980s. The high interest rates and the world recession in the aftermath of the second oil shock made the debt crisis in the developing countries a principal issue in the international economy. The external economic environment, however, turned more favorable to Korea after 1982, as oil prices and interest rates declined. It was far more favorable since 1986 with declining oil prices, a declining dollar, and declining international interest rates. While the rapid accumulation of debt had become a critical concern for the Korean economy before 1986, Korea’s possibility to be a creditor country became the issue after 1986. The drastic change in the view of the debt problem originates from the large current account surplus during 1986-89.

The current account surplus implies domestic savings over investment. The description of the current account surplus as savings minus

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[Seoul Journal of Economics 1993, Vol. 6, No. 2]
investment reflects the intertemporal rather than the static aspects of decisions on savings and investment. According to the traditional static approach, exports and imports depend on income and relative price variables. But there has recently been a growing tendency of a dynamic approach in which the anticipation of the future plays an important role in determining savings, investment, and the current account.\footnote{See Bardhan (1966), Sachs (1981), Blanchard (1983), Abel and Blanchard (1983), and Svensson (1984).} According to the latter approach, investment depends on the present value of future marginal product, while consumption is determined at the level maximizing the present value of lifetime utility under a dynamic budget constraint.

This paper traces the optimal path of foreign debt by building a dynamic model on consumption and investment. It tries to determine the optimal level of investment and growth on the basis of the Tobin's $q$-theory for a small open economy with foreign debt. It also discusses the important issues in determining the optimal consumption only under dynamic solvency constraints and finally examines the relationship among the market value of capital, the marginal product of capital, and the interest rate in Korea.

The $q$-theory stresses the dynamic characteristics of investment but does not satisfactorily explain most of the variation in investment (Abel and Blanchard 1986). It is known that the assumption of a representative economic agent with an infinite life-time is not appropriate for explaining the life-cycle of consumption (Blanchard 1984). With these limitations, we introduce a simple model that could capture dynamic aspects of consumption and investment.

The paper proceeds as follows. A basic model will be introduced in section II. Section III will analyze the effects of a change in the parameters, such as the technological progress and the interest rate, on the market value of capital and investment. Section IV examines the possibility that Korea will be a creditor country in the early 1990s. The topical interest here is the determination of the optimal consumption under the various sovereign risks on external debt. The concluding remarks are provided in section V.

II. Model

The current account balance of a country is equivalent to savings
minus investment. This section introduces a dynamic resource allocation model of savings and investment to investigate their optimal level in Korea.

This paper adopts a model of Blanchard (1983) and Cooper and Sachs (1985). The model consists of a dynamic budget constraint and a social welfare function. The household maximizes utility from (infinite) lifetime consumption under a lifetime budget constraint. The firm maximizes its value and has unrestricted access to the world capital market.

\[
\max_{(C,F)} \sum_{t=0}^{\infty} \left( \prod_{s=0}^{t} \frac{1}{1+\theta_s} \right) L_t U(C_t / L_t) \tag{1}
\]

s.t. \[D_{t+1} = C_t + I_t [1 + \varphi(I_t / K_t)] + (1+r_t)D_t - F_t \tag{2}\]

\[K_{t+1} = K_t (1-\delta) + I_t \tag{3}\]

where

- \(U\): utility function
- \(C\): consumption
- \(I\): gross fixed capital formation
- \(K\): capital stock
- \(r\): (real) interest rate
- \(\theta\): the rate of time preference
- \(D\): (real) net debt in terms of Won
- \(F\): production function of GDP
- \(L\): employment
- \(\delta\): depreciation rate of capital
- \(\varphi\): adjustment costs of investment.

