Implications of Debt Renegotiation for Optimal Bank Policy and Firm Behavior

Joonmo Cho and Ashraf Nakibullah*

This paper analyzes the problems associated with the renegotiation of debt contracts involving a bank (the lender) and a firm (the borrower) when the latter is operated by a risk averse manager. Firms undertake risky projects with loan capital borrowed from the bank. When a firm cannot pay off a loan it is technically bankrupt. Both the borrower and the lender may experience a Pareto-improvement in their positions by renegotiating the loan. By renegotiating the terms of the debt the financially distressed firm can avoid the stigmatization of bankruptcy and the bank can avoid the costs of seizing the borrower's assets. However, our main finding is that, from the bank's point of view, renegotiation as a policy of recovering loan payments may be inefficient in practice because of a) false bankruptcy claims and b) moral hazard problems associated with exposure or the firm to the risk of default. We present a solution to the false bankruptcy claim problem that involves a mixed strategy between asset seizure by the bank and debt renegotiation. We also discuss the of collateral under the mixed strategy. \( \textit{JEL Classifications: G14, G21} \)

I. Introduction

Bank loan activity and its importance in the financing of domestic U.S. business is significant. Federal Reserve Flow of Funds data for instance indicate that short-term debt for non-farm, non-financial corporations (seasonally adjusted) constituted on average 12% of total corporate funds sources for the period 1980-90. As a percentage of total debt finance, short term debt averaged about 26.5%. This fact is rein-

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forced by the studies of Duca (1992) and Duca and Vanhoose (1990) which show that bank loans and loan commitments comprise an important source of credit for domestic U.S. firms. One dimension of bank loan activity which has received only limited attention, but which is a common, known empirical phenomenon, is the revision and renegotiation process. The revision process is known to have a significant bearing on the value of the borrower (Lummer and McConnell 1989).¹ Little is known, however, about whether ex post debt renegotiation is an optimal policy from the perspective of the bank.² The analysis presented in this paper details the implications of such a bank policy for both the bank and for borrower, within a limited information setting.

This paper analyzes the problems associated with the renegotiation of debt contracts involving a bank (the lender) and a firm (the borrower) when the latter is operated by a risk averse manager. Firms undertake risky projects with loan capital borrowed from the bank. When a firm cannot pay off a loan it is technically bankrupt. Both the borrower and the lender may experience a Pareto-improvement in their positions by renegotiating the loan. By renegotiating the terms of the debt the financially distressed firm can avoid the stigmatization of bankruptcy and the bank can avoid the costs of seizing the borrower's assets. However, our main finding is that, from the bank's point of view, renegotiation as a policy of recovering loan payments may be inefficient in practice because of a) false bankruptcy claims and b) moral hazard problems associated with exposure of the firm to the risk of default.

The main focus of this paper is the optimality of renegotiation as a bank policy for recovering debt. We assume that before declaring bankruptcy, publicly, a manager must report to the banks the near bankruptcy status of the firm he operates. When the possibility of ex post renegotiation is recognized, two kinds of problems arise: false bankruptcy claims and a moral hazard problem associated with exposure of the firm to the risk of default. If renegotiation is expected in advance,

¹James (1987) and Mikkelson and Partch (1983) present evidence on the capital market's reaction to announcements of new credit agreements. The results of these papers suggest that the market views such agreements as positive news for borrower.
²For an excellent recent survey of the contemporary theory of financial intermediation, see Bhattacharya and Thakor (1992). Hart and Moore (1980) also examine the debt renegotiation problem, but do not investigate the implications of false bankruptcy claims or the moral hazard problem that arises from the connection between managerial actions and the probability of bankruptcy.
the manager of a firm may have an incentive to declare falsely that his firm is near bankruptcy. We assume that the bank cannot verify these claims.\(^3\) The inability of the bank to verify a near bankruptcy declaration may be due to unspecified information costs. Furthermore, since the managers know that the bank will lower the debt payment whenever they declare bankruptcy, their incentives to take costly actions that would reduce the probability of bankruptcy are reduced. Thus, the Pareto efficiency of the contracting arrangement may be undermined, and may even be worse than when a renegotiation policy is absent. We show that a mixed strategy of randomizing between renegotiation and asset seizure leads to a reduction in false bankruptcy claims. The mixed strategy requires the optimal collateral which circumvents the false bankruptcy claim even when the managers expect the renegotiation \textit{ex ante}. The role of collateral is different from the one in the other literature. For example, Besanko and Thakor (1987), Bester (1985, 1987) and Chan and Kanatas (1985) suggest that the outside collateral increases the punishment for default and thereby collateral enforces the selection of less-risky projects. In contrast to the above screening explanation, Bester (1994) argues that collateral is more likely to be used for financing high-risk investments. His study concludes that renegotiation may seriously undermine the role of collateral as a screening device. However, this paper provides an opposite reasoning for the role of collateral. As the collateral increases the punishment for being detected as the claimer of false bankruptcy, the incentive compatibility condition which discourages the false bankruptcy claim is satisfied even at a lower portion of renegotiation offered. Therefore, the collateral allows banks to save the seizure cost which would be paid in the absence of collateral.

