Capital, Incomes And Life Styles

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This paper proposes an economic model to analyze an interdependence among capital accumulation, sexual division of labor, and life styles in a perfectly competitive economic system. By life style we mean marital status. We are concerned with the effects of amenity levels of various life styles, working efficiency and preference on the equilibrium economic structure. We represent a dynamic model and provide the conditions for existence of equilibria. The effects of changes in some parameters on the equilibrium economic structure are examined. It is shown, for instance, that an improvement in the amenity level of women's single life style results in that more men and women live singly. (*JEL* Classification: J12)

I. Introduction

Since the end of the Second World War dramatic changes have taken place in marriage dynamics in many countries as economic conditions are improved and women play more important political and economic role. The East-Asian cultures are experiencing changes in life styles related to sexual relationships and family structure. There is a rapidly increasing literature on the issues related to family economics, sexual division of labor and dynamics of marriage (e.g., Becker, 1981, 1985). But it may be argued that a few theoretical economic models have been constructed to explain interdependence between economic development and dynamics of marriage.

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One may find various cultural, social, institutional and economic factors for explaining structural changes in life styles and male and female relationships. With a view toward understanding the dynamics of male and female relations in labor market and family, this paper tries to develop a one-sector growth model of the interdependence between economic growth and marital status within a framework of perfect competition. The analysis centers on the roles of amenity levels, propensity to save, and working efficiency of men and women in association with different life styles. The study is organized as follows. Section II defines the economic growth model with endogenous marital status. Section III discusses the conditions for existence of economic equilibria. Sections IV and V respectively examine the effects of changes in single women's amenity level and working efficiency on the economic structure and the marriage rate. Section VI concludes the study.

II. The Model

We consider an one-sector economic system. The economy is similar to the traditional one-sector neoclassical growth model. Product can be either consumed or invested. Most aspects of the model in this study are similar to the growth models with sexual division recently proposed by Zhang (1993, 1995), except that in this study marital status are endogenous. To simplify possible complexity of marriage market, family structure and endogenous population growth, we assume that the economy consists of a fixed number of identical men and a fixed number of identical women. A person may have one of two kinds of life styles, married or single. Each family consists of only two members, husband and wife. We assume monogamy. We assume that all people work in the system. Working efficiency and working hours may be different between people of the same sex when they choose different life styles. For instance, a married woman may work less hours than a single woman. We neglect transaction costs of marriage and divorce. Marriage means that a female and a male individuals decide to live together, sharing private properties and producing and consuming family services such as sex, intimate friendship, companionship and enjoy variety of life styles.

The population can thus be classified into four types, each type indexed by (j, m). The subscript indexes, j and m, indicates sex and martial status, respectively as shown in Table 1.

TABLE 1
CLASSIFICATION OF THE POPULATION

	married, $m=1$	single, $m=2$
male, $j=1$	(1, 1)	(1, 2)
female, $j=2$	(2, 1)	(2, 2)

The single commodity is selected to serve as numeraire. We define the following indexes and variables:

$\pi \equiv \{(j, m) \mid j=1, 2, m=1, 2\}$;
$K(t)$ - the total capital stock of the economy at time t;
N_j - the given population of sex j, $j=1, 2$;
$N(t)$ and $N_{j2}(t)$ - the numbers of families and single persons of sex j, $j=1, 2$, respectively;
$k(t)$ and $k_j(t)$ - the level of capital stocks owned by per family and by per single person of sex j, respectively;
$c(t)$ and $c_j(t)$ - the consumption levels of per family and per single person of sex j, respectively;
$l(t)$ and $l_j(t)$ - the lot sizes of per family and of per single person of sex j, respectively;
$s(t)$ and $s_j(t)$ - the saving made by per family and by per single person of sex j, respectively;
$F(t)$ - the output of the economy;
$w_{jm}(t)$ - the wage rates of person (j, m), $(j, m) \in \pi$; and
$r(t)$ - the rate of interest.

