Strategic Use of Delegation in Almost Strictly Competitive Games

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A social situation is *problematic* when the rivalrous players face the game characterized by competing behavior with bad outcomes. In this circumstances, players could not admit the equilibrium payoffs allocation and try to figure out the mechanisms which lead to efficiency by transforming the structure of the underlying game. This paper examines the benefit of strategic use of delegation in *almost strictly competitive game* considering two competing principal-agent pairs. The main result of this paper is that principals can realize Pareto efficient outcome in the convex hull of the feasible payoff allocation pairs when they delegate agents to play the game by proposing an incentive scheme in order to mitigate the competing structure of the original game strategically. (*JEL* Classifications: D74, C72)

I. Introduction

When a person has judicial conflicts with other person, it is common that both hire their own lawyers and let them solve the conflict on behalf of the persons concerned. In modern corporations, owners hire managers to make firms' decisions. These delegation of decision making is due to, in part, the benefit of specialization. But in some class of noncooperative game, players can reach the cooperative behavior and be better off by using the strategic delegation.

Most of the recent studies considering the separation of ownership and control paid much attention to firms' decision making and behavior based on the theory of incentives and contracts (see Holmström and Tirole 1987 and Hart and Holmström 1987). The principal-agent approach[1] provided by Holmström 1979 is widely used to analyze risk-

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sharing and asymmetric information effects between uninformed player (principal) and informed player (agent). The principal-agent relationship in a single firm can be suitably described by this approach. However, different game theoretic models are needed when more than two competing principal-agent pairs are considered simultaneously.

The international trade policy analyzed by Brander and Spencer (1983, 1985) is a case of the strategic use of delegation in which a government can transform the domestic firm's incentive by export subsidies in order to change the behavior of the other competing foreign government and its firm pair. Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987) analyze this kind of competing principal-agent pairs in an oligopoly market with restricting the incentive scheme to be a linear combination of profits and costs or sales and costs. In their works, total output in the incentive equilibrium with Cournot-quantity game is increased and price is lower by implementing delegation. But the net payoff allocation to each principal is worse than that of original game which might be played by both principals. Fershtman, Judd and Kalai (1991) construct a formal model for delegation game using pure strategies, which shares the basic direction of research with this paper. They suggest the folk theorem in delegation game such that a Pareto efficient payoff in the original game is implementable through the agents' collusive behavior.

The strategic use of delegation, however, is not always efficacious in any cases. For example, principals do not need to consider the delegation if the outcome of the original game is Pareto efficient. We examine the benefit of delegation especially focusing on the situation that the rivalrous principals face the game characterized by competing behavior with bad outcomes. A social situation is defined as *problematic* (Raub and Voss 1990) if there is a conflict between the outcomes resulting from individually rational behavior of strategically interdependent players and those outcomes which are efficient (Pareto optimal). In this situation, players could not admit the equilibrium payoffs allocation and try to figure out the mechanisms which lead to efficiency by transforming the structure of the underlying game.

In this paper, we suggest the game theoretic model of delegation as one way of preference transformation when the payoff structure of the original game has competing interest which is contrary to each princi-

[1]Principal designs a compensation scheme for agent which maximizes his net payoff subject to incentive constraints and individually rationality constraint.

pal. This *delegation game* can be formulated as a two-stage model. In the first stage, each principal offers a strategic incentive contract independently and simultaneously to his agent. And in the second stage, each agent chooses an action from his principal's strategy set so as to maximize his incentive payoff.

Our two-stage model does not assume the asymmetric information between principal and agent. Each principal knows every probability distribution which might affect payoffs when he constructs an incentive contract. This incentive contract is publicly announced and will be perfectly observable to opponent agent in the second stage. This information structure is different from Myerson's (1982) analysis in which each agent knows his own contract and makes a Nash conjecture for the other agent's contract. His emphasis is on the difficulties of establishing the general existence of equilibrium. Katz (1991) also points out that the strategic delegation would be ineffective if the delegation contracts as precommitments are unobservable. However, Fershtman and Kalai (1993) show that unobservable delegation may still be beneficial, if there is a small probability of detection and games are repeated.

The main result of this paper is that principals can realize the Pareto efficient outcome in the convex hull of the feasible payoff allocation pairs when they delegate agents to play the game by proposing an incentive scheme in order to change the competing structure of the original game strategically.

The rest of this paper is organized as follows. We define the original game and present the formal delegation game model in Section II. In Section III, we show the result by examples. All proofs are found in the Appendix.