The remainder of the paper is organized as follows. Section II contrasts the examination in this paper with related studies which also address the renegotiation problem but within different settings. Section III describes the basic model when renegotiation of debt is not expected in advance. The discussion in Section III also focuses on how the renegotiation of debt is \textit{ex post} Pareto-improving, on how the renegotiation of debt is a rent maximizing policy. Problems associated with renegotiation as a policy for recovering loan payments are analyzed in Section IV. Section V examines a mixed strategy solution to the false bankrupt-

\(^3\)The inability of the bank to verify a near bankruptcy declaration may be due to unspecified information costs.
### Table 1

**Notation**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>(\pi [\pi_L, \pi_H])</td>
<td>Gross state-contingent payoff from a unit investment in the risky project ([\text{state 1: state 2}]) where (\pi_H &gt; \pi_L).</td>
</tr>
<tr>
<td>(r)</td>
<td>Gross payoff on a unit investment in the risk-free project.</td>
</tr>
<tr>
<td>(D)</td>
<td>Loan payment due for a one unit agreement.</td>
</tr>
<tr>
<td>(\epsilon[g(\epsilon)])</td>
<td>Degree of manager disutility due to risk aversion for a unit investment in the risky project ([\text{distributed as a continuous variable over the range } (0, \infty)]).</td>
</tr>
<tr>
<td>(0[u])</td>
<td>Fixed operating cost of type 1 [type 2] firms.</td>
</tr>
<tr>
<td>(\alpha(1 - \alpha))</td>
<td>Fraction of all firms that are type 1 [type 2].</td>
</tr>
<tr>
<td>(C)</td>
<td>The size of collateral</td>
</tr>
<tr>
<td>(\delta \equiv \delta; \delta_2)</td>
<td>The level of managerial action exercised by a firm's manager ([\text{type 1 firms: type 2 firms}]). (\delta) influences the probability of bankruptcy; a manager incurs personal costs of effort.</td>
</tr>
<tr>
<td>(p(\delta) [p'(\delta) &lt; 0])</td>
<td>The probability of bankruptcy given the level of action (\delta).</td>
</tr>
<tr>
<td>(c(\delta) [c(\delta) &gt; 0])</td>
<td>The manager's personal cost from taking action (\delta).</td>
</tr>
<tr>
<td>(u_t(\delta; \epsilon))</td>
<td>Expected utility of the manager of a type (i) firm when renegotiation is not expected or allowed.</td>
</tr>
<tr>
<td>(\bar{u}_t(\delta; \epsilon))</td>
<td>Expected utility of the manager of a type (i) firm when renegotiation is permitted.</td>
</tr>
<tr>
<td>(\phi &gt; 0)</td>
<td>The cost to a borrower from the stigmatization of declaring bankruptcy.</td>
</tr>
<tr>
<td>(s)</td>
<td>The cost to the bank of seizing the assets of a firm that declares bankruptcy.</td>
</tr>
<tr>
<td>(\rho)</td>
<td>The non-pecuniary stigma cost to a borrower who declares bankruptcy falsely and is detected.</td>
</tr>
</tbody>
</table>

Section VI concludes the paper. Table 1 presents the notation used throughout the paper with a brief definition of each symbol utilized.

### II. Related Literature

In this paper we examine *ex post* Pareto-improving renegotiation within the context of credit contracting.\(^4\) In this sense, the model is in the same spirit as the models developed in Dewatripont and Maskin (1990) and Hart and Moore (1989). However, our analysis differs in important ways from the analyses presented by these authors—which

\(^4\)We borrow the concept of renegotiation from the recent literature on labor commitments (Baron and Besanko 1987 and Green and Laffont 1987). The focus of this literature has generally been on nonbinding agreements, not on Pareto-improving renegotiation moves.
we outline next.

Dewatripont and Maskin (1990) consider a firm which borrows to fund a long term project. The project generates profit over time but the firm must refinance the loan at the middle of the project's life. Ex post renegotiation (refinancing) takes place because even low quality firms will continue to generate positive profits over the entire life of the project. Since the bank cannot commit to a policy of no renegotiation in the model, low quality firms always apply for a loan. In the model we examine, the bank is willing to renegotiate the debt in order to maximize ex post loan payments recovered (as opposed to seizing the assets of the bankrupt firm—which involves real costs to the bank). Managers of financially distressed firms are willing to take the offer of a reduced loan payment because it allows their firm to avoid the stigmatization associated with bankruptcy. This stigma manifests itself as a real credit market cost. In short, ex post renegotiation takes place because of the bank's objective of loan collection maximization and the borrowers desire to minimize the stigma cost of declaring bankruptcy. However, in contrast to Dewatripont and Maskin, we also examine the implications of borrowers anticipating that renegotiation will be possible in some states of the world. This results in two problems, a) some borrowers make false bankruptcy claims to avoid debt payments, and b) moral hazard problems associated with managers not taking sufficient (costly) action to avoid bankruptcy arise.

Hart and Moore (1989) also consider a firm that borrows to invest in a long term project. Borrowers in their model cannot influence the probability of bankruptcy. They argue that the bank prefers to renegotiate the loan when the liquidation value of the assets involved is low. Because the borrower is unable to commit (credibly) to an agreement to pay a sufficient part of the asset's future value to the lender, liquidation (asset seizure by the bank) occurs even though it is inefficient. In their model the efficiency of ex post renegotiation of the loan seems to arise because the borrowers' actions in pursuing a project and their risk-taking behavior never change, regardless of whether they anticipate renegotiation or not. In the model examined here, we demonstrate that ex post renegotiation is inefficient if the possibility is anticipated because, as already mentioned, false bankruptcy claims arise along with moral hazard problems. We show that asset seizure can restore efficiency in the case of false bankruptcy claims when viewed as part of a mixed strategy of seizure and renegotiation. The structure of our model and the results developed, therefore, differ in important ways
from the models developed in Dewatripont and Maskin and in Hart and Moore.

Finally, the motive for renegotiation examined here is in contrast to the risk-sharing motive discussed for instance in Dewatripont (1989) and Gale (1991). These papers show that risk-sharing will influence the incentives to renegotiate under certain circumstances. Rather than re-illustrate these results, we choose to focus on alternative economic explanations not examined by these authors, effectively holding the risk-sharing effect fixed. In this sense our work compliments the analyses by Dewatripont and Gale on the role of risk-sharing in renegotiation.\(^5\)

III. The Model When Renegotiation Is Not Expected or Not Allowed

A. Assumptions of the Model

We keep the economic setting simple in order to highlight specific implications of debt renegotiation. Nevertheless, the setting is also rich enough to account for what have been shown to be important aspects of borrowers and lenders in other theoretical models (see Bhattacharya and Thakor 1992). As already emphasized, our objective is only to examine how renegotiation impacts the borrower and the lender. We therefore take certain aspects of the economic setting as given.

