Money, Inflation and Growth

Keith Blackburn and Victor T.Y. Hung*

Money is introduced into an endogenous growth model in which exchange requires cash-in-advance. We show that the decentralized competitive outcome is an inefficient balanced growth equilibrium in which money affects growth through two independent channels: externality in production and private transactions cost in exchange. We compute the growth (and welfare) maximizing monetary policy which trades off these two effects. We also show that, in the absence of the externality, efficiency is restorable by means of a well-known optimum money supply rule. (*JEL* Classification: E31)

I. Introduction

The "Neutrality of Money" is always a popular topic in macroeconomics. Over the years, different economists have addressed the question "Does money matter?" in different ways. There are two ways of introducing money into an economy: The first is through lump sum transfers to households. The second is through the financing of a government service used in production. Even within the context of rational expectations models a restricted context, there are many different mechanisms through which monetary policy can affect output. These occur in Lucas's (1972) signal extraction model, Taylor (1980) and Fischer's (1977) staggered contract model, Mankiw (1985) and Blanchard and Kiyotaki's (1987) menu cost model and Caplin and Leahy's (1991) buffer-stock model. All of these models focus on the temporary real effect of money. In this paper, we would like to focus only on the permanent real effect of anticipated inflation (the money growth rate). The most relevant of the established potential real effects

*Department of Economics, University of Manchester; Department of Economics and Finance, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon Tong, Hong Kong, (Tel) 852-2788-8812, (Fax) 852-2788-8806, (E-mail) efvhung@cityu.edu.hk.
[**Seoul Journal of Economics** 1996, Vol. 9, No. 2]

of anticipated inflation is the effect on the steady-state capital stock. This effect can be positive (Fischer 1979 and Tobin 1965), zero (Sidrauski's model 1967) or negative (Stockman 1981), depending on how money is introduced into the economy[1] and whether there is price rigidity in the model. Only recently the literature has started investigating how money relates to growth.

This paper straddles the literatures on endogenous growth, the real effects of anticipated inflation and the optimal quantity of money. We use a linear production technology with physical capital as the only input similar to Barro's model (1991) which generates Romer's model (1986) where the mechanism for endogenous growth is through a private-sector externality. Other mechanisms considered in the recent endogenous growth literature include human capital accumulation (King and Rebelo 1990 and Lucas 1988), fertility choices (Barro and Becker 1988 and 1989) and research and development (Grossman and Helpman 1989; Romer 1990 and Aghion and Hewitt 1992).

Endogenous models like Gomme (1993), van der Ploeg and Alogoskoufis (1994) and Ireland (1994) also address the same question of this paper how money relates with growth recently. Van der Ploeg and Alogoskoufis use Sidrauski's money-in-the utility model instead of cash-in-advance constraint to address the effect of inflation on growth. Beside, the mechanism of growth of their model is through learning by doing. Gomme's model (1994) uses the human capital accumulation in stead of capital accumulation. Ireland's model (1994) adds the feature of endogenous choice of the means of payment and focuses on the effects of growth on the monetary system.

We develop a simple model in which money matters for growth. We model money as a medium of exchange by requiring agent to use it to finance consumption and investment transactions. The simplest of transaction technologies is represented by a cash-in-advance constraint. In such a framework inflation can act like a tax on investment which discourages capital accumulation, slow down growth thus implies that money (inflation) and growth are inversely related.[2] This accords with the well-established fact that inflation and growth are negatively correlated.[3] Extending the analysis, we modify the model

[1]The effect is positive in portfolio balance models, zero in (separable) money-in-the-utility models and negative in transactions cost models.

[2]It is possible to conjecture other ways in which inflation could effect growth (e.g. by reducing the efficiency of the exchange mechanism and by creating uncertainty).

along the line with Barro (1990) by introducing a publicly-provided input to production. If the government finances this input through the issue of money, like the argument above, the financing effect of money growth on growth will be negative. But since government inputs also improve the productivity of private investment, the net effect of money growth can be positive.

In the model, we assume that the transfer and the public input to production are financed by the issue of money alone. This assumption is motivated by the results from Fischer (1982) and Giovannini and De Melo (1993). They found that in many developing countries seigniorage and financial repression represent an average of 27% of government revenue, approximate 6% of GDP. Given the fact that large proportion of government expenditure in those countries, beside defense, are devoted to infra-structure investment, health and education which are the social input to the overall production. We believe that the model may provide some insights to explain different correlations between (positive or negative) inflation and GDP growth rate amongst countries.[4]

According to the original statement of the optimum quantity of money (Friedman 1969), the rate of inflation should be set equal to the negative of the real rate of interest so that the opportunity cost of holding money (the nominal rate of interest) is equal to the social opportunity cost of supplying it (which is zero). This was later challenged on the basis of public finance (optimal taxation) considerations (Phelps 1973) and has since been the subject of further re-evaluation (Drazen 1979; Weiss 1980; Benhabib and Bull 1983 and Kimbrough 1986).[5] The externality in our model precludes a money supply rule which is capable of delivering the social optimum. The rule which maximizes growth (and also welfare) is the one which trades off the negative and positive effects of money on growth alluded to above. In the absence of the externality, however, efficiency is restorable by means of the traditional optimum-quantity-of-money rule.

