Optimizing Consumer and Excess Sensitivity

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We show that with the non-expected utility model of Epstein and Zin(1989, 1991) the excess sensitivity of consumption can be explained theoretically from the optimizing behavior through the market portfolio which includes the return on human capital. Also, using Farmer’s (1990) model where stochastic income is explicitly included and risk is neutral, we show that the excess sensitivity of consumption to income is the prediction of the model through non-zero terms of human and nonhuman wealth in the stochastic process of consumption. (JEL Classification: E21)

I. Introduction

Under the permanent income hypothesis with the expected utility model (Hall 1978), consumption lagged more than one period should have no predictive power for current consumption. Hall’s (1978) martingale property of consumption is a result of the joint hypothesis composed of the permanent income model, a time-additive ‘certainty equivalence’ expected utility specification, and rational expectations. Most empirical tests (Flavin 1981; Hall and Mishkin 1982), however, reject this implication, even though Hall (1978) suggests that the empirical evidence supports it. This phenomenon is called the excess sensitivity puzzle.¹

Against martingale without drift, there are several theories explaining the excess sensitivity of consumption to income theoretically. The key

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¹Yi (1992) shows that other consumption puzzle, the excess of consumption growth puzzle, can be explained within the non-expected utility model.

elements in explaining this puzzle are the inclusion of stochastic labor income into the model with constant fixed interest rate (Caballero 1990), liquidity constraints (Zeldes 1989), time aggregation (Christiano, Eichenbaum, and Marshall 1991), finite horizons and life-cycle savings with a constant fixed death rate and interest rate (Gali 1990), the durability of consumption goods (Mankiw 1982), and so on.

The purpose of this paper is to show that the recursive non-expected utility framework of Epstein and Zin (1989, 1991), Weil (1990), and Farmer (1990) can resolve theoretically the puzzle of the excess sensitivity of consumption to income, by showing how it may result from optimizing behavior rather than from irrational behavior under liquidity constraints.

II. Explanation of the Excess Sensitivity of Consumption to Income

A. Epstein and Zin’s Model

As a whole, the empirical performance of the time additive expected utility model is poor. So even though consumption puzzles can be explained within the expected utility model, the lack of empirical support of this model encourages one in using non-expected utility model in order to explain the excess sensitivity of consumption puzzle theoretically and empirically. Empirical part will be done later in the future. Epstein and Zin address the empirical puzzles related to the capital asset pricing models (CAPM): i) large equity premiums observed in historical data are difficult to replicate with simulated Arrow-Debreu economies where agents have expected utility (equity premium puzzle), ii) simple static CAPM is not clearly rejected in favor of intertemporal consumption CAPM (static vs. consumption CAPM puzzle), iii) asset prices are too volatile to be rational forecasts of discounted future dividends when dividends are discounted with growth rates of consumption (excess volatility of stock price puzzle), and so on.

The most convincing explanation of the excess sensitivity of consumption puzzle within the expected utility model is in terms of the liquidity constraints and it may result from irrational behavior by Keynesian ‘rule-of-thumb’ consumer. In contrast, the non-expected model may explain this puzzle by consumer’s optimizing behavior. Also several papers report that this non-expected utility model performs well empir-

The useful models for explaining the excess sensitivity of consumption to income are borrowed from Epstein and Zin (1989, 1991), Weil (1990), and Farmer (1990). First, we consider Epstein and Zin's representative agent model:

$$V(A_t, I_t) = \max u[c_t, E_t V_{t+1}]$$

$$= \max_{c_t, w_{t+1}} \left[ c_t^\rho + \beta \mu \{ V[A_{t+1}, I_{t+1}] \}^\rho \right]^{\frac{1}{\rho}},$$