We assume that the managers of firms take out loans (debt contracts) from the bank and use these funds to engage in risky projects. The bank is therefore an asset transformer in the sense of Diamond (1984). Williamson (1987) has shown that debt is the optimal contract between the bank and the firm under conditions similar to those assumed herein. We treat the debt contract as a given feature of the economic setting.

Once the payoffs from the projects are realized, borrowing firms are expected to pay their loans back. When the \textit{ex post} payoff from a project is low so that the firm is unable to repay the debt, the manager declares bankruptcy. The bank uses legal avenues to seize any payoff necessary to meet the loan payment due along with what we label the seizure cost. The seizure cost (i.e., court fees, etc.) is paid by the loser

\(^5\)Berlin and Mester (1992) also investigate issues related to renegotiation. The focus of their work is however on the value of the option to renegotiate.
(i.e., the bankrupt firm). However, if the seized amount is less than the debt due, then the bank must pay the seizure cost.\textsuperscript{6} As an alternative, the bank may renegotiate the debt. From the standpoint of the bank, the renegotiation of debt is therefore a policy of recovering the maximum principal and interest due.

The firm's manager, on the other hand, views the renegotiation of debt as a cost saving strategy. Renegotiation allows the firm to save the cost of default, which in this case is stigmatization in the credit market.\textsuperscript{7} Declaring bankruptcy is not a free lunch. We assume that if a manager declares bankruptcy, the firm is stigmatized in the credit market and must pay a greater cost in order to obtain loans in the future. It is assumed that the stigma cost varies with the credit history of the firm. The existence of a stigma cost can be justified from either a credit rationing perspective (Stiglitz and Weiss 1981, 1983; Besanko and Thakor 1987) or a pure reputation perspective (Diamond 1989).

By reducing the debt (i.e., making the debt payable), the bank can induce the managers of firms near bankruptcy to pay voluntarily, and thereby can save the seizure cost. In order to induce these managers to accept the new offer, the offer must guarantee them a greater utility than the utility they would receive under bankruptcy. By taking the renegotiated offer, a firm can avoid the stigmatization from having a bad credit history.

As mentioned above, the possibility exists for \textit{ex post} Pareto-improving renegotiation of the loan payment between the bank and a firm's manager. With a greater seizure cost, the bank is more willing to offer

\textsuperscript{6}If the bank decides not to renegotiate it has several courses of action available. These action include:

(i) The bank takes private actions to recover the debt due prior to default.
(ii) The bank takes private actions to recover the debt due after default.
(iii) The bank initiates legal action to collect the debt due after default occurs.

For instance, prior to default, the bank may threaten to repossess the associated asset or to initiate legal action. The costs incurred in the process could include the opportunity and direct costs of persuasion. After default occurs, actions such as repossession could be initiated which would involve the direct costs of recovering the asset and its ultimate disposal (which may be at a price insufficient to cover the outstanding loan and the seizure costs). If legal actions are taken the direct costs of legal services are also incurred.

\textsuperscript{7}It is possible that renegotiation itself may also involve a stigma cost. We assume that the stigma of bankruptcy exceeds the cost associated with renegotiation. To simplify the presentation but preserve the relative relation between these stigma costs, we assume that the stigma cost of renegotiation is zero.
the mitigated debt to the managers of firms near bankruptcy. Likewise, with a greater stigma cost associated with bankruptcy, managers are more willing to take the offer. Firms are assumed to have a single manager. Each manager may select a risky project which needs one unit of loan capital to implement and which has a state-contingent, net-of-fixed-cost payoff (net payoff) that is determined by both the skill level of the manager and any actions he takes; the managerial action selected determines the probability of being in a good or bad state. The loan is provided by a bank. As in the classical principal-agent setting (Harris and Raviv 1979; Holmstrom 1982; Shavell 1979; Grossman and Hart 1983), managerial actions cannot be observed by the bank, even if the state-contingent payoff can be observed ex post by both parties. The amount paid to the bank depends on the selected project's realized net payoff.

Let \( \pi \) represent the gross state-contingent payoff of the risky project. There are assumed to be two possible states of nature. In the bad state, \( \pi \) takes the low value \( \pi_l \), and in the good state it takes the higher value \( \pi_H \). As an alternative to the risky project, the manager may invest in a risk-free project which yields a constant gross payoff \( r \).\(^8\)

The firm's obligation is to pay back \( D \) units of loan capital to the bank. \( D \) is assumed to be greater than 1.\(^9\) Providing loans to firms is assumed to be profitable ex ante for the bank.

Managers are risk averse with respect to the uncertainty associated with the risky project's payoff. This view is generally consistent with a setting in which managerial income and human capital are subject to risks which depend upon the investment choices that managers make for their firms and which cannot be hedged. This applies in particular to human capital (for a review of these issues see Milgrom and Robert 1992, Ch. 13). Let \( \varepsilon \) be the degree of disutility due to risk aversion for a manager when he/she engages in the risky project. We allow for heterogeneity of risk aversion among managers by letting \( \varepsilon \) be a continuous variable distributed between 0 and infinity, where \( g(\varepsilon) \) denotes the population density of \( \varepsilon \). The density \( g(\varepsilon) \) is assumed to be continuous

\(^8\)The maximum return on the risk free project is conceptually determined by optimizing given the associated technology. In order to keep things simple we assume this problem has been solved and that the resulting gross return is \( r \).

\(^9\)We assume that \( D \) is exogenously given. Our analysis is therefore done within a partial equilibrium context. Our focus is on the renegotiation of debt, we do not examine credit market equilibrium, nor do we address the issue of why financial intermediaries exist (see Bhattacharya and Thakor 1992).
over the range of $\varepsilon$.

