Collusion, Turnover, and Efficiency Wages in Organization

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Using reputation as a self-enforcing mechanism, a dynamic model with one principal and two agents is developed to analyze the effect of collusion possibility on the structure of organization. The stage game played by the agents has a prisoner's dilemma structure, so that collusion is a dominated action. In a long-term relationship, however, the agents can collude as long as future benefits are sufficient. Without relying on any precommitment for the principal's incentive scheme, we found that costly rotation of the agents (reducing future benefits of collusion) and paying a wage higher than the market-clearing wage are optimal for the principal when monitoring technology is imperfect and the cost of replacing the agents is small. The optimal structure of organization in terms of wages and rotating schemes is also discussed. (*JEL* Classifications: L14, J41, J63)

I. Introduction

This paper studies the problem of collusion by members of an organization (eg. workers in a firm or a public policy maker and an interest group) and its effect on the organizational characteristics such as length of relationships and wage schemes. Consider collusion between a regulator and the the firm he/she is regulating. The regulator can make decisions that help the regulated firm in the expectation of future rewards, such as paid vacations or future employment. Many collusive relationships are sustained by reciprocal favors which are possible

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when the colluding parties are engaged in an on-going relationship. A long-term relationship may facilitate collusion because it increases the possibilities of getting benefits from future collusion. Knowing this, the government may want to reduce the length of this relationship (rotate the regulator) in order to deter collusion. Additionally the government may use a wage scheme to provide proper incentives for the regulator. The government can pay a bonus wage to a non-colluding regulator to deter collusion. Now it is in the government's interest to provide incentives efficiently by controlling all the available incentive schemes.

There are many forms of non-monetary side-transfers which can facilitate collusion. In our previous example of a regulator, entertainment expenses (meals, stays in resorts) or the prospect of future employment can be used as a non-monetary transfer. These non-monetary side-transfers are not simultaneous in general.[1] A party does a favor for the other party, anticipating counterfavors in the future. The collusion under such non-simultaneous side-transfers has to do with the repetition of the relationship between the colluding parties and is enforced by a sequence of favors and counterfavors. I do you a favor today because I trust you will reciprocate tomorrow. This means that reputation can be used as an important enforcement mechanism for the collusion which arises from an on-going relationship between colluding parties.

A long-term relationship, in general, facilitates the accumulation of relationship-specific assets for agents. Thus a principal may want to have a long-term relationship with agents since he has to pay search or training costs for new agents whenever he replaces the agents with relationship-specific assets. It is generally recognized that long-term contracts also involve costs in the sense that they reduce flexibility for mutually advantageous "breaches". Another important cost of long-term relationships under collusion possibility, as discussed above, is that it increases the incentive to collude because the potential gains from collusion are larger.[2]

[1]Even if it is simultaneous, collusion can emerge from an on-going relationship. Thus, we can extend our analysis for this case also.

[2]Short-term relationship may also be optimal in organizations plagued by the ratchet effect. Assuming a credible commitment to switching agents between jobs, Ickes and Samuelson (1987) show that job transfers break the link between current performance and future incentive schemes, and hence remove the incentive-stifling implications of the ratchet effects. For more details of the benefits of and the costs of the long-term relationships, see also Hart and Holmstrom (1987).

Using traditional contract theory, Tirole (1986) analyzes the effect of collusion possibility on the wage scheme with a three-tier principal/supervisor/agent model in which a collusion side contract is enforceable by the third party and the form of side-transfer is monetary. Monetary transfers are used for expositional convenience (in order to use standard economic analysis) even though most covert transfers are nonmonetary. Tirole (1986, 1990) also provides some intuition as to how the possibility of collusion may induce short-term relationships in an organization where efficiency alone might dictate long-term relationships.

As Tirole (1990) points out, the limitation of the traditional approach in modeling collusion is that it presumes that agents can make a binding side contract which realizes any gain from trade regardless of whether it is legal or illegal. It is doubtful that courts would enforce any side contract whose purpose was to facilitate collusive behavior. That is, the contract may not be enforceable because of legal restrictions even though it is verifiable. To avoid this criticism, we model collusive behavior that is sustainable through a self-enforcing mechanism. The enforcement of illegal side contracts in general relies on (among other things) non-judicial mechanisms such as reputation. This self-enforcing approach suggests that collusion is linked with ongoing relationships.

The literature on efficiency wages deals with the effect of the individual moral hazard problem on the wage. Shapiro and Stiglitz (1984) argue that a firm may have an incentive to pay more than the "going wage" in order to induce its workers not to shirk. Our model shows that for a collusion problem, a principal has a similar incentive to use a wage premium to ameliorate collusion. If agents are detected colluding and are fired, they will suffer a penalty of the forgone higher wage they would have received if they remained in the organization. In a world of imperfect monitoring the fear of losing the above market wage premium provides an incentive not to collude. Thus it is optimal for the principal to pay a wage higher than the going wage even if it is costly.

This paper analyzes the effect of collusion possibility on organizational characteristics such as the length of the relationship and the form of the wage scheme. We model collusion with a stochastically repeated game which provides a self-enforcing collusion. Furthermore, we don't assume precommitment to the principal's intertemporal incentives scheme since the principal's reputation is also used as an enforcement mechanism for the implicit contract between the principal and the agents. We allow the principal to choose not only a wage scheme but also the length of the relationship; they both serve to control the benefits of collusion. By choosing two instruments (or structural aspects of an organization) optimally, the principal can do strictly better than he can had he employed only one of the two instruments. This suggests that we need to look at the full array of structural components of an organization to understand the effects of collusion.

The wage plays two roles in our model; (1) it serves as an incentive device as does an efficiency wage and (2) it provides ex-post incentives for not colluding. In addition to wages, costly rotation of agents (or, alternatively terminating the relationship without cause) is found to be part of an optimal employment scheme for the principal when there is a severe monitoring problem. This means the length of the relationship will be shortened under the possibility of collusion.

In addition to these basic results, we derive many testable implications from comparative statics exercises. First, when the principal faces collusion possibility between the skilled agents, the wage scheme becomes a more cost effective instrument for the principal than the rotation scheme. Thus the principal will provide an incentive for the agent with high replacement cost by using wage scheme relatively more. Second, with less precise monitoring technology or high opportunity of the agents doing a favor to each other, there are two effects that the principal has to consider: (1) The rotation scheme becomes more effective than the wage since the principal is more likely to pay wage to the colluding agents. (2) On the other hand, the benefits from collusion increase for the agents. Thus the principal must reward the non-colluding agents more and rotate the agents more frequently to deter collusion. Combining these two effects, the principal will rotate the agents more frequently but the effect on the wage depends on the relative magnitude of these two effects.

The rest of the paper is organized as follows. Section II describes the model. In Section III, the equilibrium of the model is characterized. The results on the optimal structure of organization are reported in Section IV. The concluding section summarizes the results and discusses possible extensions.

II. The Model

Consider an organization in which there is one principal and two agents who jointly take actions for the principal. At the beginning of a period, nature chooses one of three states, ϕ_1, ϕ_2 and ϕ_3 with probability r, r and (1-2r) respectively. Once state ϕ_j is realized, agents play the chosen state game. In state ϕ_1, agent 1 can (1) be honest to principal or (2) "cheat" to do a favor for agent 2. Symmetrically in state ϕ_2, agent 2 can (1) be honest or (2) cheat to do a favor for agent 1. Nobody can do a favor in state ϕ_3. The following payoff matrices summarize the one-period game described above.

i) state ϕ_1:

		Agent 2
		H
Agent 1	C	$-d, c, B$
	H	$0, 0, G$

ii) state ϕ_2:

		Agent 2	
		C	H
Agent 1	H	$c, -d, B$	$0, 0, G$

iii) state ϕ_3:

		Agent 2
		H
Agent 1	H	$0, 0, G$

where c and d are the non-wage payoff to the agents $c > d > 0$. $G > B$ are the one-period payoff to the principal and $G - B > c - d$.