We define the qualified labor inputs, N^*, of the economy as follows

$$N^* = z_{11}N + z_{21}N + z_{12}N_{12} + z_{22}N_{22} \tag{1}$$

The parameters, z_{jm}, are the given levels of working efficiency of person (j, m), $(j, m) \in \pi$. The parameters are aggregated measurements of human capital level (such as education and skill levels) and working time. For simplicity of analysis, we assume z_{jm} to be constant. It can be shown that it is not conceptually difficult to treat z_{jm} as endogenous variables (Zhang, 1993, 1995).

The production function of the production sector is specified as follows:

$$F=K^{\alpha}N^{*\beta},\ \ \alpha+\beta=1,\ \ \alpha,\ \beta>0. \tag{2}$$

The marginal conditions for the industrial sector are given by

$$r=\alpha F/K,\ w_{jm}/z_{jm}=\beta F/N^{*},\ (j,\ m)\in\pi \tag{3}$$

A. Families' Behavior

we specify land revenue as follows. We assume that each person owns $L/(N_1+N_2)$ amount of land, where N_1+N_2 is the population and L is the land used for housing of the economy. It is assumed that it is impossible to sell land but it is free to rent one's own land to others. Under this assumption, the land revenue per person is given as follows

$$\underline{r}=l_0 R, \tag{4}$$

where $l_0\equiv L/(N_1+N_2)$.

Let us denote $y(t)$ the income of per family and $y_j(t)$ the incomes of per single person of sex j, respectively. As income sources consist of land revenue, interest payment and wages, we have

$$y=2\underline{r}+rk+w_{11}+w_{21},\ y_j=\underline{r}+rk_j+w_{j2},\ j=1,\ 2. \tag{5}$$

We assume that the utility level of a married person of sex j is dependent on the consumption level, $c/2$, of per member of the family, the lot size, $l/2$, of per member of the family, and wealth, $(k+s-\delta_0 k)/2$, of per member of the family, where δ_0 is the fixed depreciation rate of capital. Here, we assume that a married couple always jointly owns the family's property. Similar to the utility function used in Zhang (1995), we specify the utility functions $U_{j1}(t)$ of a married person of sex j as follows:

$$\begin{gathered}U_{j1}(t)=A_{j1}(l/2)^{\eta_{j1}}(c/2)^{\xi_{j1}}\{(k+s-\delta_0 k)/2\}^{\lambda_{j1}},\ \ j=1,\ 2,\\ \eta_{j1},\ \xi_{j1},\ \lambda_{j1}>0,\ \ \eta_{j1}+\xi_{j1}+\lambda_{j1}=1,\end{gathered} \tag{6}$$

in which the parameters, η_{j1}, ξ_{j1} and λ_{j1} are person $(j,1)$'s propensity to consume lot size, to consume goods, and to hold wealth, respectively. In (6), A_{j1} is the amenity level of person $(j,1)$. For simplicity, we assume A_{j1} to be constant. This assumption is strict. For instance, it is quite reasonable to assume that a man's amenity level, A_{11}, of a married life may be affected by the developmental stage of the society.

In a traditional intertemporal framework, person (j, k) maximizes

$$\int_0^\infty U_{jk}(C_{jk}) \exp(-\rho t)\, dt,$$

subject to the dynamic budget constraint of capital accumulation. In the above formula, it is assumed that utility is additional over time. Although one may add capital and money over time, it is a very strict requirement to add utility over time. It is not reasonable to add happiness over time since one cannot accurately measure (relative) happiness of any special person at two different points of time. It is obvious that our formula does not involve this issue.

We assume that the family utility level, U, is dependent on the husband's utility level and wife's utility level in the following way:

$$U(t) = U_{11}^{\theta} U_{21}^{1-\theta}, \quad 1 > \theta > 0, \tag{7}$$

where θ is a parameter determined by the "relative weight" of the husband and wife in the family decision making. A family's financial budget constraint is given by:

$$Rl + c + s = y. \tag{8}$$

A family maximizes its utility level given by (7) subject to (6) and (8). A unique solution of the optimal problem is given by

$$Rl = \eta \Omega, \ c = \xi \Omega, \ s = \lambda \Omega - \delta k. \tag{9}$$

where

$$\Omega \equiv y + \delta k, \ \delta \equiv 1 - \delta_0, \ \eta \equiv \theta \eta_{11} + (1-\theta)\eta_{21},$$
$$\xi \equiv \theta \xi_{11} + (1-\theta)\xi_{21}, \ \lambda \equiv \theta \lambda_{11} + (1-\theta)\lambda_{21}.$$

