Two-Stage Cournot Oligopolies with Industry-wide Externalities

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A two-stage Cournot oligopoly with cost functions involving externalities is formulated. A unique subgame perfect Cournot-Nash Equilibrium is proven to exist under a set of reasonable assumptions. The existence proof consists of solving two fixed-point problems for the industry output. The perfect equilibrium is then compared with one-shot Cournot oligopoly equilibrium. The effects of entry are also analyzed. Our result will be illustrated using linear cost and demand functions. (*JEL* classification: L13)

I. Introduction

Any firm in perfectly or imperfectly competitive situation engages in productive activity over many periods. It is quite common that in a two-stage oligopoly where any firm produces over two consecutive periods, its second-period cost function depends not only on its second-period output but also on its first-period one or on the industry total output. The productivity of labor of any firm in an industry may increase if production is repeated over time. The firm-specific externalities originating from the firm's learning by doing has been introduced into oligopoly models by Fudenberg and Tirole (1983), Bulow *et al.* (1985) and Slade (1989).[1] However, externalities due to repetitive production may some-

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[1]Slade (1989) was concerned with two-stage Cournot oligopoly with firm specific externalities from an algorithmic point of view. She reformulated a two-stage Cournot oligopoly (or more generally, two-stage dynamic games) as a game

times be industry-wide. If industry total output in the first-period increases, the inputs necessary for the industry's product may be supplied more cheaply in the second period because of economies of scale in the input producing industry, causing a downward shift of the second-period cost function for any firm. An increase in the first-period industry total output increases the total demand for the input, leading to its higher price because of its scarcity. This in turn will cause an upward shift of the second-period cost function for any firm. In other case, expansion of an industry in the first-period may induce the industry to devote its effort to educate its labor more systematically. This may increase the labor's second-period productivity and cause a downward shift of the second-period cost function. We can conceive of many other cases of industry-wide externalities, where any firm's second-period industry cost function is affected by the first-period total output.

In this paper we will be concerned with a two-stage oligopoly with industry-wide externalities and without product differentiation. In Section II, we will present a novel proof of the existence of a unique subgame perfect (Cournot-Nash) equilibrium for our two-stage oligopoly model under a set of general assumptions regarding the inverse market demand function and firm's cost function. Our proof will consist of solving a two-stage fixed-point problem for the first-period industry total output. We will then compare the equilibria for one-short and two-stage Cournot oligopolies. In Section III we will illustrate, on the basis of a linear inverse demand function, the validity of our assumptions used in establishing our main results in Section II. Our analysis in Section II will enable us easily to clarify the effects of entry on our two-stage Cournot oligopoly. This is the task of Section IV. The final section concludes.

II. Model and Unique Subgame-Perfect Equilibrium

Let n be the number of firms in Cournot oligopoly without product

with a fictitious aggregate payoff function of which maximizing condition is identified with the equilibrium condition for the original two-stage Cournot oligopoly (or two-stage dynamic games). However, she did not analyze in detail the existence of the two-stage Cournot-Nash equilibrium. Furthermore, her model was based on a linear market demand function. In Bulow et al (1985), the existence problem was not analyzed except for a simple numerical example.

differentiation. Furthermore, let x_{1i} and x_{2i}, be firm i's first-and second-period outputs, respectively. If industry-wide externalities exist, firm i's first-and second-period cost functions are given by

$$C^{1i} = C^{1i}(x_{1i}),\ i = 1, 2, \ldots, n \tag{1}$$

and

$$C^{2i} = C^{2i}(x_{1i}, Q_1),\ i = 1, 2, \ldots, n, \tag{2}$$

respectively, where $Q_1 \equiv \sum_{j=1}^{n} x_{1j}$ is the first-period industry output. The cost function (2) shows that firm i's second-period cost is affected not only by its second-period output but also by the first-period industry output. We define the externalities to be positive if $(\partial C^{2i})/(\partial Q_1) < 0$ and $(\partial^2 C^{2i})/(\partial Q_1 \partial x_{2i})$ < 0, and negative if both inequalities are reversed.[2] If $Q_2 \equiv \sum_{j=1}^{n} x_{2j}$ is the second-period industry output, firm i's second period profit function is defined by

$$\pi^{2i} \equiv x_{2i} f(Q_2) - C^{2i}(x_{2i}, Q_1),\ i = 1, 2, \ldots, n, \tag{3}$$

where $p = f(\cdot)$ with $f' < 0$ is the product price as a decreasing function of the industry output. This inverse demand function is assumed to hold for both periods. Maximizing (3) with respect to x_{2i} and assuming an interior maximum, we get

$$\frac{\partial \pi^{2i}}{\partial x_{2i}} = f(Q_2) + x_{2i} f'(Q_2) - \frac{\partial C^{2i}}{\partial x_{2i}} = 0,\quad i = 1, 2, \ldots, n. \tag{4}$$