Notice that doing a favor is costly since $-d < 0$ and the payoff from honest behavior is 0, the market clearing wage after normalization. Thus the agents are indifferent between holding the job without cheating and receiving wage 0 from the market.[3] Imagine that one-period game is played. It is obvious that the agents will be honest in ϕ_1, ϕ_2 and ϕ_3, which yields the first best outcome G for the principal. Thus there is no need for the principal to change the structure of his organization. Now imagine that this game is infinitely repeated assuming no structural change of the organization. It is easy to show that the agents can support collusion (i.e. cheat in ϕ_1 and ϕ_2) as a subgame perfect equilibrium outcome if their discount factor is sufficiently close to 1. Consider following grim trigger strategy for the agents: (1) cheat as long as there has been collusion, and (2) be honest if there was a deviation from col-

[3]The participation constraint for the agents is not binding at equilibrium of this model. We focus on the incentive constraint from now on.

lusion. The agents have an incentive to collude with the above grim trigger strategy if and only if $\delta_A > d/\{r(c-d)+d\}$. The agents can collude via a sequence of favors and counterfavors, which are harmful to the principal. Thus, self-enforcing collusion can emerge unless the principal changes the structure of his organization.[4]

If the principal has a perfect monitoring technology and it is not costly to find new agents, then the threat of firing cheating agents will be credible and the principal can implement the first best outcome. It is, however, costly in general to fire a cheating agent because of positive replacement cost (search or training cost of new agents) and monitoring technologies are usually poor. If the replacement cost is small and the monitoring technology is precise enough, the firing scheme may be sufficient for the principal to implement his first best outcome. For higher replacement cost or less precise monitoring technology, a firing scheme (which is not credible) alone will not implement the first best outcome. Now the principal may want to introduce other structural components in order to achieve his second best outcome.

There can be many structural components which the principal can use to control the incentives of agents who face collusion opportunities. One example might be that the principal can pay an extra bonus wage to an honest agent. By introducing a bonus wage scheme, the principal can change one period payoffs for the agents thus providing an incentive not to cheat. However, this scheme may be too costly for the principal.[5] We will, in this paper, consider an additional structural component that the principal can use. Knowing that collusion emerges from the on-going relationship between the agents, the principal may want to reduce the length of relationships between them. That is, the principal may rotate (or terminate the relationship randomly between) the agents without cause at the end of each period. It is in the principal's interest to provide incentives efficiently for the agents by choosing these two incentive schemes optimally. Notice that, like a firing scheme

[4]The agents in our model can live forever. If the agents can live only finitely many period, the agents' collusion may not be sustainable. The argument of backward induction in finitely repeated game may make it unnecessary for the principal to change the structure of his organization. However, we can think of the agents' discount factor as the probability of continuing the game one more period. Under this interpretation, our results can still provide insightful implications.

[5]For the one period total payoff under a bonus wage scheme with imperfect technology, look at Section II-B of this paper.

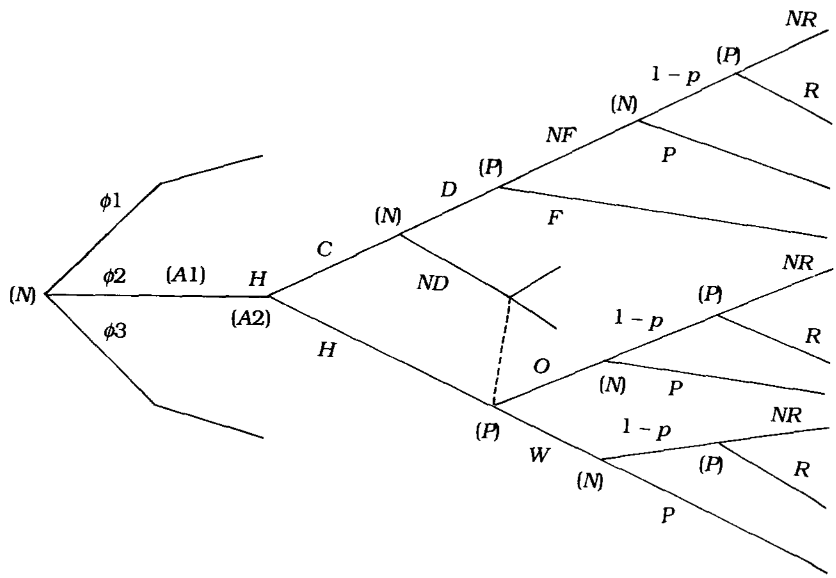

FIGURE 1
IMPERFECT MONITORING

in the previous paragraph, the credibility of the principal's incentive instruments is important in any dynamic relationship and should be satisfied in equilibrium.

Now we want to introduce the extensive form game with which we will analyze the effect of collusion possibility on the structure of organization. At period 0, the principal chooses a stationary wage scheme w and the length of relationship measured by the probability p of the agent being retained in the organization one more period.[6,7] Given this

[6]We can imagine that the principal and the agents have public randomizing device which draws a lottery of rotation. The principal chooses the probability for this randomizing device at period 0. Alternatively, we can imagine that the principal has a continuum of pairs of agents and that the agents can observe at the end of each period the portion of the agents that are rotated (of the agents whose relationships are terminated randomly) and also can identify the rotated agents.

[7]We consider only stationary instruments for the principal, that is, instruments that do not depend on the history (especially the past outcome for the principal) and time t. Allowing more general instruments for the principal may capture many other interesting issues in this dynamic principal agents relationships. However, stationary instruments are sufficient to analyze the effects of collusion possibility on the wage and the length of the relationships. For more

structure of organization, agents start working for the principal where interactions between principal and agents in a period are as represented in Figure 1.[8]

If an agent did not do a favor when he could or did a favor without detection, the principal can pay the wage w chosen in period 0 or pay nothing.[9] As long as the agents were not detected, the principal cannot fire the agents with cause. He can, however, rotate agents (or randomly terminate the relationship) without cause with probability $1 - p$. If an agent were detected cheating with exogenous probability q, the principal can choose whether to fire the agents or not. Even if the agents were not fired, the principal can decide whether he will rotate the agents if he wishes. If the agents are retained or new agents are hired, they play the same one period game in the next period.

We summarize the notation and assumptions that we use in this paper.

Notation

ϕ_1 : state, with probability r, where agent 1 can do a favor for agent 2

ϕ_2 : state, with probability r, where agent 2 can do a favor for agent 1

ϕ_3 : state, with probability $(1 - 2r)$, where neither agent can do a favor

ω : sub-state realized by monitoring technology when an agent cheats in ϕ_1 or ϕ_2

$\omega \equiv D$ if there is detection with probability q

$\omega \equiv ND$ if there is no detection with probability $1 - q$

$1 - p$: probability of agents being rotated at the end of each period

δ_A and δ_p : discount factor for agents and principal respectively

$w_i(\phi_j, \omega)$: stationary wage rate for agent i in state (ϕ_j, ω) $i = 1,2$, $j = 1,2,3$, $\omega = D, ND$

α : replacement cost (search or training cost)

Assumptions

[A1] $r \neq 0$ is exogenously given and constant over time.

detail, see Section II-A and (1) of Section V in this paper.

[8]Figure 1 is for the case of a bonus wage scheme. However, we can imagine a similar extensive form with more general wage scheme. Look at Section II-A for various wage schemes and Section II-B for the one period payoff under a bonus wage scheme.