II. Delegation Game

There are two principals who face the game which is called the original game. A strategic form of the original game is as follows:

$$\Gamma^p = (P, (S_i)_{i \in p}, (\pi_i)_{i \in p}),$$

where $P = (p_1, p_2)$ is a set of two principals and, for each i in P, S_i is a nonempty set of strategies for principal i. For the set of strategy profiles $S = S_1 \times S_2$, the payoff π_i to each principal is described as a function from S to $\mathcal{R}$.

The strategy profile s^* in S is a *Nash equilibrium* of Γ^p if $\pi_i(s_i^*, s_j^*) \geq$

$\pi_i(s_i, s_j^*)$ for every $s_i \in S_i$ and $i, j = 1, 2(i \neq j)$. We consider the original game Γ^p for which there is at least one pure strategy Nash equilibrium throughout this paper.

Now let us assume that the original game is *almost strictly competitive* as introduced by Aumann (1961). First, recall that a *twisted equilibrium* for a game Γ^p is any strategy $\hat{s} \in S$ such that for $i, j = 1, 2(i \neq j)$ and for any $s_j \in S_j$

$$\pi_i(\hat{s}) \leq \pi_i(\hat{s}, s_j).$$

In two-person games, a twisted equilibrium is such that neither player can decrease the other player's payoff by a unilateral change of strategy. A game is called *almost strictly competitive* if (i) the set of payoff vectors to Nash equilibrium points equal to the set of payoff vectors to twisted equilibrium points, and (ii) the set of Nash equilibrium points intersects the set of twisted equilibrium points.

***Proposition* 1**

Assume the original game Γ^p is almost strictly competitive. Then there is a unique Nash equilibrium payoff $\pi^* = (\pi_1^*, \pi_2^*)$. (Aumann 1961)

Now we define the associated *delegation game* played by two competing principal-agent pairs. This delegation game is formulated as a two-stage game. In the first stage, principals offer compensation schemes independently and simultaneously to their agents. These contracts are publicly announced and perfectly observable to all agents. In the second stage, each agent plays the original game, choosing his principal's strategy so as to maximize his incentive provided in the first stage. In this case, agents are allowed direct communication between themselves but cannot make binding commitments.

Let us define a delegation game in strategic form associated with the original game Γ^p as follows:

$$\Gamma = ((P, A), (W, C), (\pi^p, \pi^a)).$$

The set of players $(P, A) = (p_1, p_2, a_1, a_2)$ denotes two principals and two agents. There does not exist a common agency problem, i.e., p_i cannot offer compensation scheme to a_j where $i \neq j$. Moreover, neither principals p_i and p_j nor the agents a_i and a_j contract between themselves. Each agent has a positive reservation value v, which is the smallest incentive that makes him take the compensation scheme and participate in the game.

The set of strategies of principal i is defined as follows:

$$W_i = \{w_i\colon \mathcal{R} \to \mathcal{R}^+ \mid w_i \text{ is a monotone increasing function}\}.$$

The compensation scheme w_i is based on his principal's gross profit realized by the delegated agents' choices. It is assumed that each principal is fully committed to the compensation scheme and these are common knowledge when the agents make their choices. Moreover, there is neither a moral hazard problem nor a chance for renegotiation between the principal and agent.

The strategy set of agent i is defined $C_i = \{c_i\colon W \to S_i\}$ in which the element c_i is a response function from the set of incentive contract combinations $W = W_1 \times W_2$ to the principal i's original strategy set S_i of Γ^p.

For any strategy profile (w_1, w_2, c_1, c_2), π_i^p and π_i^a are payoff functions to principal i and agent i, respectively.

$$\pi_i^p(w_1, w_2, c_1, c_2) = \pi_i(c_1(w_1, w_2), c_2(w_1, w_2))$$

and

$$\pi_i^a(w_1, w_2, c_1, c_2) = w_i(\pi_i(c_1(w_1, w_2), c_2(w_1, w_2))),$$

where π_i is a payoff function in the original game Γ^p. Notice that π_i^p is a gross payoff realized by agents' choices. Thus, the principal i gets this gross payoff nets the incentive he pays to agent i.

Now we define the agents' game $\Gamma^a(w)$ induced by $w = (w_1, w_2)$ which is a subgame of Γ as follows:

$$\Gamma^a(w) = (A, (C_i)_{i \in A}, (\pi_i^a)_{i \in A}),$$

where A is a set of two agents, C_i is a set of strategy and π_i^a is a payoff function as defined in Γ.