A family's capital accumulation is given by : $dk/dt = s - \delta_0 k$, i.e.,

$$dk/dt = \lambda \Omega - k. \tag{10}$$

B. Behavior of Single Men and Women

We specify the utility functions $U_j(t)$ of a single person of sex j as follows :

$$U_{j2} = A_{j2} l_j^{\eta_{j2}} c_j^{\xi_{j2}} (k_j + s_j - \delta_0 k_j)^{\lambda_{j2}},$$
$$\eta_{j2}, \ \xi_{j2}, \ \lambda_{j2} > 0, \ \eta_{j2} + \xi_{j2} + \lambda_{j2} = 1, \ j = 1, 2. \tag{11}$$

It should be remarked that amenity levels that the same sex obtains from different life styles may be quite different. For instance, a man's amenity level may be much different between unmarried and married life styles. It is quite significant to examine whether or not the social and economic system may sustain such an equilibrium at which some men will be unmarried and some other men will be married, even though single men and married men are identical in the individual characteristics. We show that such an equilibrium may indeed exist. It should also be remarked that although we treat A_{jm} as parameters, they are actually affected by many factors. For instance, as consequences of female liberation and other social and economic changes, some "social burdens" on women who choose single life style may be greatly relaxed. The reduced social burden for women to choose single life style tends to increase the amenity level, A_{22} (in comparison to A_{21}). We will examine the impact of change in A_{22} on the system.

A single person $(j, 2)$'s budget constraint is given by

$$Rl_j + c_j + s_j = y_j. \tag{12}$$

A single person $(j, 2)$ maximizes the person's utility level, $U_j(t)$, subject to the above budget constrain. The optimal solution is given by

$$Rl_j = \eta_{j2}\,\Omega_j, \quad c_j = \xi_{j2}\,\Omega_j, \quad s_j = \lambda_{j2}\,\Omega_j - \delta\, k_j, \tag{13}$$

where $\Omega_j(t) \equiv y_j + \delta\, k_j$. A person $(j, 2)$'s capital accumulation is given by : $dk_j/dt = s_j - \delta_0 k_j$, i.e.,

$$dk_j/dt = \lambda_{j2}\,\Omega_j - k_j, \quad j = 1, 2. \tag{14}$$

C. Market Balance Conditions

As the population of each sex is homogenous, the utility level that persons of the same sex should be identical between different life styles, i.e.,

$$U_{j1} = U_{j2}, \text{ for } 0 < N,\ N_{j2} < N_j, \quad j = 1, 2. \tag{15}$$

The above conditions are held for $N \neq 0$ and $N_{j2} \neq 0$.

We assume that capital stocks and labor force are fully employed. The assumption that each sex is fully employed is represented by

$$N + N_{j2} = N_j. \tag{16}$$

By the definition of K, k, and k_{1j}

$$K = kN + k_{12}N_{12} + k_{22}N_{22}, \tag{17}$$

is held. The output is used either for consumption or investment, i.e.,

$$(c + s)N + (c_1 + s_1)N_{12} + (c_2 + s_2)N_{22} = F. \tag{18}$$

The land constraint is given by

$$lN + l_1N_{12} + l_2N_{22} = L. \tag{19}$$

We have thus built the model. There are 32 endogenous variables, K, N^*, F, l, k, c, s, y, r, R, N, U, l_j, k_j, c_j, s_j, y_j, N_j, w_{jm} and U_{jm}, j, $m = 1, 2$. We now provide conditions for existence of equilibria.

