Choosing Roles in a Model of Quality Differentiation

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This paper examines games involving quality differentiation in a sequential duopoly model and points to the leadership solutions. It also deals with the issue of choosing roles (leader or follower) of the firms in quality and price choice game. In this paper, we show that (i) the leadership solutions are the subgame perfect equilibria and simultaneous price competition is in either case of Stackelberg leadership solutions not, and that (ii) the sustaining leadership solution is a unique subgame perfect equilibrium if the diversity of tastes is sufficiently large. (*JEL* Classifications: D43, L13)

I. Introduction

Several authors have examined the issue of vertical (quality) differentiation in the context of simultaneous price choice. The Nash equilibria in simultaneous-move games turn out to have several interesting properties, the principle of product differentiation. (For details, see Gabszewicz and Thisse 1979, 1980; Shaked and Sutton 1982, 1983; Tirole 1988; Choi and Shin 1992 among others.) However, these papers fail to explain a frequently observed phenomenon of a sequential behavior where some firms choose their optimal price after observing others' price choices. Casual observations tend to support the sequential situation over the simultaneous move story. (See, for example, Prescott and Visscher 1977; Eaton and Ware 1987; Anderson and Engers 1992 among others).[1] In this paper, we investigate games of quality differentiation in the context of sequential model and deal with the issue of

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choosing roles (leader or follower) in quality and price choice game.

Specifically, the purpose of this paper is two folds. First, this paper extends the analysis of Tirole (1988) to sequential games with duopoly model and points to the explicit leadership solutions. This paper also compares the conventional simultaneous price competition solution with two leadership solutions: the sustaining leadership and the reversed leadership. The former represents that the first mover in the quality choice stage is price leader and thus, moves firstly in the price choice stage, while the other firm chooses its own price as a follower, having observed the leader's price level. The latter is the reversed sequential game, in which the first mover in quality choice is a price follower.

Second, we make an inquiry that under what circumstances firms agree on the choice of roles of leader and follower in the sequential duopoly model. In order to investigate the market role of firms within a larger game structure, this paper constructs a game of choosing roles of firms in price choice stage, as in Dowrick (1986). This paper provides that a key determinant for choosing roles is consumers' taste parameter for quality. In particular, this paper shows that the sustaining leadership solution is a unique subgame perfect equilibrium if the diversity of tastes is sufficiently large and simultaneous price competition is in either case of Stackelberg leadership solutions not Nash equilibria.

II. The Model

There are two firms, 1 and 2, which produce distinct goods, sold at prices p_1 and p_2 respectively. Each firm's product is associated with its quality level $s_i \in [\underline{s}, \bar{s}]$ where $\bar{s} > \underline{s}$ $(i = 1, 2)$. We assume that there are no production costs for analytic simplicity.

Let the consumer's preferences be described by $U(\tilde{\theta}) = \tilde{\theta}s_i - p_i$ if the consumer consumes one unit (of quality $s_i > 0$) and pays price p_i (> 0), and by 0 otherwise. U should be the surplus derived from the consumption of the good, which depends on the taste parameter $\tilde{\theta}$.[2] The

[1]The Cournot game with simultaneous model is more applicable when information (observation) lags are long relative to realization lags, whereas the Stackelberg game with sequential model applies when the reverse holds. See Anderson and Engers (1992).

[2]All consumers prefer higher quality at a given price, but a consumer with higher $\tilde{\theta}$ is willing to pay more for higher quality.

parameter $\tilde{\theta}$ of tastes for quality is distributed uniformly over the interval $[\theta - 1, \theta]$ with unit density. We make the following two assumptions:[3]

Assumption 1
$1 < \theta < 2.$

Assumption 2
$[(3 - \theta)/(3\theta - 1)]\ \bar{s} < \underline{s} < \bar{s}.$

Assumption 1 says that the amount of consumer heterogeneity is sufficient and thus it ensures the interior solutions of the analysis. Assumption 2 ensures that the market is covered in price equilibrium. Namely, we assume that each consumer only buys one of two goods so as to look for the equilibria in which the market is covered and both firms compete for consumers.[4] Notice that in Assumption 2, $(3 - \theta)/(3\theta - 1)$ decreases as θ increases (especially, this approaches to 1 when θ is close to 1 and to 1/5 when θ is close to 2). This implies that as θ increases, the difference between maximum quality ($\bar{s}$) and minimum quality ($\underline{s}$) increases, Therefore, a larger θ has an interpretation of a greater diversity of tastes on quality[5] or a greater distribution of consumers' valuation on quality.[6]

Consider a situation that firm 1 and 2 play a three stage game. In the first stage, firm 1 chooses s_1 from the interval $[\underline{s}, \bar{s}]$. In the second stage, firm 2 chooses s_2 from the interval $[\underline{s}, s_1]$, having observed s_1.