Managers also have heterogenous entrepreneurial skills. We assume that a manager's entrepreneurial skill determines the fixed cost of the firm he operates. The manager knows his skill level while the bank does not. For simplicity, there are two types of firms, where type is determined by the entrepreneurial skill of the firm's manager. The managers of the type 1 firms are assumed to have better skills than those of the type 2 firms. As a result, the fixed costs of type 1 and type 2 firms are assumed to be $0$ and $u > 0$, respectively. The fractions of type 1 and 2 firms in the population are $\alpha$ and $(1 - \alpha)$ respectively. The parameter $\alpha$ is assumed to be independent of the level of managerial risk aversion $\varepsilon$.

If the firm's net payoff falls below its debt due, the manager declares bankruptcy. Suppose that $(\pi_L - D) < u < (\pi_H - D)$, then a type 2 firm's net payoff is below its debt level in the bad state (i.e., when $\pi_L - u < D$). The managers of these firms therefore declare bankruptcy in the bad state.

If bankruptcy is declared, the bank is assumed to have the authority to seize any income above the firm's fixed cost necessary to meet the debt due. The bank incurs the seizure cost $s$ in the process.\(^\text{10}\)

Declaring bankruptcy results in future costs for the defaulting firm that are associated with the stigmatization of the event. It seems plausible to assume that this stigma cost varies with the credit history of the firm. Most existing models of reputation (see Diamond 1989) suggest that the stigma effect decreases as the credit history of the firm increases.

We begin by examining optimal project selection when renegotiation is not expected or not allowed. We then investigate the implications of renegotiation. After receiving a bankruptcy notice, the bank is allowed to renegotiate the debt. That is, bank may offer a one-time, take-it-or-leave-it alternative debt payment, but may also choose to make no offer. This permits us to focus our attention on the Pareto improvement associated with debt renegotiation when default occurs.

**B. Optimal Managerial Action and Risky Project Selection**

Managers take actions which influence the chances of their firm's

\(^{10}\) We assume the bank uses legal means to obtain the authority required to seize the borrower's assets. The bank must pay the seizure cost if the seized amount is less than the initial debt.
success. Define \( p(\delta) \) as the probability of receiving a low payoff for a given level of action \( \delta \). We assume that \( p'(\delta) < 0 \).\(^{11}\) That is, as a higher level of action is chosen, the probability of receiving a low (high) payoff decreases (increases).

The bank cannot observe \( \delta \). Let \( c(\delta) \) be the cost to the manager of choosing level of action \( \delta \). It is assumed that \( c'(\delta) > 0 \). That is, personal cost increases with the level of action. Tension therefore exists between the benefits and costs of managerial action (from the manager's perspective).

Managers are risk averse, and their risk aversion influences project selection. The expected utility of a manager who chooses the risky project and action level \( \delta \) is denoted by \( v_i (i = 1, 2) \) where \( i \) refers to the classification of entrepreneurial skills described earlier. We continue to assume that \( \pi_L - D < u < \pi_H - D \). With this assumption it follows that

\[
\begin{align*}
    v_1 (\delta; \epsilon) &= p(\delta)(\pi_L + |1 - p(\delta)|\pi_H - D - \epsilon - c(\delta) \\
    v_2 (\delta; \epsilon) &= p(\delta)(-\phi) + |1 - p(\delta)|(\pi_H - D - u) - \epsilon - c(\delta).
\end{align*}
\]

Equations (1) and (2) are in certainty equivalent form. A type 1 firm can always pay back the debt regardless of the state. However, if a type 2 firm has a low payoff, the manager declares bankruptcy since the fixed cost causes the net payoff to be less than the debt payment due. Any payoff above the fixed cost \( u \) is seized by the bank. The parameter \( \phi (> 0) \) (2) is the cost from the stigmatization of declaring bankruptcy.

The managers of type \( i \) firms will choose the risky project when

\[
v_i (\delta; \epsilon) > r - D \quad \text{for } i = 1, 2.
\]

In (3), \( (r - D) \) is the net-of-debt payoff of the risk-free project. In terms of \( \epsilon \), (3) becomes

\[
\epsilon_i (\delta) > \epsilon \quad \text{for } i = 1, 2
\]

where

\[
\begin{align*}
    \epsilon_1 (\delta) &= \pi_H - p(\delta)(\pi_H - \pi_L) - r - c(\delta) \\
    \epsilon_2 (\delta) &= \pi_H - p(\delta)(\pi_H - D - u + \phi) - (r + u) - c(\delta).
\end{align*}
\]

If a manager's disutility from risk aversion is between 0 and a critical value \( \epsilon^*_i (\delta) \), then the manager will choose the risky project.

Let \( \delta_i (i = 1, 2) \) be the manager's optimal action in the type \( i \) firm. The

\(^{11}\) \( x'(\cdot) \) denotes the derivative of \( x(\cdot) \) with respect to its argument.
selected \( \delta_1 \) therefore maximizes \( v_1 (\delta_2, \varepsilon) \) and fulfills (6) and (7)

\[
H (\delta) = (\pi_H - \pi_L)
\]

where \( H(\delta) = -\frac{C'(\delta)}{p'(\delta)} \).

The second order condition is always guaranteed because \( H' (\delta) > 0 \). From (6) and (7), two cases present themselves.

**Case 1**: \( \phi < (D + u - \pi_L) \)

For managers of type 2 firms, the declaration of bankruptcy is a way to avoid a portion of the debt due (i.e., \( D + u - \pi_L \)). However, the market punishes bankrupted firms with the stigma cost \( \phi \). When the exempted portion of the debt due is greater than the stigma cost, the managers of type 2 firms have a greater incentive to select an action which exposes the firm to a greater risk of default as compared to the managers of type 1 firms (i.e., \( \delta_1 > \delta_2 \)). This occurs because bankruptcy declaration permits evasion of a portion of the debt due.

**Case 2**: \( \delta > (D + u - \pi_L) \)

When the stigma cost is greater than the exempted portion of the debt due, the managers of type 2 firms have an incentive to expend greater effort the managers of type 1 firms (i.e., \( \delta_1 < \delta_2 \)). This occurs because the managers of type 2 firms are penalized more severely by the stigmatization of bankruptcy if the bad state occurs.