This simple model of equation (1) to (3) can determine optimal consumption and investment as well as foreign debt in a small open economy. The model comes from economic reasoning about the foreign borrowing from the perspective of central planners so that it may not reflect the actual determinants of complicated borrowing strategies. For the planner’s perspective to be consistent with the laissez-faire approach to foreign borrowing, the following necessary conditions must hold. Both the households and firms have perfect foresight (or rational expectations under uncertainty). The social welfare function, \(U\), should be the representative household’s utility function. The private marginal

\[This\ simple\ model\ does\ not\ consider\ the\ monetary\ impacts\ on\ prices\ or\ the\ role\ of\ the\ relative\ prices.\ These\ aspects\ are\ considered\ in\ Bruno\ (1976)\ and\ Svensson\ and\ Razin\ (1983).\ Kharas\ and\ Shishido\ (1986)\ has\ made\ a\ dynamic\ optimization\ model\ of\ foreign\ borrowing\ considering\ changes\ of\ relative\ prices.\]
product of capital is no different from the social marginal product of capital. Also, the domestic and world interest rate should be equalized through "free" access to the world capital market. In addition, for the above model to be sound, the country must face no risk, such as liquidity constraints or repudiation risk, other than insolvency risk.\textsuperscript{3}

The social welfare function or the representative household's utility function is assumed to be a function of per capita consumption. The time preference rate is assumed to vary over time. Equation (1) defines the present value of lifetime utility.

Equation (2) is a dynamic budget constraint and is transformed as follows, in the case of the constant interest rate:

\[
\frac{D_t}{(1+r)^t} = D_0 + \sum_{s=0}^{t-1} \frac{1}{(1+r)^{t-s}} [C_s + I_s (1 + \varphi_s) - F_t].
\]

The transversality condition is assumed to avoid the Ponzi-game:

\[
\lim_{t \to \infty} \frac{D_t}{(1+r)^t} = 0. \tag{4}
\]

Then we get the following:

\[
(1+r)D_t = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} [F_{t+s} - C_{t+s} - I_{t+s} (1 + \varphi_{t+s})]. \tag{5}
\]

The solvency constraint is implied by equation (4) since the discounted sum of current and future trade balances matches the principal plus the interest payments on the existing debt according to equation (5).

In equation (5), the trade balance is equal to the domestic output minus the sum of consumption and investment, but the adjustment costs or the installation costs of investment have been explicitly considered. The adjustment cost, \( \varphi \), is assumed to be a linear function of the ratio of investment to capital stock:

\[
\varphi(I_t/K_t) = \phi \cdot I_t/K_t. \tag{6}
\]

The adjustment costs make total investment expenditures greater than the actual amount of capital formation, thus making the marginal product of capital adjust gradually rather than instantaneously to equal the world cost of capital. On the other hand, the discrepancy

\textsuperscript{3}For a more complicated model that includes liquidity constraints and repudiation risk, see Eaton and Gersovitz (1981), Sachs and Cohen (1983), Cooper and Sachs (1985), and Krugman (1985).
between the current account balance and the actual change in the foreign debt could diminish by accounting for the adjust costs of investment.

The production function, $F_t$, is assumed homogeneous of degree one under Harrod-neutral technological progress:

$$F_t = A [K_t]^{a} [L_t \cdot \prod_{s=0}^{t} (1 + \beta_s)]^{1-a}, \quad (7)$$

where $F_t$: GDP
- $A$: constant
- $K_t$: capital stock at the beginning of period $t$
- $L_t$: average employment
- $\beta_t$: technological progress embodied to labor
- $a$: share of capital.

The optimality conditions are:

$$\mu_t' = U' = (1 + \theta)(1 + r) \mu_{t-1}, \quad (8)$$

$$\lambda_t = - \mu_t (1 + 2\phi \cdot L_t/K_t), \quad (9)$$

$$\lambda_t (1 - \phi) = (1 + \theta) \lambda_{t-1} + \mu_t F_{K_t}' + \phi \cdot (L_t/K_t)^2. \quad (10)$$

where $U' = \partial U / \partial (C_t / L_t)$
- $\mu_t$ = marginal utility of wealth (dynamic Lagrange multiplier attached to equation (2))
- $\lambda_t$ = shadow value of capital (dynamic Lagrange multiplier attached to equation (3)).

Consumption is characterized by equation (8), where the two parameters of interest rate and time preference rate play important roles. If both are the same, the preferred path of per capita consumption is constant. But, time preference rate is assumed to be a function of technological progress, for convenience, to make consumption per capita grow at the rate of technological progress:

$$\theta_t = r_t + \epsilon \beta_t, \quad \epsilon_t = U' / U' \cdot (C_t / L_t), \quad (10)$$

where $\epsilon = -1$, when $U$ is logarithmic.