$$\text{s.t. } A_{t+1} = (A_t - c_t) w_t^R,$$

where \( u[\cdot, \cdot, \cdot] \) is a Kreps and Porteus-type aggregator function, \( V_t = V(A_t, I_t) \) is value function at time \( t \), \( \mu(V_{t+1}) = [E_t(V_{t+1})]^\alpha \) is certainty equivalent function, \( c_t \) is consumption, \( A_t \) is wealth, \( w_t \) and \( R_t (= 1 + r_t) \) are \( P \)-vector of portfolio weights and \( P \)-vector of random gross real returns on individual assets, with the typical element \( w'_t \) and \( R'_t \) respectively, \( I_t \) is the information set available to the consumer in the planning period, \( \beta (1) \) is the discount factor, \( \alpha (1) \) is the risk aversion parameter, \( \rho (1) \) is the intertemporal substitution parameter.

The optimization problem characterized in equation (1) can be solved by using the information that the aggregator function is homothetic and the certainty equivalent function is linear homogeneous. By homotheticity of aggregator function, there exists a function \( H(I) \) such that

$$V(I_t, A_t) = H(I) A_t.$$

Using the fact that the value function is homothetic, and \( \mu(dx_{t+1})^\rho = d^\rho \mu(x_{t+1})^\rho \), where \( d \) is nonstochastic, the equation (1) can be rewritten as follows:

$$V_t(A_t, I_t) = \max_{c_t, w_{t+1}} \left[ c_t^\rho + \beta \mu \{ V_{t+1}[(A_t - c_t) w_t^R, I_{t+1}] \}^\rho \right]^{\frac{1}{\rho}}$$

$$= \max_{c_t, w_{t+1}} \left[ c_t^\rho + \beta \mu \{ H(I_{t+1}) w_t^R \}^\rho \right]^{\frac{1}{\rho}}.$$

Deriving the Euler equations from equation (1) is equivalent to deriving them from the following two separate optimization problems with

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4The coefficient of relative risk aversion is defined as \( \text{RRA} = 1 - \alpha \), and the elasticity of intertemporal substitution is defined as \( \sigma = 1/(1 - \rho) \).
respect to consumption and portfolio weights:

\[ H(I_t)A_t = \max_{c_t}[c_t^\rho + \beta(A_t - c_t)^\gamma \mu^\rho]^{\frac{1}{\rho}} \] (4)

and

\[ \mu^* = \max_{w_t} [H(I_{t+1})w_t^\gamma R_t] \] (5)

From the first order condition with respect to consumption and portfolio weights, we have the following equations directly from Epstein and Zin (1989, 1991):

\[ \beta^\gamma E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\gamma(\rho-1)} M_t^{\gamma(\rho-1)} \right] = 1, \] (6)

\[ E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\gamma(\rho-1)} M_t^{\gamma-1}(R_t^k - R_t^j) \right] = 0, \] (7)

\[ k = 1, \ldots, P, \quad j = 1,\ldots,P, \quad k \neq j, \]

where \( \gamma = \alpha/\rho \) and \( M_t (= w_t^\gamma R_t) \) is the rate of return on the market portfolio. Summing \((7) \times w_t^k\) over \(k (= 1,\ldots,P)\), multiplying the resulting equation by \(\beta^\gamma\) and subtracting it from (6) (i.e., \((6) - \beta^\gamma \sum_{k=1}^{P} [(7) \times w_t^k])\), we get the equation (8):

\[ E_t \left[ \beta^\gamma \left( \frac{c_{t+1}}{c_t} \right)^{\gamma(\rho-1)} M_t^{\gamma-1}R_t^j \right] = 1, \quad j = 1,\ldots,P. \] (8)

We will concentrate on equation (8) later on, as the validity of equation (6) and (7) implies the validity of equation (8). Several special cases are worth noting. When \( \alpha = \rho \) \((\gamma = \alpha/\rho = 1)\), the equation (8) reduces to the Euler equation of the CRRA (constant relative risk aversion) expected utility model. In particular, when \( \alpha = \rho = 0 \), the equation (8) reduces to the Euler equation of the logarithmic expected utility model. And when \( \alpha = 0 \) and \( \rho \neq 0 \) \((\gamma = 0)\), the equation (8) reduces to the Euler equation of the non-expected utility model with logarithmic risk preferences.