Using equation (6) and (7), we have \( (\pi_H - \pi_L) p' (\delta_1) + c' (\delta_1) = 0 \) and

\[
(\pi_H - D - u + \phi) p' (\delta_2) + c' (\delta_2) = 0.
\]

Then, one may show that (see the Appendix)

\[
\frac{de_1 (\delta_1)}{d\pi_L} > 0 \quad \frac{de_1 (\delta_1)}{d\pi_H} > 0 \quad \frac{de_1 (\delta_1)}{dr} < 0
\]

\[
\frac{de_2 (\delta_2)}{d\pi_H} > 0 \quad \frac{de_2 (\delta_2)}{dD} > 0 \quad \frac{de_2 (\delta_2)}{d\phi} < 0
\]

\[
\frac{de_2 (\delta_2)}{du} < 0.
\]

Let \( F (\delta_1, \delta_2) \) denote the population proportion of firms who choose the risky project. Then

\( (D + u - \pi_L) \) is always positive by the assumption that \( (\pi_H - D) < u < (\pi_L - D) \).
\[ F(\delta_1, \delta_2) = \alpha \int_0^{\epsilon_1(\delta_1)} g(\epsilon) d\epsilon + (1 - \alpha) \int_0^{\epsilon_1(\delta_1)} g(\epsilon) d\epsilon. \] (11)

Comparative statics of (11) provide the following:

\[
\frac{dF(\delta_1, \delta_2)}{d\pi_t} = \alpha g(\epsilon_1) p(\delta_1) > 0
\] (12)

\[
\frac{dF(\delta_1, \delta_2)}{d\pi_H} = \alpha g(\epsilon_1)[1 - p(\delta_1)] + (1 - \alpha) g(\epsilon_2)[1 - p(\delta_2)] > 0
\] (13)

\[
\frac{dF(\delta_1, \delta_2)}{dr} = -(1 - \alpha) g(\epsilon_1) < 0
\] (14)

\[
\frac{dF(\delta_1, \delta_2)}{dD} = (1 - \alpha) g(\epsilon_2) p(\delta) > 0
\] (15)

\[
\frac{dF(\delta_1, \delta_2)}{d\phi} = -(1 - \alpha) g(\epsilon_2) p(\delta) < 0
\] (16)

\[
\frac{dF(\delta_1, \delta_2)}{du} = (1 - \alpha) g(\epsilon_2)[p(\delta) - 1] < 0.
\] (17)

From (8), (9), (10) and (12), (13), (14), (15), (16), (17), the following results are obtained.

**Result 1.** As the payoffs ($\pi_t$, $\pi_H$) increase, even managers with high risk aversion are likely to choose a risky project, and overall, more managers will choose risky projects.

**Result 2.** As the size of a firm's debt increases, declaring bankruptcy as a way of avoiding the debt claim becomes more tempting. The stigma effect from being bankrupted is perceived as a less severe punishment by managers. Hence, more firms will choose the risky project.

**Result 3.** As the stigma effect from bankruptcy increases, firms whose managers are characterized by high risk aversion are likely to choose the risk free project, and overall fewer firms will choose the risky project.

**Result 4.** As the return from the risk free project increases, fewer firms will choose the risky project.

**Corollary 1.** The future cost from the stigma effect (i.e., $\phi$) may change with a firm's credit history. If $\phi$ decreases with the credit history of a firm over time, a firm with a long credit history is more likely to choose a risky project than a firm with a short credit history.
Corollary 1 follows directly from Result 3.

C. Pareto-Improving Renegotiation

Once type 2 firms receive the low payoff (as a result of the bad state), their managers declare bankruptcy unless the bank offers a one-time debt reduction which makes the debt payable. Before declaring bankruptcy publicly, it is assumed that the manager must report the near-bankruptcy status of his firm to the bank.

The bank has an incentive to renegotiate the debt of financially distressed firms that are near bankruptcy. By mitigating the debt (i.e., by making the debt payable), the bank may be able to recover more because the seizure cost can be avoided. Of course, in order to induce the managers of type 2 firms to take the offer, the new offer must guarantee them a greater utility than their utility under bankruptcy. In the case examined here, a possibility exists for ex post Pareto-improving renegotiation of the debt due between the managers of type 2 firms and the bank.

Proposition 1

If (i) the renegotiation of a firm's debt can be executed privately between the bank and the managers of type 2 firms, and (ii) the cost of seizing the payoff of type 2 firms is sufficiently large, then there exists an opportunity for ex post Pareto-improving renegotiation between the managers of type 2 firms and the bank when the bad state (state 1) occurs.

An intuitive explanation of this proposition is as follows. When the bank has the authority to initiate the renegotiation of the debt, it will initially offer an alternative debt level equal to the total net payoff earned by a type 2 firm.\textsuperscript{13} The maximum payable on the debt of type 2 firms in the bad state is \(\pi - u\). By offering a reduced payment, the bank induces the managers to make a voluntary payment and thus can save the seizure cost \(s\). Likewise, the managers may accept the offer because the firm avoids the stigmatization from declaring bankruptcy. By accepting the renegotiated debt, the managers of type 2 firms can avoid the future cost \(\phi\) associated with the stigmatization.

\textsuperscript{13}The bank is a profit maximizer. With a sufficiently large \(D\), the bank will lend money to the managers even if it expects some bankruptcies in advance.
Therefore, as a result of the renegotiation, both the managers of type 2 firms and the bank are better off.

Here *ex post* Pareto-improving renegotiation takes place because of the bank's desire to save the seizure cost and because of the managers' wish to avoid the stigmatization from declaring bankruptcy. In general, the incentive of the bank to offer a revised debt increases with the seizure cost, and the managers' incentive to accept the offer increases with the future cost $\phi$. With greater values of $s$ and $\phi$, the renegotiation is more attractive for both parties. Even when the seizure cost is zero, a renegotiated offer still makes managers of type 2 firms better off because they can avoid the stigmatization of bankruptcy. However, without any positive seizure cost the bank has no incentive to offer the renegotiation opportunity because the maximum payable debt is recovered anyway.