Thus consumption in efficiency units is constant:

$$C_t = \rho \cdot L_t \cdot \prod_{s=0}^{t} (1 + \beta_s). \quad (11)$$
Equation (9) and (10) show that investment depends only on the variables on the production side, such as technological progress and adjustment costs, not on the debt level or social utility function. In other words, investment depends on the conditions of production. Once the investment has been decided, the household decides its utility-maximizing level of consumption under the dynamic budget constraint.

Equation (9) and (10) are modified, when \( q_t = -\lambda_t/\mu_t \), into the following:

\[
q_t = 1 + 2\phi \cdot I_t/K_t. \tag{12}
\]

\[
q_t(1 - \delta) = (1 + r_i)q_{t-1} - [F_{K_t} + \phi \cdot (I_t/K_t)^2]. \tag{13}
\]

As \( (1 + r_i)/(1 - \delta) > 1 \) for all \( t \), \( q_t \) is the present value of the future total marginal product of capital which is the sum of the future marginal product of capital and the marginal reduction in adjustment costs due to a unit increase in capital.\(^4\) The discount rate is the sum of the interest rate on loans and the depreciation rate of capital. For simplicity, the discount rate is assumed to be constant in equation (14):

\[
q_t = \frac{1}{1 - \delta} \sum_{s=1}^{\infty} \frac{1}{(1+r)^s} \left[ F_{K_t} + \phi \cdot (I_t/K_t)^2 \right]_{t+s}. \tag{14}
\]

While \( q_t \) in equation (14) represents the present value of the total marginal products, \( q_t \) in equation (12) represents the marginal cost of investment. Pulling these together, investment should take place until the marginal cost of investment is equal to the present value of the total marginal products.

In all, the optimal solution must satisfy equation (11), (12), and (13) under the solvency constraint of equation (5). Plugging equation (11) into equation (5) makes consumption per efficiency unit a function of net wealth:

\[
\rho = \frac{\sum_{s=0}^{\infty} \left[ F_{t+s} - I_{t+s}(1+\varphi_{t+s}) \right]}{(1+r)^s} - (1+r)D_t.
\tag{15}
\]

Once investment and production are determined from equation (12)

\(^4\)The marginal \( q \) is assumed to equal the average \( q \), unless noted otherwise. Hayashi (1982) proved that the two are equivalent in the perfect market.
and (13), the household maximizes consumption within the limits of
the present value of lifetime resources. Consumption depends posi-
tively on lifetime consumable resources (income minus investment) and
negatively on existing debt.

III. Dynamic Effects of Changes in Parameters

We examine the effects of changes in parameters on investment and
growth as well as on the market value of capital. The parameters intro-
duced in the model are the growth rate of employment ($\lambda$), the tech-
nological progress($\beta$), the coefficient of adjustment cost($\phi$), the interest
rate ($r$), the depreciation rate($\delta$), and the share of capital in output($\alpha$).
In order to investigate the effect of a parameter change, we first analyze
the dynamic steady-state properties of the state variables of the model.

Equation (7), (12), and (13) are a dynamic system in ($q_t$, $k_t$) or ($\chi_t$, $k_t$),
where $k_t$ is the capital stock per efficiency unit and $\chi_t$ is the ratio of
investment to capital stock. It is useful to characterize the model in
terms of ($\chi_t$, $k_t$) as in the following where the parameters are assumed
not to change over time.

$$q_t = 1 + 2\phi \chi_t$$

$$2\phi \chi_t (1 - \delta) = r + \delta + 2\phi \chi_{t-1} (1 + r) - (\alpha A k_t^{\alpha - 1} + \phi \chi_t^2)$$

$$k_{t+1} (1 + \eta) (1 + \beta) = k_t (\chi_t + 1 - \delta)$$

Then we derive the steady-state values of $\chi_t$ and $k_t$:

$$\bar{\chi} = (1 + \eta) (1 + \beta) - (1 - \delta)$$

$$\bar{k} = \frac{(1 + 2\phi \bar{\chi} (r + \delta) - \phi \bar{\chi}^2)^{1 - \alpha}}{\alpha A}$$

The steady-state value of Tobin's $q$, which is defined as the ratio of
the market value of capital to the replacement cost of capital, is a posi-
tive function of the investment-capital ratio. The investment-capital
ratio($\chi_t$) is also a positive function of the growth rate of employment,
the technological progress, and the depreciation rate. On the other
hand, the steady-state value of capital per efficiency unit is a positive
function of the steady-state value of $\chi_t$, and a negative function of the
interest rate.