Now we must introduce the following assumptions

$$f'(Q_2) + x_{2i} f''(Q_2) < 0,\quad i = 1, 2, \ldots, n. \tag{5}$$

$$f'(Q_2) < \frac{\partial^2 C^{2i}}{\partial x_{2i}^2},\quad i = 1, 2, \ldots, n. \tag{6}$$

The assumption (5) requires that firm i's second period marginal revenue, $f(Q_2) + x_{2i} f'(Q_2)$, be decreasing with respect to any other firm's output for all i. A similar assumption has been widely used in analyzing the existence and stability of one-shot Cournot oligopoly equilibri-

[2] In Bulow *et al.* (1985), externalities are firm specific, so instead of $(\partial C^{2i})/(\partial Q_1)$ < 0 and $(\partial^2 C^{2i})/(\partial Q_1 \partial x_{2i}) < 0$, they have $(\partial C^{2i})/(\partial x_{1j}) < 0$ and $(\partial^2 C^{2i})/(\partial x_{1j} \partial x_{2i}) < 0$. According to them, joint economies exist if $(\partial^2 C^{2i})/(\partial x_{1j} \partial x_{2i}) < 0$ and joint diseconomies exist if inequality is reversed. Extending Bulow et al, we can say that industry-wide joint economies or diseconomies exist depending on whether $(\partial^2 C^{2i})/(\partial Q_1 \partial x_{2i}) < 0$ or $(\partial^2 C^{2i})/(\partial Q_1 \partial x_{2i}) > 0$.

um (see Okuguchi 1976, and Okuguchi and Szidarovszky 1990). Since $f'(Q_2) < 0$, (6) holds if the second-period marginal cost of any firm is constant or increasing for an arbitrary Q_1, (see also Okuguchi 1976, and Okuguchi and Szidarovszky 1990, for the corresponding assumption for one-shot Cournot oligopoly). Under (5) and (6) the second order condition is satisfied. Totally differentiating (4), we have

$$\left(f'(Q_2) - \frac{\partial^2 C^{2i}}{\partial x_{2i}^2}\right)dx_{2i} + (f'(Q_2) + x_{2i}f''(Q_2))dQ_2 + \left(\frac{\partial^2 C^{2i}}{\partial Q_1 \partial x_{2i}^2}\right)dQ_1 = 0, \tag{7}$$

$$i = 1, 2, \ldots, n.$$

Equations (4) and (7) together yield

$$x_{2i} = \varphi^{2i}(Q_1, Q_2), \quad i = 1, 2, \ldots, n, \tag{8}$$

where in the light of (5) and (6)

$$\varphi_1^{2i} = \frac{\partial \varphi^{2i}}{\partial Q_1} \gtreqless 0 \ \text{according to} \ \frac{\partial^2 C^{2i}}{\partial Q_1 \partial x_{2i}} \lesseqgtr 0, \quad i = 1, 2, \ldots, n. \tag{9}$$

$$\varphi_2^{2i} = \frac{\partial \varphi^{2i}}{\partial Q_2} < 0, \quad i = 1, 2, \ldots, n. \tag{10}$$

Given Q_1, the second period equilibrium industry output is given as a solution of a fixed point problem,

$$Q_2 = \sum_{i=1}^{n} \varphi^{2i}(Q_1, Q_2) \equiv \varphi^2(Q_1, Q_2) \tag{11}$$

where , if $\dfrac{\partial \varphi^2}{\partial Q_1} \equiv \varphi_1^2, \ \dfrac{\partial \varphi^2}{\partial Q_2} \equiv \varphi_2^2, \ \dfrac{\partial \varphi_i^2}{\partial Q_j} \equiv \varphi_{ij}^2, \ i, j = 1, 2,$

we have

$$\frac{dQ_2}{dQ_1} = \frac{\varphi_1^2}{1 - \varphi_2^2} \gtreqless 0 \ \text{according to} \ \frac{\partial^2 C^{2i}}{\partial Q_1 \partial x_{2i}} \lesseqgtr 0, \quad i = 1, 2, \ldots, n. \tag{12}$$

$$\frac{d^2 Q_2}{dQ_1^2} = (1 - \varphi_2^2, \varphi_1^2)\begin{pmatrix} \varphi_{11}^2 & \varphi_{12}^2 \\ \varphi_{21}^2 & \varphi_{22}^2 \end{pmatrix}\begin{pmatrix} 1 - \varphi_2^2 \\ \varphi_1^2 \end{pmatrix} \tag{13}$$

As it stand, the sign of $(d^2Q_2)/(dQ_1^2)$ is indeterminate.