Equation (8) plays an important role in explaining the excess sensitivity of consumption to income in some restrictive way. To address this consumption puzzle, we should reinterpret the wealth evolving equation, since labor income is not explicitly included in the wealth evolving equation. First, we may assume that labor income is non-stochastic and there is a safe asset. The sequence of labor income can
then be discounted back to the initial period and treated as part of the initial nonhuman wealth. In this case, the consumer can move resources across periods in this safe asset. Burbidge (1988) shows, from simulation results with a three-period, three-state, two-asset model, that once the labor income is introduced into the model, Epstein and Zin's (1989, 1991) Euler equations hold only when the entire labor income arrives in the first period. Otherwise, some of the gross return on the portfolio might not be well defined; i.e., some of them may be negative for a certain distribution of rate of return.

Second, we may assume that labor income is stochastic and diversifiable. Labor income can be treated as a stochastic dividend for human capital. We can think of an agent as selling shares to his human capital, so the problem can be recast in terms of portfolio choice without labor income. As is shown by Epstein and Zin (1988), a shadow price and a shadow return can be computed for this asset in equilibrium.

The Euler equation (8) serves as a basis in explaining the consumption puzzles. Let

\[ Y_{t+1} = \left( \frac{c_{t+1}}{c_t} \right)^{\gamma(p-1)} M_t^{-1} R_t \]

and

\[ y_{t+1} = \ln(Y_{t+1}) \]

Then we can express equation (8) as equation (9), by shifting \( \beta^\gamma \) to the RHS of (8).

\[ E_t[Y_{t+1}] = (1 + \delta)^\gamma. \]  \hspace{0.5cm} (9)

where \( \beta = 1/(1 + \delta) \). We assume the joint lognormality between consumption growth, the return on the portfolio and the return on individual asset. Given that \( \ln(Y) \) is normal with mean \( m \) and variance \( \sigma^2 \), then \( E[Y] = e^{m + \sigma^2/2} \). We can express LHS of equation (9) as equation (10),

\[ E_t[Y_t] = e^{m + \sigma^2/2} \]  \hspace{0.5cm} (10)

where \( m = E_t[\ln(Y_{t+1})] \), i.e., \( \ln(Y_{t+1}) = m + \epsilon_{t+1} \) and where \( \epsilon_{t+1} = \ln(Y_{t+1}) - E_t[\ln(Y_{t+1})] \) and \( \epsilon_{t+1} \sim N(0, \sigma^2) \). Combining (9) and (10) and taking the logarithm, we get

\[ m + \frac{\sigma^2}{2} = \gamma \ln(1 + \delta) \]  \hspace{0.5cm} (11)
Using \( m = \ln \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\gamma(p-1)} M_t^{-1} R_t^j \right] \) and solving equation (11) for \( \ln \left( \frac{c_{t+1}}{c_t} \right) \), equation (8) can be expressed as equation (12), via equation (11),

\[
\ln \left( \frac{c_{t+1}}{c_t} \right) = \frac{-\delta}{1-\rho} + \frac{\sigma^2}{2\gamma(1-\rho)} + \frac{1}{\gamma(1-\rho)} \ln(R_t^j) \\
+ \frac{\gamma - 1}{\gamma(1-\rho)} \ln(M_t) + \frac{1}{\gamma(\rho-1)} \epsilon_{t+1}, \quad (12)
\]

where \( M_t = \omega_t R_t \) and \( R_t = 1 + r_t \).

Rearranging equation (10), we can obtain

\[
\ln(c_{t+1}) = \frac{-\delta}{1-\rho} + \frac{\sigma^2}{2\gamma(1-\rho)} + \ln(c_t) + \frac{1}{\gamma(1-\rho)} \ln(R_t^j) \\
+ \frac{\gamma - 1}{\gamma(1-\rho)} \ln(M_t) + \frac{1}{\gamma(\rho-1)} \epsilon_{t+1}, \quad (13)
\]

where \( j = 1,...,P \).

