Growth, Taste Change and the Sexual Division of Labor

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This paper proposes an economic model to analyze a dynamic interaction between economic growth, preference change and sexual division of labor in a perfectly competitive economic system. We show how sexual division of labor between working outside and staying at home are changeable in a sexually just labor market. Conditions for existence of equilibria and stability are provided. Effects of changes in some parameters on the equilibrium economic structure are examined. (*JEL* Classification: E13)

I. Introduction

It is a commonplace to argue that there are dynamic interactions between capital accumulation, preference change, sexual division of labor, time distribution between working and leisure. Yet there are only a few theoretical economic models which explicitly take account of these dynamic interactions within a single theoretical framework. The purpose of this study is to solve the issue by proposing a growth model with endogenous sexual division of labor and taste changes.

Any social and economic activity requires time. How time distribution between pure leisure, children care, home production, working and other activities is affected by and affects economic growth is obviously an important issue. Over the years there have been a number of attempts to modify neoclassical consumer theory to deal with economic issues about family structure, working hours and the valuation of travelling time (Becker 1976; Chiappori 1988, 1992; Folbre 1986; Mills and Hamilton 1985). There is an increasing amount of economic literature

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about, for instance, sexual division of labor, marriage and divorce, and decision makings on children number. It has been argued that increasing returns from human capital accumulation is a powerful force creating a division of labor in the allocation of time between male and female population (e.g., Becker 1985). There are studies on the relationship between economic growth and family distribution of income (e.g., Fei, Ranis and Kuo 1978). There are also studies on female labor supply. Woman chooses levels of market time on the basis of wage rates and incomes. Life time variation in costs and opportunities—due to children, unemployment of the spouse, and general business cycle variation—influences the timing of female labor participation (e.g., Mincer 1962; Smith 1977; Heckman and Macurdy 1980). The relationship between home production and non-home production and time distribution has been investigated (e.g., Lancaster 1966, 1971). Possible sexy discrimination in labor markets have also attracted much attention from economists (e.g., Becker 1957; Cain 1986; Lazear and Rosen 1990). The gain from marriage may be reduced as people become rich and educated. The growth in female population's earning power may raise the forgone value of their time spent at child care, education and other household activities, which may reduce the demand for children and encourage a substitution away from parental. Divorce rates, fertility, and labor participation rates may interact in much more complicated ways. Decision makings on family size is extremely complicated (e.g., Weiss and Willis 1985). Irrespective of so many studies on the complexity of family as a subsystem of economic production, yet family economics, swept into a pile labeled economic demography or labor economics, is often relegated to a somewhat corner of the mainstreams of economic growth and development.

The main purpose of this study is to propose a simple model on the basis of neoclassical growth theory and family economics to examine the complexity of dynamic interaction between economic growth, preference change, time distribution between work and leisure (time at home) of male and female population. Interdependence of growth and taste change has been investigated in the literature of economic dynamics (Uzawa 1968; Wan 1970; Boyer 1978; Shi and Epstein 1993). It should be remarked that as these studies introduce taste changes within the framework of optimal growth model, it is difficult to get explicit conclusions. This study tries to solve the issue of taste change within a different framework as just mentioned.

The main contribution of this study is that we propose a compact theoretical framework to analyze dynamic interactions between capital accumulation, preference change, sexual division of labor, time distribution between working and leisure. The framework synthesizes various ideas in family economics and growth theory. The study is organized as follows. Section II defines the model with preference dynamics. Section III provides conditions for existence of equilibria and stability. Section IV examines the impact of changes in husband's human capital on the economic structure. Section V concludes the study.

II. The Model

We consider an economic system similarly to the one-neoclassical growth model (e.g., Solow 1956; Swan 1956; Burmeister and Dobell 1970). Only one commodity is produced in the system. The commodity is assumed to be composed of homogeneous qualities, and to be produced by employing two factors of production—labor and capital.

To simplify possible complexity of family structure and endogenous population growth, similarly to Zhang (1993) we specify the population and family structure in this study as follows. We assume that the population consists of N identical families. Each family consists of four members—father, mother, son and daughter. The total population is equal to $4N$. There is division of labor in the family. The children consume goods and accumulate knowledge through education. The parents have to do home work and find job for family's living. The father and mother may either do home work or do business. The working time of the father and the mother may be different. We assume that working time of the two adults are determined by maximizing the family's utility function subject to the family and the available time constraints. We omit any possibility of divorce. We assume that the young people get educated before they get married and join labor market. We assume that the husband and wife pass away at the same time. When the parents pass away, the son and the daughter respectively find their marriage partner and get married. The property left by the parents is shared equally by the two children. The children are educated so that they have the same level of human capital as their parents when they get married. When a new family is formed, the young couple join the labor market and have the two children. As all the families are identical, the family structure is invariant over time under these assumptions.