The paper is organized as follows. Section II sets out the growth

[3]See, for example, Fischer (1991).

[4]In Gomme's paper, he provides a histogram of correlations between annual inflation and real GDP growth rates for 82 countries. Only 20 exhibit a positive correlation.

[5]The public finance argument is that inflation should be chosen optimally like any other distortionary tax by equating the marginal distortion of the last unit of tax revenue across different tax bases.

model with a cash-in-advance constraint and conditions of the balanced growth equilibrium are illustrated in Section III. In Section IV, we show that the market outcome is inefficient and growth is inversely related to money growth. In Section V, we introduce the public input to production and obtain the effect of money finance on the decentralized equilibrium of the model. Section VI contains a discussion of the properties of the equilibrium. The conclusion is in Section VII.

II. An Endogenous Growth Model with Cash-in-advance Constraint

We consider an artificial economy which is populated by a large number of infinitely-lived identical agents. Each agent produces and consumes a single storable commodity. The decision problem for the representative producer-consumer is

$$\text{Max}\sum_{t=0}^{\infty}(1+\rho)^{-1}\log(c_t), \tag{1}$$

$$\text{s.t.}\quad c_t + i_t + \frac{M_t + B_t}{P_t} = y_t + \frac{M_{t-1}+\tau_t}{P_t} + \frac{(1+R_{t-1}B_{t-1})}{P_t}, \tag{2}$$

$$i_t = k_{t+1} - (1-\delta)k_t, \tag{3}$$

$$\frac{M_{t-1}+\tau_t}{P_t} \geq c_t + i_t, \tag{4}$$

where c_t is consumption, y_t is the total output, i_t is investment, k_t is the (beginning of period t) capital stock, M_t denotes (end of period t) nominal money balances, B_t denotes (end of period t) nominal private loans, τ_t is a (beginning of period t) lump sum monetary transfer, P_t is the price level and R_t is the gross nominal rate of interest between t and $t+1$.

Equation (1) is the intertemporal utility function which depends on life time consumption. The parameter $\rho \geq 0$ is the subjective rate of time preference. We know that the logarithmic momentary utility function possesses the usual curvature properties and satisfies the Inada conditions. Equation (2) is the budget constraint which defines the feasible allocations of total real resources between consumption and savings. Equation (3) defines investment, or capital accumulation, where $\delta \in (0, 1)$ is the rate of depreciation.

Equation (4) specifies the transactions technology which is the key ingredient of the model. For the analysis in the text, we assume a cash-in-advance constraint which requires that purchases of consumption and investment goods be financed by post-transfer money holdings at the beginning of the period. The assumption is essential because this implicitly introduces the transaction demand for money in the model. In the Appendix we generalize the analysis to the case of a more flexible transactions technology.

Following Barro (1990) and Barro and Sala-i-Martin (1990), we assume that inputs of production include government expenditure as well as private capital. Output is $y_t = AF(k_t, g_t) = Af(k_t/g_t)g_t$, where $A > 0$ is a technological shift parameter. Production depends on privately-owned capital and a complementary public service.[6] These inputs exhibit constant returns jointly and decreasing returns separately. A Cobb-Douglas representation is $F(\cdot) = k_t^{\alpha} g_t^{1-\alpha}$ or $f(\cdot) = (k_t/g_t)^{\alpha}$. For the special case of a linear technology without the public service, $\alpha = 1$ and $F(\cdot) = k_t$ or $f(\cdot) = k_t/g_t$, it is similar to the constant-returns-to-capital production function in Rebelo's paper (1991).

III. Balanced Growth Equilibrium

The economy as a whole is subject to three equilibrium conditions. Equilibrium in the (private) loan market implies $B_t = 0$ for all t. Equilibrium in the money market is given by $M_t = H_t$ for all t, where H_t denotes nominal money supply. Equilibrium in the goods markets is given by

$$c_t + i_t + g_t = y_t \tag{5}$$

for all t, which is merely the aggregate resource constraint. Together with the private sector's and government's budget constraints, these define the aggregate consistency conditions. As usual, they are linearly dependent so that one of them may be ignored. It is legitimate to study the economy along the balanced growth path. Since there are no transitional dynamics, the economy is always on this path. Thus, all variables start at some initial value depending on the initial capital stock and grow thereafter at a common constant rate. We compute the steady state growth equilibrium of the decentralized economy through the optimization of the representative household.