Definition 1

A strategy profile c^* in $C = C_1 \times C_2$ is a *Nash equilibrium* of $\Gamma^a(w)$ induced by $w = (w_1, w_2) \in W$ if

$$\pi_i^a(w, c_i^*, c_j^*) \geq \pi_i^a(w, c_i, c_j^*),$$

for every c_i in C_i and $i, j = 1, 2 (i \neq j)$. And define the *set of Nash equilibrium choices* of agents' game $\Gamma^a(w)$ denoted by $\mathcal{NE}(w)$ as follows:

$$\mathcal{NE}(w) = \{c^*(w) \in S \mid c^* \text{ is a Nash equilibrium of } \Gamma^a(w)\}.$$

Definition 2

A strategy profile c° in $C = C_1 \times C_2$ is a *locally competitive equilibrium* of $\Gamma^{a}(w)$ induced by $w = (w_1, w_2) \in W$ if

$$\pi_i^a(w, c_i, c_j^{\circ}) \leq \pi_i^a(w, c_i^{\circ}, c_j^{\circ}) \leq \pi_i^a(w, c_i^{\circ}, c_j),$$

for every $c_i \in C_i$, $c_j \in C_j$ and $i, j = 1, 2 (i \neq j)$. And define the *set of locally competitive equilibrium choices* of $\Gamma^{a}(w)$ denoted by $\mathcal{LCE}(w)$ as follows:

$$\mathcal{LCE}(w) = \{c^{\circ}(w) \in S \mid c^{\circ} \text{ is a locally competitive equilibrium of } \Gamma^{a}(w)\}.$$

Proposition 2

Assume the original game Γ^{p} is almost strictly competitive in pure strategies. Let $\pi^* = (\pi_1(s^*), \pi_2(s^*))$ be the unique equilibrium payoff corresponding to the Nash equilibrium $s^* \in S$ of Γ^{p}. Then,

1. $s^* \in \mathcal{LCE}(w)$ for every $w \in W$.
2. every $c^*(w) \in \mathcal{NE}(w)$ is weakly Pareto superior to any $c^{\circ}(w) \in \mathcal{LCE}(w)$.

Proof: See Appendix.

The Nash equilibrium of Γ^{p} is preserved as a Nash equilibrium choice of $\Gamma^{a}(w)$ by the monotonicity of compensation scheme. It is also a locally competitive equilibrium choice of agents by the first part of Proposition 2. Moreover, if there are other Nash equilibrium choices of $\Gamma^{a}(w)$, agents might have a chance to choose them because those Nash equilibrium choices are weakly Pareto superior to s^* from the agent's perspectives. However, we do not know how agents choose those strategies and whether principal will be Pareto improved due to agents' Pareto superior Nash equilibrium choices. The only intuition of the above proposition is that agents may possibly choose a strategy profile which is different from the Nash equilibrium of Γ^{p} if $\mathcal{NE}(w)$ is not a singleton.

Definition 3

Any probability distribution over the set of Nash equilibrium choices of $\Gamma^{a}(w)$ is a *perfectly correlated equilibrium*[2] of agents' game induced by w. That is, μ_w is a *perfectly correlated equilibrium* induced by w if

[2]Maskin and Tirole (1987) defined the same equilibrium concept in which they are concerned with a competitive economy rather than a well-defined game.

$$\mu_w \in \Delta\mathcal{NE}(w) = \{\mu_w : \mathcal{NE}(w) \to [0,1] \mid \sum_{s \in \mathcal{NE}(w)} \mu_w(s) = 1\}.$$

Correlated equilibrium[3] is the relevant solution concept for game communication is possible, but in which no contracts are binding. Since agents are allowed to communicate directly with each other, not through a mediator, the only self-enforcing plans that the agents can coordinate would be randomizations among the Nash equilibrium choices of $\Gamma^a(w)$. In this case, the perfectly correlated equilibrium payoff vectors lie in the convex hull of the Nash equilibrium payoffs of $\Gamma^a(w)$. Bernheim, Peleg and Whinston (1987) develop a similar concept of *Coalition-Proof Nash equilibrium* that is designed to capture the notion of an efficient self-enforcing agreement for environments with unlimited, but nonbinding, pre-play communication. In two player games, the set of Coalition-Proof Nash equilibria is the same as the set of Nash equilibria which are not Pareto dominated by any other Nash equilibrium. Therefore, the definition of perfectly correlated equilibrium is also a Coalition-Proof Nash equilibrium when we construct the model in such a way that agents maximize the expected sum of their payoffs.