III. Existence of Equilibria

This section is concerned with conditions for existence of equilibria of the dynamic system. By (10) and (14), at an equilibrium

$$\lambda\Omega = k, \quad \lambda_{j2}\Omega_j = k_j, \quad j = 1, 2, \tag{20}$$

are held, Substituting (20) into (9) and (13) yields

$$Rl = \eta k / \lambda, \quad c = \xi k / \lambda, \quad s = \delta_0 k, \tag{21}$$

$$Rl_j = \eta_{j2}k_j / \lambda_{j2}, \quad c_j = \xi_{j2}k_j / \lambda_{j2}, \quad s_j = \delta_0 k_j. \tag{22}$$

Substituting (21) and (22) into (6) and (11) and then using (15), we obtain

$$k_j / k = a_j(R)^{\eta_j}, \tag{23}$$

where $\eta_j \equiv \eta_{j2} - \eta_{j1}$ and

$$a_j \equiv (A_{j1}/2A_{j2})(\eta^{\eta_{j1}} \xi_j^{\xi_{j1}} \lambda^{(\lambda_{j1}-1)} \lambda_{j2}^{(1-\lambda_{j2})})1/\eta_{j2}^{\eta_{j2}} \xi_{j2}^{\eta_{j2}}.$$

Substituting (21) and (22) into (18) and (19), we obtain

$$\begin{gathered}\delta_1 kN + \delta_{12}k_{12}N_{12} + \delta_{22}k_{22}N_{22} = F, \\ \eta kN / \lambda + \eta_{12}k_1N_1 / \lambda_{21} + \eta_{22}k_2N_2 / \lambda_{22} = RL,\end{gathered} \tag{24}$$

where $\delta_1 \equiv \xi / \lambda + \delta_0$ and $\delta_{j2} \equiv \xi_{j2} / \lambda_{j2} + \delta_0$. By (20), (5) and the definitions of Ω and Ω_j, we get

$$(1/\lambda - \delta - r)k = 2\underline{r} + w_{11} + w_{21}, \quad (1/\lambda_{j2} - \delta - r)k_j = \underline{r} + w_{j2}, \quad j = 1, 2. \tag{25}$$

By (25), we directly have

$$k_j/k = (\underline{r}+w_{j2})(1/\lambda - \delta - r)/(1/\lambda_{j2} - \delta - r)(2\underline{r}+w_{11}+w_{21}). \tag{26}$$

By (2) and (3), we get

$$r = \alpha/\Lambda^{\beta}, \quad w_{jm} = \beta z_{jm}\Lambda^{\alpha}, \tag{27}$$

where $\Lambda \equiv K/N^{*}$. By (26), (27), (4) and (23), we get

$$(\underline{r}+\beta z_{j2}\Lambda^{\alpha})(g\Lambda^{\beta}-1)/(g_j\Lambda^{\beta}-1)(2\underline{r}+\beta z\Lambda^{\alpha}) = a_j^{*}\,\underline{r}^{\,\eta_j}, \tag{28}$$

where $z \equiv z_{11} + z_{21}$, $g \equiv (1/\lambda - \delta)/\alpha$, $g_j \equiv (1/\lambda_{j2} - \delta)/\alpha$ and

$$a_j^{*} \equiv a_j/l_0^{\,\eta_j}, \quad j = 1, 2.$$

By (25), we see that it is necessary to require :

$$\Lambda^{\beta} > \max\{1/g, 1/g_1, 1/g_2\}. \tag{29}$$

The inequalities guarantee that k and k_j are positive. The two equations in (28) include two variables, $\underline{r}$ and Λ. It is difficult to guarantee the existence of solutions.

We now discuss how we may solve other variables for a given solution, $\underline{r}$ and Λ, of (28). By (27), we directly solve r and w_{jm} as functions of $\underline{r}$ and L. By (25) and $\underline{r}=l_0R$, we solve k, k_{j2} and R as functions of $\underline{r}$ and Λ. By (21) and (22), we solve l, c_{11}, c_{21}, s, l_{j2}, c_{j2} and s_{j2}. By (16) and (19), we solve

$$N = (L - l_{12}N_1 - l_{22}N_2)/(l - l_{12} - l_{22}), \; N_{j2} = N_j - N. \tag{30}$$

The above equations determine the marriage rate of each sex. It should be noted that we don't examine corner solutions of the equilibrium problem. For instance, it is quite possible that all the population of some sex live married life. Summarizing the above discussion, we get the following results.