**Corollary 2.** If $\phi$ decreases with the credit history of the firm over time, *ex post* Pareto-improving renegotiation is more likely to take place between the bank and a firm with a short credit history rather than with a firm that has a long credit history.

An implication of most existing models of reputation (see, especially Diamond 1989), is that the stigma effect decreases with the credit history of a firm over time. If this is the case, then Corollary 2 predicts that *ex post* Pareto-improving renegotiation is more likely to take place between the bank and a firm with a short credit history.

**IV. Problems with Debt Renegotiation**

The previous analysis explained why the bank and the borrower have an incentive to renegotiate. However, under certain conditions a policy of debt renegotiation may not result in a Pareto improvement, or, may not be a policy which recovers the maximum payment on the debt. There are two problems which may arise. The first problem is false reporting of potential bankruptcy. The second is a moral hazard problem: Managers have reduced incentives to take costly actions which could reduce the probability of bankruptcy when they expect that debt renegotiation will be allowed.
A. False Bankruptcy Declarations

If the manager of a type 1 firm observes that type 2 firms have the option to renegotiate, he will have an incentive to claim (falsely) that the repayment of his firm's debt will force the firm into bankruptcy. If the bank knows the type of each firm, the managers of type 1 firms could not make such false claims. However, the problem of a false bankruptcy claim will occur when the bank cannot sort out whether the claim is true or false \textit{ex ante}. In this case, the bank's expected gain from renegotiation is:

\begin{align*}
G &= sF_2 - |D - (\pi_L - u)| F_1, \\
F_1 &= \alpha \left\{ \int_0^{\delta_1} g(\epsilon) d\epsilon \right\} p(\delta_1), \quad \text{(18)} \\
F_2 &= (1 - \alpha) \left\{ \int_0^{\delta_2} g(\epsilon) d\epsilon \right\} p(\delta_2).
\end{align*}

The first term on the right hand side of (18) is the expected savings from avoiding the seizure cost, and the second term is the expected loss of debt due from managers of type 1 firms who falsely claim that they will be forced to declare bankruptcy. Of course the sign of $G$ can be negative.

Suppose the managers choose the project and the level of action without knowing that the opportunity to renegotiate the debt is available.\textsuperscript{14} Then, after observing the state, they recognize the bank's incentive to renegotiate. With $F_1$ and $F_2$ fixed, the partial derivatives of $G$ are:

\begin{align*}
\frac{dG}{ds} &= F_2 > 0 \\
\frac{dG}{dD} &= -F_1 > 0 \\
\frac{dG}{du} &= -F_1 > 0 \\
\frac{dG}{d\pi_L} &= F_1 > 0.
\end{align*}

The above results can be explained as follows.

\textsuperscript{14}If managers expect renegotiation before receiving a loan, they may change their decisions about the projects to be selected or adjust the action level if they choose a risky project. These problems are examined in Section IV.
**Result 5.** When the seizure cost is large, renegotiation is more likely to be an attractive policy for the bank even in the face of a false bankruptcy claim.

**Result 6.** When the debt level (or fixed cost) is large, the managers of the type 1 firms are more tempted to make a false bankruptcy claim. As a result, renegotiation is less likely to be an attractive policy for the bank.

**B. A Moral Hazard Problem: Managerial Actions and the Probability of Default**

Suppose that managers expect the bank to renegotiate if they declare bankruptcy. The managers thereby perceive that the expected debt payment is less than what is contractually due. Furthermore, the stigmatization from declaring bankruptcy is not perceived to be a cost because renegotiation will always protect the firms in near bankruptcy from bearing it.

A moral hazard problem therefore exists when the renegotiation of debt is expected. Managers have a reduced incentive to take costly actions that would reduce the probability of bankruptcy. With the expectation of debt renegotiation, the expected utility of the managers of type $i$ firms changes from $v_i(\delta; \varepsilon)$ in (1) and (2) to $\hat{v}_i$ in (19) and (20). A type $i$ firm's manager now chooses an action to maximize $\hat{v}_i(\delta; \varepsilon)$, where

\[
\hat{v}_1(\delta; \varepsilon) = p(\delta)u + (1 - p(\delta))(\pi_H - D) - \varepsilon - c(\delta) \tag{19}
\]

\[
\hat{v}_2(\delta; \varepsilon) = (1 - p(\delta))(\pi_H - D - u) - \varepsilon - c(\delta). \tag{20}
\]

If the bad state occurs, managers of type 1 firms will falsely declare that their firms are near bankruptcy because renegotiation will reduce their obligation from $D$ to $\pi_H - u (< D)$. The type 1 firm's net-of-debt payoff will therefore increase from $\pi_H - D$ to $u$. On the other hand, the managers of type 2 firms will no longer consider the stigma effect of bankruptcy to be a cost because renegotiation will allow them to avoid incurring it. The managers in both types of firms will now choose $\delta$ to maximize (19) and (20) which gives

\[H(\delta) = (\pi_H - D - u)\]

where $H(\delta) = -c'(\delta)/p'(\delta)$ and $H'(\delta) > 0$. 

**Proposition 2**

When the possibility of debt renegotiation exists it leads to a moral hazard problem because managers have incentives to expend less effort trying to avoid bankruptcy. Specifically, if the possibility of renegotiation is expected in advance, the probability of bankruptcy increases. This moral hazard problem is exacerbated by (i) a greater debt (i.e., a greater $D$), (ii) a greater fixed cost for type 2 firms (i.e., a greater $u$) or (iii) a greater stigma cost from bankruptcy (i.e., a greater $\phi$).