Given the delegation game Γ and perfectly correlated equilibrium μ_w of $\Gamma^a(w)$ induced by w which maximizes the expected sum of the agents' payoffs, let us define the expected payoff functions to principal i and agent i in von Neumann-Morgenstern utility scale as follows:

$$U_i^p(w, \mu_w) = \sum_{s \in \mathcal{NE}(w)} \mu_w(s)\pi_i(s) - \sum_{s \in \mathcal{NE}(w)} \mu_w(s) w_i(\pi_i(s))$$

and

$$U_i^a(w, \mu_w) = \sum_{s \in \mathcal{NE}(w)} \mu_w(s) w_i(\pi_i(s)),$$

[3]Following Aumann (1974,1987), a *correlated strategy* for a set of players, given any strategic form game $\Gamma^p = (P, S, \pi)$, is any probability distribution over the set of possible combinations of pure strategies that these players can choose in Γ^p. That is, for $S = S_1 \times S_2$,

$$\mu_w \in \Delta(S) = \{\mu : S \to [0,1] \| \sum_{s \in S} \mu(s) = 1\}.$$

We call μ a correlated equilibrium of Γ^p if $\mu \in \Delta(S)$ and

$$\sum_{s \in S} \mu(s)\pi_i(s) \geq \sum_{s \in S} \mu(s)\pi_i(e_i, s_j)$$

for any e_i in S_i and $i, j = 1, 2 (i \neq j)$.

where π_i is a payoff function of Γ^p and $\mathcal{NE}(w)$ is the set of Nash equilibrium choices of $\Gamma^a(w)$. In the first stage of the delegation game Γ, principal i offers a compensation scheme to his agent. If the principal i decides to delegate, then the agent i's expected payoff should not be less than his reservation value. The principal i, finally, gets the payoff realized in agents' game nets the incentive he pays to agent i.

Definition 4

The compensation function $w_i \in W_i$, $i = 1,2$, is a *strategic forcing contract* if $w_i(\pi_i) = 0$ for $\pi_i \leq \pi_i^*$ and $U_i^a(w, \mu_w) \geq v$ where π_i^* is the unique Nash equilibrium payoff of Γ^a, μ_w is a perfectly correlated equilibrium of Γ^a which maximizes the expected sum of the agents' payoffs and v is a positive reservation value of agent i.

This nonnegative monotone increasing contract can be interpreted as one in which the agent is paid according to some prespecified schedule depending on the outcome which will be realized by agents' choices. In cases of Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), the payoff realized in the agents' game is less than the Nash equilibrium payoff in the original game. Principals, however, would not consider the opportunity of delegation if the expected payoff realized by agents' choices is not larger than the Nash equilibrium payoff π^* of original game. When the realized outcome is unacceptable to principal (the expected payoff realized in agents' game is lower than the original game's Nash equilibrium payoff), his payment zero can be interpreted as dismissal of the agent or principal's commitment not to delegate. This compensation scheme is called *strategic* because principals could mitigate their competing behavior by transforming the payoffs structure. More specifically, each principal could transform the preference equivalently for every payoff below the prespecified disagreeable level. It is also called a *forcing* contract because, by the satisfaction of the individual rationality condition, agents always take the compensation schemes and join the game.

Now we define the equilibrium of delegation game Γ supported by a noncooperative subgame perfect equilibrium concept and state the main result.

Definition 5

$(w_1^*, w_2^*, \mu_{w^*}^*)$ is a *subgame perfect equilibrium* of delegation game Γ with respect to the strategic forcing contract if

1. For every pair of strategic forcing contract $w = (w_1, w_2)$, μ^*_w is a perfectly correlated equilibrium of agents' game $\Gamma^a(w)$ induced by w which maximizes the expected sum of the agents' payoffs.

2 $U^p_i(w^*_i, w^*_j, \mu^*_{w^*}) \geq U^p_i(w_i, w^*_j, \mu^*_{(w_i, w^*_j)})$ for every strategic forcing contract wi and $i, j = 1, 2 (i \neq j)$.