A change in the interest rate or the capital share affects the steady-
state value of capital per efficiency unit, but does not affect the steady-
state market value of capital.

The dynamic paths of $\chi_t$ and $k_t$ are illustrated in Figure 1.

Since $k_{t+1} - k_t = [k_t/(1 + \delta)(1 + \beta)] [\chi_t + (1 - \delta) - (1 + \delta)(1 + \beta)]$ in equation (17), $\dot{k} = 0$ locus should be horizontal at the steady state level of $\bar{\chi}$. The slope of the $\dot{k} = 0$ locus varies according to the value of $\chi_0$, and is shown in Figure 1.\(^5\)

The steady-state is a saddle point equilibrium. To reach the steady state, $\chi_t$ and $k_t$ should be located on a downward sloping saddle path. Along the saddle path, a rise (fall) in the $q_t$ value is accompanied by a decline (increase) in the capital per efficiency unit.

Now, we examine the effect of a change in technological progress. Technological progress here incorporates not only the progress in production technology but also the extrinsic changes in production due to changes in world economy, weather, or tax rates.

The accelerated technological progress raises the steady-state value of $\bar{\chi}$ and $\bar{k}$, leading to an upward shift of the saddle path as in Figure 2. The immediate impacts on the state variables depend on whether it is anticipated or not and whether it is permanent or temporary. We investigate the cases of the permanent (anticipated or unanticipated)

\(^5\)A detailed derivation appears in Appendix A of Abel and Blanchard (1983).
Figure 2
THE EFFECT OF THE RAPID TECHNOLOGICAL PROGRESS
(a) Unanticipated case

(b) Anticipated case

FIGURE 3
THE EFFECTS OF INCREASES IN INTEREST RATE(θ) OR IN THE ADJUSTMENT COST OF INVESTMENT(φ)
changes.\textsuperscript{6}

As shown in Figure 2a, an unanticipated rise in technological progress abruptly increases the market value of capital or investment from $A$ to $B$ on a new saddle path. The jump in investment to point $B$ subsequently leads to a gradual increase in capital and the gradual decrease in the value of capital. The new equilibrium is obtained at $C$.

In the case of an anticipated rise in technological progress, the market value of capital immediately increases to $B'$ when technological progress is anticipated, and then moves to $B''$ on the saddle path until it actually occurs, at which time it moves to point $C$ along the saddle path, the new equilibrium.

The effect of a change in the employment growth rate is similar to the above, so analysis of this case is omitted.

Next, the effect of an increase in the interest rate is presented in Figure 3. An increase in the interest rate diminishes $\bar{k}$ without affecting $\bar{X}$, so the saddle path shifts downward. With a shock, both the value of capital and investment fall immediately.

An increase in the adjustment cost has the same impact as an increase in the interest rate.

The effect of an increase in the depreciation rate on investment is ambiguous. If the steady-state value of $\bar{k}$ changes very little, investment is likely to increase.

\textbf{IV. Long-Term Prospects}

This section discusses the long-term prospects for the Korean economy, using a simple model in section II. Since it is impossible to have quantitative prospects without the specification of a model and its parameters, we start with a set of parameter values that envision the long-term prospects. Although a change in parameter value produces the different outcome, the model provides a benchmark to understanding the optimal future direction of the Korean economy.

\textsuperscript{6}Summers (1981) calculated the equilibrium path in the case of temporary changes. This is done using the algorithm for multiple shooting developed in Lipton, Poterba, Sachs and Summers (1982). For lack of such an algorithm, this paper calculates only the impacts of unanticipated permanent changes in parameters.
# Table 1

**The Benchmark Assumptions on Parameters**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Employment growth rate (l)</td>
<td>2.90</td>
<td>2.70</td>
<td>2.00</td>
<td>1.44</td>
<td>1.26</td>
<td>1.20</td>
</tr>
<tr>
<td>2. Technological progress (β)</td>
<td>4.00</td>
<td>4.00</td>
<td>4.01</td>
<td>4.30</td>
<td>4.47</td>
<td>4.58</td>
</tr>
<tr>
<td>3. Adjustment cost coefficient (ϕ)</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>4. Interest rate (r)</td>
<td>11.57</td>
<td>7.58</td>
<td>6.19</td>
<td>5.46</td>
<td>5.18</td>
<td>5.06</td>
</tr>
<tr>
<td>5. Depreciation rate (δ)</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>6. Capital share (α)</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Note: 1. The unit is percentage except for ϕ and α.