[3]The similar assumptions applied to the model of vertical differentiation are introduced in Tirole (1988: 296-8).

[4]Notice that the market-covered assumption is in fact feasible when $1 < \theta < 2$. That is, $p_2/s_2 \leq \theta - 1$ for all cases described below. Choi and Shin (1992), however, examined the other case where the firms do not cover the market in the context of Tirole's model of simultaneous price competition.

[5]In Section III, we show that maximum principle of product differentiation holds, i.e., $s_1 = \bar{s}$ and $s_2 = \underline{s}$ for all cases. Under the assumption that market is covered, this implies that when θ is large, tastes are sufficiently diverse so that the difference of qualities of products is sufficiently large. See discussions in Section IV.

[6]An interesting reinterpretation of θ is the inverse of the marginal rate of substitution between income and quality rather than as a taste parameter. That is, wealthier consumers with a higher θ have a lower "marginal utility of income" or, equivalently, a higher "marginal utility of quality." For this point, see Tirole (1988: 96-7).

Quality choice is costless. In the third stage of price choice, firms choose the roles of price leader and follower, having observed the choices of s_1 and s_2. In this moment, we consider a game where each firm announces independently its choice of role, as follower of leader, and then acts in a prescribed manner.[7] If firm i chooses to be a leader, it commits itself to setting the leadership price p_i; if it chooses to be a follower, it commits itself to following its rival's price decision p_j where $j \neq i$, by making the decision $p_i = R_i\ (p_j)$, which defines its own reaction function.

There are four possible outcomes to the third stage. If firms choose adverse roles, the two Stackelberg solutions will emerge. One is the sustaining leadership solution where firm 1 is price leader and firm 2 is follower. The other is the reversed leadership solution where firm 2 firstly chooses p_2 and then, having observed p_2, firm 1 chooses p_1 sequentially. If both desire to be leaders, the situation is Stackelberg warfare. However, if both choose to follow, the result is the conventional simultaneous game solution.

III. The Analysis

In solving the game, consider the demand faced by each firm. By the rules of the game, we have $\underline{s} \leq s_2 \leq s_1$. A consumer with parameter $\tilde{\theta}$ is indifferent between the two brands if and only if $\tilde{\theta}s_1 - p_1 = \tilde{\theta}s_2 - p_2$. This yields the following demand functions:

$$D_1(p, s) = \theta - \frac{p_1 - p_2}{s_1 - s_2}, \tag{1}$$

$$D_2(p, s) = \theta - \frac{p_1 - p_2}{s_1 - s_2} - \theta + 1, \tag{2}$$

where $p = (p_1, p_2)$ and $s = (s_1, s_2)$. Since costs are zero, the profit function for firm i, $\pi_i\ (p, s)$, is given by $p_iD_i\ (p, s)$. Taking s_1 and s_2 as given, the reaction functions are obtained from the first order conditions as follows:

$$p_1 = R_1(p_2) = \frac{p_2 - \theta\delta}{2}, \tag{3}$$

[7]This game structure is similar to that in Dowrick (1986).

$$p_2 = R_2(p_1) = \frac{p_1 + (1-\theta)\delta}{2}, \tag{4}$$

where $\delta \equiv s_1 - s_2$. Now, we have four outcomes:

***Case A.* Simultaneous Nash Solution:**

When both firms choose to be followers, simultaneous Nasg equilibrium satisfies $p_i^a = R_i\,(p_j^a)$ and yields the following results:

$$p_1^a = \frac{1+\theta}{3}\delta, \quad p_2^a = \frac{2-\theta}{3}\delta$$

$$D_1^a = \frac{1+\theta}{3}, \quad D_2^a = \frac{2-\theta}{3}$$

$$\pi_1^a = \frac{(1+\theta)^2}{9}\delta, \quad \pi_2^a = \frac{(2-\theta)^2}{9}\delta$$