We assume that there is no sexual discrimination in labor market.

Here, by justice in labor market we mean that any worker is paid according to marginal value of "qualified labor". Markets are characterized by perfect competition. The total labor input, $N^*(t)$, at time t is defined by

$$N^* = z_1 \breve{t}_1 N + z_2 \breve{t}_2 N, \tag{1}$$

where $\breve{t}_1(t)$ and $\breve{t}_2(t)$ are the working time of husband and wife, respectively. The two parameters, z_1 and z_2, are levels of human capital of the male and female population, respectively. From the economic literature on endogenous knowledge (e.g., Zhang 1993), we see that it is conceptually not difficult to treat human capital as endogenous variables. As our modeling framework is already very complicated, for simplicity of analysis, we assume z_1 and z_2 to be constant at this initial stage.

We specify production function of the economy as follows

$$F = K^{\alpha} N^{*\beta}, \quad \alpha + \beta = 1, \ \alpha, \ \beta > 0, \tag{2}$$

where $F(t)$ is the output level, $K(t)$ is the total capital stocks of the economy, and α and β are parameters.

The industrial sector maximizes its profit, $F - rK - w_1 \breve{t}_1 N - w_2 \breve{t}_2 N$, where r is the rate of interest and $w_1(t)$ and $w_2(t)$ are, respectively, the wage rates per unity of working time of husband and wife. We assume that the working time is determined by family. The marginal conditions are given by

$$r = \frac{\alpha F}{K}, \ w_1 = \frac{\beta z_1 F}{N*}, \ w_2 = \frac{\beta z_2 F}{N*}. \tag{3}$$

From (3), the ratio of the wage rates per unity of time between husband and wife is given by: $w_1 / w_2 = z_1 / z_2$ The ratio is independent of capital stock and production scale, only dependent on the ratio of human capital. When $z_1 = z_2$, the husband and wife have the identical wage rate per unity of time.

Let Y denote the total income of the economy. The net income, Y/N, of each family consists of the wage incomes and the interest payment for family's capital. The net income at any point of time satisfies

$$Y = rK + w_1 \breve{t}_1 N + w_2 \breve{t}_2 N = F, \tag{4}$$

where we use (3) and $\alpha + \beta = 1$.

Let us denote $S(t)$ the savings made by the population at time t. We assume that we can find an aggregated utility function for a typical family. As each member of the family has his/her own utility function, how the family game (i.e., the distribution of consumption among the family members in this study) is actually played is very complicated. We assume the existence of a utility function which represents "collective preference" of the family members. Let us denote T_0 the total available time (which is assumed to be equal between the two sexes). The time constraint requires that the amounts of time allocated to each specific use add up to the time available:

$$\breve{T}_j + \breve{t}_j = T_0, \quad j = 1, 2, \tag{5}$$

where $\breve{T}_1$ and $\breve{T}_2$ are husband's leisure time (time at home) and wife's leisure time, respectively.

Let $C(t)$ and $S(t)$ denote the total levels of consumption and savings made by the economy at time t. We assume that family's utility level is dependent on husband's leisure time, $\breve{T}_1$, wife's leisure time, $\breve{T}_2$, family's temporary consumption level, $C(t)/N$, and family's net wealth, $(K + S + \delta_0 K)/N$, where δ_0 is the fixed depreciation rate of capital. For simplicity of analysis, we specify a typical family's utility function as follows:

$$\begin{gathered} U(t) = \breve{T}_1^{\sigma_1} \breve{T}_2^{\sigma_2} C^{\xi} \frac{(K + S - \delta_0 K)^{\lambda}}{N^{\lambda+\xi}}, \\ \sigma_1(t), \sigma_2(t), \xi(t), \lambda(t) > 0, \quad \sigma_1 + \sigma_2 + \xi + \lambda = 1. \end{gathered} \tag{6}$$

In this study by taste change we mean change in $\sigma_1(t)$, $\sigma_2(t)$, $\xi(t)$, and $\lambda(t)$. For convenience of interpretation, we call σ_1, σ_2, ξ, and λ, respectively, the family's propensities to use husband's time at home (or leisure time), to use wife's time at home, to consume goods, and to hold wealth.