Firm i's total profit over two periods are given by

$$\begin{aligned} \pi^i &\equiv \pi^{1i} + \pi^{2i} \\ &= \{f(Q_1)x_{1i} - C^{1i}(x_{1i})\} + f(Q_2(Q_1))\varphi^{2i}(Q_1, Q_2(Q_1)) \\ &\quad - C^{2i}(\varphi^{2i}(Q_1, Q_2(Q_1)), Q_1), \quad i = 1, 2, \ldots, n, \end{aligned} \tag{14}$$

where we have assumed away the discount factor, as it does not affect the essential analysis that follows. The first order condition for maximizing π^i with respect to x_{1i} is

$$\frac{\partial \pi^i}{\partial x_{1i}} = f(Q_1) + x_{1i} f'(Q_1) - C_x^{1i} + A_i(Q_1) = 0, \quad i = 1, 2, \ldots, n, \tag{15}$$

where

$$A_i(Q_1) \equiv f'(Q_1)\varphi^{2i} \frac{dQ_2}{dQ_1} + f(Q_1)\left(\varphi_1^{2i} + \varphi_2^{2i} \frac{dQ_2}{dQ_1}\right) - \left\{C_x^{2i}\left(\varphi_1^{2i} + \varphi_2^{2i} \frac{dQ_2}{dQ_1}\right) + C_Q^{2i}\right\}, \quad i = 1, 2, \ldots, n, \tag{16}$$

$$\frac{dC^{1i}}{dx_{1i}} \equiv C_x^{1i}, \frac{\partial C^{2i}}{\partial x_{2i}} \equiv C_x^{2i}, \frac{\partial C^{2i}}{\partial Q_1} \equiv C_Q^{2i}, \quad i = 1, 2, \ldots, n.$$

The second order condition is

$$\frac{\partial^2 \pi^i}{\partial x_{1i}^2} = f'(Q_1) + x_{1i} f''(Q_1) + f'(Q_1) - C_{xx}^{1i} + A_i'(Q_1) < 0, \quad i = 1, 2, \ldots, n. \tag{17}$$

Before going further, we introduce three additional assumptions

$$f'(Q_1) + x_{1i} f''(Q_1) < 0, \; i = 1, 2, \ldots, n, \tag{18}$$

$$f'(Q_1) < C_{xx}^{1i}, \; i = 1, 2, \ldots, n, \tag{19}$$

$$A_i'(Q_1) \leq 0, \; i = 1, 2, \ldots, n, \tag{20}$$

Assumptions (18) and (19) correspond to (5) and (6), respectively. Since $f' < 0$, we know that (15) holds under (18), (19) and (20). Solving (15) with respect to x_{1i}, we have

$$x_{1i} = \varphi^{1i}(Q_1), \quad i = 1, 2, \ldots, n, \tag{21}$$

where in the light of (18) – (20),

$$\frac{dx_{1i}}{dQ_1} = -\frac{f' + x_{1i} f'' + A_i'(Q_1)}{f' - C_{xx}^{1i}} < 0, \quad i = 1, 2, \ldots, n. \tag{22}$$

Assumptions (18) and (20) yield[3]

[3]Note that

$$\beta_{1i} \equiv f' + x_{1i} f'' + f''\varphi^{2i}\left(\frac{dQ_2}{dQ_1}\right)^2 + f'\varphi^{2i} \frac{d^2 Q_2}{dQ_1^2} + f'\left(\varphi_1^{2i} + \varphi_2^{2i} \frac{dQ_2}{dQ_1}\right)\frac{dQ_2}{dQ_1} + \left(\varphi_1^{2i} + \varphi_2^{2i} \frac{dQ_2}{dQ_1}\right)\left(f' \frac{dQ_2}{dQ_1} - C_{xQ}^{2i}\right)$$

$$\beta_{1i} \equiv f' + x_{1i}f'' + A_i'(Q_1) < 0,\ i = 1, 2, \ldots, n. \tag{23}$$

We note here that

$$\frac{\partial^2}{\partial x_{1j}\partial x_{1i}}(\pi^i(x_{11}, x_{12}, \ldots, x_{1n})) = \beta_{1i}, \quad i = 1, 2, \ldots, n. \tag{24}$$

Under (23), any firm's rate of change of its total profit with respect to its first-period output as a function of all firms' first-period output is a decreasing function of any other firms first-period output, thus in our two-stage Cournot oligopoly, any two firms' first-period outputs are strategic substitutes in the extended sense[4] of Bulow et al (1985).

The first-period equilibrium industry output is identical to the fixed point of $\varphi^1(Q_1)$, namely the solution of

$$Q_1 = \sum_{i=1}^{n} \varphi^{1i}(Q_1) \equiv \varphi^1(Q_1), \tag{25}$$

where in view of (21),

$$\frac{d\varphi^1}{dQ_1} < 0. \tag{26}$$

Assume that

$$\varphi^1(0) > 0.^5 \tag{27}$$

The curve for $\varphi^1(Q_1)$, which takes a positive value for $Q_1 = 0$, is down-

$$+\left(\varphi_1^{2i} + \varphi_{21}^{2i}\frac{dQ_2}{dQ_1} + \varphi_2^{2i}\frac{d^2Q_2}{dQ_1^2}\right)(f - C_x^{2i}) - C_{QQ}^{2i}, \quad i = 1, 2, \ldots, n.$$

This expression is needed in Section V to evaluate β_{1i} for a simple example.