The martingale property of consumption does not hold if the coefficients of individual rates of return and market portfolio are not zero. Of course, if we assume a time-varying rate of return in the expected utility case, it can explain the excess sensitivity of consumption. Although we assume \( \delta = r \), the martingale property of consumption is violated because of nonzero portfolio return (\( \ln M_t \)) and individual return (\( \ln R_t^j \)) term with \( \gamma \neq 1 \). Note that if \( \gamma = 1 \) (expected utility case) and \( \delta = r \), we get the martingale property of consumption.

We focus on the coefficient of the portfolio return. If we introduce the stochastic income such that human capital yields a stochastic dividend and let \( j = P \) denote this asset, labor income has some explanatory power on consumption indirectly through the market portfolio. This is a partial and indirect explanation of the excess sensitivity of consumption to income. It is worth noting that if \( \gamma = 1 \), the equation (13) reduces to that of the expected utility model and so the above explanation of excess sensitivity of consumption to income is no longer valid except when \( j = P \). The sign of \( \gamma \) can be positive or negative, so the market portfolio and individual return term explaining the sensitivity puzzle can be positive or negative. In Zeldes (1989), his estimate on individual
return term is sometimes negative, although insignificant. This fact may suggest that the expected utility model may be misspecified.

B. Farmer’s model

So far, we treat income in a restrictive way. In the expected utility model the more general case where there are both interest uncertainty and income uncertainty, can have a closed form solution for the consumption function only when labor income is diversifiable, or when the model consists of quadratic utility and linear constraints.

Farmer (1988, 1990) shows that the closed form solution for the consumption function can be obtained only if the agent is indifferent to the income risk \(\alpha = 1\), risk neutrality, while maintaining the intertemporal aspects of choice (which he calls RINCE (Risk Neutrality and Constant Elasticity of substitution) preferences). In the RINCE preferences, the value function is linear in wealth, the individual is indifferent to income fluctuation, and the individual is indifferent to the composition of his portfolio if all assets pay the same expected return. Thus, the RINCE preferences are not useful for analyzing the portfolio choice problem, but is still useful in examining the intertemporal aspect of the choice problem.

From the following framework

\[
V_t = \max_{c_t} \left[ c_t^\rho + \beta (E_t[V_{t+1}]^F)^\rho \right],
\]

\[s.t. \quad A_{t+1} = R_t A_t + y_t - c_t,
\]

he obtains the optimal consumption function,5

\[c_t = G(Q_t) W_t, \quad t = 1, 2, \ldots, T,
\]

subject to the following equations:

\[G(Q_t) = (1 + \beta^{1-\rho} Q_t^{1-\rho})^{-1},\]

\[Q_t = E_t[R_{t+1} F(Q_{t+1})],\]

\[F(Q_t) = (1 + \beta^{1-\rho} Q_t^{1-\rho})^{-\rho},\]

\[W_t = R_t A_t + H_t,\]

\[H_t = y_t + E_t \left[ H_{t+1} \frac{F(Q_{t+1})}{Q_t} \right],\]

\[5\text{See Appendix for details.}\]
where $Q_t$ is a sequence of interest rate, $F(Q_{t+1})/Q_t$ acts as a stochastic discount rate on the future income, $W_t$ is the total wealth, $H_t$ is the human wealth, and $y_t$ is labor income. Farmer’s (1990) model restricts the parameters of Epstein and Zin (1989, 1991) and Well (1990), while extending their models by including labor income into the wealth evolving equation explicitly.