## A. Benchmark Parameters

The parameters in the model introduced above are the growth rate of employment, l, technological progress, β, the coefficient of adjustment costs, ϕ, the interest rate, r, the depreciation rate of capital, δ, and the share of capital in production, α. They are specified in Table 1. The depreciation rate is assumed to be 5% per annum. The coefficient of the adjustment cost function is 1.5. The capital share is 35%. The annual average growth rate of employment is lowered from 2.7% for 1987-91 to 1.4% for 2000-10. Technological progress is accelerated from 4.0% to 4.3%, while the interest rate drops from 7.6% to 5.5% for the corresponding period. Table 1 also includes the parameter values between 1970-87 which are needed to apply the model to past achievements.

The values in Table 1 are based on the actual performance and the generally accepted view for the future of the Korean economy. The growth rate of employment is adopted from the Sixth Five-Year Plan. The depreciation rate of 5% is the annual average of capital depreciation used for the estimation of capital stock in the KDI quarterly macroeconomic model.  

On account of the model's characteristics, the real interest rate must

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7Kwack (1985) and Pyo (1987) estimated Korea's annual depreciation rate to be 7.6% and 7.2% respectively. We obtained 5% depreciation rate because we
TABLE 2
THE COST FOR FOREIGN BORROWING

(annual average, %)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Curb-Market rate</td>
<td>40.1</td>
<td>41.4</td>
<td>30.5</td>
<td>24.4</td>
<td>23.2</td>
</tr>
<tr>
<td>2. Euro-Dollar Interest rate</td>
<td>7.9</td>
<td>9.5</td>
<td>13.0</td>
<td>9.5</td>
<td>6.7</td>
</tr>
<tr>
<td>3. Depreciation rate2</td>
<td>9.3</td>
<td>4.7</td>
<td>8.5</td>
<td>5.9</td>
<td>1.3</td>
</tr>
<tr>
<td>4. GDP Deflator inflation</td>
<td>18.8</td>
<td>20.9</td>
<td>8.5</td>
<td>4.0</td>
<td>1.4</td>
</tr>
<tr>
<td>5. Gap between domestic and foreign interest (1-2-3)</td>
<td>2.30</td>
<td>27.3</td>
<td>9.1</td>
<td>9.0</td>
<td>15.1</td>
</tr>
<tr>
<td>6. Real interest rate for foreign borrowing (2+3-4)</td>
<td>-1.7</td>
<td>-6.7</td>
<td>13.0</td>
<td>11.4</td>
<td>6.6</td>
</tr>
</tbody>
</table>

Note: 1. Three-Month.

be equal to the proper cost of foreign borrowing. This is, however, hard to measure since a large gap between domestic and foreign interest rates would bring about a large spread and limited opportunities for foreign borrowing. The real interest rate on foreign borrowing as defined in the last row of Table 2 could be used, but it is negative in 1970's because of high domestic inflation and a large gap between foreign interest rates and domestic curb-market rates. For these reasons, the benchmark real interest rate in Table 1 is somewhat arbitrarily adjusted to decrease gradually and be equal to the real interest rates for foreign borrowing during 1984-85 in Table 2.

Both the rate of technological progress and the coefficient of the adjustment cost function are chosen in consideration of the actual consumption and investment. The share of capital of 35% is obtained by estimating the production function during 1970-87 assuming technological progress of 4% per annum during that period.8

estimated the depreciation of building and structure to be lower than their calculations.