Since a family consists of several members and each member has his/her own utility function, family's behavior should be analyzed as the result of all members' rational decisions. The "collective" utility function should be analyzed within a framework which explicitly takes accounts of interactions within family's members (e.g., Becker 1976; Heckman and Macurdy 1980; Chiappori 1988). Here, for simplicity of analysis, we neglect issues about possible conflicts and inequality among family members. It should be noted that for simplicity we assume a passive role of children in the family decision. It is quite possible to enrich the model by assuming that consumption affects the human capital and varies over time, even though the model may become analytically too complicated.

It should be emphasized that in this study, wealth is treated from broad social and cultural perspectives. We assume that an increase in wealth tends to increase utility level of a typical household. Wealth may practically be accumulated for different reasons such as the capitalist spirit, old age consumption, providing education for children, power and social status (see also, e.g., Modigliani 1986; Ram 1982; Gersovitz 1988). Obviously, those different reasons determine preference structures.

In a traditional intertemporal framework, the household problem is to maximize

$$\int_0^\infty U(C)\exp(-\rho t)dt,$$

subject to the dynamic budget constraint of capital accumulation. In the above formula, there are two assumptions. The first is that utility is additional over time. Although one may add capital and money over time, it is a very strict requirement to add utility over time. It is not reasonable to add happiness over time, even though one may reasonably assume preference structures to be invariant over certain period of time in economic analysis. The second is that the depreciation parameter, ρ, is meaningless if utility is not additional over time. In our approach, we take account of social and cultural factors which affect saving behavior by the preference parameter, λ, at each point of time.

In the traditional studies of growth and taste change (e.g., Uzawa 1968; Wan 1970; Boyer 1978; Shi and Epstein 1993), by taste change it means changes of ρ. In this study, by taste change we mean changes in σ_1, σ_2, ξ, and λ.

Each family makes decision on the four variables, $\breve{T}_1$, $\breve{T}_2$, C/N and S/N, at any point of time. The financial budget constraint is given by:

$$C + S = rK + w_1\breve{t}_1N + w_2\breve{t}_2N.$$

We rewrite the above constrain as follows:

$$C + S + w_1\breve{T}_1N + w_1\breve{T}_2N = rK + w_1T_0N + w_2T_0N. \tag{7}$$

Each family maximizes $U(t)$ subject to the time and budget constrains, (5) and (7). The optimal problem has the following unique solution:

$$\breve{T}_j = \frac{\sigma_j\Omega}{w_jN}, \quad C = \xi\Omega, \quad S = \lambda\Omega - (1 - \delta_0)K. \tag{8}$$

where

$$\Omega(t) \equiv rK + w_1T_0N + w_2T_0N + (1 - \delta_0)K. \tag{9}$$

In the remainder of this study, we neglect depreciation of capital, i.e., $\delta_0 = 0$. It can be seen that this will not affect our main conclusions.

From (3) and (8), the ratio, $\breve{T}_1/\breve{T}_2$, of time at home of husband and wife is given by:

$$\frac{\breve{T}_1}{\breve{T}_2} = \frac{\sigma_1 z_2}{z_1\sigma_2}. \tag{10}$$

The ratio, $\breve{T}_1/\breve{T}_2$, is positively related to the ratio, σ_1/σ_2, of the family's propensities to use husband's and wife's leisure time and negatively to the ratio, z_1/z_2, of husband's and wife's human capital levels. If $z_1 = z_2$ and $\sigma_1 = \sigma_2$, husband and the wife spend the same time at home. If human capital is sexually identical, i.e., $z_1 = z_2$, the sex with higher family's propensity to stay at home will stay at home longer than the other sex. If the family's propensities to use the leisure time are sexually identical, i.e., $\sigma_1 = \sigma_2$, the sex with higher level of human capital will work longer time than the other sex.

The capital accumulation is given by: $dK/dt = S - \delta_0K$. Substituting S in (8) into the above equation yields

$$\frac{dK}{dt} = \lambda\Omega - K. \tag{11}$$

As product is either invested or consumed,

$$C + S = F, \tag{12}$$

holds.