***Proposition* 3**

Assume the symmetric original game Γ^p is problematic and almost strictly competitive in pure strategies. Then, there exists a subgame perfect equilibrium of delegation game Γ with respect to the strategic forcing contract in which the gross payoff allocation to the principals realized in agents' game is Pareto efficient in the convex hull of the feasible payoff allocation pairs of Γ^a.

***Proof*:** See Appendix.

When players are confronted with competing interest which is contrary to each other, they try to mitigate the competing structure of the game. But, in some cases, it is hard to compromise the conflict with each other as a rival and finally inefficient outcomes are incurred as a result. By Proposition 3, in this case, players (principals) can get Pareto efficient outcomes in the convex hull of the feasible payoff allocation pairs through the strategic use of delegation.

Fershtman, Judd and Kalai (1991) use the definition of *implementation* and assume the *mutual rationality condition* for agents (i.e., agents will choose the Nash equilibrium point collusively among the multiple Nash equilibria of agents' game) to disregard the usual difficulty of dealing with multiple Nash equilibria as well as a choice of Pareto dominated equilibrium in the agents' game. However, we apply the different solution concept allowing the self-enforcing coordination between agents. That is, agents can freely discuss their strategies with each other before they make choices, but agreements among the agents are meaningless unless they are self-enforcing .

It is worth to note that the principal's strategic advantage by delegation is due to the commitment power through the compensation scheme. But this contract constitutes genuine commitment whenever it is observable to rivals and is credible. Therefore, the assumption that the compensation scheme between a principal and his agent is observable to the other players is central to the result. Katz (1991) points out that the lack of observability, and hence the lack of credibility, of such contracts may undermine their strategic role. Similarly, Bagwell (1993) examines the strategic role of commitment when other players observe the contract imperfectly, and shows that the advantage of committing to a course of action is eliminated when other players observe this commitment with even the slightest amount of imprecision.

III. Examples

***Example* 1:** Convex feasible set of the original game

Table 1 describes the prisoners' dilemma game as an original game. The strategy profile (Y_1, Y_2) is a unique Nash equilibrium as well as a unique twisted equilibrium which gives payoffs allocation (0, 0) that is not efficient. Thus, the original game played by two principals is almost strictly competitive and problematic.

Now both principals delegate agents with following compensation scheme.

$$w_i^*(\pi_i) = \begin{cases} v & \text{if } \pi_1 \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

The managers' payoffs based on the above strategic forcing contract are shown in Table 2.

There are multiple Nash equilibria in this agents' game. But by allowing direct pre-play communication, both agents choose the self-enforcing correlated strategy $\mu(X_1, X_2) = 1$ which maximizes the expected sum of the agents' payoffs. The agents realize the Pareto efficient payoff allocation of the original game, and thus the principals would not play the original game by themselves and delegate agents as long as their reservation value is less than 2.

***Example* 2:** Nonconvex feasible set of the original game

Table 3 describes also the prisoners' dilemma game as an original game. The payoff allocation (2, 2) by the strategy profile (X_1, X_2) is Pareto efficient but it is not in the Pareto frontier of the convex hull of the feasible payoff allocation pairs. As was in Example 1, (0, 0) is the unique equilibrium payoff allocation of this problematic and almost strictly competitive game.

Now both principals delegate agents with following compensation scheme.

TABLE 1
CONVEX ORIGINAL GAME

	Principal 2	
Principal 1	X_2	Y_2
X_1	2, 2	-1, 3
Y_1	3, -1	0, 0

TABLE 2
AGENTS' GAME

	Agent 2	
Agent 1	X_2	Y_2
X_1	v, v	0, v
Y_1	v, 0	0, 0

TABLE 3
NONCONVEX ORIGINAL GAME

	Principal 2	
Principal 1	X_2	Y_2
X_1	2, 2	-1, 7
Y_1	7, -1	0, 0

$$w_i^*(\pi_i) = \begin{cases} 2v & \text{if } \ \pi_1 \geq 3 \\ 0 & \text{otherwise} \end{cases}$$

The managers' payoffs based on the above strategic forcing contract are shown in Table 4.

There are three Nash equilibrium choices in this agents' game, i.e., (X_1, Y_1), (Y_1, X_2) and (Y_1, Y_2). But agents choose $\mu(X_1, Y_2) = \mu(Y_1, X_2) = 1/2$ as a self-enforcing correlated strategy which maximizes the expected sum of the agents' payoffs. Hence, the agents realize the Pareto efficient payoff allocation in the convex hull of the feasible payoff pairs of the original game and thus the principals would delegate agents as long as their reservation value is less than 3.