The first-order conditions for the representative agent's optimization problem are

[6]For a discussion of this, see Barro (1990).

$$c_t^{-1} = \phi_t + \lambda_t, \tag{6}$$

$$(1+\rho)^{-1}\left\{A\frac{\partial f(x_{t+1})}{\partial x_{t+1}} + 1 - \delta\right\}\lambda_{t+1} + (1+\rho)^{-1}(1-\delta)\phi_{t+1}$$
$$= \lambda_t + \phi_t;\ \ x_{t+1} \equiv \frac{k_{t+1}}{g_{t+1}}, \tag{7}$$

$$(1+\rho)^{-1}\left(\frac{\lambda_{t+1}+\phi_{t+1}}{P_{t+1}}\right) = \frac{\lambda_t}{P_t}, \tag{8}$$

$$\frac{(1+\rho)^{-1}(1+R_t)\lambda_{t+1}}{P_{t+1}} = \frac{\lambda_t}{P_t}, \tag{9}$$

$$\phi_t\left(\frac{M_{t-1}+\tau_t}{P_t} - c_t - i_t\right) = 0,\ \ \phi_t \geq 0,\ \ \frac{M_{t-1}+\tau_t}{P_t} - c_t - i_t \geq 0, \tag{10}$$

where λ_t is the multiplier on the budget constraint and ϕ_t is the multiplier on the cash-in-advance constraint. Equation (6) states that the marginal utility of consumption is equal to the marginal cost of consumption (which is the marginal value of an additional unit of money). Equation (7) states that the marginal value of an additional unit of capital (which is the value of output it produces next period plus the value of having $1 - \delta$ units leftover after next period) is equal to the marginal cost of that additional unit. Equation (8) (equation (9)) states that the marginal value of an additional unit of money (loans) at the beginning of the next period is equal to the marginal cost of that additional unit. Equation (10) gives the complementary slackness conditions for the liquidity constraint.

To solve for the balanced growth equilibrium of the economy, we proceed as follows. From equations (8) and (9), we obtain $\phi_{t+1} = R_t\lambda_{t+1}$ which shows that the cash-in-advance constraint is binding if $R_t > 0$. Assuming this to be the case, equations (6)-(8) may be written more compactly as

$$c_t^{-1} = v_t, \tag{11}$$

$$(1+\rho)^{-1}(1+\delta)v_{t+1} + (1+\rho)^{-1}A\frac{\partial f(x_{t+1})}{\partial x_{t+1}}\lambda_{t+1} = v_t;\ \ x_{t+1} \equiv \frac{k_{t+1}}{g_{t+1}}, \tag{12}$$

$$\frac{(1+\rho)^{-1}v_{t+1}}{P_{t+1}} = \frac{\lambda_t}{P_t}, \tag{13}$$

where $v_t = \lambda_t + \phi_t$. from equation (11), we have $c_{t+1}/c_t = v_t/v_{t+1}$. Define $(1 + \gamma) = c_{t+1}/c_t$ as the (gross) growth rate of consumption in the steady-state. Then $(1 + \gamma) = v_t/v_{t+1}$. Similarly, define $(1 + \pi) = P_{t+1}/P_t$ as the gross rate of inflation in the steady-state. Together with $(1 + \gamma) = v_t/v_{t+1}$ and (13), we obtain λ_t/v_t as the steady-state constant

$$\frac{\lambda_t}{v_t} = \{(1 + \rho)(1 + \gamma)(1 + \pi)\}^{-1}, \tag{14}$$

that can be substituted with $(1 + \gamma) = v_t/v_{t+1}$ into equation (12) to arrive at

$$(1 + \gamma) = (1 + \rho)^{-1}\left[A\frac{\partial f(x)}{\partial x}\{(1 + \rho)(1 + \gamma)(1 + \pi)\}^{-1} + 1 - \delta\right] \; ; \; x \equiv \frac{k}{g}, \tag{15}$$

where k/g is the ratio of capital to government expenditure in the steady state. In the following sections, we consider two kinds of inflationary policy: simple monetary transfer policy and money finance government policy, one can see that money has an adverse effect on growth unless it is used to finance some complementary government inputs in production.

IV. Monetary Transfer in an Endogenous Growth Model

Assuming that monetary transfer is issued at the rate of μ and there is no productive government service,[7] $g_t = 0$ and $\alpha = 1$,[8] it follows that

$$M_t = (1 + \mu)M_{t-1}, \tag{16}$$

To determine the rate of inflation, observe that equilibrium money balances satisfy the quantity theory: $M_t/P_t = Ak_t$. This follows from equation (2) with $B_t = B_{t-1} = 0$ and $(M_{t-1} + \tau_t)/P_t = c_t + i_t$ in equilibrium. Since output $Ak_t = y_t = c_t + i_t$, capital, output and consumption all grow at the same steady-state rate of $1 + \gamma$. The economy is always on this balanced growth path and all variables are determined once the initial value of capital is known. It follows from the quantity theory equation that $(1 + \pi) = (1 + \mu)(1 + \gamma)^{-1}$.