$$s_1^a = \bar{s} \quad s_2^a = \underline{s}$$

***Case B.* Sustaining Leadership Solution:**

Firm 1 (first-mover in quality choice stage) is price leader and firm 2 is price follower. Then, taking firm 2's reaction function, $R_2(p_1)$ in (4) as given, firm 1 chooses p_1 which maximizes $\pi_1\,(p,\,s)$. Firm 2 can then observe firm 1's price choice and choose its own profit-maximizing price level. The results are as follows:

$$p_1^b = \frac{1+\theta}{2}\delta, \quad p_2^b = \frac{3-\theta}{4}\delta$$

$$D_1^b = \frac{1+\theta}{4}, \quad D_2^b = \frac{3-\theta}{4}$$

$$\pi_1^b = \frac{(1+\theta)^2}{8}\delta, \quad \pi_2^b = \frac{(3-\theta)^2}{16}\delta$$

$$s_1^b = \bar{s} \quad s_2^b = \underline{s}$$

***Case C.* Reversed Leadership Solution:**

Firm 2 (second-mover in quality choice stage) is price leader and firm 1 is price follower. Then, firm 2 chooses p_2 which maximizes $\pi_2\,(p,\,s)$, taking $R_1\,(p_2)$ in (3) as given. Firm 1 can then observe firm 2's price choice and choose its own profit-maximizing price level. The result are as follows:

$$p_1^c = \frac{2+\theta}{4}\delta, \quad p_2^c = \frac{2-\theta}{2}\delta$$

$$D_1^c = \frac{2+\theta}{4}, \quad D_2^c = \frac{2-\theta}{4}$$

$$\pi_1^c = \frac{(2+\theta)^2}{16}\delta, \quad \pi_2^c = \frac{(2-\theta)^2}{8}\delta$$

$$s_1^c = \bar{s} \quad s_2^c = \underline{s}$$

Case D. **Stackelberg Warfare:**

When both firms choose to lead, each firm maximizes its own profit, $\pi_i\ (p,\ s)$, independently, taking $R_j\ (p_i)$ as given.[8] The results are as follows:

$$p_1^d = \frac{1+\theta}{2}\delta, \quad p_2^d = \frac{2-\theta}{2}\delta$$

$$D_1^d = \frac{1}{2}, \quad D_2^d = \frac{1}{2}$$

$$\pi_1^d = \frac{1+\theta}{4}\delta, \quad \pi_2^d = \frac{2-\theta}{4}\delta$$

$$s_1^c = \bar{s} \quad s_2^c = \underline{s}$$

Using the results in each case, we have the following payoffs table.

THE PAYOFFS OF GAME

		Firm 2: Leader	Firm 2: Follower
Firm 1	Leader	$\left(\pi_1^d = \frac{1+\theta}{4}\delta,\ \pi_2^d = \frac{2-\theta}{4}\delta\right)$	$\left\{\pi_1^b = \frac{(1+\theta)^2}{8}\delta,\ \pi_2^b = \frac{(3-\theta)^2}{16}\delta\right\}$
	Follower	$\left\{\pi_1^c = \frac{(2+\theta)^2}{16}\delta,\ \pi_2^c = \frac{(2-\theta)^2}{8}\delta\right\}$	$\left\{\pi_1^a = \frac{(1+\theta)^2}{9}\delta,\ \pi_2^a = \frac{(2-\theta)^2}{9}\delta\right\}$

[8]In general, the outcomes of Stackelberg warfare cannot be deterministic. In this paper, as in Dowrick (1986), we assume that each firm solves its objective, expecting the other firm behaves a follower. Therefore, the payoff of leader-leader may be somewhat unrealistic and be mutually inconsistent. Instead, one may introduce the war of attrition to explain the firms' fights for rents. See Tirole (1988: 311-4). For such a price war to take place, we can expect that firms earn no *ex ante* rents and thus, discussions in the next section are not affected.

IV. Some Discussions

Based on the above analysis and payoff table, we can find some observations as follows: First, at the choice of quality, we still obtain the maximum principle of product differentiation, i.e., $s_1 = \overline{s}$ and $s_2 = \underline{s}$ for all configurations. This follows that maximum principle can relax price competition between firms as far as the market is covered and thus, the low-quality firm gains from reducing its quality to the minimum because this softens price competition.[9]

Second, the leadership in quality choice, which makes firm 1 choose higher quality, ensures higher profit. This implies that the first-mover advantage always exists, i.e., $\pi_1 > \pi_2$ for all solutions since $s_1 = \overline{s} > s_2 = \underline{s}$.[10] Therefore, if one of the firms entered the market firstly, that firm would choose higher quality and the other choose lower quality. This suggests the possibility of both firms trying to be first (preemption game) in the choice of quality. Notice also that, however, this is not always true for the price choice game.