8Park and Lee (1986) showed that it was hard to find stable relationships among the aggregates of production, labor, and capital in Korea because of the drastic structural changes and increased social overhead capital. Nonetheless, we estimated the production function using the dummy variable:

\[
\log (GDP/L_t) = 0.15 + 0.38 \log (K_t/L_t) + 0.13 D80
\]

\( R^2 = 0.92, \) D.W = 1.56, D80: Dummy for 1980-87, ( ): t-values.
### Table 3
**Long-Term Prospects**

<table>
<thead>
<tr>
<th>Year</th>
<th>Growth rate (%)</th>
<th>$k_t^1$</th>
<th>$\chi_t^2$</th>
<th>$q_t$</th>
<th>Marginal product of capital</th>
<th>$\rho^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GDP</td>
<td>Investment</td>
<td>Consumption</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>8.4</td>
<td>9.5</td>
<td>6.9</td>
<td>3.70</td>
<td>0.16</td>
<td>1.48</td>
</tr>
<tr>
<td>1990</td>
<td>8.0</td>
<td>8.7</td>
<td>6.7</td>
<td>3.99</td>
<td>0.15</td>
<td>1.46</td>
</tr>
<tr>
<td>1995</td>
<td>7.2</td>
<td>7.1</td>
<td>6.2</td>
<td>4.67</td>
<td>0.14</td>
<td>1.42</td>
</tr>
<tr>
<td>2000</td>
<td>6.6</td>
<td>6.3</td>
<td>5.8</td>
<td>5.26</td>
<td>0.13</td>
<td>1.38</td>
</tr>
<tr>
<td>2010</td>
<td>6.1</td>
<td>5.9</td>
<td>5.8</td>
<td>5.96</td>
<td>0.12</td>
<td>1.35</td>
</tr>
<tr>
<td>2020</td>
<td>5.9</td>
<td>5.9</td>
<td>5.8</td>
<td>6.25</td>
<td>0.11</td>
<td>1.33</td>
</tr>
<tr>
<td>2030</td>
<td>5.8</td>
<td>5.8</td>
<td>5.8</td>
<td>6.37</td>
<td>0.11</td>
<td>1.33</td>
</tr>
<tr>
<td>2040</td>
<td>5.8</td>
<td>5.8</td>
<td>5.8</td>
<td>6.42</td>
<td>0.11</td>
<td>1.33</td>
</tr>
</tbody>
</table>

Note: 1. Capital per efficiency unit. The unit of $k_t$, $\chi_t$, and $\rho$ depends on the units of capital stock, consumption and persons employed. Here capital and consumption are in 1980 billion won. The employed are in thousand persons.

2. Investment/capital.

3. Calculated by equation (14) assuming the net debt at the end of each period is zero.

### B. Investment and Growth

Given the parameter values in Table 1, the benchmark simulation results which use 1987 values for the variables as a starting point are reported in Table 3.\(^9\)

The simulated GDP growth rate declines from 8.4% in 1988 to 5.8% in 2030 and the growth rate of fixed investment declines from 9.5% to 5.8% for the same year. These long-term prospects for the growth of GDP and fixed investment are similar to those found in other studies such as a study by the Future Industry Research Group, which was commissioned by the government and established at the Korea Development Institute in 1986 for more than a year (Korea Development Institute 1987).

After 2030, the variation of the state variables, $\chi_t$ and $k_t$, is very small, indicating that the economy has reached a steady-state. $\chi_t$ decreases and $k_t$ increases until they reach the steady state. This means that the economy is initially located on the left side of the downward sloping saddle path and moves toward the equilibrium according

\(^9\)We used FORTRAN to solve equation (12) and (13) on investment.
### Table 4
**Effects of a Change in Adjustment Cost of Investment**  
\( \phi = 0.75 \)

<table>
<thead>
<tr>
<th>Year</th>
<th>Growth rate (%)</th>
<th></th>
<th></th>
<th>Marginal product of capital</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GDP</td>
<td>Investment</td>
<td>Consumption</td>
<td>( k^1 )</td>
<td>( \chi^2 )</td>
</tr>
<tr>
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Note: 1. Capital per efficiency unit. The unit of \( k, \chi \) and \( \rho \) depends on the units of capital stock, consumption and persons employed. Here capital and consumption are in 1980 billion won. The employed are in thousand persons.

2. Investment/capital.

3. Calculated by equation (14) assuming the net debt at the end of each period is zero.

To Figure 1. The steady-state values of \( \bar{\chi} \) and \( \bar{k} \) are 0.11 and 6.4, respectively.\(^{10}\) The market value of capital, \( q_t \), decreases gradually with \( \chi_t \) decreasing and the marginal product of capital decreases gradually with \( k_t \) increasing. Consequently, the steady-state value of \( q \) and the marginal product of capital is 1.33 and 0.12, respectively.