***Proposition* 1**

If (28) have solutions, $\underline{r}$ and Λ, which satisfy (29) and N given by (30) satisfies $0<N<\min\{N_1, N_2\}$, then the dynamic system has at least one equilibrium. The number of equilibria is equal to the number of meaningful solutions of (28). Moreover, the equilibrium values of the variables are given by the following procedure :

$\underline{r}$ and Λ by (28) $\rightarrow$ $R=\underline{r}/l_0 \rightarrow r$ and w_{jm}, j, $m=1, 2$, by (27) $\rightarrow$ k and k_{j2} by (25)$\rightarrow l$, c and s by (21) $\rightarrow l_j$, c_j and s_j by (22) $\rightarrow N$ and N_{j2}

by (30) $\rightarrow y$ and y_j by (5) $\rightarrow F$ by (24) $\rightarrow K$ by (3) $\rightarrow N^*$ by (1) $\rightarrow U_{j2}$ by (11) $\rightarrow U_{j1}$ by (6) $\rightarrow U$ by (7).

Since it is difficult to interpret the conditions for existence of equilibria, for illustration we specify values of some parameters. We make the following requirements :

$$\eta_1 = \delta_0 = 0, \ \eta_2 < 0, \ \lambda_{11} = \lambda_{12} = \lambda_{21} = \lambda_{22}, \qquad (31)$$
$$2a_1 - 1 > 0, \ z_{12} > a_1 z, \ N_1 = N_2.$$

The requirements, $N_1 = N_2$ and $\delta_0 = 0$, simply mean that the population of two sexes are equal and capital depreciation is neglected. The condition, $\eta_1 = 0$, implies that the single man's propensity to consume lot size is equal to the married man's. The condition, $\eta_2 < 0$, implies that the single woman's propensity to consume lot size is lower than the married woman's. The condition, $2a_1 - 1$, is rewritten as $A_{11} > A_{12}$, which means that a man gets higher amenity level in married life style than in single life style. The condition, $z_{12} > a_1 z > z/2$, says that a single man's working efficiency level is higher than the average working efficiency level of husband and wife. We see that it is quite acceptable to assume (31). We now show that (31) guarantees the existence of a unique equilibrium of the dynamic system (with some additional requirements discussed below).

By $\lambda_{11} = \lambda_{12}$, we have $\lambda = \lambda_{11}$. Under (31), we solve (28) as follows :

$$\Lambda^{\alpha} = \underline{r}^* / \underline{r}_0 > 0, \ \underline{r} = \underline{r}^* > 0, \qquad (32)$$

where

$$\underline{r}_0 \equiv (z_{12} - a_1 z)\beta / (2a_1 - 1), \ r^* \equiv \{\underline{r}_0 + \beta z_{22}) / a_2^*(2\underline{r}_0 + \beta z)\}^{1/\eta_2}.$$

We now discuss under what conditions (29) and $N < \min\{N_1, N_2\}$ are satisfied. Under (31), $g = g_1 = g_2$ is held. Hence, (29) can be rewritten as

$$(\underline{r}^* / \underline{r}_0)^{\beta/\alpha} > 1/g_1 = \alpha\lambda_{12} / (\eta_{12} + \xi_{12}). \qquad (33)$$

For illustration, we require $\underline{r}^* \geq 1$, i.e.,

$$\underline{r} + \beta z_{22} \leq a_2^* \ (2\underline{r}_0 + \beta z).$$

This requirement is satisfied, for instance, if $a_2^* \geq 1/2$ and $a_2^* z \geq z_{22}$. The conditions, $a_2^* \geq 1/2$ and $a_2^* z \geq z_{22}$, can be similarly interpreted as the conditions, $(a_1^* =)\, a_1 > 1/2$ and $z_{12} > a_1 z$ in (31).

In the case of $\underline{r}^* \geq 1$, the inequality, (33), is satisfied if

$$(2a_1 - 1) / \beta(z_{12} - a_1 z) > \{\alpha \lambda_{12} / (\eta_{12} + \xi_{12})\}^{\alpha/\beta}. \tag{34}$$

For instance, in the case of $\alpha = \beta$ and $\eta_{12} + \xi_{12} = 4\lambda_{12}$, then the inequality, (34) is guaranteed if $2a_1 - 1 > (z_{12} - a_1 z)/16$. We see that it is possible to find some acceptable conditions for (32) to satisfy (29).