[9]We consider a discrete wage payment, i.e., paying w or paying nothing. This is without loss of generality since the optimal deviation for the principal is paying nothing.

[A2] At period 0, the risk-neutral principal chooses p and $w_i(\phi_j, \omega)$ where $i = 1, 2, j = 1, 2, 3, \omega = D, ND$. For the purpose of the wage scheme, the principal must treat identical agents equally. That is, $w_1(\phi_1, D) = w_2(\phi_2, D)$, $w_1(\phi_1, ND) = w_2(\phi_2, ND)$, $w_1(\phi_2, D) = w_2(\phi_1, D)$, $w_1(\phi_2, ND) = w_2(\phi_1, ND)$ and $w_1(\phi_3) = w_2(\phi_3)$. The principal has to pay constant replacement cost α in the next period whenever he replaces the agents, and he replaces both agents whenever he does. The principal can observe the realization of states, but he cannot distinguish between an honest agent and an agent not detected cheating.

[A3] Agents are risk-neutral and there is a gain for agents from a collusion. when agents are replaced, they will get the minimum reservation wage 0 starting next period.

[A4} A cheating can be detected with exogenous probability q if an agent cheats.

[A1] implies that the agents are playing a possibly repeated stochastic game. We will have similar results even without symmetry, i.e., with prob$(\phi_1) \neq$ prob(ϕ_2). [A2] describes assumptions regarding the principal. The risk neutrality of the principal and the agents reduces the problem to one of incentives without the complication of risk-sharing. We allow only a symmetric wage scheme for the principal. The assumption of firing both agents is for simplicity only. Firing only the agent who cheated in this period reduces the replacement cost by half without altering our qualitative results. [A3] simply assumes that there is a non-trivial collusion problem. More specific conditions are discussed later. Also note that the past history of an agent doesn't affect the minimum reservation wage which is normalized to be 0; i.e., it is the same irrespective of whether he was fired with cause of rotated without cause. [A4] describes the monitoring technology; we might have assumed alternative monitoring technologies, such as a symmetric one, without altering our qualitative results.

A. Wage Schemes

The wage schemes available to the principal may depend on many factors such as the observability or verifiability of realization of states. Legal restrictions also affect the enforceability of the wage scheme. In light of this, we define several wage schemes.

***Definition* 1:** Partially Contingent Wage Scheme & Constant Wage Scheme

A *partially contingent wage scheme (PCWS)* is a wage scheme which depends only on the realization of ϕ_j's.

A *constant wage scheme (CWS)* is a partially contingent wage scheme scheme which is constant over states ϕ_j's.

Definition 2: Fully Contingent Wage Scheme & Bonus Wage Scheme

A *fully contingent wage scheme (FCWS)* is a wage scheme which depends on the realizations of both ϕ_j's. and ω.

A *bonus wage scheme (BWS)* is a fully contingent wage scheme in which $w_i(\phi_j, \omega) = 0$ for all i, j, ω except the cases of $i = j$, $i = 1.2$ and $\omega =$ ND.

Notice that a fully contingent wage scheme is not literally fully contingent since the wage does not depend on the history (in particular, it is independent of the past outcomes for the principal) and time t.[10] That is, we allow the principal to choose a stationary wage scheme in which an agent's reward depends only on the state this period. Under the fully contingent wage scheme, agents will receive $w_i(\phi_j, ND) \geq 0$ in this period if they were not detected cheating or were honest. If detected cheating, they will receive $w_i(\phi_j, D) \geq 0$ in this period. Notice that the principal can the wage at his discretion after he observes the monitoring result in this period. We can imagine that the wage is paid at the end of the period. However, the principal has to pay the same wage under the partially contingent wage scheme. That is, the principal cannot make the wage payment depend on the outcome of monitoring technology in this period. We can think of this case as the wage being paid before the monitoring outcome is realized or as there being some legal restrictions on wage schemes.

The analysis under the fully or partially contingent wage schemes may involve huge complications because of high dimensionality. However, it can be shown that those complications are not necessary. That is, the analysis can be done under either constant or bonus wage

[10]We suppress the observability of the one-period outcome to the principal. Outcome is realized at infinity so that principal has no information until the end. Alternatively we can interpret the principal's discount factor δ_P as the probability that the game will continue one more period. Under this interpretation, the principal will observe the final outcome in finite time with probability 1. We can imagine the following examples: (1) Mining: digging many holes but final outcome is observable at the end, (2) Agriculture harvest is observable at the end. Note that the monitoring probability can be viewed as reflecting any information that the principal randomly receives.

scheme.[11] From now on, we will focus on bonus wage scheme only. The qualitative results do not change under the constant wage scheme. The next sub-section describes the one period payoff under a bonus wage scheme with imperfect monitoring technology, given that the principal will pay the bonus wage w. Paying a bonus w, the principal can manipulate not only the agents' ex-post incentives by changing the one-period payoff but also their ex-ante incentives by controlling the future benefits from collusion. The payoff structure changes if the principal behaves differently.

B. One-period Total Payoff under a Bonus Wage Scheme

i) state ϕ_1:

		Agent 2
		H
Agent 1	C	$-d+(1-q)w,\ c,\ B-(1-q)w$
	H	$0+w,\ 0,\ G-w$

ii) state ϕ_2:

		Agent 2	
		C	H
Agent 1	H	$c,$ $-d+(1-q)w,$ $B-(1-q)w$	$0,$ $0+w,$ $G-w$

iii) state ϕ_3:

		Agent 2
		H
Agent 1	H	$0,\ 0,\ G$

III. Characterization of Perfect Bayesian Equilibrium

The equilibrium concept we use in this paper is perfect Bayesian

[11]The equivalence results between wage schemes are available from the author upon request.

equilibrium (in the spirit of sequential equilibrium by Kreps and Wilson (1982)) which is a set of strategies and beliefs such that, at any stage of the game, strategies are optimal given the beliefs, and the beliefs are consistent with the strategies in the Bayesian sense.[12] In our one-period extensive form game, for example, as in Figure 1, there is one information set for agent 2 and are five information sets for the principal after nature chooses ϕ_2 at the beginning of a period. We should check, for the given beliefs, the sequential rationality of the agents' and principal's strategies in each information set and the consistency of the beliefs. Notice that this one-period stochastic stage game is infinitely repeated after the principal chooses a bonus wage w and a probability p at period 0. Therefore, the sequential rationality and the consistency must be satisfied in each information set of the infinitely many period.

We are interested in the pure strategy equilibrium in which the agents' collusion is deterred by the principal and the agents collude most severely if possible. We will first classify the set of possible histories in Section III-A and in Section III-B characterize the range of exogenous variables in which the equilibrium of our model provides a non-trivial outcome. The specification of the perfect Bayesian equilibrium is reported in Section III-C.

A. History and Value Functions

The principal and the agents can have potentially infinitely lived relationships. However, depending on the firing and rotation decision, the relationship between the principal and the agents can be terminated. If the principal terminates the relationship, he must hire new agents. Given the complicated structure of the whole game, the principal and the agents can conceivably choose very complicated history dependent strategies. However, the following classification of the set of possible histories are found to be sufficient to find a perfect Bayesian equilibrium in which collusion is deterred by the principal and the agents collude most severely if possible.

We say a history satisfies PR (Principal's Reputation) if the principal has, so far,

(1) fired agents whenever there was detection,

(2) paid bonus w whenever there was no detection or the agents were

[12]Adopting stronger refinements such as coalition-proofness or renegotiation-proofness may provide a more accurate characterization of rational intelligent behavior in our model.

honest when they could cheat, and

(3) rotated agents whenever nature gave a chance with probability $1 - p$ chosen by the principal at period 0.