The competitive equilibrium endogenous growth rate of the economy is obtained by combining $(1 + \pi) = (1 + \mu)(1 + \gamma)^{-1}$ with equation (15):

$$(1 + \gamma) = (1 + \rho)^{-1}\,[A\{(1 + \rho)(1 + \mu)\}^{-1} + 1 - \delta], \tag{17}$$

[7]Since government expenditure are now wasteful, we assume that $g_t = 0$. Agents are assumed to receive money in the form of lump-sum transfer.

[8]It is shown that production function with a constant return to scale to the accumulated factor can generate endogenous growth, see Barro and Sala-i-Martin (1994).

where $\partial f/\partial k = 1$. There are two important properties of this equilibrium. The first is that real growth γ is inversely related to money growth μ, $d\gamma/d\mu < 0$. The second is that the equilibrium is inefficient, the efficient outcome being given by $(1 + \gamma^*) = (1 + \rho)^{-1} (A + 1 - \delta) > 1 + \gamma$.[9]

The inverse relationship between real growth and nominal growth is explained as follows. A higher rate of monetary expansion leads to a higher rate of inflation which induce agents to economize on money balances. Since money is used to purchase both consumption and investment goods, there is a fall in investment and a fall in the steady-state growth rate. In short, inflation acts like a tax on both money and capital. The implied negative correlation between inflation and growth is recognized as one of the stylized facts in the growth literature.[10]

The difference between the competitive and socially-optimal equilibria depends solely on the term $\{(1 + \rho)(1 + \mu)\}^{-1}$. The difference is eliminated by setting $(1 + \mu) = 1/(1 + \rho)$ which implies $(1 + \pi) = (A + 1 - \delta)^{-1}$ and $R_t = 0$. This is our re-statement of the traditional optimum-quantity-of-money rule: set the rate of monetary growth equal to the negative of the rate of time preference so that the nominal rate of interest is equal to zero.

As indicated earlier, the essential, but trivial, requirement for our results is that money must be used to finance capital (as well as consumption) expenditure. We have modeled this in the simplest possible way by imposing a cash-in-advance constraint. We have no reason to believe that more general transactions technologies would not deliver similar results and we provide support for this presumption in the Appendix. In the following section, we introduce a public input to production and study how "money finance" affects the growth.

V. Money Finance in an Endogenous Growth Model

We assume that $\tau = 0$, and the government finances its expenditure, i.e. $g_t \neq 0$, by issuing money. Its budget constraint is $g_t = (H_t - H_{t-1})/P_t$, where H_t denotes nominal money supply. At steady state, $c_{t+1}/c_t = y_{t+1}/y_t = k_{t+1}/k_t = i_{t+1}/i_t = g_{t+1}/g_t = 1 + \gamma$ and $k_{t+1}/g_{t+1} = k/g$ for all t. After some substitution, we obtain a similar equation to (15). To determine the rate of inflation, again we use the credit market equilibrium: $B_t = 0$,

[9]This is obtained as the solution to the problem of maximizing (1) subject to (3) and (5).

[10]See, for examples, Prescott (1990) and Fischer (1991).

the quantity equation $M_t/P_t = y_t$ and the assumption that money is issued at the rate of μ. We again obtain $(1 + \pi) = (1 + \mu)(1 + \gamma)^{-1}$.

To determine the steady state ratio k/g, use equation $y_t = Af(k_t/g_t)g_t$ and $c_t + i_t = M_{t-1}/P_t$ in conjunction with $g_t = \mu M_{t-1}/P_t$ (from government budget constraint $g_t = (H_t - H_{t-1})/P_t$ and money market equilibrium $H_t = M_t$):

$$\frac{k}{g} = f^{-1}\left(\frac{1+\mu}{A\mu}\right). \tag{18}$$

Substituting equation (18) and $(1 + \pi) = (1 + \mu)(1 + \gamma)^{-1}$ into equation (15) gives us our basic result

$$(1+\gamma) = (1+\rho)^{-1}\left[A\frac{\partial f(x)}{\partial x}\{(1+\rho)(1+\mu)\}^{-1} + 1 - \delta\right]; x \equiv \frac{k}{g} = f^{-1}\left(\frac{1+\mu}{A\mu}\right), \tag{19}$$

which is the competitive equilibrium balanced growth rate γ. In the above expressions, the term in the brace bracket measures the private marginal return to capital.

The social optimum of the model is the equilibrium that would be chosen by a central planner. The planner maximizes equation (1) subject to $y_t = Af(k_t/g_t)g_t$, (3) and the resource constraint, $c_t + i_t + g_t = y_t$. It is straightforward to show that the solution is

$$(1+\gamma^*) = (1+\rho)^{-1}\left\{A\frac{\partial f(x)}{\partial x} + 1 - \delta\right\} \; ; \; x \equiv \frac{k}{g}, \tag{20}$$

$$A\left\{f(x) - x\frac{\partial f(x)}{\partial x}\right\} = 1 \; ; \; x \equiv \frac{k}{g}. \tag{21}$$

Equation (21) defines the socially optimal steady-state growth rate. The term in $\{\cdot\}$ is the social return to capital. Equation (22) is the condition for productive efficiency of public input.