**Proof:** First, note that $\delta_1$ fulfills $H(\delta_1) = (\pi_{i1} - \pi_i)$ where $H(\delta) = -c' (\delta / p' (\delta)$, and $\hat{\delta}$ fulfills $H(\hat{\delta}) = (\pi_{i1} - D - u)$. Note also that $(\pi_{i1} - D - u) < \pi_{i1} - \pi_i$ because $u > \pi_{i1} - D$. It is then clear that

$$H(\delta_1) - H(\hat{\delta}) = (-\pi_i + D + u) > 0.$$  \hspace{1cm} (21)

Thus, $\hat{\delta} < \delta_1$ because $H(\delta)$ increases with $\delta$. From (21), as $D$ or $u$ increases, $(\delta_1 - \hat{\delta})$ decreases. Therefore, with a greater $D$ or $u$ the managers of type 1 firms have a greater incentive to expend less effort.

Second, note that $\delta_2$ fulfills $H(\delta_2) = (\pi_{i1} - D - u + \phi)$. It is then clear that

$$H(\hat{\delta}) - H(\delta_2) = \phi > 0.$$  \hspace{1cm} (22)

Thus we also have $\hat{\delta} < \delta_2$. Furthermore, with a greater $\phi$, the managers of type 2 firms also have a greater incentive to expend less effort. Therefore, the moral hazard problem is greater the greater is $D$, $u$ or $\phi$.

**Q.E.D.**

**Corollary 3.** If the stigma cost $\phi$ decreases with the credit history of a firm over time, and if the renegotiation of debt is expected in advance, then the moral hazard problem is more likely to exist for firms with a short credit history.

**C. Self-Selection of Risky Projects**

If debt renegotiation is expected in advance, managers are more likely to choose the risky project. When renegotiation is expected, the managers of type $i$ firms will choose the risky project if

$$\hat{v_i} (\hat{\delta}; \varepsilon) > r - D \text{ for } i = 1, 2,$$

or, in terms of $\varepsilon$, if

$$\varepsilon_i (\hat{\delta}) > \varepsilon \text{ for } i = 1, 2,$$

where
\[ \varepsilon_1 (\delta) = \pi_H - p (\delta) (\pi_H - D - u) - r - c (\delta) \]
\[ \varepsilon_2 (\delta) = \pi_H - p (\delta) (\pi_H - D - u) - (r + u) - c (\delta). \]

The managers of a type \( i \) firm accept the renegotiation offer only if
\[ \hat{v}_i (\delta; \varepsilon) > v_i (\delta; \varepsilon) \quad \text{for} \ i = 1, 2 \]
which implies that
\[ \varepsilon (\delta) > \varepsilon_i (\delta). \]

Therefore, if renegotiation is expected in advance, even managers with risk aversion between \( \varepsilon_i (\delta) \) and \( \varepsilon_i (\delta) \) will choose the risky project.\(^{15}\)

Therefore, the risky project is more attractive to managers if renegotiation is expected in advance than when it is not expected. This can be explained as follows. When the managers of the type 1 firms choose between a risky and a risk free project, they will take into account the fact that they can evade a portion of the debt due by falsely declaring that they are near bankruptcy. These managers are therefore more likely choose a risky project than when the possibility of renegotiation is not expected in advance. Similarly, when the managers of the type 2 firms choose their project, they will take into account the fact that renegotiation will prevent them from being stigmatized in the bad state. These managers are therefore also more likely to choose a risky project than when renegotiation is not expected in advance. Overall, more managers will expose their firms to the risky project.

V. Reducing False Bankruptcy Declarations

In Appendix 2 and 3, it is demonstrated that the imposition of collateral dilutes the incentives or the moral hazard problem and of self-selection of risky projects. Here we concentrate on the role of collateral which reduces false bankruptcy declarations.

Type 1 borrowers always have the ability to repay \( D \); even in the bad state type 1 firms generate \( \pi_L = D \). Type 2 borrowers only generate \( \pi_L - u \) in the bad state, and thus do not have sufficient funds available to repay the debt owed, \( D \). Let \( \pi_L - u = D_1 (< D) \). The bank therefore has several options for recovering the debts owed by Type 2 borrowers.

\(^{15}\)The managers with risk aversion between \( \varepsilon_i (\delta) \) and \( \varepsilon_i (\delta) \) (\( i = 1, 2 \)) will choose the risky project if they do not expect that renegotiation will be allowed.
First, the bank can minimize seizure costs by renegotiating the debts of all borrowers who make a bankruptcy claim, allowing each to pay \( D_1 \). This option (as already suggested above) increases the incentives for type 1 borrowers to make false bankruptcy declarations in the bad state. Second, the bank can attempt to seize \( D \) from every borrower who submits a bankruptcy claim. This policy will permit the bank to identify the type of every borrower submitting a bankruptcy claim, but the bank's seizure costs will be maximized. The third option is a mixed strategy with the provision of collateral. Specifically, the bank can select a policy of randomly choosing between renegotiation and seizure but requires the collateral before initiating the credit contract. The bank will select how to randomize based upon an objective of minimizing "expected" seizure cost, subject to a self-selection constraint which will force borrowers to only declare that bankruptcy is eminent when this is in fact the case. The program solved by the bank is given by

\[
\begin{align*}
\min_{\tau} & \quad (1 - \tau) s \\
\text{s.t.} & \quad \tau(D - D_1) - (1 - \tau) \{ \rho \} \geq C,
\end{align*}
\]

where \( \rho \) is the non-pecuniary stigma cost to a borrower of declaring bankruptcy falsely and being detected as a consequence of the seizure action taken by the bank, and \( C \) is the amount of collateral required. Thus \( \rho \) is a cost for cheating and being detected. This is like a reputation cost and conceptually should be imposed by the capital market. The solution to the program is

\[
\tau^* = \frac{(\rho + C)}{D - D_1 + \rho}
\]

(23)