Similarly we say a history satisfies AC (Agents' Collusion) if the agents have, so far,

(1) cheated whenever they could.

We denote by NPR the history where PR is violated (i.e., principal has no reputation). Similarly, we denote by NAC the history where AC is violated (i.e., agents are not in collusive phase). Having defined these variables, we can completely partition the set of all histories into the following 4 histories, (PR, AC), (PR, NAC), (NPR, AC) and (NPR, NAC). Any information set of the whole game has a history in one of the 4 sets described above.

Notice that PR is publicly observed by both the principal and the agents. Thus, there is no private information about the principal's reputation. On the other hand, there can be a difference between the agents' private history and the publicly observed history concerning the agents' collusion under this classification of histories. For example, if the agents have colluded so far without ever being detected, the agents have a private history AC which is different from the public history NAC perceived by the principal. Since the principal cannot directly observe one-period outcomes, there is no way that the principal can infer the agents' private history which may be different from the public history. Thus the principal's strategy only depends on the publicly observed history. Furthermore it turns out that the principal cares only about the public history concerning his reputation but not the public history concerning agents' collusion. (See the specification of the principal's equilibrium strategies in Section III-C). Thus the above classification of histories is sufficient to find a perfect Bayesian equilibrium of our interest.

Classifying histories in this manner, we can calculate the expected value to the principal and the agents of playing the continuation game given each history in steady state. For example, $EV_p(\text{PR, NAC}) = \{G - 2rw - \delta_p(1 - p)\alpha\}/(1 - \delta_p)$ represents the expected value to the principal in the continuation game in which the principal maintains his reputation and agents are honest in every period.[13] Similarly $EV_A(\text{PR, NAC}) = rw/(1 - p\delta_A)$ is the expected value to the agents in the same continua-

[13]Notice that we abuse the notation of NAC and NPR.

tion game. These value functions will be used to calculate the payoffs to the principal and agents in the next two sub-sections. Other value functions of the principal and agents for each history can also be calculated and are reported in Appendix.

B. Incentive Constraints and Sequential Rationality Constraints

This section describes the agents' incentive constraints and the principal's sequential rationality constraints which should be satisfied in the perfect Bayesian equilibrium of our interest. The behavior specification in the continuation game is a part of the perfect Bayesian equilibrium which will be discussed later in Section III-C.

A) Agents' Incentive Constraints

Consider agents (new or old) who choose to be honest of cheat when their private history was (PR, AC) and when the principal has an incentive to maintain his reputation in the future. Furthermore the agents will be honest as long as the principal maintains his reputation. They will choose to be honest if the following inequality is satisfied.

$$\begin{aligned}(\mathrm{IC}^A)\text{: } 0 + w + p\delta_A EV_A\,(\mathrm{PR, NAC}) &\geq -d + (1-q)\,w \\ &+ (1-q)\,p\delta_A EV_A\,(\mathrm{PR, AC}) \\ \text{or } w &\geq w^{IC}\,(p\,;\,q,\,r)\end{aligned}$$

The LHS represents the payoff when the agent chooses to be honest. The agent will receive a bonus w and the private history becomes (PR, NAC). The RHS is the payoff to the agent when he chooses to cheat. He will be paid w when he is not detected and the agents' private history stays (PR, AC). Notice that the agents can get EV_A (PR, AC) only if there in no detection and no rotation with probability $(1-q)p$. This is true because the principal has an incentive to maintain his reputation, so that he will fire the agents if there is a detection and rotate the agents if nature gives a chance.

Now consider agents (new or old) who choose to be honest or cheat when their private history was (NPR, AC) and the principal has no incentive to maintain his reputation in the future. Furthermore an agent will cheat as long as both agents have cheated so far under NPR and will be honest if there is any deviation from collusion. They will choose to cheat if $-d + \delta_A EV_A(\mathrm{NPR, AC}) > 0 + \delta_A EV_A\,(\mathrm{NPR, NAC})$.[14] If this

[14]We assume the agents will be honest if this holds as an equality.

inequality is not satisfied, the agents do not have an ex-post incentive to collude. In order to have a non-trivial problem, we assume $\delta_A > d/\{r(c - d) + d\}$ so that this inequality is satisfied. This implies w^{IC} $(p = 1 ; q, r) > 0$ for q sufficiently small and notice that EV_A (NPR, AC) > EV_A (NPR, NAC) since $c - d > 0$ and $r \neq 0$.

B) Principal's Sequential Rationality Constraints

Assume that the one-period payoff to the principal with an honest agent, G, is sufficiently bigger than B, the one-period payoff with a cheating agent, so that EV_P (PR, NAC) > EV_P (NPR, AC). Let's consider a principal who makes a decision of firing cheating agents when the public history in the previous period was (PR, NAC) but becomes (PR, AC) after detection in this period. If he fires the agents, he can maintain his reputation and the agents will be honest as the principal maintains his reputation. If he does not, he loses his reputation forever and the agents will collude thereafter unless one of the agents deviates from the collusion. Furthermore, the principal has no incentive to maintain his reputation once he has lost it.

$$(\mathrm{SR}^F)\text{: } -\delta_P\alpha + \delta_P EV_P\,(\mathrm{PR, NAC}) \geq \delta_P EV_P\,(\mathrm{NPR, AC})$$
$$\text{or } w \leq w^F\,(p\,;\,\alpha, r) = (G - B) - \alpha\,(1 - p\delta_P)/2r.$$

The LHS represents the payoff to the principal when he fires cheating agents and maintains his reputation. The principal must search, with cost α in the next period for new agents who will be honest. The RHS is the payoff to the principal when he does not fire the cheating agents and loses his reputation forever, which will bring about the agents' collusion.[15] If the bonus wage w and the length of relation measured by p satisfies (SR^F), then the principal has an incentive to maintain his reputation by firing the agents. The sequential rationality constraint for the principal regarding rotation of agents is the same as (SR^F).

Now consider a principal who makes a decision of paying a bonus wage w when the public history is (PR, NAC). If he pays the bonus w, he can maintain his reputation and the agents will be honest forever. If he does not, the agents will collude thereafter unless one of the agents

[15]This is true for both old and new agents. If we allow negotiation between the principal and new agent, the continuation game payoff for the principal may change. For example, if the principal fires the cheating agents one period later and can negotiate with new agents, the sequential rationality constraint for firing scheme may change for the principal. However this does not affect the qualitative results of this paper.

deviates from collusion and the principal will have no incentive to maintain his reputation in the future.

$$(\mathrm{SR}^w)\colon -w + \delta_p EV_p\,(\mathrm{PR,\ NAC}) \geq \delta_p EV_p(\mathrm{NPR,\ AC})$$

$$\text{or} \quad w \leq w^w(p;\alpha,r) = \delta_p \frac{2r(G-B) - \delta_p(1-p)\alpha}{1-\delta_p + 2r\delta_p}.$$

If (w,p) satisfies (SR^w), then the principal has an incentive to maintain his reputation by paying w.

C) Principal's Period 0 Problem

Let $F(\alpha, q, r)$ represent the set of contract (w, p) which satisfies IC^A, SR^F, and SR^w. The set $F(\alpha, q, r) = \{(w, p) \mid w \geq 0, 0 \leq p \leq 1$, and IC^A, SR^F, and SR^w are satisfied$\}$ is the set of credible contracts for given a, q and r from which the principal can choose optimal (w, p) to maximize his expected payoff $EV_p(\mathrm{PR, NAC})$ at period 0. The following two lemmas characterize the set $F(\alpha, q, r)$ and $EV_p(\mathrm{PR, NAC})$. All lemmas are proved in Appendix.