[4]According to Bulow et al (1985), any two firms' outputs are strategic substitutes if any firm's marginal profit in one period decreases in the event of an increase in any other firm's output in the same period.

[5]Let the value for $Q = 0$ of the middle expression in (14) be

$$g_i(x_{1i}) \equiv f(0) + x_{1i}f'(0) - C_x^{1i} + A_i(0).$$

Then the sign of $g_i(0)$ is, in general, indeterminate. However, if $f(0)$ is sufficiently large and if, in addition, $C_x^{1i}(0) \geq 0$, we have $g_i(0) > 0$. According to assumption (19),

$$g_i'(x_{1i}) \equiv f'(0) - C_{xx}^{1i} < 0.$$

Hence x_{1i} which satisfies $g_i(x_{1i}) = 0$ must be positive for any i. This implies that $\varphi^{1i}(0) > 0$, hence the validity of assumption (27).

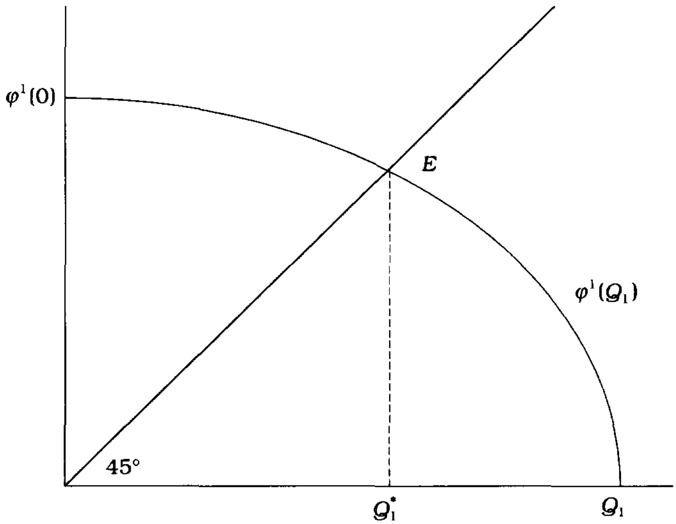

Figure 1
First-Period Equilibrium

ward-sloping because of (26). Therefore, it has a unique intersection with a 45 degree line emanating from the origin, as shown in Figure 1. Hence (25) has a unique solution which is identical to the first-period equilibrium industry output.

We now turn to the problem of comparing this subgame-perfect Cournot-Nash equilibrium for two-stage oligopoly and the Cournot-Nash equilibrium for one-shot oligopoly. This comparison turns out to depend crucially on the sign of $A_i\ (Q_1)$, which is rewritten[6] in view of

[6]The derivation of (28) is as follows: Taking into account

$$\frac{\partial \pi^{2i}}{\partial x_{2j}} = \varphi^{2i} f', \quad j \neq i, \quad \frac{\partial \pi^{2i}}{\partial x_{2i}} = f + \varphi^{2i} f' - C_x^{2i},$$

$$f - C_x^{2i} = -x_{2i} f' \quad \text{(see (4))}$$

we get

$$\sum_{j=1}^{n} \frac{\partial \pi^{2i}}{\partial x_{2j}} \left(\frac{\partial \varphi^{2j}}{\partial Q_1} + \frac{\partial \varphi^{2j}}{\partial Q_2} \frac{dQ_2}{dQ_1} \right) = \varphi^{2i} f' \sum_{j=1}^{n} \left(\frac{\partial \varphi^{2j}}{\partial Q_1} + \frac{\partial \varphi^{2j}}{\partial Q_2} \frac{dQ_2}{dQ_1} \right)$$

$$+ (f - C_x^{2i}) \left(\frac{\partial \varphi^{2i}}{\partial Q_1} + \frac{\partial \varphi^{2i}}{\partial Q_2} \frac{dQ_2}{dQ_1} \right)$$

$$= \varphi^{2i} f' \frac{dQ_2}{dQ_1} - \varphi^{2i} f' \left(\varphi_1^{2i} +_2^{2i} \frac{dQ_2}{dQ_1} \right)$$

(14), the definition of π^i, as a function of Q_1 alone as

$$\begin{aligned} A_i(Q_1) &= \frac{\partial \pi^{2i}}{\partial Q_1} + \sum_{j=1}^{n} \frac{\partial \pi^{2i}}{\partial x_{2j}} \left(\varphi_1^{2j} + \varphi_2^{2j} \frac{dQ_2}{dQ_1} \right) \\ &= -C_Q^{2i} + f'\varphi^{2i} \left\{ (1 - \varphi_2^{2i}) \frac{dQ_2}{dQ_1} - \varphi_1^{2i} \right\}, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{28}$$