Third, in the price choice stage, given that one firm is acting as leader, the other's optimal choice is to follow, and *vice versa*. Thus, only two leadership solutions must be subgame perfect equilibria in the price choice stage.[11] The intuition for this result is as follows: When one firm is expected to be leader, the other firm will choose to follow in order to avoid Stackelberg warfare and thus, gain higher profit. On the other hand, when one firm is acting as a follower, the other firm can arbitrarily determine its optimal price at the equilibrium price when he is a follower. This implies that to lead cannot be worse than to follow and thus, he will choose to be a leader. Consequently, both firms choose adverse roles and two leadership solutions will emerge.

[9]Choi and Shin (1992) point put that, in a duopoly model of simultaneous price choice, if the firms do not cover the market, then the maximum principle does not hold. But they show that the principle of differentiation is robust.

[10]This also implies that even though the game rule in quality choice stage is changed to that firm 2 may choose s_2 from the interval $[\underline{s}, \overline{s}]$ rather than $[\underline{s}, s_1]$ in the second stage, it is more profitable to choose $\underline{s}$ by the maximum principle. For other analyses of first-mover advantages under duopoly with complete information, see Gal-Or (1985) and Dowrick (1986).

[11]Notice that (i) $\pi_2^b > \pi_2^d$ and $\pi_1^c > \pi_1^d$, which implies that Stackelberg warfare is not an equilibrium of the game and that (ii) $\pi_1^b > \pi_1^a$ and $\pi_2^c > \pi_2^a$, which means that the simultaneous price competition is also not.

This result is somewhat similar to that of Dowrick (1986: 258). Despite a few similarities, there is a major difference between this paper and Dowrick's: Whereas this paper considers the choice of roles in the context of heterogeneous products with different qualities, Dowrick only analyzed the game of choosing roles without considering quality choice. Without predetermined asymmetry between firms, he shows that if both firms have upward-sloping reaction functions, then each firm prefers leadership if the other will comply. Thus, we can anticipate that there is always a conflict over the choice of roles in his model. However, this paper considers the possibility of asymmetry in products' quality and shows that there is an obvious solution of the subgame perfect equilibrium for some parameter, which will be examined in the next. On balance, this paper emphasizes the choice of price leadership of the quality leader, which does not arise in Dowrick's model.

Finally, for some value of θ, it is of interest that the first-mover in the quality choice may want to be the first in the price choice. Namely, $\pi_2^b > \pi_2^c$ and $\pi_1^b > \pi_1^c$ if $\theta > \sqrt{2}$. Here, θ represents taste parameter or consumers' valuation on quality. Therefore, when taste parameter is sufficiently large (i.e., consumers have greater diversity on quality or there are consumers with greater intensity on quality), it turns out that the sustaining leadership solution is a unique subgame perfect equilibrium in the game of choosing roles.

On the other hand, when θ is small, the first-mover in quality may not want to be the first in the price choice. In particular, when $1 < \theta \leq \sqrt{2}$, there are two leadership solutions in the subgame perfect equilibria. This possibility of multiple equilibria yields mixed-strategy equilibria.[12]

The economic intuition of this observations is as follows: We have shown that the first-mover in quality choice produces a product with higher quality and earns higher profit. This follows that firm 1 has leading power on choosing roles in price choice and firm 2 with lower quality will find its optimal choice according to the behavior of firm 1.

[12]While firm 1 chooses price leader with probability $r_1^L = (\pi_2^c - \pi_2^a)/(\pi_2^b + \pi_2^c - \pi_2^a - \pi_2^d) = 2\,(2 - \theta)^2/[2(2 - \theta)^2 + 9\,(\theta - 1)^2]$ and price follower with probability $r_1^F = (\pi_2^b - \pi_2^d)/\,(\pi_2^b + \pi_2^c - \pi_2^b - \pi_2^d) = 9(\theta - 1)^2/[2(2 - \theta)^2 + 9(\theta - 1)^2]$, firm 2 chooses price leader with probability $r_2^L = (\pi_1^b - \pi_1^a)/(\pi_1^b + \pi_1^c - \pi_1^a - \pi_1^d) = 2(1 + \theta)^2/[2(1 + \theta)^2 + 9\theta]$ and price follower with probability $r_2^F = (\pi_1^c - \pi_1^d)/(\pi_1^b + \pi_1^c - \pi_1^a - \pi_1^d) = 9\theta/[2(1 + \theta)^2 + 9\theta]$. Notice that $r_1^L >$ (<) r_1^L when $\theta <$ (>) $(5 + 3\sqrt{2})/7$ and $r_2^L < r_2^F$ for all values of θ.