***Lemma* 1**

(1) There exists α, q and r such that $F(\alpha, q, r)$ is non-empty.
For given α, q and r, $F(\alpha, q, r)$ is closed and bounded.

(2) $F(\alpha, q, r) \subset F(\alpha', q, r)$ if $\alpha > \alpha'$ and $F(\alpha, q, r) \subset F(\alpha, q', r)$ if $q < q'$.

(3) $F(\alpha, q, r)$ is convex for q sufficiently small.

***Lemma* 2**

(1) $\partial EV_p(\mathrm{PR, NAC})/\partial p > 0$, $\partial EV_p(\mathrm{PR, NAC})/\partial w < 0$.

(2) $\mathrm{MRS}_{(p\ w)} = \alpha\delta_p/2r$ $(= \partial w^F(p)/\partial p > 0)$.

From lemmas 1 & 2, $F(\alpha, q, r)$ is non-empty and compact and $EV_p(\mathrm{PR, NAC})$ is quasi-concave in (w, p). Thus, the following principal's period 0 optimization problem (P^0) is well defined and let (w^*, p^*) be the solution of this problem.

$$(P^0)\colon \max_{(w, p)} EV_p(\mathrm{PR, NAC}), \text{ s.t. } (w, p) \in F(\alpha, q, r)$$

***Lemma* 3**

Suppose $\delta_A > d/\{r(c-d) + d\}$ and q sufficiently small, then the following hold:

(1) $\partial w^{IC}(p; q, r)/\partial p > 0$, $\partial^2 w^{IC}(p; q, r)/\partial p^2 > 0$.

(2) $w^* = w^{IC}(p^*; q, r)$.

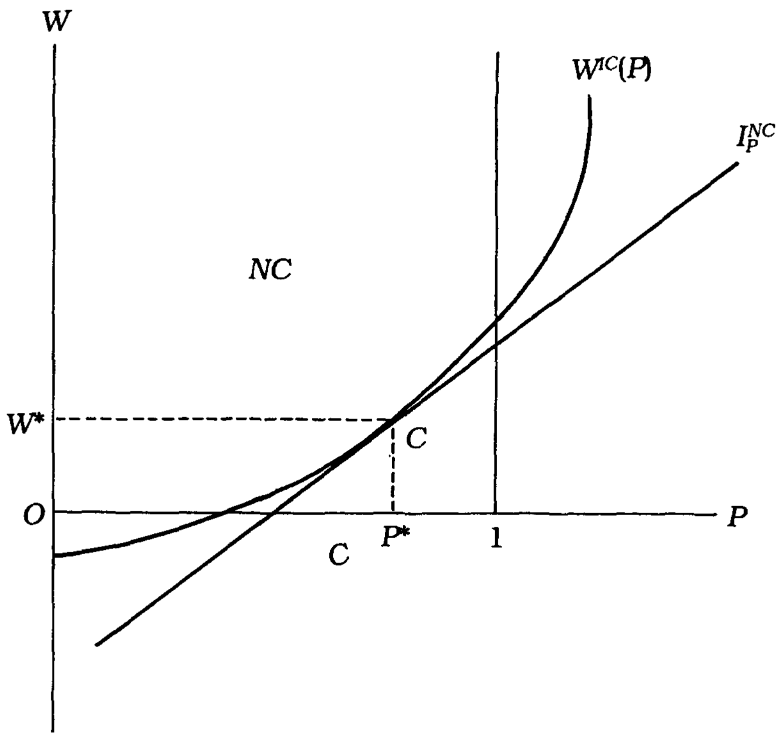

FIGURE 2
EQUILIBRIUM WITH NO COLLUSION

(3) There exists (α, q, r) such that (P^0) has an interior solution.

Look at Figure 2 in which $w^{IC}(p; q, r)$ from the agents' incentive constraint and the principal's indifference line are drawn in (w, p) space. When the monitoring technology is very poor (q is small), the principal must reduce the benefits of collusion by choosing a low p if he is to deter collusion with a low wage; i.e., $w^{IC}(p; q, r)$ is upward sloping. The strict convexity of $w^{IC}(p; q, r)$ comes from the fact that the principal must compensate the agents with a higher wage as he increases p since collusion becomes more attractive to the agents as p increases when q is small. However, the principal's MRS between p and w is independent of the level of p. If the agents have an ex-post incentive to collude (i.e., $\delta_A > d/\{r(c - d) + d\}$ and q is sufficiently small), then (IC^A) is binding at the optimal solution.

Poor monitoring technology and a relatively small replacement cost make paying a positive bonus and rotating the agents optimal even when costly. Thus the optimal structure of organization is such that the principal pays a wage higher than the "going wage" and the length of relationship will be shortened under the possibility of collusion. Once we have an interior solution, we can conduct many interesting comparative statics concerning the optimal structure of organization. We report these results later in Section IV.

C. *Perfect Bayesian Equilibrium*

Section III-B characterizes the agents incentive constraints and the principal's sequential rationality constraints with some behavior specification in the continuation games. We next show that it is part of a perfect Bayesian equilibrium. For this purpose, we denote G_F, G_W, and G_R as the principal's information set in a period in which he must make firing, wage payment and rotation decisions respectively. Similarly, let ϕ_1 and ϕ_2 denote the information set for agent 1 and agent 2 respectively in a given period.

Proposition 1

Suppose (α, q, r) is such that the principal's period 0 problem (P^0) is well defined. The following strategy profiles and beliefs will then constitute the perfect Bayesian equilibrium which induces the outcome in which the principal maintains his reputation and the agents are honest always.[16]

A. Strategy profiles:

(1) Under PR, Principal: fires the agents in G_F,
pays w^* in G_w and
rotates the agents in G_R.
Agent 1: is honest in ϕ_1 under both AC and NAC.
Agent 2: is honest in ϕ_2 under both AC and NAC.

(2) Under NPR, Principal: does not fire the agents in G_F,
pays nothing in G_W and

[16]For simplicity, we do not specify period 0 choice by the principal and the behaviors of the principal and the agents along the subgame which starts with $(w, p) \neq (w^*, p^*)$. We can, however, extend the definition so PR of that the principal can maintain his reputation if (w, p) satisfies IC^A. Notice that this organization will emerge if and only if $EV_P(\mathrm{PR, NAC})|_{(w^*, p^*)} \geq \alpha$.

does not rotate the agents in G_R.

Agent 1: i) cheats in ϕ_1 under AC,

ii) is honest in ϕ_1 under NAC.

Agent 2: i) cheats in ϕ_2 under AC,

ii) is honest in ϕ_2 under NAC.

B. Beliefs:

(1) prob(H) = 1 in G_W and G_R if PR in the previous period.

(2) prob(H) = 0 in G_W and G_R if NPR in the previous period.

Proof: The consistency of the beliefs is obvious and we will just prove the sequential rationality of the strategy profiles given the beliefs.

(1) Under PR, consider the principal's problem in G_F. The public history was (PR, NAC) in the previous period but now it is (PR, AC) since there has been detection in this period. If he fires the agents, then he expects $-\delta_P\alpha + \delta_P EV_P$(PR, NAC) and if not, $0 + \delta_P EV_P$(NPR, AC). Since $(w^*, p^*) \in F(\alpha, q, r)$, SR^F is satisfied. Thus, firing the agents is optimal for the principal. A similar argument can be applied in G_W and G_R.