Due to the adjustment costs of capital, it takes a long period for the marginal product of capital to be equal to the interest rate (plus depreciation rate). Even in the steady state, the two would not be equal. Table 3 shows that the difference is about 4% points in 1988, but it decreases to 2% points in the steady state. Because of the adjustment costs of investment, investment did not expand in the steady state to the point where the marginal product of capital equals the interest rate.

The marginal product of capital in the steady state is obtained by transforming equation (20). The steady-state marginal product of capital is

\(^{10}\)The units of \( \chi_t \) and \( k_t \) depends on the units of capital stock and investment. See the footnote in Table 3.
\[ \alpha A k^{-\sigma - 1} = (r + \delta) + \phi \bar{\lambda}[2(r + \delta) - \bar{\lambda}], \]  

(21)

in which the marginal product of capital becomes greater than the sum of interest rate and depreciation rate as long as \(2(r + \delta) > \bar{\lambda}.\)\(^{11}\)

The more interesting results on the Tobin's q and marginal product of capital are presented in both Table 3 and the Appendix.

But the question of whether the benchmark parameter values in Table 1 are appropriate still remains. A higher technological progress rate, a lower interest rate or a lower adjustment cost of investment than that found in Table 1 results in higher investment and growth rates. For example, the lower the coefficient of the adjustment cost function, the smaller the gap between the marginal product and interest rate by equation (21) and the higher the investment, contrary to Figure 3. Suppose the appropriate level of the coefficient of the adjustment cost function is 0.75. To accept this value, needless to say, sufficient justification should be provided, in particular with regard to past performance. Table 4 shows the influence of this change on investment, consumption, and the growth rate. The steady-state value of \(\bar{\lambda}\) does not change and that of \(\bar{k}\) increases. However, \(\chi_t\) immediately rises to move to a new saddle path, thus investment in 1987 rises 24%. Afterwards, the investment grows at a lower rate compared with Table 3. GDP grows faster due to an increase in capital stock. The gap between the marginal product of capital and the interest rate (plus the rate of depreciation) narrows by 1% point.

C. Optimal Consumption and Foreign Debt

In addition to the determination of investment and growth, the determination of optimal consumption is crucial to finding the optimal surplus or optimal debt level. In the previous section where we were only concerned about the country's long-run capacity to service its debt, we modelled external borrowing as a formal problem of dynamic resource allocation. If the country's creditworthiness is so great that there is no problem with foreign debt management except for insolvency risk, the

\(^{11}\)In developing countries, the private sector saves a fraction of aftertax income rather than optimizing intertemporally, because of an imperfect capital market. The government taxes to augment private savings but the constraints on taxing lead to overconsumption, underinvestment, and overborrowing. Thus the marginal product of capital would be kept higher than the cost of capital (Cooper and Sachs 1985).
goal of becoming a creditor country in the long-run is not the best policy to maximize social welfare, as shown in equation (15). Assuming that borrowing is only bound by the country's long-run capacity to service its debt, the steady-state value of private consumption per efficiency unit ($\rho$) can rise higher than 1.22 according to Table 3. The last column of Table 3 shows the maximum private consumption per efficiency unit calculated by equation (15) that can make the net debt stock disappear at the end of each period. If $\rho = 1.22$, the current account shows a deficit of 14% of GDP in 1987 and turns into a surplus after 2020.

Such a level of current account deficit may be considered too high because of many risks other than insolvency. Various opinions on the disutility of foreign debt give different levels of optimal consumption. A number of studies have focused on the optimal size of the current account and foreign debt by applying the dynamic resource allocation model as above. Among them, Blanchard (1983) looked into the debt problem of Brazil using the same methodology as in this study and finds that the Brazilian debt is not a problem in terms of solvency. But the seriousness of Brazil's debt problem is widely recognized for reasons of liquidity constraints, repudiation risk, and even political risk. Therefore, Blanchard (1983) extends the objective function to include a "disutility of debt."