We now specify possible dynamics of preference. As $\sigma_1 + \sigma_2 + \xi + \lambda = 1$ holds at any point of time, it is sufficient to be concerned with three variables of the four. For simplicity, we assume the following relationships between σ_1, σ_2 and λ

$$\sigma_1 = \sigma_0 - h_\sigma\sigma_2, \ \lambda = \lambda_0 + h_\lambda\sigma_2, \quad 0 < \sigma_0, \lambda_0 < 1, \tag{13}$$

where σ_0, λ_0, h_σ and h_λ are parameters. At $\sigma_2 = 0$, $\sigma_1 = \sigma_0$ and $\lambda = \lambda_0$. It is necessary to require $0 < \sigma_0, \lambda_0 < 1$. The parameters, h_σ and h_λ, may be either positive or negative. In the case of $h_\sigma > 0$ (< 0), an increase in the family's propensity, σ_2, to use wife's time at home reduces (increases) the family's propensity, σ_1, to use the husband's time at home. In the case of $h_\sigma > 0$ (< 0), an increase in σ_2 reduces (increases) σ_1. In the case of $h_\lambda >$ (< 0), an increase in the family's propensity, σ_2, to use the wife's time at home increases (reduces) the family's propensity, λ, to hold wealth. From the definitions of h_σ and h_λ, it may be reasonable to argue that h_σ and h_λ may be dependent on living conditions and family wealth. As our model is already very complicated, for simplicity we assume h_σ and h_λ to be constant and to be positive in this study. By $\sigma_1 + \sigma_2 + \xi, + \lambda = 1$ and (13),

$$\xi = 1 - \sigma_0 - \lambda_0 - (1 + h_\lambda - h_\sigma)\, \sigma_2, \tag{14}$$

holds. We require: $1 > \sigma_0 + \lambda_0$. For $1 > \sigma_1, \sigma_2, \xi, \lambda > 0$ to be satisfied, it is necessary to add some constraints on the parameters, σ_0, λ_0, h_λ, and h_σ. We will discuss the matter later on. By (13) and (14), we solve σ_1, ξ and λ as functions of σ_2 at any point of time. We now specify a possible dynamics of wife's propensity, σ_2, to stay at home.

We assume that wife's propensity, $\sigma_2(t)$, to stay at home is affected by the capital stock, $K(t)/N$, and the net income, $Y(t)/N$, as follows

$$\frac{d\sigma_2}{dt} = \theta\{G(K,Y) - \sigma_2\}, \quad \sigma_2 \geq 0, \quad \infty > \theta \geq 0, \tag{15}$$

where θ is a positive parameter and G is a continuous function of K and Y. If $\theta = 0$, σ_2 is constant. This case means that taste is not affected by current living conditions and wealth accumulated. If $\theta \rightarrow +\infty$, $G(K, Y) = \sigma_2$ is held at any point of time (except some singular points, e.g., Chow and Hale 1982; Haken 1983; O'Malley 1988; Kevorkian and Cole 1981; Zhang 1991). This implies that taste is quickly adapted to living conditions and wealth accumulated.

Although we may generally argue that the family's propensity to use the wife's time at home is affected by wealth accumulated and current living conditions, it is not easy to generalize meaningful functional form of G. For simplicity, this study specifies G as follows

$$G = \frac{\theta_1}{(1 + \theta_2 K^a Y^b)}, \quad \theta_1 > 0, \quad \theta_2 > 0. \tag{16}$$

The requirements of $\theta_2 > 0$ guarantees $G < \theta_1$. If $a >$ (<) 0, then the family's propensity to use the wife's time at home tends to be reduced (increased) as the family's wealth is increased. If $a = 0$, the term G is not affected by K. If $b >$ (<) 0, then the family's propensity to use the wife's time at home tends to be reduced (increased) as the income, Y, is increased. If $b = 0$, the term G is not affected by Y. In this study, we will not specify whether a and b are positive or negative as it is difficult to judge whether an increase in wealth and income will certainly increase or decrease wife's propensity to stay at home. We will examine what will happen to the system when a and b are taken on various values.

By (15) and (16), we may require: $0 < \sigma_2(t) < \theta_1$.It is reasonable to require that for $0 < \sigma_2(t) < \theta_1$, $\sigma_1(t) > 0$, $\lambda(t) > 0$ and $\xi(t) > 0$. By (13) and (14), this is guaranteed if the parameters, λ_0, σ_0, h_σ and h_λ, in (13) satisfy

$$\lambda_0 > 0,\ \sigma_0 > h_\sigma\theta_1,\ 1 - \sigma_0 - \lambda_0 > \max\{0,\ (1 + h_\lambda - h_\sigma)\theta_1\}. \tag{17}$$

We have thus built the model. The system has 17 variables, K, F, Y, C, S, U, r, ξ, λ, w_j, σ_j, $\breve{T}_j$ and $\breve{t}_j (j = 1, 2)$. It contains the same number of independent equations. We now examine properties of the dynamic system.