VI. Effects of Inflation with the Public Externality

A straightforward comparison of equation (19) and (21) reveals that the competitive equilibrium is inefficient. The reason is that the private return to capital is less than the social return to capital for the same k_t/g_t. Money-finance policy of the complementary public input distorts the portfolio allocation between investments and money holdings through the cash-in-advance constraint. However, it provides part, but not the whole, of the explanation for the inefficiency. As we shall see shortly, the inefficiency remains in the absence of the externality. Under such circumstances, decentralized choices continue to deliver a (Pareto) suboptimal equilibrium as the solution to a second-best problem.

The effect of money on growth in the competitive equilibrium is ambiguous. Money has both negative and positive effects on growth. From equation (19), we find

$$\frac{d\gamma}{d\mu} \gtreqless 0 \Leftrightarrow \frac{\partial}{\partial x} f\left\{f^{-1}\left(\frac{1+\mu}{A\mu}\right)\right\} + \frac{\frac{\partial^2}{\partial x^2} f\left\{f^{-1}\left(\frac{1+\mu}{A\mu}\right)\right\}}{\frac{\partial}{\partial x} f\left\{f^{-1}\left(\frac{1+\mu}{A\mu}\right)\right\} A\mu^2} \gtreqless 0 \ ; \ x \equiv \frac{k}{g}, \quad (22)$$

which has the following interpretation. The first term on the right-hand-side is positive and reflects the negative effect of money on growth. This operates through the transactions cost mechanism and allude to in Section IV. A higher rate of monetary expansion leads to higher inflation which induces agents to economize on money holdings, lower capital stock and growth. In this way, inflation acts like a tax on both money and capital. The second term on the right-hand-side is negative and reflects the positive effect of money on growth. This operates through the production technology. The engine of growth in the economy is the money-financed public service which affects production. Higher monetary growth implies more of this service and higher real growth. It will be useful to express the above condition in an alternative form:

$$\frac{d\gamma}{d\mu} \gtreqless 0 \Leftrightarrow Af(x) + \frac{\left\{\frac{\partial f(x)}{\partial x}\right\}^2}{f(x)\frac{\partial^2 f(x)}{\partial x^2}} \gtreqless 1 \ ; \ x \equiv f^{-1}\left(\frac{1+\mu}{A\mu}\right). \quad (23)$$

For the Cobb-Douglas technology, this implies that $d\gamma/d\mu > 0$, if and only if $\mu < (1-\alpha)/\alpha$. In case of $\mu = (1-\alpha)/\alpha$, $d\gamma/d\mu = 0$. The condition is equivalent to share of government in output $g/y < 1-\alpha$, the share of government input in production. As in Barro (1990), therefore, the effect of government action on growth depends on the size of the government relative to its share in output. If the government sets a expenditure ratio g/y below $1-\alpha$, an increase in output by one unit will result households receiving additional income, which is net of the inflationary tax associated with the increase in g_t (or μ_t) and their private marginal product of capital. This induces an incentive for the private sector to expand output through investment. As a result, the growth rate increases. In the opposite, if the government sets a low expenditure ratio g/y above $1 - \alpha$, an increase in output by one unit will result households receiving less income, which is net of the inflationary tax associated with the increase in g_t (or μ_t) and their private marginal product of capital. This discourages the private sector to expand output through investment. As a result, the growth rate decreases.

Given the above, it is possible to say something about optimal monetary policy. A growth maximizing government will trade off the negative and positive effects of money on growth such that the right-hand side of either of the above conditions holds with equality. This implies $d\gamma/d\mu = 0$. For the Cobb-Douglas technology, the optimal monetary policy is to set $\mu = (1 - \alpha)/\alpha$. Equivalently, $g/y = 1 - \alpha$ which states that the government should set its share of output equal to the share that it would obtain if the public service was supplied competitively. Of course, there is no reason for a benevolent government to maximize growth. What it should maximize is the welfare of the representative household. In this model, however, the two problems are equivalent. To see this, write equation (19) with $g_t/k_t = f^{-1}\{(1 + \mu)/\mu A\}$ as $1 + \gamma = (1 + \rho)^{-1}\{(1 + \rho)^{-1}\alpha\mu^{-1}(g_t/k_t) + 1 - \delta\}$ and resource constraint as $\mu^{-1}(g_t/k_t) = c_t/k_t + \gamma + \delta)$.[11] Combine these expressions and set $t = 0$ to obtain an initial value for consumption,

$$c_0 = \frac{\{1 - \alpha(1 + \rho)^{-2}\}(1 + \gamma) - (1 + \rho)^{-1}\{1 - \alpha(1 + \rho)^{-1}\}(1 - \delta)}{\alpha(1 + \rho)^{-2}} k_0. \tag{24}$$

Substitute this into the intertemporal utility function, $U = \Sigma_{t=0}^{\infty} (1 + \rho)^{-1} u\{(1 + \gamma)^t c_o\}$, and observe that U is increasing in γ. Hence, anything which maximizes growth also maximizes welfare.