By dropping the representative agent assumption, he applies his model to the overlapping generation framework and shows that he can get the closed form relationship between aggregate consumption at two consecutive periods in stochastic environment. Now using Farmer’s model we want to show that the excess sensitivity of consumption to income is the prediction of the model (i.e., the result of consumer’s optimizing behavior), while maintaining the representative agent assumption. As Farmer (1990) shows, using the wealth evolving equation in (14) and total wealth equation in (19), if the sequences $|R_t|$ and $|y_t|$ are independently and identically distributed and are independent of each other, the human wealth ($H_t$) can be expressed by

$$H_t = y_t + E_t \left[ H_{t+1} \frac{F(Q_{t+1})}{Q_t} \right]$$

$$= y_t + \frac{E_t(H_{t+1})}{E_t(R_{t+1})}. \tag{21}$$

Substituting for $H_{t+1}$ ($i = 1, 2, ..., T$) repeatedly, we get

$$H_t = y_t + \frac{E_t(y_{t+1})}{E_t(R_{t+1})} + \frac{E_t(y_{t+2})}{E_t(R_{t+1})E_t(R_{t+2})} + ...$$

$$+ \frac{E_t(y_T)}{E_t(R_{t+1})E_t(R_{t+2})...E_t(R_T)}. \tag{22}$$

Farmer (1988) applies his RINCE model to overlapping generation framework where each generation faces the same terminal date, irrespective of date of birth. For the case of unit elasticity of substitution ($\rho = 0$), from the aggregate consumption $c_t = \alpha(R_t, A_t, H_t)$, aggregate human wealth $H_t = n_t y_t + 1/g E_t[H_{t+1}$ ($F(Q_{t+1})/Q_t$)], and aggregate nonhuman wealth $A_{t+1} = R_t A_t + n_t y_t - c_t$, where $n_t$ is the number of individuals alive at $t$ and $g$ is population growth factor ($g = 1/(1 + p)$, $p$ is population growth rate), if $R_t$ is independent of $c_t$ the Euler equation is, using equations (16)-(20),

$$c_t = \frac{1}{g-\alpha} E_t \left[ \frac{c_{t+1}}{R_{t+1}} \right] + \left[ \frac{g-1}{g-\alpha} \right] Y_t,$$

where $v_i = (ac_i^\beta + \beta(E_tu_{t+1}))^{1/\beta} \left|_{\rho = 0} \right.$

$$= c_i^\beta (E_tu_{t+1})^\beta (\alpha + \beta = 1) \text{ and } Y_t = R_t A_t + n_t y_t.$$

Unless $g = 1$, we have another version of the Euler equation which can explain the excess sensitivity of consumption to income.
If $y_t$ and $R_t$ are nonstochastic, equation (22) reduces to the familiar definition of human wealth. Subtracting $15 \times G(Q_{t+1}) \times \mathbb{E}_t(R_{t+1})$ from (15) (with one period forward) $\times G(Q_t)$, we get the following difference equation for optimal consumption,\(^7\)

\[
\begin{align*}
\dot{c}_{t+1} &= \left[ \frac{G(Q_{t+1})}{G(Q_t)} \mathbb{E}_t(R_{t+1}) \right] \mathbb{E}_t(c_{t+1}) + G(Q_{t+1})R_{t+1}A_{t+1} \\
&\quad - R_t \mathbb{E}_t(R_{t+1})A_t - \mathbb{E}_t(R_{t+1})y_t \right] + G(Q_{t+1}) \sum_{i=1}^{T} \epsilon_{t+i},
\end{align*}
\]

where

\[
\begin{align*}
\epsilon_{t+1} &= y_{t+1} - \mathbb{E}_t(y_{t+1}), \quad \epsilon_{t+2} = \frac{\mathbb{E}_{t+1}(y_{t+2})}{\mathbb{E}_{t+1}(R_{t+2})} - \frac{\mathbb{E}_t(y_{t+2})}{\mathbb{E}_t(R_{t+2})},
\end{align*}
\]

and

\[
\begin{align*}
\epsilon_{t+i} &= \frac{\mathbb{E}_{t+i}(y_{t+i})}{\mathbb{E}_{t+i}(R_{t+i-1} \cdots R_{t+1} R_{t+1})} - \frac{\mathbb{E}_t(y_{t+i})}{\mathbb{E}_t(R_{t+i-1} \cdots R_{t+1} R_{t+1})},
\end{align*}
\]

\[i = 3, \ldots, T, \quad j = i - 2.\]

From (23), we can show that lagged human and nonhuman wealth terms have some predictive power for consumption in a nonlinear way.