III. Properties of the Dynamic System

This section examines properties of the dynamic system.

First, by (8), (9) and (6),

$$C + S = (\xi + \lambda)(rK + w_1T_0N + w_2T_0N) - (\sigma_1 + \sigma_2)K, \tag{18}$$

holds. Substituting (12) and (3) into (18) yields

$$H(N^*) \equiv F - (\xi + \lambda)(\alpha + \beta z_0/N^*)F + (\sigma_1 + \sigma_2)K = 0, \tag{19}$$

where $z_0 \equiv T_0N(z_1 + z_2)$. It is direct to show that for any given $K > 0$ and $\sigma_m > \sigma_2 > 0$, the function, $H(N^*)$, has the following properties: $H(0) < 0$, $H(\infty) > 0$ and $dH/dN^* > 0$ for any $N^* > 0$. This implies that for any given $K > 0$ and $\sigma_m > \sigma_2 > 0$ the equation, $H(N^*) = 0$, has a unique positive solution, $N^*(t) = N^*(K, \sigma_2)$. By (1) and (5), we solve

$$\begin{aligned} \breve{T}_j &= \frac{(z_0 - N^*)\sigma_j}{z_jN(\sigma_1 + \sigma_2)}, \\ \breve{t}_1 &= \frac{(\sigma_2T_0z_1N - \sigma_1T_0z_2N + \sigma_1N^*)}{z_1N(\sigma_1 + \sigma_2)}, \\ \breve{t}_2 &= \frac{(\sigma_1T_0z_2N - \sigma_2T_0z_1N^* + \sigma_2N^*)}{z_2N(\sigma_1 + \sigma_2)}. \end{aligned} \tag{20}$$

It is necessary to require $\breve{T}_j \geq 0$ and $\breve{t}_j \geq 0$. As $1 + (\sigma_1 + \sigma_2)K/F = (\xi + \lambda)(\alpha + \beta z_0/N^*)$, $\alpha + \beta = 1$ and $\xi + \lambda < 1$, we have $z_0 > N^*$. This guarantees $\breve{T}_j > 0$. By (20), $\breve{t}_1 > 0$ if $N^* > (\sigma_1z_2 - \sigma_2z_1)T_0N/\sigma_1$. In the case of $z_2/\sigma_2 <$

z_1/σ_1, $\breve{t}_1 > 0$. If the ratio, z_1/σ_1, of husband's human capital and the family's propensity to use the husband's time at home is larger than that of the wife's, then the husband certainly works outside. If $N^* \leq (\sigma_1 z_2 - \sigma_2 z_1)T_0 N/\sigma_1$, then $\breve{t}_1 = 0$. The husband stays at home. Similarly, we may discuss $\breve{t}_2$. As $N^* > 0$, at least one adult from a family works outside.

We have the following lemma.

***Lemma* 1**

For any given $K(t) > 0$ and $0 \leq \sigma_2(t) < \sigma_m$ at any point of time, other variables are uniquely given as functions of $K(t)$ and $\sigma_2(t)$ by the following procedure

α_1 and λ by (12) $\rightarrow \xi$ by (13) $\rightarrow N^*$ by (19) $\rightarrow F$ by (2) $\rightarrow Y = F \rightarrow \breve{T}_j$ and $\breve{t}_j$, $j = 1, 2$, by (20) $\rightarrow r$ and w_j by (3) $\rightarrow C$ and S by (8) $\rightarrow U$ by (6).

By (11), (14), and (15), we have

$$\frac{dK}{dt} = \lambda\Omega(K, \sigma_2) - K,$$
$$\frac{d\sigma_2}{dt} = \theta\left(\frac{\theta_1}{1 + \theta_2 K^a Y^b} - \sigma_2\right), \tag{21}$$

where $\Omega(K, \sigma_2)$ is given by (9). The two-dimensional dynamics determine the two variables, $K(t)$ and σ_2(t), over time. The other variables are uniquely determined by lemma 1 at any point of time. This implies that in order to analyze dynamic properties of the economic system, it is sufficient to examine (21).