It is of interest to compare the results above with those obtained when money affects growth solely through the transactions technology. In this case, as in Section IV, there is no public input to production which depends linearly on capital alone. As before, we have shown that $1 + \gamma < 1 + \gamma^*$ due the presence of cash-in-advance constraint. Thus, the competitive equilibrium remains inefficient even in the absence of the public externality. In Section IV, $d\gamma/d\mu < 0$ unambiguously which is different from here. This follows immediately from the fact that money affects growth solely through the exchange mechanism. This result

[11]Use $\partial f(x)/\partial x/f(x) = \alpha x$; $x = k_t/g_t$ and $g_t = \mu M_{t-1}/P_t = \mu(y_t - g_t) = \mu\{Af(k_t/g_t)\} - 1\}g_t$.

illustrates the fact that whether monetary policy can promote growth depends on how the government uses its seigniorage.

VII. Conclusions

There is a strong presumption that macroeconomic policies matter for economic growth. This presumption is supported by an overwhelming body of empirical evidence but has received relatively attention at the theoretical level except those by Gomme (1993), van der Ploeg and Alogoskoufis (1994) and Ireland (1994). The purpose of the present paper is to introduce another dimension on the topic by constructing a simple model economy in which money and monetary policy do, indeed, matter for growth.

Of the assumptions that kept the analysis tightly-focused, we may single out three for further consideration. The first is the assumption that the only means of government finance is money. A potentially rewarding avenue for future research would be to explore the implications of other means of finance. Of particular interest would be the introduction of government borrowing and the relationship between debt, deficits and development. What matters for our own results is that at least some government revenue is raised though seigniorage. The second important assumption is that we treat the money supply as exogenous. Relaxing this assumption would almost certainly change our results and complicate the analysis.[12] The third assumption is that agents must have cash-in-advance of transacting in both consumption and investment goods. We have no reason to believe that our results would be significantly altered by adopting a more general transactions technology, like the one in the Appendix. Provided only that capital cannot be traded without cost, there is potential for money to affect capital accumulation and growth. This transactions cost effect is independent of the public externality effect in production. Even in the absence of this externality, there is still a mechanism through which money can affect growth.

[12] Buiter (1982) has shown that increases in government expenditure through money financing will have a temporary effect on output even in a zero effect model like Sidrauski's model.

Appendix

We present a generalization of the analysis in the text to the case of a more flexible transactions technology according to which money is a means of freeing a scarce resource (time) which must be used in transacting. In here, we assume $g_t = 0$ and $\alpha = 1$. This version of the model has the additional implication of an inverse relationship between money growth and velocity.

The decision problem of the representative agent is

$$\mathrm{Max} \sum_{t=0}^{\infty} (1+\rho)^{-t} u(c_t, l_t), \tag{A1}$$

$$\text{s.t.} \quad c_t + i_t + \frac{M_t + B_t}{P_t} = y_t + \frac{M_{t-1} + \tau_t}{P_t} + \frac{(1 + R_{t-1} B_{t-1})}{P_t} \tag{A2}$$

$$i_t = k_{t+1} - (1-\delta) k_t, \tag{A3}$$

$$s_t + l_t = 1, \tag{A4}$$

$$s_t = s(m_t); \quad m_t = \frac{M_{t-1}}{P_t (c_t + i_t)}. \tag{A5}$$

Equation (A1) defines intertemporal utility which now depends on lifetime consumption and leisure l_t. We assume that $u(c_t, l_t)$ possesses the usual curvature properties and satisfies the Inada conditions. We also demand that $u(c_t, l_t)$ be such as to generate no long-run trend in l_t. This require either $u(c_t, l_t) = \{c_t^{1-\sigma}/(1-\sigma)\} v(l_t)$ or $u(c_t, l_t) = \log(c_t) + v(l_t)$. For simplicity, we choose the latter. Equations (A2) and (A3) are the same as equation (2) and (3). Equation (A4) defines the time constraint which states that total time (normalized to unity) is allocated between leisure and time spent transacting (s_t). The transaction technology is given in equation (A5) and states that the transactions time required for each unit of consumption and investment depends on the ratio of money holdings to nominal expenditure. We make the standard assumptions, $s'(m_t) \leq 0$ for $m_t < m^*$ and $s'(m_t) \geq 0$ for $m_t > m^*$ where $m_t < \infty$, $s''(m_t) \geq 0$, $s'(0) = -\infty$ and $s'(m^*) = 0$. In this version of the model, it is essential for our result that time must be spent on transacting in both consumption and capital goods. The equilibrium conditions for the economy as a whole are given by $B_t = B_{t-1} = 0$ and equations (5) and (16).