Notice that in the self-selection constraint the borrower recognizes that renegotiation is possible with probability \( \tau^* \) but that seizure (and consequently identification of true type) occurs with probability \((1 - \tau^*)\). The borrower trades off the gain from renegotiation \((D - D_1)\) against the cost of a false declaration. However, the expected gain from false bankruptcy should be decreased so as to be smaller than or equal to the imposed collateral, which is the incentive compatibility condition. The solution to the bank's problem produces the strategy \( \tau^* \) which is optimal in terms of maximizing bank profits. The extent to which the problem is totally eliminated depends upon the sizes of \( D - D_1 \), and \( \rho \).
VI. Conclusion

This paper has considered the implications of the bank debt renegotiation problem for both banks and borrowing firms. Firms are managed by a single risk averse manager. Managers invest funds borrowed from a bank in risky projects. When the payoffs on these projects are realized, managers of the firms are required to pay the debt back to the bank. Firms are classified by the entrepreneurial skills of their managers. The managers of type 1 firms are assumed to have superior entrepreneurial skill as compared to those of type 2 firms. There are two states of nature. The managers of type 2 firms declare bankruptcy in the bad state unless the bank offers a one time debt reduction. We assume that if a manager declares bankruptcy, the firm she manages is stigmatized in the credit market and must pay a greater cost in order to obtain loans in the future.

Under these circumstances we show that if renegotiation of the debt can be executed privately between the bank and the manager of a type 2 firm that will otherwise be in bankruptcy, and if the cost to the bank of the bank of seizing income is sufficiently large, then there exists the opportunity for ex post Pareto-improving renegotiation of the debt between the manager and the bank. However, we also show that renegotiation as a bank policy for recovering debt may not be efficient if renegotiation is expected in advance. There are two problems which may arise. The first problem is false claims of potential bankruptcy. The second problem is a moral hazard problem. Managers have reduced incentives to engage in costly entrepreneurial actions that would lower the probability of bankruptcy. We show that the false bankruptcy claim problem can be reduced if the bank declares a policy of randomizing between two actions—renegotiation of the debt and asset seizure. Such mixed strategy requires the collateral which circumvents the false bankruptcy claim even when the managers expect the renegotiation ex ante. As the collateral increases the punishment for being detected as the claimer of false bankruptcy, the incentive compatibility condition which discourages the false bankruptcy claim is satisfied even at a lower portion of renegotiation offered. Therefore, the collateral allows banks to save the seizure cost which would be paid in the absence of collateral. Other roles of collateral which dilutes the moral hazard problem and the self-selection of risky projects.
Appendix

1. The partial derivatives of $\varepsilon_1 (\delta_1)$ and $\varepsilon_2 (\delta_2)$ are as follows:

\[
\frac{d\varepsilon_1(\delta_1)}{d\pi_L} = p(\delta_1) - \frac{d\delta_1}{d\pi_L} [(\pi_H - \pi_L)p'(\delta_1) + c'(\delta_1)]
\]

\[
\frac{d\varepsilon_1(\delta_1)}{d\pi_H} = (1 - p(\delta_1)) - \frac{d\delta_1}{d\pi_H} [(\pi_H - \pi_L)p'(\delta_1) + c'(\delta_1)]
\]

\[
\frac{d\varepsilon_1(\delta_1)}{d\pi_H} = -1 - \frac{d\delta_1}{d\pi_H} [(\pi_H - \pi_L)p'(\delta_1) + c'(\delta_1)]
\]

\[
\frac{d\varepsilon_2(\delta_2)}{d\pi_2} = (1 - p(\delta_2)) - \frac{d\delta_2}{d\pi_2} [(\pi_H - D - u + \phi)p'(\delta_2) + c'(\delta_2)]
\]

\[
\frac{d\varepsilon_2(\delta_2)}{dD} = p(\delta_2) - \frac{d\delta_2}{dD} [(\pi_H - D - u + \phi)p'(\delta_2) + c'(\delta_2)]
\]

\[
\frac{d\varepsilon_2(\delta_2)}{d\phi} = -p(\delta_2) - \frac{d\delta_2}{d\phi} [(\pi_H - D - u + \phi)p'(\delta_2) + c'(\delta_2)]
\]

\[
\frac{d\varepsilon_2(\delta_2)}{du} = p(\delta_2) - 1 - \frac{d\delta_2}{du} [(\pi_H - D - u + \phi)p'(\delta_2) + c'(\delta_2)]
\]

Using $\pi_L, \pi_H, p'(\delta_1) + c'(\delta_1) = 0$, the signs of these partials are easily determined and are shown as equation (8), (9), (10) of the text.

2. The role of collateral as a remedy for the moral hazard problem

If we incorporate the collateral $C$ into (7), it is obtained that

\[ H(\delta) = (\pi_H - D - u + \phi + C) \]  

(7)

Since $H'(\delta) > 0$, the optimal value of $\delta$ increases with $C$.

3. The role of collateral as a remedy for the self-selection for the risky projects

As we incorporate $C$ into (19) and (20),

\[
\hat{v}_1(\delta; \varepsilon) = p(\delta)(u - C) + [1 - p(\delta)](\pi_H - D) - \varepsilon - c(\delta) \quad (19')
\]

\[
\hat{v}_2(\delta; \varepsilon) = [1 - p(\delta)](\pi_H - D - u) - \varepsilon - c(\varepsilon - p(\delta)C) \quad (20')
\]

Then $\varepsilon_1 (\delta)$ and $\varepsilon_2 (\delta)$ become

\[
\varepsilon_1 (\delta) = \pi_H - p(\delta)(\pi_H - D - u + C) - r - c(\delta)
\]

\[
\varepsilon_2 (\delta) = \pi_H - p(\delta)(\pi_H - D - u + C) - (r + u) - c(\delta)
\]
Then, it is clear that $\varepsilon_1 (\hat{\delta})$ and $\varepsilon_2 (\hat{\delta})$ in the above are smaller than $\varepsilon_1 (\hat{\delta})$ and $\varepsilon_2 (\hat{\delta})$ respectively when no collateral is imposed. Therefore, as the collateral is imposed, a fewer managers tend to choose risk projects.

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References


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