An equilibrium of (20) is determined by

$$\lambda\Omega = K, \ \theta_1 = \sigma_2 + \sigma_2\theta_2 K^a Y^b. \tag{22}$$

By (8), $\Omega = K/\lambda$, (12) and $F = K^\alpha N^{*\beta}$,

$$N^* = \left(\frac{\xi}{\lambda}\right)^{\frac{1}{\alpha}} K, \tag{23}$$

holds. Substituting $Y = F$ and (23) into the second equation in (22), we solve

$$K = \left(\frac{\theta_1 - \sigma_2}{\sigma_2\theta_2}\right)^{\frac{1}{a+b}} \left(\frac{\xi}{\lambda}\right)^{\frac{-b\beta}{\alpha(a+b)}} \tag{24}$$

Substituting (9) into $\Omega = K/\lambda$ and using (23) and (24), we have

$$H^*(\sigma_2) \equiv H_0(\sigma_2) - \beta z_0 \lambda^{\frac{a+\alpha b}{\alpha(a+b)}} = 0, \tag{25}$$

in which

$$H_0(\sigma_2) \equiv \xi^{\frac{\alpha\beta}{\alpha(a+b)}} (\sigma_1 + \sigma_2 + \beta\xi) \left(\frac{\theta_1 - \sigma_2}{\sigma_2 \theta_2} \right)^{\frac{1}{a+b}}, \tag{26}$$

and σ_1, ξ and λ are given in (13) and (14). In the case of $a + b > 0$, $H^*(0) > 0$ and $H^*(\theta_1) < 0$; in the case of $a + b < 0$, $H^*(0) < 0$ and $H^*(\theta_1) > 0$. This implies that the equation, $H^*(\sigma_2) = 0$, has at least one solution in the interval of $0 < \sigma_2 < \theta_1$. Taking derivatives of H^* with respect to σ_2 yields

$$(a+b)H'^* = -H_0 \left\{ (1 + h_\lambda - h_\sigma) \frac{\alpha\beta}{\alpha\xi} + (a+b) \frac{\alpha h_\sigma + \beta h_\lambda - \alpha}{\sigma_1 + \sigma_2 + \beta\xi} + \frac{\theta_1}{\sigma_2(\theta_1 - \sigma_2)} \right\}$$
$$-\beta h_\lambda z_0 \lambda^{\frac{\alpha\beta}{\alpha(a+b)}} \frac{a + \alpha b}{\alpha}, \tag{27}$$

where we use (13) and (14). Since $H^*(0) > (<)\ 0$ and $H^*(\theta_1) < (>)\ 0$ in the case of $a + b > (<)\ 0$, we conclude that $H^*(\sigma_2) = 0$ has a unique solution in the interval of $0 < \sigma_2 < \theta_1$ if the right-hand side of (27) is negative for $0 < \sigma_2 < \theta_1$. Otherwise, the problem may have multiple solutions. As a and b may either be positive or negative, it is not easy to generally judge the sign of $H^{*\prime}$. For any given equilibrium value of σ_2, by (23) and the procedure in lemma 1 we uniquely determine all the other variables.

Summarizing the above discussion, we get the following proposition.

***Proposition* 1**

The dynamic system has at least one equilibrium. If the right-hand side of (27) is negative, then the system has a unique equilibrium.

It should be remarked that we don't provide stability conditions as it is difficult to explicitly interpret the results. The eigenvalues can be calculated directly from (19)-(22).

In the remainder of this study, we examine effects of changes in some parameters on the equilibrium structure. As shown below, conclusions are dependent on the sign of $H^{*\prime}$ at the equilibrium. In order to determine the sign of $H^{*\prime}$, we add constrains on some parameters. For simplicity, in the remainder of this study we assume

$$a > 0, \;\; b > 0, \;\; 1 + h_\lambda \geq h_\sigma, \;\; \alpha h_\sigma + \beta h_\lambda \geq \alpha. \tag{28}$$

The requirement of $a > 0$ and $b > 0$ implies that an increase in family's wealth or net income tends to reduce the family's propensity to use the wife's time at home. By (11), the meanings of the parameters, h_σ and h_λ, are given by

$$\frac{d\sigma_1}{d\sigma_2} = -h_\sigma, \;\; \frac{d\lambda}{d\sigma_2} = h_\lambda .$$

If we assume that the change rates of the family propensities to use the wife's and the husband's time at home are equal (but in opposite directions), i.e, $h_\sigma = 1$, then $1 + h_\lambda \geq h_\sigma$ and $\alpha h_\sigma + \beta h_\sigma \geq \alpha$ hold. It is not very strict to require (28). It must be emphasized that it is very difficult to specify a taste structure dynamics in general terms as the issue is too complicated.