If the industry-wide positive externalities as we have defined exist, $C_Q^{2i} < 0$, $\varphi_1^2 > 0$ and $\varphi_1^2 = (dQ_2)/(dQ_1) > 0$ (see (9) and (12)). In this case the sign of A_i is indeterminate. In the case of industry-wide negative externalities, $C_Q^{2i} > 0$, $\varphi_1^2 < 0$ and $(dQ_2)/(dQ_1) < 0$, resulting also in the indeterminacy of the sign of A_i. If industry-wide externalities are absent, we have $A_i = 0$. In the last expression of (28), the first term refers to the effect on firm i's second-period cost of a change in the first-period industry output due to a change in firm i's first-period output. The second term refers to the total effect of the same change on firm i's second-period profits. The sign of the second effect is indeterminate. However, if industry-wide externalities are positive and the first effect dominates the second one, A_i becomes positive. If, on the other hand, industry-wide externalities are negative and the first term dominates the second one, A_i is negative.[7]

$$= \varphi^{2i} f'(1 - \varphi_2^{2i}) \frac{dQ_2}{dQ_1} - \varphi^{2i} f'\varphi_1^{2i}.$$

This last expression together with $(\partial\pi^{2i})/(\partial Q_1) = -C_Q^{2i}$ leads to (28).

[7]As an extreme case, let the second term vanishes, that is,

$$\frac{dQ_2}{dQ_1} = \frac{\varphi_1^{2i}}{1 - \varphi_2^{2i}}.$$

However, we have from (12)

$$\frac{dQ_2}{dQ_1} = \frac{\varphi_1^{2}}{1 - \varphi_2^{2}}.$$

These two equations simultaneously holds if $n = 1$. Hence monopoly leads to

$$A \gtreqless 0 \quad \text{according to} \quad \frac{\partial^2 C^2}{\partial x_1 \partial x_2} \gtreqless 0,$$

where we have omitted superscript and subscript denoted by i. Next consider a symmetric case where firms have identical cost functions. Then $A_i\,(Q_1)$ is evaluated at the equilibrium as

$$A_i(Q_1) = -C_Q^{2i} + \varphi^{2i} f' \frac{n-1}{n} \frac{\varphi_1^2}{1 - \varphi_2^2}$$

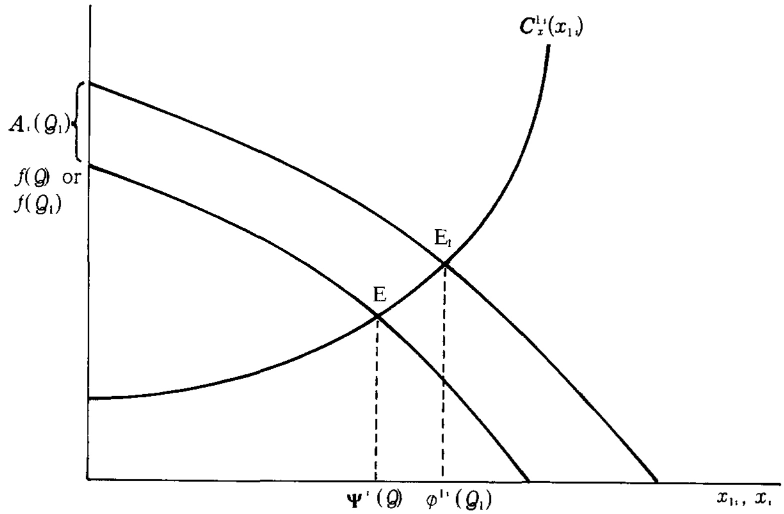

FIGURE 2
CASE OF $A_i\,(Q_1) > 0$

Let Q_1^* be the equilibrium first-period industry output in the two-stage oligopoly and let Q^* be the equilibrium industry output for one-shot Cournot oligopoly. In order to compare Q_1^* and Q^* we have to consider two cases according to the sign of $A_i\,(Q_1)$. Let $x_i = \psi^i\,(Q)$ be the solution of the first order condition for one-shot Cournot oligopoly as given by

$$f(Q) + x_i f'(Q) - C_x^{1i}(x_i) = 0, \quad i = 1, 2, \ldots, n, \tag{29}$$

where $Q \equiv \sum_{j=1}^{n} x_j$, the relationship between φ^{1i} and ψ^i is depicted either in Figure 2 when $A_i\,(Q_1) > 0$, or in Figure 3 when $A_i\,(Q_1) < 0$. Hence the

the sign of the second term being negative, zero or positive according as $(\partial^2 C^{2i})/(\partial Q_1 \partial x_{2i}) \lesseqgtr 0$. Even in this symmetric case, the sign of A_i is indeterminate. We can, however, get a more definite result if we let

$$C^{2i} \equiv C\,(Q_1)\,x_{2i}.$$

In this case, A_i turns out to be

$$A_i(Q_1) = \frac{-C'(Q_1)\varphi^{2i}}{n(f' + x_{2i}f'') + f'}\,\{f' + (f' + Q_2 f'')\}.$$

Assume the industry marginal revenue to be strictly decreasing $(2f' + Q_2 f'' < 0)$ and industry-side positive externalities to prevail $(C' < 0)$, then we get $A_i > 0$.