Many characteristics of the economy determines how much it may consume, invest, and borrow. As the current account surplus materialized in 1986 and expanded until 1988 in Korea, many people could safely assess that Korea had a structural surplus in the current account and it could be a creditor country in the early 1990s. In contrast, Balassa and Williamson (1987) argued that Korea should eliminate its recent current account surplus and maintain 1 to 2 billion dollars current account deficit for the longer term. This is because the marginal product of capital in Korea is much higher than the international interest rate, external conditions will remain favorable to the Korean economy, and there are few problems in terms of solvency. However, none of these arguments are based on a rigorous model or quantitative analysis.

Leaving aside the complicated issues on optimal consumption, we calculate the level of consumption that can make the net debt at the end of each year be zero, as shown in the last column of Table 3. Table 3 suggests that Korea could be a creditor country in the first half of 1990s if consumption in efficiency units ($\rho$) is maintained at slightly less than one and investment and growth are determined according to
the model. Considering that the actual value of $\rho$ is 0.91 in 1987, consumption in efficiency units could be increased by a little less than 10% over that in 1987 to be a creditor country until 1995. Therefore our simple model of consumption and investment does not negate that Korea could optimally be a creditor country in the early 1990s.

V. Concluding Remarks

Since the current account of a country is represented as savings minus investment, the optimal amount of current account surplus or foreign debt can be determined by estimating the optimal size of investment and consumption. Balassa and Williamson (1987) argued that Korea should preferably maintain a current account deficit of 1 to 2 billion dollars per year because the rate of return on capital is higher than the interest rate on foreign borrowing, the favorable external conditions are likely to continue and creditworthiness is high.

This paper intended to establish a simple model of consumption and investment that could incorporate their arguments and assess them quantitatively. Investment and growth are traced on the basis of Tobin's $q$-theory. The parameter values such as technological progress, interest rate, and adjustment cost of investment are determined in consideration of the Korea's past performance and various long-term plans that contain a consensus view.

According to the results, the $q$ value is estimated to be greater than 1, reflecting the fact that the marginal product of capital is higher than the interest rate on foreign borrowing in Korea. With a decrease in the investment-capital ratio and an increase in capital per efficiency unit, $q$ and the marginal product of capital fall. The margin of productivity over the interest rate also declines gradually. The incorporation to the model of the adjustment cost of investment, the imperfection of capital markets, and tax limits in developing countries negates a simple neoclassical proposition that investment should expand until the marginal product of capital is equal to the interest rate.

Issues concerning the optimal consumption were a focal point of the paper. The level of consumption that satisfies the dynamic budget constraint is not desirable given many risks related to the huge foreign debt. Aside from addressing this complicated issue, the paper attempted to estimate the level of consumption that can assure Korea to be a net creditor in the early 1990s. The paper showed that it could be optimal for Korea to be a net creditor country in the first half of 1990s.
OPTIMAL CURRENT ACCOUNT

TABLE A1
Tobin's q, Marginal Product of Capital, and Interest Rate (1970-87)

<table>
<thead>
<tr>
<th>Year</th>
<th>Tobin's q</th>
<th>Marginal product of capital (%) (A)</th>
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Appendix

Tobin's q, Marginal Product of Capital, and Interest Rate in Korea

As the state variable $x_t$ decreases and $k_t$ increases over 1970-87, both $q$ and marginal product of capital decrease as shown in the Appendix Table A1.

The marginal product of capital decreases from 30% in 1970 to 17% in 1987. This result is not much different from Hong (1987)'s estimate of the average rate of return on investment in manufacturing. For example, during 1972-76, the marginal product of capital averages 25% while the average rate of return on investment is estimated at 23% in Hong (1987). During 1977-79, the former is 22% and the latter is 23%.

When the interest rate for discounting future returns on investment is assumed as in the Appendix Table A1, the gap between marginal productivity and the interest rate narrows from 10% points in 1970 to 4% points in 1987. The $q$ value decreases from 1.62 to 1.49 during the
same period.

This result could be compared to others. Dailami (1986) calculated the average $q$ of Korea's nonfinancial private corporate sector for 1963-83 using data on corporate dividends and the book value of corporate debt. He found that $q$ increases until the middle of 1970s and then decreases. It is greater than 1 over the whole period and its highest value is 1.53 in 1976.

References


Korea Development Institute, *Challenge for the Furture*, 1987.