From (27), we conclude that under (28), $H^{*\prime} < 0$. This also guarantees that the system has a unique equilibrium.

IV. Husband's Human Capital, z_1

This section is concerned with effects of changes in husband's human capital, z_1, on the equilibrium structure. Taking derivatives of (28) with respect to z_1 yields

$$-H^{*\prime}\frac{d\sigma_2}{dz_1} = -\beta T_0 N \lambda^{\frac{(a+\alpha b)}{\alpha(a+b)}} < 0, \tag{29}$$

in which $H^{*\prime}$ is negative under (28). An increase in the husband's human capital, z_1, reduces the family's propensity, σ_2, to use the wife's time at home. By (13) and (14),

$$\begin{aligned} &\frac{d\sigma_1}{dz_1} = -h_\sigma \frac{d\sigma_2}{dz_1} > 0, \;\; \frac{d\lambda}{dz_1} = h_\lambda \frac{d\sigma_2}{dz_1} < 0. \\ &\frac{d\xi}{dz_1} = -(1 + h_\lambda - h_\sigma)\frac{d\sigma_2}{dz_1} > 0, \end{aligned} \tag{30}$$

hold. As the husband's human capital is increased, the family's propensity, σ_1, to use the husband's time at home is increased, the family's propensity, λ, to hold wealth is reduced, and the family's propensity, ξ, to consume goods is increased.

Taking derivatives of (24) and (23) with respect to z_1, we get

$$\frac{\alpha(a+b)}{K}\frac{dK}{dz_1} = \left\{(1+h_\lambda - h_\sigma)\frac{b\beta}{\xi} - \frac{\alpha\theta_1}{\sigma_2(\theta_1-\sigma_2)} + \frac{b\beta h_\lambda}{\lambda}\right\}\frac{d\sigma_2}{dz_1},$$

$$\frac{\alpha(a+b)}{N^*}\frac{dN^*}{dz_1} = -\left\{\frac{\alpha\theta_1}{\sigma_2(\theta_1-\sigma_2)} + \frac{(a+\alpha b)(1+h_\lambda-h_\sigma)}{\xi} + \frac{(a+\alpha b)h_\lambda}{\lambda}\right\}\frac{d\sigma_2}{dz_1} > 0. \tag{31}$$

The total human capital, N^*, is increased. If $(1 + h_\lambda - h_\sigma)/\xi + h_\lambda/\lambda < (>)$ $\alpha\theta_1/b\beta\sigma_2(\theta_1 - \sigma_2)$, then the level of capital stocks, K, is increased (reduced). By (2) and $Y = F$

$$\frac{\alpha(a+b)}{F}\frac{dF}{dz_1} = -\left\{\frac{\alpha\theta_1}{\sigma_2(\theta_1-\sigma_2)} + (1+h_\lambda-h_\sigma)\frac{a\beta}{\xi} + \frac{a\beta h_\lambda}{\lambda}\right\}\frac{d\sigma_2}{dz_1} > 0,$$
$$\frac{dY}{dz_1} = \frac{dF}{dz_1} > 0, \tag{32}$$

hold. The total output, F, and the net income, Y, are increased. Taking derivatives of (3) with respect to z_1 yields

$$\frac{\alpha}{\beta r}\frac{dr}{dz_1} = -\left(\frac{1+h_\lambda-h_\sigma}{\xi} + \frac{h_\lambda}{\lambda}\right)\frac{d\sigma_2}{dz_1} > 0,$$
$$\frac{1}{w_1}\frac{dw_1}{dz_1} = \frac{1}{z_1} + \left(\frac{1+h_\lambda-h_\sigma}{\xi} + \frac{h_\lambda}{\lambda}\right)\frac{d\sigma_2}{dz_1}, \tag{33}$$
$$\frac{1}{w_2}\frac{dw_2}{dz_1} = \left(\frac{1+h_\lambda-h_\sigma}{\xi} + \frac{h_\lambda}{\lambda}\right)\frac{d\sigma_2}{dz_1} < 0.$$