The first-order conditions for the above problem are

$$u'(c_t) + \frac{v'(l_t)s'(m_t)m_t}{c_t + i_t} = \psi_t, \tag{A6}$$

$$(1+\rho)^{-1}\left\{(A+1-\delta)\psi_{t+1} - \frac{(1-\delta)v'(l_{t+1})s'(m_{t+1})m_{t+1}}{c_t + i_t}\right\} = \psi_t - \frac{v'(l_t)s'(m_t)m_t}{c_t + i_t}, \tag{A7}$$

$$(1+\rho)^{-1}\left\{\frac{\psi_{t+1} - v'(l_{t+1})s'(m_{t+1})}{P_{t+1}}\right\} = \frac{\psi_t}{P_t}, \tag{A8}$$

$$\frac{(1+\rho)^{-1}(1+R_t)\psi_{t+1}}{P_{t+1}} = \frac{\psi_t}{P_t}, \tag{A9}$$

where ψ_t is the multiplier on the budget constraint. Equation (A6) defines the marginal utility of consumption which is the sum of two components—the direct (positive) effect on utility of an additional unit of consumption and the indirect (negative) effect on utility of extra expenditure which increases transactions time. Equation (A7) states that the marginal benefit of an additional unit of capital (which is the marginal value of output it produces next period plus the marginal direct and indirect values or having $1 - \delta$ units leftover after next period) is equal to the marginal cost of that additional unit (which is the marginal utility cost of foregone current consumption plus the marginal cost of currently transacting in capital). Equation (A8) states that the marginal benefit of an additional unit of money (which includes the marginal value of a reduction in current transactions time) is equal to the marginal cost of that additional unit. Equation (A9) gives the equalization of the marginal benefit and marginal cost of an additional unit of loans.

Using $u(c_t) = \log(c_t)$, $c_t + i_t = Ak_t$ and $c_t/Ak_t = (A - \gamma - \delta)/A$ (where γ is the net growth rate), equations (A6)-(A8) can be rewritten as

$$A + v'(l_t)s'(m_t)m_t(A - \gamma - \delta) = Az_t, \tag{A10}$$

$$\begin{aligned}(1+\rho)^{-1}\{A(A+\gamma-\delta)z_{t+1} - (1-\delta)v'(l_{t+1})s'(m_{t+1})m_{t+1}(A-\gamma-\delta)\}\\ = (1+\gamma)\{Az_t - v'(l_t)s'(m_t)m_t(A-\gamma-\delta)\},\end{aligned} \tag{A11}$$

$$(1+\rho)^{-1}\{Az_{t+1} - v'(l_{t+1})s'(m_{t+1})(A-\gamma-\delta)\} = (1+\gamma)(1+\pi)Az_t, \tag{A12}$$

where $z_t = \psi_t c_t$ and $1 + \pi = P_{t+1}/P_t$. Along the balanced growth path, $l_t = l_{t+1} = l$ and $s_t = s_{t+1} = s$. From equation (A5), therefore, $s(m_{t+1})/s(m_t) = 1$ with $m_t = M_{t-1}/(P_t A k_t)$. Hence, at the steady state where $m_{t+1} = m_t$, we have $(1+\pi) = (1+\mu)(1+\gamma)^{-1}$ and $z_t = z_{t+1}$. Substituting these results into equations (A10)-(A12) delivers

$$v'(l)s(m)m(A-\gamma-\delta)\{(1+\delta)-(1+\rho)^{-1}(1+\delta)\} \\ = (1+\gamma)-(1+\rho)^{-1}(A+1-\delta)\}Az, \tag{A14}$$

$$(1+\rho)^{-1}v'(l)s'(m)(A-\gamma-\delta) = \{(1+\rho)^{-1}-(1+\mu)\}Az. \tag{A15}$$

Equations (A13) and (A15) may each be combined with equation (A14) to produce two loci in (m, γ) space:

$$v'\{1-s(m)\}s'(m)m = \frac{(1+\rho)(1+\gamma)-(A+1-\delta)}{A-\gamma-\delta}, \tag{A16}$$

$$m = \frac{(1+\gamma)(1+\rho)-(A+1-\delta)}{\{1-(1+\rho)(1+\mu)\}\{(1+\gamma)(1+\rho)-(1-\delta)\}}. \tag{A17}$$

Together, these relationships determine the equilibrium growth rate (γ) and the velocity (m). The assumption $s'(m) + ms''(m) > 0$ is a sufficient condition for the schedule defined by equation (A16) to be upward sloping. The schedule defined by equation (A17) is unambiguously downward sloping. Schedule (A17) shifts to the left as μ is increased. Hence, an increase in the rate of monetary expansion reduce both the growth rate and velocity.

The socially optimal growth rate is again given by $(1+\gamma^*) = (1+\rho)^{-1}(A+1-\delta)$. This is obtained as the solution to the problem of maximizing equation (A1) subject to (A3)-(A5) and (16). A property of the solution is that $s'(m) = 0$: intuitively, a social planner will generate that level of transactions balances at which the marginal reduction in transactions cost is equal to zero. Inspection of equation (A16) and (17) reveals that the inefficiency of the decentralized equilibrium is removed as before by setting $(1+\mu) = (1+\rho)^{-1}$, implying $s'(m) = 0$, $(1+\pi) = (A+1-\delta)^{-1}$ and $R_t = 0$.