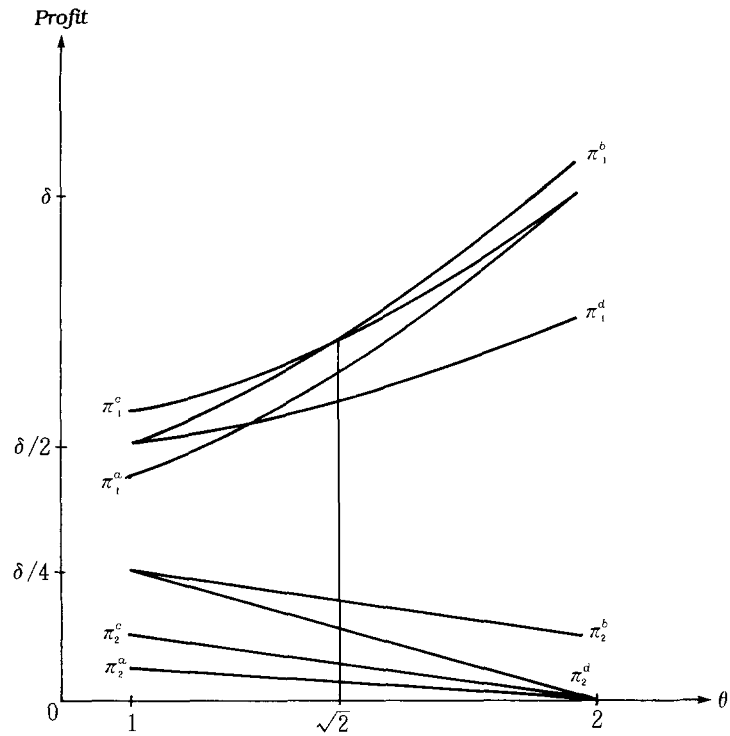

Figure 1

However, the behavior of firm 1 is directly affected by market conditions of consumers' tastes. When θ is sufficiently large, then market for quality is sufficiently diverse (or there are many consumers with a larger valuation on quality). We can interpret this situation that market demand is sufficiently large irrespective of price difference in the marginal sense (or demand is less elastic). Thus, choosing one of leadership solutions, in this case, firm 1 wants to be a leader rather than to be a follower in order to set higher price and earn higher profit. This sticks firm 2 in a follower. Meanwhile, when θ is sufficiently small, market for quality is not sufficiently diverse or products are not sufficiently differentiated, i.e., market demand is more elastic to price difference. This situation induces firm 1 to choose a role of price follower rather than price leader. However, whether the reversed leadership is fulfilled is not pre-determined since firm 2 does not also want to be a leader.

This explanation is described in Figure 1. While firm 1 earns the largest profit in the reversed leaderships (π_1^c) when $1 < \theta < \sqrt{2}$, he earns that in the sustaining leaderships (π_1^b) when $\theta \geq \sqrt{2}$. On the other hand firm 2 always earns the largest profit in the sustaining leaderships (π_2^b) for all θ. Hence, when $\theta \geq \sqrt{2}$, both firms agree on the sustaining leaderships. However, when $1 < \theta < \sqrt{2}$, both firms do not want to be in the position of leader and they want to be followers.[13] This is because profit increases as its rival's price increases when θ is small and products are not sufficiently differentiated.

IV. Summarizing Remarks

This paper have investigated games of quality differentiation in a sequential duopoly model and dealt with the issue of choosing roles (leader or follower) in quality and price choice game. In this paper, we show that leadership solutions are the subgame perfect equilibria and simultaneous price competition is in either case of Stackelberg leadership solutions not. We also provide that a key determinant for the choice of role in price competition is the taste parameter for quality. In particular, we show that the sustaining leadership solution is a unique subgame perfect equilibrium if the diversity of consumers' valuation on quality is sufficiently large.

We believe that the fundamental insight of the paper is robust enough to survive any reasonable relaxations of assumptions. Specific details of the results, however, should be interpreted with qualifications. A promising extension of the present paper would be to analyze the case that the supposed market is not covered and to examine its effects.

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[13]In this point, see footnote 12.

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