III. Conclusion

By using the general recursive framework of Epstein and Zin (1989, 1991) and Well (1990), we can show that the excess sensitivity of consumption can be explained theoretically from the optimizing behavior through the market portfolio which includes the rate of return on human capital. Furthermore, using Farmer's (1990) model where stochastic labor income is explicitly included, we are able to show that the excess sensitivity of consumption to income is the prediction of the model through non-zero terms of human and nonhuman wealth in stochastic process of consumption.

The future direction of research will be to check if the excess sensi-

\(^7\)Flavin (1981) follows the similar approach from consumption function to the stochastic process of consumption. She shows that the certainty equivalence formulation of expected utility framework gives us a random walk consumption process, which shows no excess sensitivity of consumption to income in a theoretical setting.
tivity puzzle of consumption can be also explained empirically as an optimizing behavior. We hope that this line of research will be made later in the future as a companion paper of this one.

Appendix

The linearity of consumption function \( c_t = G(Q_t)W_t \) can be derived as follows:

Taking expectation in both side of value function and using equation (14)-(20), we get

\[
E_{t-1}(V_t) = E_{t-1}[F(Q_t)R_t(R_{t-1}A_{t-1} + y_{t-1} - c_{t-1})] + E_{t-1}[F(Q_t)H_t]
\]
\[
= E_{t-1}[Q_{t-1}(W_{t-1} - H_{t-1} + y_{t-1} - c_{t-1})] + E_{t-1}[(H_{t-1} - y_{t-1})Q_{t-1}]
\]
\[
= Q_{t-1}(W_{t-1} - c_{t-1})
\]

From equation (14)

\[
V_{t-1} = \max_{c_{t-1}}^1 c_{t-1}^{\rho} + \beta[ E_{t-1}(V_t)^{\rho} ]^{\frac{1}{\rho}}
\]
\[
= \max_{c_{t-1}}^1 c_{t-1}^{\rho} + \beta[ Q_{t-1}(W_{t-1} - c_{t-1})^{\rho} ]^{\frac{1}{\rho}}
\]

The first order condition is

\[
\frac{1}{\rho} [ \frac{1}{\rho} - 1 ] c_{t-1}^{\rho-1} + \beta \rho [ Q_{t-1}(W_{t-1} - c_{t-1})^{\rho-1}]^{\rho} = 0
\]

\[
c_{t-1}^{\rho-1} = Q_{t-1}^{\rho-1} \beta Q_{t-1}(W_{t-1} - c_{t-1})^{\rho-1}
\]

\[
c_{t-1} = Q_{t-1}^{\rho-1} \beta^{\rho-1} Q_{t-1}(W_{t-1} - c_{t-1})^{\rho-1}
\]

\[
= \beta^{\rho-1} Q_{t-1}^{\rho-1} W_{t-1} - \beta^{\rho-1} Q_{t-1}^{\rho-1} c_{t-1}
\]

Therefore the consumption function is linear in total wealth,

\[
c_{t-1} = \frac{1}{1 + \beta^{\rho-1} Q_{t-1}^{\rho-1}} W_{t-1}
\]

\[
= (1 + \beta^{1-p} Q_{t-1}^{\rho})^{\frac{1}{p}} W_{t-1}
\]

\[
= G(Q_{t-1})W_{t-1}
\]
where \( G(t) = \frac{1}{(1 + \beta^{-1}G^{p}_{t-1})^{-1}} \).

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