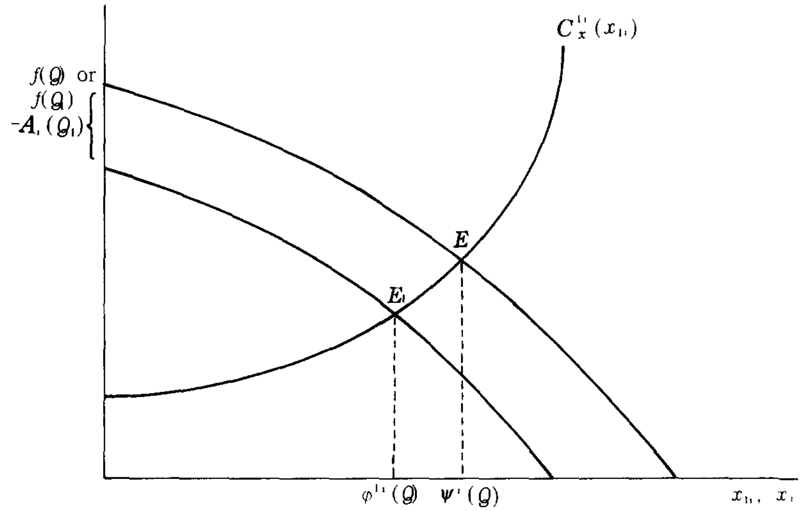

Figure 3
Case of $A_i\,(Q_1) < 0$

first-period industry output Q_1^* and one-shot oligopoly equilibrium output Q^* are comparable as in Figure 4 when $A_i\ (Q_1) > 0$ for all i or in Figure 5 when $A_i\ (Q_1) < 0$ for all i, namely

$$Q^* \lesseqgtr Q_1^* \text{ according to } A_i\,(Q_1) \gtreqless 0 \text{ for all } i. \tag{30}$$

Unfortunately, as we have seen above, the sign of A_i is indeterminate when positive or negative industry-wide externalities exist. In the absence of externalities, $A_i = 0$.

Let Q_2^* be the second-period industry output in two-stage oligopoly, then (30) together with (12) and (22) leads to the following assertion.

$$\begin{aligned} &Q^* \lesseqgtr Q_1^* \quad \text{and} \quad Q^* \leq Q_2^* \quad \text{according to} \quad A_i(Q_1) \gtreqless 0 \\ &\text{and} \quad \frac{\partial^2 C^{2i}}{\partial Q_1 \partial x_{2i}} \lesseqgtr 0 \quad \text{for all } i. \end{aligned} \tag{31}$$

We can explain this result intuitively as follows. If industry-wide externalities are sufficiently positive so that $A_i > 0$, an increase in any firm's first-period output, hence an increase in the industry output, leads to a decrease in any firm's cost in the second-period, which in turn leads to an increase in its second-period profit even if its second-period out-