TABLE 4
AGENTS' GAME

	Agent 2	
Agent 1	X_2	Y_2
X_1	v, v	0, $2v$
Y_1	$2v$, 0	0, 0

Appendix: Proofs

Proof of Proposition 2

A. Since the original game Γ^p is almost strictly competitive, the Nash equilibrium $s^* \in S$ corresponding to the unique Nash equilibrium payoff $\pi^* = \pi(s^*)$ satisfies

$$\pi_i(s_i, s_j^*) \leq \pi_i(s_i^*, s_j^*) \leq \pi_i(s_i^*, s_j), \tag{1}$$

for every $s_i \in S_i$, $s_j \in S_j$ and $i, j = 1, 2$ $(i \neq j)$.

By the monotonicity of $w_i \in W_i$, the Nash equilibrium s^* of Γ^p is preserved as a Nash equilibrium choice of $\Gamma^a(w)$, i.e., $s^* \in \mathcal{NE}(w)$. Let $\delta_i \in C_i$ be the agent i's response function satisfying

$$\delta_i(w) \in \mathcal{NE}(w) \quad \text{and} \quad \delta_i(w) = s_i^*.$$

Then, by the monotonicity of w_i, (1) provides, for player 1,

$$w_1(\pi_1(s_1, \delta_2(w))) \leq w_1(\pi_1(\delta_1(w), \delta_2(w))) \leq w_1(\pi_1(\delta_1(w), s_2)),$$

for every pair of $w \in W$ and every $s_i \in S_i$ $(i = 1, 2)$. This implies, for every $c_i \in C_i$ $(i = 1, 2)$,

$$\pi_1^a(w, c_1, \delta_2) \leq \pi_1^a(w, \delta_1, \delta_2) \leq \pi_1^a(w, \delta_1, c_2).$$

Similarly

$$\pi_2^a(w, \delta_1, c_2) \leq \pi_2^a(w, \delta_1, \delta_2) \leq \pi_2^a(w, c_1, \delta_2).$$

Therefore, $\delta(w) = (\delta_1(w), \delta_2(w)) \in \mathcal{LCE}(w)$.

B. $c^\circ(w) \in \mathcal{LCE}(w)$ implies c° is a twisted equilibrium of $\Gamma^a(w)$. Thus, it satisfices;

$$\pi_1^a(w, c_1^\circ, c_2^\circ) \leq \pi_1^a(w, c_1^\circ, c_2^*) \text{ and } \pi_2^a(w, c_1^\circ, c_2^\circ) \leq \pi_2^a(w, c_1^*, c_2^\circ).$$

Since c^* is a Nash equilibrium of $\Gamma^a(w)$, it satisfies;

$$\pi_1^a(w, c_1^\circ, c_2^*) \leq \pi_1^a(w, c_1^*, c_2^*) \text{ and } \pi_2^a(w, c_1^*, c_2^\circ) \leq \pi_2^a(w, c_1^*, c_2^*).$$

Therefore, by adding up together

$$\pi_1^a(w, c^\circ) \leq \pi_1^a(w, c^*) \text{ and } \pi_2^a(w, c^\circ) \leq \pi_2^a(w, c^*).$$

Q.E.D.

Proof of Proposition 3

Let F be the feasible payoff allocation pairs of Γ^p, ∂F be the associated Pareto frontier of F. Similarly, let $\hat{F}$ be the convex hull of F, and $\partial\hat{F}$ be the Pareto frontier of $\hat{F}$.

For $\bar{\pi} = (\alpha, \alpha) \in \partial\hat{F}$, define a compensation function $w_i^* \in W_i$ as follows:

$$w_i^*(\pi_i) = \begin{cases} v(1+\varphi) & \text{if } \pi_i \geq \alpha \\ 0 & \text{otherwise} \end{cases}$$

for

$$\varphi = \begin{cases} 0 & \text{if } \bar{\pi} \geq \partial F \\ 1 & \text{otherwise.} \end{cases}$$

For every pair of strategic forcing contract w, let us define μ_w^* induced by w as follows:

$$\mu_w^* \in \operatorname{argmax}_{\mu_w} \sum_{s \in \mathcal{NE}(w)} \mu_w(s)\{w_1(\pi_1(s)) + w_2(\pi_2(s))\},$$

where $\mu_w \in \Delta\mathcal{NE}(w) = \{\mu_w\colon \mathcal{NE}(w) \to [0, 1] \mid \Sigma_{s \in \mathcal{NE}(w)} \mu_w(s) = 1\}$. Then, by construction, μ_w^* is a perfectly correlated equilibrium of $\Gamma^a(w)$ induced by a strategic forcing contract w that maximizes the expected sum of the agents' payoffs.