(Received March, 1995; Revised May, 1996)

References

Aghion, P., and Hewitt, P. "A Model of Growth through Creative Destruction." *Econometrica* 60 (1992): 323-51.

Barro, R.J. "Government Spending in a Simple Model of Endogenous Growth." *Journal of Political Economy* 98 (1990): 103-25.

____________. "Economic Growth in a Cross Section of Countries." *Quarterly Journal of Economics* 106 (1991): 407-43.

____________, and Becker, G. "A Reformulation of the Economic Theory of

Fertility." *Quarterly Journal of Economics* 103 (1988): 1-25.

__________. "Fertility Choice in a Model of Economic Growth." *Econometrica* 57 (1989): 481-501.

Barro, R.J., and Sala-I-Martin, X. "Public Finance in Models of Economic Growth." *Review of Economic Studies* 59 (1992): 645-61.

__________. *Economic Growth*. New York: McGraw-Hill, 1994.

Benhabib, J., and Bull, C. "The Optimum Quantity of Money: A Formal Treatment." *International Economic Review* 24 (1983): 101-11.

Blanchard, O., and Kiyotaki, N. "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review* 77 (1987): 647-66.

Buiter, W.H. "Deficits Crowding Out and Inflation: The Simple Analysis." *LSE* mimeo, 1982.

Caplin, A., and Leahy, J. "State-Dependent Pricing and the Dynamics of Money and Output." *Quarterly Journal of Economics* 106 (1991): 683-708.

Drazen, A. "The Optimal Rate of Inflation Revisited." *Journal of Monetary Economics* 5 (1979): 231-48.

Fischer, S. "Long Term Contracts, Rational Expectations, and the Optimal Money Supply Rule." *Journal of Political Economy* 85 (1977): 163-90.

__________. "Anticipations and the Non-neutrality of Money." *Journal of Political Economy* 87 (1979): 225-52.

__________. "Seigniorage and the Case for a National Money." *Journal of Political Economy* 90 (1982): 295-313.

__________. "Growth, Macroeconomics and Development." *National Bureau of Economic Research*, Working Paper No. 3702, 1991.

Friedman, M "The Optimum Quantity of Money." in M. Friedman, *The Optimum Quantity of Money and Other Essays*. Chicago: Aldine, 1969.

Giovannini, A., and De Melo, M. "Government Revenue from Financial Repression." *American Economic Review* 83 (1993): 953-63.

Gomme, P. "Money and Growth Revisited." *Journal of Monetary Economics* 32 (1993): 51-77.

Grossman, G.M., and Helpman, E. "Product Development and International Trade." *Journal of Political Economy* 97 (1989): 1261-83.

Ireland, P. "Money and Growth: An Alternative Approach." *American Economic Review* 84 (1994): 47-65.

King, R.G., and Rebelo, S. "The Public Policy and Economic Growth: Developing Neo-classical Implications." *Journal of Political Economy* 98 (1990): 126-50.

Lucas, R.E. "Expectations and the Neutrality of Money." *Journal of Economic Theory* 77 (1972): 721-54.

__________. "On the Mechanics of Economic Development." *Journal of Monetary Economics* 22 (1988): 3-42.

Mankiw, G. "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly." *Quarterly Journal of Economics* 100 (1985): 529-39.

Phelps, E.S. "Inflation in the Theory of Public Finance." *Swedish Journal of Economics* 75 (1973): 67-82.

van der Ploeg, F., and Alogoskoufis, G.S. "Money and Endogenous Growth." *Journal of Money, Credit and Banking* 26 (1994): 771-91.

Prescott, E.C. "Business Cycles: Real Facts and a Monetary Myth." *Federal Reserve Bank of Minneapolis Quarterly Review* (Spring 1990): 3-18.

Rebelo, S. "Long-run Policy Analysis and Long-run Growth." *Journal of Political Economy* 99 (1991): 500-21.

Romer, P. "Increasing Returns and Long Run Growth." *Journal of Political Economy* 94 (1986): 1002-37.

__________. "Endogenous Technological Change." *Journal of Political Economy* 98 (1990): S71-S102.

Sidrauski, M. "Inflation and Economic Growth." *Journal of Political Economy* 75 (1967): 798-810.

Stockman, A.C. "Anticipated and the Capital Stock in a Cash-in-Advance Economy." *Journal of Monetary Economics* 8 (1981): 387-94.

Taylor, J. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88 (1980): 1-24.

Tobin, J. "Money and Economic Growth." *Econometrica* 33 (1965): 671-810.

Weiss, L. "The Effects of Money Supply on Economic Welfare in the Steady State." *Econometrica* 48 (1980): 565-76.