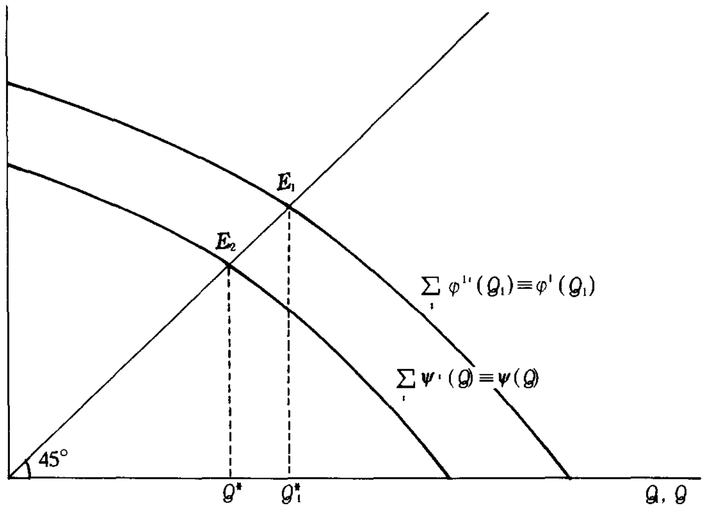

FIGURE 4
CASE OF $A_i(Q_1) > 0$

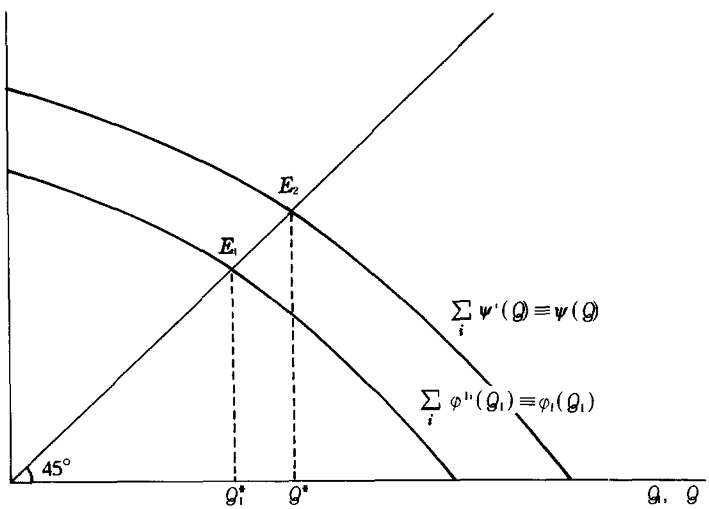

FIGURE 5
CASE OF $A_i(Q_1) < 0$

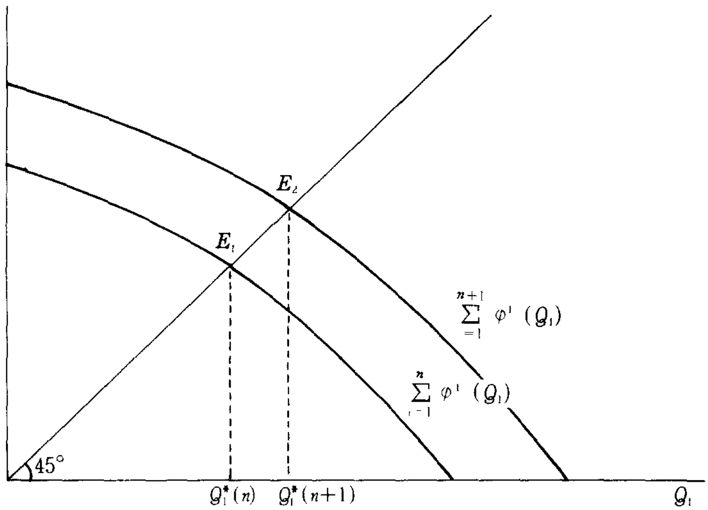

FIGURE 6

ENTRY AND FIRST-PERIOD OUTPUT

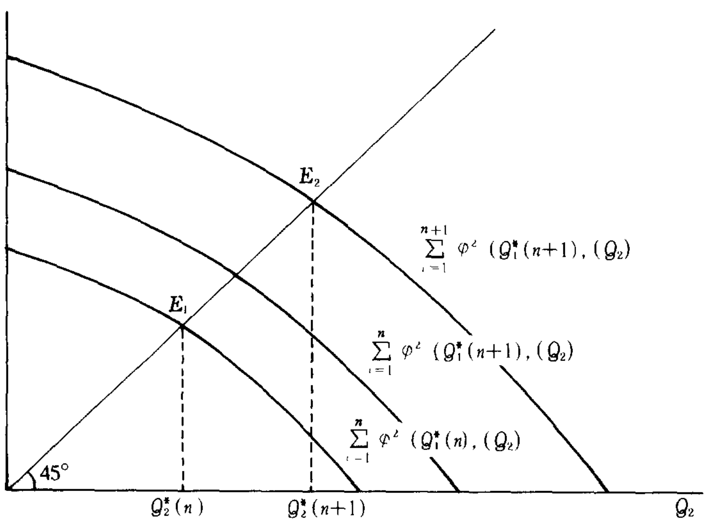

FIGURE 7

ENTRY AND SECOND-PERIOD OUTPUT WHEN $\frac{\partial^2 C^{2i}}{\partial Q_1 \partial x_{2i}} < 0$ FOR ALL i

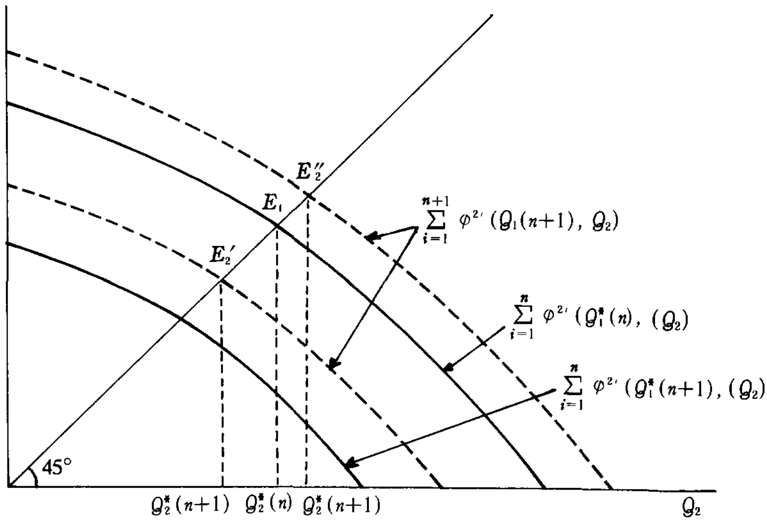

FIGURE 8

ENTRY AND SECOND-PERIOD OUTPUT WHEN $\dfrac{\partial^2 C^{2i}}{\partial Q_1 \partial x_{2i}} > 0$ FOR ALL i