Now we check whether w^*, defined above, is a strategic forcing contract and next we will show w_i^* is indeed a best response strategy to principal i. Since Γ^p is problematic and almost strictly competitive, the unique Nash equilibrium payoff is not Pareto efficient. Thus, we have, $w_i^*(\pi_i) = 0$ for every $\pi_i \leq \pi_i^*$ $(\leq \alpha)$.

Case 1: Suppose $\bar{\pi} = (\alpha, \alpha) \in \partial F$. then,

$$w_i^*(\pi_i) = \begin{cases} v & \text{if } \pi_i \geq \alpha \\ 0 & \text{otherwise.} \end{cases}$$

By the symmetry of Γ^p, there exists a (pure) Pareto efficient strategy profile $\bar{s}$ such that $\pi_1(\bar{s}) = \pi_2(\bar{s}) = \alpha$. Let $\delta = (\delta_1, \delta_2) \in C$ be the pair of agents' response functions such that $\delta(w^*) = \bar{s}$. In this case, the existence of such δ is easily verified by following construction:

$$\delta(w^*) = \begin{cases} \bar{s} & \text{if } \bar{s} \in \mathcal{NE}(w^*) \\ s^* & \text{otherwise.} \end{cases}$$

Now it is enough to show that $\bar{s} \in \mathcal{NE}(w^*)$ and individual rationality condition. The agents' payoffs allocation corresponding to $\bar{s}$ is (v, v), which is the largest compensation from w^* when $\bar{\pi} \in \partial F$. Neither agent could gain by unilateral deviating from $\delta(w^*) = \bar{s}$. Thus, $\bar{s} \in \mathcal{NE}(w^*)$ and obviously $\mu^*_{w^*}(\bar{s}) = 1$ since it is a perfectly correlated equilibrium which maximizes the expected sum of the agents' payoffs. And for individual rationality condition,

$$U^a_i(w^*, \mu^*_{w^*}) = w^*_i(\pi_i(\bar{s})) = w^*_i(\alpha)\ v.$$

Therefore, w^* is a strategic forcing contact.

Case 2: Suppose $\bar{\pi} = (\alpha, \alpha)$ is not in the ∂F. Then,

$$w^*_i(\pi_i) = \begin{cases} 2v & \text{if } \pi_i \geq \alpha \\ 0 & \text{otherwise.} \end{cases}$$

By the symmetry of Γ^p, there exist (pure) Pareto efficient strategies (t, r) and (r, t), for $t \neq r$, such that $\pi(t, r) \in \partial F$ and $\pi(r, t) \in \partial F$. Without loss of generality, let $\pi_i(t, r) > \pi_j(t, r)$ for $i, j = 1, 2$ $(i \neq j)$, then $\pi_i(r, t) > \pi_j(t, r)$. Now define $a = 1/2\{\pi_i(t, r) + \pi_i(r, t)\}$, then $\bar{\pi} = (\alpha, \alpha) \in \partial\hat{F}$ and we have $\pi_i(t, r) > \alpha > \pi_i(r, t)$. Thus, w^* provides

$$w^*_i(\pi_i(t, r)) = 2v \text{ and } w^*_i(\pi_i(r, t)) = 0.$$

Since the agents' game induced by this w^* is also symmetric and $2v$ is the largest compensation, (t, r) and (r, t) are in $\mathcal{NE}(w^*)$. Moreover, $\mu^*_{w^*}(t, r) = \mu^*_{w^*}(r, t) = 1/2$. Thus, we have

$$U^a_i(w^*, \mu^*) = \frac{1}{2}\{w^*_i(\pi_i(t, r)) + w^*_i(\pi^p_i(r, t))\} = v,$$

which satisfies individual rationality condition.

Now the only remaining to show is that w^*_i is indeed the best response strategy to principal i. This is obvious because when w^* is offered, agents' perfectly correlated equilibrium $\mu^*_{w^*}$ realizes Pareto efficient payoff allocation in $\partial\hat{F}$. This implies any principal i cannot be better off from deviation.

Q.E.D.

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