By (21), (22) and $\underline{r} = l_0 R$, we get

$$l - l_1 - l_2 = (\eta - k^*)k/\lambda R,\ L - l_1 N_1 - l_2 N_2 = (2\lambda \underline{r}/k - k^*)kN_1/\lambda R, \tag{35}$$

where

$$k^* \equiv \eta_{12} k_1 / k + \eta_{22} k_2 / k.$$

For $N_1 > N > 0$, it is sufficient to require

$$1 > (2\lambda \underline{r} / k - k^*) / (\eta - k^*) > 0. \tag{36}$$

By (32), (25) and (27), we have

$$\begin{aligned} k^* &= \{h_{12}(\underline{r}_0 + \beta z_{12}) + \eta_{22}(\underline{r}_0 + \beta z_{22})\} / (2\underline{r}_0 + \beta z), \\ 2\lambda \underline{r}/k &= 2\alpha\lambda \underline{r}_0(g^{\beta} - 1)/ \Lambda^{\beta}(2\underline{r}_0 + \beta z). \end{aligned} \tag{37}$$

In the case of $\eta < 2\lambda \underline{r} / k < k^*$ or in the case of $\eta > 2\lambda \underline{r} / k > k^*$, $N_1 > N > 0$ are satisfied. By (37), the conditions of $\eta > 2\lambda \underline{r} / k > k^*$ are guaranteed if

$$\begin{aligned} \beta \eta z / 2\underline{r}_0 + \alpha \lambda / \Lambda^{\beta} &> \xi, \\ 2\alpha\lambda \underline{r}_0(g\Lambda^{\beta} - 1) / \Lambda^{\beta} &> \eta_{12}(\underline{r}_0 + \beta z_{j2}) + \eta_{22}(\underline{r}_0 + \beta z_{22}), \end{aligned} \tag{38}$$

are satisfied. It is difficult to interpret the above conditions. We may similarly discuss other cases of possible parameter combinations.

***Corollary* 1**

We assume that (31), (34), (38), $a_2^* \geq 1/2$, $a_2^* z \geq z_{22}$ are satisfied. The dynamic system has a unique equilibrium. The equilibrium values of the variables are given by the procedure in Proposition 1.

From the above discussion, we see that only the conditions given by (38) are difficult to interpret. In the remainder of this study, for convenience of illustration we always assume that the requirements in Corollary 1 are satisfied. We examine the effects of changes in some parameters on the equilibrium.

IV. The Amenity Level of Women's Single Life style

This section is concerned with the impact of changes in the amenity level, A_{22}, of women's single life style on the marriage rate and economic structure.

Taking derivatives of (32) with respect to A_{22} yields

$$(1/\underline{r})d\underline{r}/dA_{22} = (1/R)dR/dA_{22} = 1/\eta_2 A_{22} < 0,$$
$$(\alpha/\Lambda)d\Lambda/dA_{22} = 1/\eta_2 A_{22} < 0 \tag{39}$$

where we use $\underline{r} = l_0 R$ and $\eta_2 < 0$. The land rent, R, and land revenue, $\underline{r}$, per capita are reduced.

By (27),

$$(1/r)dr/dA_{22} = -(\beta/\Lambda)d\Lambda/dA_{22} > 0,$$
$$(1/w_{jm})dw_{jm}/dA_{22} = (\alpha/\Lambda)d\Lambda/dA_{22} < 0, \quad j,\ m = 1,\ 2, \tag{40}$$

are held. As single women's amenity level is increased, the rate of interest, r, is reduced and the wage rates, w_{jm}, of all the groups are reduced.

By (25), $y = (1/\lambda - \delta)k$ and $y_j = (1/\lambda_{j2} - \delta)k_{j2}$, we obtain

$$(1/k)dk/dA_{22} = (1/y)dy/dA_{22} = (1/k_j)dk_j/dA_{22} = (1/y_j)dy_j/dA_{22}$$
$$= \{(\alpha g\Lambda^{\beta} - 1)/\Lambda(g\Lambda^{\beta} - 1)\}d\Lambda/dA2_{22}. \tag{41}$$