(1-1) It is optimal for agent 1 to be honest in ϕ_1 under (PR, AC) because (w^*, p^*) satisfies IC^A. Honesty is also optimal under (PR, NAC) since $0 + w + p\delta_A EV_A$ (PR, VAC) $> -d + (1 - q)\, w + (1 - q)p\delta_A EV_A$ (PR, NAC). Symmetry assures the optimality for agent 2 in ϕ_2.

(2) Under NPR, the principal, in G_F, expects $0 + \delta_P EV_P$ (NPR, AC) if he does not fire agents and $-\delta_P\alpha + \delta_P EV_P$ (NPR, AC) if he does fire agents. Thus it is optimal for the principal not to fire the agents under NPR. We can apply the same arguments in G_W and G_R. Notice that this argument can be applied even if the continuation game is such that there is no collusion.

(2-1) Now consider the agents' strategy under NPR. We should check the optimality of the agents' strategy under AC and NAC. Because of symmetry, we check only the optimality for agent 1.

(i) Under AC, agent 1 can get $-d + \delta_A EV_A$ (NPR, AC) if he cheats in ϕ_1. If he is honest, he will get $0 + \delta_A EV_A$ (NPR, NAC). Since $\delta_A > d/\{r(c - d) + d\}$, cheating is optimal for agent 1 in ϕ_1.

(ii) Under NAC, being honest gives $0 + \delta_A EV_A$(NPR, NAC) to agent 1 in ϕ_1 and cheating gives $-d + \delta_A EV_A$ (NPR, VAC). Since the expected value of the continuation game is the same, being honest is optimal for agent 1 under NAC in ϕ_1.

Q.E.D.

IV. Optimal Structure of Organization

We can interpret replacement cost and imperfect monitoring technology as imperfections in organization which may induce the principal to adjust the structure of his organization and achieve his second best outcome. There may be many structural aspects that the principal can adjust to control agents' incentives efficiently. We consider explicitly in this paper two aspects of organization structure, namely the wage scheme and the length of the relationship in terms of the probability of rotation. If we have an interior solution for the principal's period 0 problem (P^0), then rotating agents (or terminating the relationship randomly) with cost and paying wages higher than the going rate are better for the principal than either no rotation or no payment.

Any change in the exogenous variables such as α, q and r may affect the solution of the principal's problem (P^0). Especially the change in some of the variables will affect the agents' incentive constraint (IC^A). The manners in which (IC^A) is affected are two-fold. The first is the *direct effect* which measures the degree that the change in the exogenous variables strengthens or relaxes the agents' incentive constraint. This amounts to shifting $w^{IC}(p; q, r)$ up or down in Figure 2. The second is the *indirect effect* which represents the change in the relative effectiveness of p and w as incentive instruments. This can be represented by the change of the the slope of $w^{IC}(p; q, r)$ in Figure 2.

Now we want to introduce the notion of a normal instrument and an inferior instrument. A *normal instrument* is an incentive instrument which is used more as the underlying incentive problem becomes severe. Similarly, an *inferior instrument* is an incentive instrument which is used less as the underlying incentive problem becomes severe. In terms of the direct effect discussed above, a normal instrument has a positive direct effect and an inferior instrument has a negative direct effect.

Assuming an interior solution, we provide simple but interesting comparative statics results that are proved in Appendix.

Proposition 2

Suppose that the princpal's period 0 problem (P^0) has an interior solution and w and p are normal incentive instruments, then the following holds:

(1) As α or δ_p increase, the optimal p^* and w^* increase.

(2) As q decreases or as δ_A or r increase, the optimal p^* decreases but the effect on w^* depends on the relative magnitude of the direct and indirect effects.

As the replacement cost increases or as the principal values future benefits more highly, the optimal structure or organization is such that the principal wants to pay honest agents more today and increase the likelihood of the agents receiving future reward if they stay longer in the organization without cheating. This is true because the principal is now willing to pay a high wage to increase p so that he can reduce the high rotation cost. It becomes more obvious from Figure 2. As α and δ_P increase, the principal's indifference line becomes steeper without altering $w^{IC}(p; q, r)$. Thus the optimal p^* and w^* increase. We can reinterpret this result. Let α represent the amount of the agents' relation-specific asset, which amounts to a replacement cost for the principal. The principal may use different incentive schemes for agents who are different in terms of their asset accumulation. That is, the principal replaces the novice agents more often and pays a low wage but it is optimal for the principal to provide incentives for the skilled agents through higher wages rather than by rotating them.

In order to analyze the effects of changes in q, δ_A and r on (p^*, w^*), we have to study their effects on (IC^A) and $MRS_{(p,w)}$. Look at Figure 2 again. The change in q and δ_A do not affect the principal's relative valuation between p and w. The principal, however, is more likely to pay the bonus wage w as r increases. Thus the principal is now willing to pay less w than before for the same change in p in order to be on the same indifference line. The effects of the change in q, δ_A and r on (IC^A) are two-fold. For example, the decrease in q affects $w^{IC}(p; q, r)$ directly by shifting it up (direct effect) and it also affects indirectly by making $w^{IC}(p; q, r)$ steeper (indirect effect).

With less precise monitoring technology, the principal rotates the agents more frequently. This is true because as q decreases the degree that the bonus wage w can differentiate colluding and non-colluding agents becomes smaller but the degree that the length of relation p can differentiate becomes larger. Thus rotation becomes a more effective incentive instrument. On the other hand, as the monitoring technology becomes less precise, the benefits from collusion increase for agents. The principal must reward the honest agents more and rotate them more frequently to deter collusion since w and p are normal incentive, instruments. That is, it becomes harder to deter agents' collusion.

Combining these two effects, the principal now has more incentive to rotate the agents if he has less precise monitoring technology. The effect on w^* is not obvious, however. The principal may have to pay a higher wage because of the direct effect, even if he can relax (IC^A) due to the indirect effect that the length of relationship p now becomes a more efficient incentive instrument as q decreases. The final effect on the equilibrium wage depends on the relative magnitude of the direct and indirect effects.

We can interpret the effect of the change of δ_A and r in the same fashion. For example, if we compare the structure of public and private organization, we see more frequent rotation in public organization because it is more likely for the agents in public organization to have an opportunity to do a favor (i.e., high r). Finally, the increase in r reinforces the effect on p since it reduces the principal's MRS between p and w.

Proposition 2 provides rich testable implications regarding the relation between the structure of an organization and the underlying incentive problems. Specifically, some structural components of organization can be used more efficiently to control certain incentive problems. When we consider designing the structure of an organization, we have to consider the relative effectiveness of incentive instruments which depends on the precision of monitoring technology, etc.

V. Concluding Remarks

We have used a dynamic one-principal two-agent model to analyze the effect of possible collusion on the wage and the length of relationships in an organization. Without relying on enforceability for agents' collusion and precommitment for the principal's incentive scheme, we showed that for a range of (α, q, r), it is optimal for the principal to pay higher wages than the going rate and rotate the agents without cause in order to deter collusion. Furthermore, we must consider all the structural components and their effectiveness as incentive instruments in order to understand the underlying incentive problems in an organization. As our concluding remarks, we want to discuss the potential extensions.

(1) The principal's instruments, the wage and the rotation schemes, have a stationary structure in that they do not depend on history (especially the past output to the principal) and time t. Allowing the wage to depend on time t, for example, may increase the payoff the payoff to the principal, especially if the principal and the agents have

differential access to the capital market. Such an extension might capture the idea of performance bonds or retirement funds in the efficiency wage literature.