put does not change, However a decrease in any firm's second-period marginal cost induces it to increase its second-period output. Hence Q_1^* > Q^* and $Q_2^* > Q^*$. The case of negative externalities may be similarly explained. Of course, if $A_i = 0$, we have $Q_1^* = Q^*$ and $Q_2^* = Q^*$.

III. An Example

Let us now consider a simple example to show an applicability of the result stated in (30). Consider the following inverse demand and cost functions,

$$p = a - bQ,\ a > 0,\ b > 0.$$
$$C^{1i}(x_{1i}) = c^{1i}x_{1i},\ i = 1, 2, \ldots, n.$$
$$C^{2i}(x_{2i}, Q_1) = C^{1i}(x_{2i}) - \delta Q_1 x_{2i},\ i = 1, 2, \ldots, n.$$

A small amount of calculation yields

$$\varphi_1^{2i} = \frac{\delta}{b},\quad \varphi_1^{2i} = -1,$$

$$\varphi_{11}^{2i} = \varphi_{12}^{2i} = \varphi_{21}^{2i} = \varphi_{22}^{2i} = 0,$$

for all i, which in turn leads to

$$\frac{dQ_2}{dQ_1} = \frac{n\delta}{b(n+1)}, \quad \frac{d^2Q_2}{dQ_1^2} = 0.$$

Hence for all i,

$$\beta_{1i} = -\left\{b + \frac{(n-1)\delta^2}{b(n+1)^2}\right\} < 0.$$

$$A_i(Q_1) = \frac{2\delta x_{2i}}{n+1} > 0.$$

Thus we claim that

$$A_i(Q_1) \gtreqless 0 \text{ according to } \frac{\partial^2 C^{2i}}{\partial Q_1 \partial x_{2i}} = -\delta \lesseqgtr 0 \text{ or } \delta \gtreqless 0.$$

In our example, $(\partial C^{2i})/(\partial Q_1) = -\delta x_{2i}$. Hence $Q_1^* > Q^*$ when industry-wide externalities are positive and $Q_1^* < Q^*$ if they are negative.

IV. Entry

In this section we analyze the effect of entry on the subgame-perfect Cournot-Nash industry equilibrium outputs, Q_i^* and Q_2^*. We assume that all assumptions in Section II are valid also for an entrant.

Regardless of the type of the externalities, the first-period equilibrium is determined as in Figure 6, where $Q_1^*(n)$ and $Q_1^*(n+1)$ are the equilibrium industry outputs before entry and after entry, respectively. As Figure 6 shows, $Q_1^*(n) < Q_1^*(n+1)$. Let $Q_2^*(n)$ and $Q_2^*(n+1)$ be the second-period industry outputs before and after entry, respectively. In Figure 7 where $A_i(Q_1) > 0$ for all i, we have $Q_2^*(n) \lesseqgtr Q_2^*(n+1)$ as shown in Figure 8. It is well known for classical Cournot oligopoly without externalities that $Q^*(n) < Q^*(n+1)$ under assumptions corresponding to (18) and (19). This means that $f(Q^*(n)) > f(Q^*(n+1))$, that is, the classical Cournot oligopoly is quasi-competitive. The quasi-competitiveness holds also for the first stage of two-stage Cournot oligopoly with positive or negative industry-wide externalities.

[8]Note that the same relationship is true even if C_x^{1i} is constant or decreasing.

V. Conclusion

In this paper we have proven the existence of a unique Cournot-Nash equilibrium for a two-stage Cournot oligopoly with industry-wide externalities and without product differentiation. If industry-wide externalities exist, each firm's second-period cost function depends on its second-period output as well as on the first-period industry output. The first-period equilibrium industry output is larger (smaller) than that for one-shot Cournot oligopoly if positive (negative) industry-wide externalities are sufficient so that $A_i(Q_1) > 0$ $(A_i(Q_1) < 0)$ for all i. The second-period equilibrium industry output is unambiguously larger in the presence of industry-wide externalities. As noted in footnote 7, the sign of A_i is indeterminate even if firms symmetric. However, if the inverse demand and cost functions are specified as in Section III, it turns out that $A_i(Q_1) > 0$ $(A_i(Q_1) < 0)$ if industry-wide externalities are positive (negative). Finally, we have clarified the effect of entry. We have found that the first stage in two-stage Cournot oligopoly is quasi-competitive, which does not necessarily hold in the second period.

(Received January, 1995; Revised November, 1995)

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