In the case of $\alpha g\Lambda^{\beta} > (<)\ 1$, we have $dk/dA_{22} < (>) 0$ and $dk_j/dA_{22} < (>) 0$. Taking derivatives of (21) and (22) yields

$$(1/l)dl/dA_{22} = (1/l_j)dl_j/dA_{22} = -\beta/\alpha\eta_2 A_{22}(g\Lambda^{\beta} - 1) > 0,$$
$$(1/c)dc/dA_{22} = (1/c_j)dc_j/dA_{22} = \{(\alpha g\Lambda^{\beta} - 1)/\Lambda(g\Lambda^{\beta} - 1)\}d\Lambda/dA_{22}. \tag{42}$$

The lot sizes, l and l_j, per family, single man and per single woman are increased. In the case of $\alpha g\Lambda^{\beta} > (<) 1$, we have $dc/dA_{22} < (>) 0$ and $dc_j/dA_{22} < (>) 0$.

By (30) and (35), we have.

$$N = (2\lambda\underline{r}/k - \eta)N_1/(\eta - k^*) + N_1.$$

Taking derivatives of the above equation and $N_{j2} = N_j - N$, we get

$$dN/dA_{22} = \{2\alpha\beta\lambda\underline{r}_0 N_1/\Lambda^{1+\beta}(2\underline{r}_0 + \beta z)(\beta - k^*)\}d\Lambda/dA_{22} < 0,$$
$$dN_{j2}/dA_{22} = -dN/dA_{22} > 0, \tag{43}$$

in which (37) and $\eta > 2\lambda\underline{r}/k > k^*$ are used. As the amenity life of women's single life style is improved, the marriage rate is reduced. In other words, more men and women live in unmarried life. By (1), $K = \Lambda N^*$ and $F = \Lambda^{\alpha} N^*$

$$dN^*/dA_{22} = (z - z_{12} - z_{22})dN/dA_{22},$$
$$(1/K)dK/dA_{22} = (1/\Lambda)d\Lambda/dA_{22} + (1/N^*)dN^*/dA_{22}, \tag{44}$$
$$(1/F)dF/dA_{22} = (1/\Lambda)d\Lambda/dA_{22} + (1/N^*)dN^*/dA_{22},$$

are held. If the sum, z, of the husband and wife's working efficiency is larger than the sum, $z_{12}+z_{22}$, of a single woman and a single man, then the qualified labor force, N^*, the total capital stocks, K, and the output, F, of the economy are reduced. In the case of $z < z_{12} + z_{22}$, then N^* is increased and K and F may be either increased or reduced. Summarizing the discussion in this section, we get the following proposition.

Proposition 2

Let the assumptions in Corollary 1 be satisfied. Then an increase in the amenity level, A_{22}, of women's single life style has the following impact on the unique equilibrium economic structure :

(1) the marriage rate, N, is reduced;

(2) in the case of $z > z_{12} + z_{22}$, the qualified labor force, N^*, the total capital stocks, K, and the output, F, of the economy are reduced; in the case of $z < z_{12} + z_{22}$, N^* is increased and K and F may be either increased or reduced;

(3) the capital stocks, k and k_j, and the net incomes, y and y_j, of per family, single man and single woman are increased (reduced) in the case of $\alpha g \Lambda^{\beta} <(>) 1$;

(4) the lot sizes, l and l_j, of per family, single man and single woman are increased; the consumption levels, c and c_j, of per family, single man and single woman are reduced (increased) in the case of $\alpha g \Lambda^{\beta} >(<) 1$;

(5) the rate of interest, r, is increased, and the land rent, R, the land revenue, $\underline{r}$, from land ownership, and the wage rates, w_{jm}, of men and women are reduced.

It should noted that the above conclusions are obtained under strict requirements. It can be seen that even in our simple model it is quite difficult to explicitly judge the effects of changes in parameters.