Furthermore, consider the following finite termination scheme without randomness: $p_t = 1$ if $t \leq T$ and $p_t = 0$ if $t > T$. This scheme will, for sufficiently large T, provide an outcome for the principal which is ε-close to the first best outcome.[17] Under this scheme, finite termination will break down the agents' collusion. However, we can use the arguments by Benoit and Krishna (1985) in order to have a non-trivial collusion problem. If the stage game has several equilibria, it is possible to punish an agent for deviating in the next-to-last period by specifying that if he does not deviate the static equilibrium he prefers will occur in the last period, and that deviations lead to the static equilibrium he likes less. Another modification of the model, in line of Kreps-Milgrom-Roberts-Wilson (1982), can be to introduce a small probability that the agents get extra satisfaction from cheating, so long as the other agent has cheated for them in the past.

There have been many experimental studies of games in which the participants are told that the horizon has been set at a fixed finite point, and there is an unique stage-game equilibrium. In such experimental studies of the prisoner's dilemma, subjects do in fact tend to cooperate in many periods, despite what the theory predicts.[18] We believe that the qualitative results of this paper do not change even if we allow more general instruments for the principal. It might be interesting to see whether simple stationary instruments are sufficient for the principal to achieve the incentive efficient outcome.

(2) This paper characterizes an equilibrium for a certain range of exogenous parameters (α, q, r). Finding all the equilibria for every value of (α, q, r) may allow us to characterize optimal incentive schemes in general. It may also allow us to understand how the division of rents from this dynamic principal-agents relationship depends on the replacement cost, monitoring technology, etc. We can think that these exogenous variables affect the bargaining power of the principal and agents.

(3) We interpret the replacement cost α as a training or search cost that the principal has to pay. Another interpretation is that it repre-

[17]I would like to thank Steven A. Matthews for pointing out this scheme.

[18]For more detailed experimental studies, see Axelrod (1984) and Rapport and Chammah (1965).

sents a degree of relationship-specific asset accumulation by the agent which may increase as the agent stays longer in an organization. It would be nice to find the characteristics of the optimal incentive schemes for the principal and how the division of the rents from this dynamic relationship would be affected when agents can accumulate relationship-specific assets over time. In this line, another dynamic extension might be to study how the optimal employment scheme may look like if we assume that the probability of an agent's having a chance to do a favor to the other, r, increases as the agents stay longer in an organization.

Furthermore it would be desirable to build a model in which the agent's ex-ante decision of relation-specific asset investment can be analyzed. Since the agent's ex-ante decision will depend on the employment schemes (of course upon the credibility of such schemes) and the degree of asset specificity, the analysis may involve some complications but it will provide fruitful results. Unless the standard contract theory is used, we expect the model to be built on a reasonable extensive form game which captures the actual process of ex-ante investment decisions and the employment scheme.

(4) Assuming risk-neutrality for both the agents and the principal, we have shown that a principal-agent problem can arise solely from the issue of group incentives. The risk-neutrality assumption simplifies the analysis significantly since it reduces the problem to one of incentives without the complication of risk-sharing. Allowing risk-aversion, for example, for the agents will provide many interesting implications about the tradeoff between incentive provision and risk-sharing at the cost of tractablity of the model.

Appendix

(1) Expected Value Functions: We calculate value functions for principal and agents in each steady state of history.

(i) Agents:

$$EV_A(\mathrm{PR, AC}) = \{r(c-d) + r(1-q)w\}/(1 - p\delta_A + 2rqp\delta_A)$$
$$EV_A(\mathrm{PR, NAC}) = rw/(1 - p\delta_A)$$
$$EV_A(\mathrm{NPR, AC}) = r(c-d)/(1 - \delta_A)$$
$$EV_A(\mathrm{NPR, NAC}) = 0$$

(ii) Principal:

$$EV_P(\text{PR, AC}) = \frac{\{G - 2r(G-B) - 2r(1-q)w - \delta_p(1-p+2rqp)\alpha\}}{(1-p\delta_p)}$$

$$EV_P(\text{PR, NAC}) = \frac{\{G - 2rw - \delta_p(1-p)\alpha\}}{(1-\delta_p)}$$

$$EV_P(\text{NPR, AC}) = \frac{\{G - 2r(G-B)\}}{(1-\delta_p)}$$

$$EV_P(\text{NPR, NAC}) = G/(1-\delta_p)$$

(2) Proof of Lemmas and Proposition 2:

The following facts are useful in proving Lemma 1, 3 and Proposition 2. We skip their proofs since they are obvious. From now on, we will denote EV_A(PR, AC)and EV_A(PR, NAC) with EV_A^C and EV_A^{NC} respectively.

Facts:

(F1) $\partial EV_A^C/\partial p > 0$, $\partial EV_A^{NC}/\partial p > 0$, $\partial EV_A^C/\partial w > 0$, $\partial EV_A^{NC}/\partial w > 0$, $\partial EV_A^C/\partial q < 0$, $\partial EV_A^{NC}/\partial q = 0$.

(F2) $EV_A^C > EV_A^{NC}$, $\partial EV_A^C/\partial p > \partial EV_A^{NC}/\partial p$ for sufficiently small q, $\partial EV_A^C/\partial w < \partial EV_A^{NC}/\partial w$.

(F3) $\partial^2 EV_A^C/\partial p^2 > \partial^2 EV_A^{NC}/\partial p^2 > 0$ for sufficiently small q, $\partial^2 EV_A^{NC}/\partial w\partial p > \partial^2 EV_A^C/\partial w\partial p > 0$.

(F4) $\partial w^F(p)/\partial q = \partial w^W(p)/\partial q = 0$, $\partial w^{IC}(p)/\partial q < 0$ for all p such that $w \geq 0$. $\partial w^F(p)/\partial \alpha < 0$, $\partial w^W(p)/\partial \alpha < 0$, $\partial w^{IC}(p)/\partial \alpha = 0$ for all p. $\partial w^F(p)/\partial r > 0$, $\partial w^W(p)/\partial r > 0$, $\partial w^{IC}(p)/\partial r > 0$ for all p.

(F5) $\partial w^F(p)/\partial p > \partial w^W(p)/\partial p > 0$.

***Proof of Lemma* 1**

(1) Consider $\alpha = 0$, $q = 1$ and $r \neq 0$, then $F(\alpha = 0, q = 1, r \neq 0)$ is non-empty. Since SR^F, SR^W, IC^A, $w \geq 0$ and $0 \leq p \leq 1$ are weak inequalities, $F(\alpha, q, r)$ is closed and bounded.

(2) If $\alpha > \alpha'$, then $w^F(p; \alpha, r) < w^F(p; \alpha', r)$ and $w^W(p; \alpha, r) < w^W(p; \alpha', r)$ for all p. $w^{IC}(p; q, r)$ is not affected by α. Thus $F(a, q, r) \subset F(\alpha', q, r)$. Similarly, $w^F(p; \alpha, r)$ and $w^W(p; a, r)$ are not affected by q but $w^{IC}(p; q, r) > w^{IC}(q; q', r)$ for $q < q'$ and all p such that $w \geq 0$. Thus $F(a, q, r) \subset F(\alpha, q', r)$ if $q < q'$.

(3) Since $w^F(p; \alpha, r)$ and $w^W(p; \alpha, r)$ are linear in p and $w^{IC}(p; q, r)$ is convex as q becomes small from (1) of Lemma 3, $F(\alpha, q, r)$ is a convex set.

Q.E.D.

***Proof of Lemma* 2**

It is obvious.

Proof of Lemma 3

(1) Let $w(p)$ the value of w as a function of p which satisfies (IC^A) as an equality. Then $w(p)$ satisfies $-d + (1-q)w(p) + (1-q)\, p\delta_A EV_A(\mathrm{PR, AC}) = 0 + w(p) + p\delta_A EV_A(\mathrm{PR, NAC})$. As mentioned above, we denote $EV_A(\mathrm{PR, AC})$ and $EV_A(\mathrm{PR, NAC})$ with EV_A^C and EV_A^{NC}.