The rate of interest, r, is increased and the wife's wage rate, w_2, is reduced. As $1/z_1 > 0$ and the other term in the right-hand side of the equation for dw_1/dz_1 is negative, dw_1/dz_1 may be either positive or negative. By (25), (27), and (29), we may rewrite dw_1/dz_1 in (33) as follows

$$-\frac{z_1\lambda^{1-\frac{\beta a}{\alpha(a+b)}}H^{*\prime}}{\beta w_1}\frac{dw_1}{dz_1} = z_0\left\{\frac{\alpha h_\sigma + \beta h_\lambda - \alpha}{\sigma_1+\sigma_2+\beta\xi} + \frac{\theta_1}{\sigma_2(\theta_1-\sigma_2)(a+b)}\right\}$$

$$+\{\beta a z_1 + (a + \alpha b) z_2\} \frac{h_\lambda T_0 N}{\lambda \alpha (a+b)} \tag{34}$$
$$+\{z_2 a\beta + (a\beta - \alpha a - \alpha b) z_1\} \frac{(1 + h_\lambda - h_\sigma) T_0 N}{\xi \alpha (a+b)}$$

If $z_2 a\beta + (a\beta - \alpha a - \alpha b) z_1 \geq 0$, then $dw_1/dz_1 > 0$. Otherwise, the husband's wage rate may be either increased or decreased. By $\lambda\Omega = K$ and (8), we have $\breve{T}_j = \sigma_j K/\lambda w_j N$ and $C = \xi K/\lambda$. Taking derivatives of these equations with respect to z_1 yields

$$\frac{\alpha(a+b)}{\breve{T}_1} \frac{d\breve{T}_1}{dz_1} = -\frac{\alpha(a+b)}{z_1} - \left\{ \frac{(1 + h_\lambda - h_\sigma)(\alpha a + \alpha b - b\beta)}{\xi} + \frac{(a+b)\alpha h_\sigma}{\sigma_1} \right.$$
$$\left. + \frac{\alpha\theta_1}{\sigma_2(\theta_1 - \sigma_2)} + \frac{(2\alpha a + 2\alpha b - b\beta) h_\lambda}{\lambda} \right\} \frac{d\sigma_2}{dz_1},$$
$$\frac{\alpha(a+b)}{\breve{T}_2} \frac{d\breve{T}_2}{dz_1} = \left\{ \frac{(1 + h_\lambda - h_\sigma)(b\beta - \alpha a - \alpha b)}{\xi} \right.$$
$$\left. + \frac{\alpha(a+b)(\theta_1 - \sigma_2) - \alpha\theta_1}{\sigma_2(\theta_1 - \sigma_2)} + \frac{(b\beta - 2\alpha a - 2\alpha b) h_\lambda}{\lambda} \right\} \frac{d\sigma_2}{dz_1}, \tag{35}$$
$$\frac{\alpha(a+b)}{C} \frac{dC}{dz_1} = -\left\{ \frac{(\alpha a + \alpha b - b\beta)(1 + h_\lambda - h_\sigma)}{\xi} + \frac{\alpha\theta_1}{\sigma_2(\theta_1 - \sigma_2)} \right.$$
$$\left. + \frac{(\alpha a + \alpha b - b\beta) h_\lambda}{\lambda} \right\} \frac{d\sigma_2}{dz_1}.$$

In the case of $\alpha a + \alpha b \geq b\beta$, the level of family consumption, C, is increased. The husband's and the wife's time, $\breve{T}_1$ and $\breve{T}_2$, at home may be either increased or decreased.

V. Concluding Remarks

This study suggested an one-sector growth model with endogenous capital and preference changes to examine dynamic interdependence of economic growth and sexual division of labor. We were mainly concerned with behavior of the dynamic system when female population takes part in labor market. We showed that the dynamic system may have either a unique or multiple equilibria. We examined the effects of changes in the husband's human capital on the equilibrium economic structure and sexual division of labor.

We may further examine dynamic behavior of the model, for instance, by simulation. Although we defined a dynamic growth model, our analysis was mainly concerned with the equilibrium structure of the economic system. It is significant to simulate the model to see how parameters may affect dynamic features, such as growth rate at any point of time and sexual division of labor. This can be carried out with computer. We may extend the model in different ways. For instance, it is significant to introduce multiple sectors into the economy. We may examine effects of changes in some other parameters, such as population, marginal productivity, on the long-run economic growth and sexual division of labor. It is also important to examine behavior of the system when there is strict sexual division of labor, such as husband works and woman stays at home, between male and female population (i.e., some "corner solutions" in the system).

The study may be extended in some other directions. For instance, we may introduce endogenous knowledge into the system (e.g., Zhang, 1992, 1993). This study did not specify how family game is actually played. We assumed an ideal family structure. Some people may actually be unmarried for whole life. A couple may divorce, which simply implies non-existence of family utility function.

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