(i) Taking total differentiation, we can find

$$\left\{\delta_A(1-q)p\frac{\partial EV_A^C}{\partial w} - \delta_A p\frac{\partial EV_A^{NC}}{\partial w} - q\right\}w'(p)$$
$$= \delta_A\{EV_A^{NC} - (1-q)EV_A^C\} + \delta_A p\left\{\frac{\partial EV_A^{NC}}{\partial p} - (1-q)\frac{\partial EV_A^C}{\partial p}\right\}, \tag{A1}$$

i.e., LHS$\cdot w'(p)$ = RHS. Notice that LHS < 0 always and RHS < 0 for q sufficiently small from (F1) and (F2) of the Facts. Thus, $w'(p) > 0$ for q sufficiently small.

(ii) In order to check convexity of $w(p)$, take total differentiation of (A1) again. Then we can show that

$$\left\{\delta_A(1-q)\frac{\partial EV_A^C}{\partial w} - \delta_A p\frac{\partial EV_A^{NC}}{\partial w} - q\right\}w''(p)$$
$$= 2\delta_A w'(p)\left[\left\{\frac{\partial EV_A^{NC}}{\partial w} - (1-q)\frac{\partial EV_A^C}{\partial w}\right\} + p\left\{\frac{\partial^2 EV_A^{NC}}{\partial w\partial p} - (1-q)\frac{\partial^2 EV_A^C}{\partial w\partial p}\right\}\right]$$
$$+\delta_A\left[2\left\{\frac{\partial EV_A^{NC}}{\partial p} - (1-q)\frac{\partial EV_A^C}{\partial p}\right\} + p\left\{\frac{\partial^2 EV_A^{NC}}{\partial p^2} - (1-q)\frac{\partial^2 EV_A^C}{\partial p^2}\right\}\right],$$

i.e., LHS$\cdot w''(p) = 2\delta_A w'(p)\cdot$RHS$_1$ + RHS$_2$. Notice that LHS < 0 and RHS$_1$ > 0 always since $\partial EV_A^C/\partial w < \partial EV_A^{NC}/\partial w$ and $\partial^2 EV_A^{NC}/\partial w\partial p > 0$ from (F2) and (F3) of the Facts. Furthermore, $\lim_{q\to 0}$ RHS$_1$ = 0 and $\lim_{q\to 0}$ RHS$_2$ = $-2\delta_A{}^2 r(c-d)/(1-\delta_A p)^2 < 0$. Thus $w''(p) > 0$ for q sufficiently small.

(2) Notice that $(w = 0,\ p = 1) \notin F(\alpha, q, r)$ since $w^{IC}(\mathrm{p} = 1, q, r) > 0$. Suppose not, i.e., $w^* \neq w^{IC}(p^*, q, r)$. Then either $w^* < w^{IC}(p^*, q, r)$ or $w^* > w^{IC}(p^*, q, r)$. If $w^* < w^{IC}(p^*, q, r)$, then it is contradiction to $(w^*, p^*) \in F(\alpha, q, r)$. If $w^* > w^{IC}(p^*, q, r)$, then consider $w^* - \varepsilon$. This will increase $EV_P(\mathrm{PR, NAC})$ with $(w^* - \varepsilon, p^*) \in F(\alpha, q, r)$. Contradiction.

(3) This is obvious since $\mathrm{MRS}_{(p,w)} = \alpha\delta_P/2r$ from Lemma 2 and (F4) of the Facts.

Q.E.D.

Proof of Proposition 2

(1) This is obvious since $\partial w^{IC}(p)/\partial\alpha = \partial w^{IC}(p)/\partial\delta_P = 0$ for all p and $MRS_{(p,w)} = \alpha\delta_P/2r$.

(2) We'll prove the statement only for q. Let $w(q; q^*)$ be the value of w at p^* as a function of q which satisfies (IC^A) as an equality. Then $w(q; p^*)$ satisfies $-d + (1 - q)w(q; p^*) + \delta_A(1 - q)p^*EV_A^C = 0 + w(q; p^*) + \delta_A p^*EV_A^{NC}$. Taking total differentiation, we can find

$$\left\{\delta_A(1-q)p^*\frac{\partial EV_A^C}{\partial w} - \delta_A p^*\frac{\partial EV_A^{NC}}{\partial w} - q\right\}\frac{\partial w(q;p^*)}{\partial q}$$

$$= w'(q;p^*) + \delta_A p^*\,\partial EV_A^C - \delta_A(1-q)p^*\frac{\partial EV_A^C}{\partial q}$$

i.e., $LHS\partial w(q; p^*)/\partial q = RHS_3$. Since $LHS < 0$ always and $RHS_3 > 0$ for $w \geq 0$ from (F1) of the facts, $\partial w(q; p^*)/\partial q < 0$. Since $\partial EV_A^C/\partial q < 0$ from (F1), imprecise monitoring technology makes it hard for the principal to deter the agents' collusion. Principal has to reward the honest agents more in terms of (p, w). This is the direct effect.

Similarly let $w'(q; p^*)$ be the slope of w^{IC} at p^* as a function of q which satisfies (A1). Taking total differentiation again, we can find

$$\left\{\delta_A(1-q)p^*\frac{\partial EV_A^C}{\partial w} - \delta_A p^*\frac{\partial EV_A^{NC}}{\partial w} - q\right\}\frac{\partial w'(q;p^*)}{\partial q}$$

$$= -w'(q;p^*)\left\{\delta_A(1-q)p^*\frac{\partial\left(\frac{EV_A^C}{\partial w}\right)}{\partial q} - \delta_A p^*\frac{\partial EV_A^{NC}}{\partial w} - 1\right\}$$

$$-\delta_A(1-q)\frac{\partial EV_A^C}{\partial q} + \delta_A EV_A^C - \delta_A(1-q)p^*\frac{\partial\left(\frac{\partial EV_A^C}{\partial p}\right)}{\partial q} - \delta_A p^*\frac{\partial EV_A^{NC}}{\partial p}$$

i.e., $LHS\partial w'(q; p^*)/\partial q = RHS_4$. Since $\partial(\partial EV_A^C/\partial w)/\partial q < 0$, $\partial EV_A^C/\partial q < 0$ and $\partial(\partial EV_A^C/\partial p)/\partial q < 0$, $RHS_4 > 0$. Thus $\partial w'(q; p^*)/\partial q < 0$.

Notice that $\partial EV_A^C/\partial p > \partial EV_A^{NC}/\partial p$ for sufficiently small q, $\partial EV_A^C/\partial w < \partial EV_A^{NC}/\partial w$ from (F2) and both $\partial EV_A^{NC}/\partial p$ and $\partial EV_A^{NC}/\partial w$ do not depend on q. However, $\partial(\partial EV_A^C/\partial w)/\partial q < 0$ and $\partial(\partial EV_A^C/\partial p)/\partial q < 0$. Thus as q decreases the difference between $\partial EV_A^C/\partial p$ and $\partial EV_A^{NC}/\partial p$ increases but the difference between $\partial EV_A^C/\partial w$ and $\partial EV_A^{NC}/\partial w$ decreases. This implies that rotation becomes more efficient incentive scheme than the wage scheme as the monitoring technology becomes imprecise. This is the indirect effect. Notice that $MRS_{(p,w)}$ is independent of q.

Combining these two effects we can find p^* decreases as q decreases but the effect on w^* is not clear. The proof for δ_A and r involves the same step.

Q.E.D.

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