V. The Working Efficiency of Single Women

This section is concerned with the impact of changes in the working efficiency, z_{22}, of single women on the marriage rate and economic structure. By (32) and $r = l_0 R$

$$(1/\underline{r})d\underline{r}/dz_{22} = (1/R)dR/dz_{22} = 1/\eta_2(r_0 + \beta z_{22}) < 0,$$
$$(\alpha/\Lambda)d\Lambda/dz_{22} = 1/\eta_2(r_0 + \beta z_{22}) < 0, \quad (45)$$

are held. The land revenue, $\underline{r}$, per capita and the land rent, R, are reduced. Taking derivatives of (27) with respect to z_{22} yields

$$(\Lambda/\beta r)dr/dz_{22} = -d\Lambda/dz_{22} > 0,$$
$$(\Lambda/\alpha w_{jm})dw_{jm}/dz_{22} = d\Lambda/dz_{22} < 0, \ (j, m) = (1, 1), (1, 2), (2, 1), \quad (46)$$
$$(1/w_{22})dw_{22}/dz_{22} = 1/\eta_2(\underline{r}_0 + \beta z_{22}) + 1/z_{22}.$$

The rate of interest, r, is increased and the wage rates, w_{1j}, and w_{21}, of married men, single men and married women are reduced. The wage rate, w_{22}, of single women may be either increased or decreased.

By (25), $y = (1/\lambda - \delta)k$ and $y_j = (1/\lambda_{j2} - \lambda)k_{j2}$,

$$(1/k)dk/dz_{22} = (1/y)dy/dz_{22} = (1/k_1)dk_1/dz_{22} = (1/y_1)dy_1/dz_{22}$$
$$= \{(\alpha g\Lambda^{\beta} - 1)/\Lambda(g\Lambda^{\beta} - 1)\}d\Lambda/dz_{22}, \ (1/k_2)dk_2/dz_{22} \quad (47)$$
$$= (1/y_2)dy_2/dz_{22} = \{(\beta\eta_2 + 1)\alpha g\Lambda^{\beta} - 1 - \alpha\beta\eta_2\}/\alpha\eta_2(g\Lambda^{\beta} - 1)(\underline{r}_0 + \beta z_{22}),$$

are held. The capital stocks, k and k_1, and incomes, y and y_1, of per family and single men are increased (reduced) in the case of $\alpha g\Lambda^{\beta} <$ $(>)1$. The sign of dk_2/dz_{22} and dy_2/dz_{22} is the opposite to that of the term, $(\beta\eta_2 + 1)\alpha g\Lambda^{\beta} - 1 - \alpha\beta\eta_2$. By (21) and (22)

$$(1/l)dl/dz_{22} = (1/l_1)dl_1/dz_{22}$$
$$= \{(\alpha g\Lambda^{\beta} - 1)/\Lambda(g\Lambda^{\beta} - 1)\}d\Lambda/dz_{22} - 1/\eta_2(\underline{r}_0 + \beta z_{22}), \ (1/l_2)dl_2/dz_{22} \quad (48)$$
$$= (\beta\eta_2 - 1)/\eta_2(\underline{r}_0 + \beta z_{22}) + \{(\alpha g\Lambda^{\beta} - 1)/\Lambda(g\Lambda^{\beta} - 1)\}d\Lambda/dz_{22}$$
$$(1/c)dc/dz_{22} = (1/c_j)dc_j/dz_{22} = \{(\alpha g\Lambda^{\beta} - 1)/\Lambda(g\Lambda^{\beta} - 1)\}d\Lambda/dz_{22}.$$

are held. The impact on the marriage rate is given by

$$dN/dz_{22} = \{2\alpha\beta\lambda\underline{r}_0 N_1/\Lambda^{1+\beta}(2\underline{r}_0 + \beta z)(\eta - k^*)\}d\Lambda/dz_{22} +$$
$$(2\lambda\underline{r}/k - \eta)\beta\eta_{22}N_1/(\eta - k^*)^2(2\underline{r}_0 + \beta z), \quad (49)$$
$$dN_{j2}/dz_{22} = -dN/dz_{22}.$$

We see that the marriage rate may be either increased or reduced when the working efficiency of single women is improved. By (1), $K = \Lambda N^*$ and $F = \Lambda^{a} N^*$

$$dN^*/dz_{22} = (z - z_{12} - z_{22})dN/dz_{22} + N_{22},$$

$$(1/K)dK/dz_{22} = (1/\Lambda)d\Lambda/dz_{22} + (1/N^*)dN^*/dz_{22}, \qquad (50)$$

$$(1/F)dF/dz_{22} = (1/\Lambda)d\Lambda/dz_{22} + (1/N^*)dN^*/dz_{22},$$

are held. It is difficult to explicitly determine the signs of the effects of changes in single women's working efficiency on the total capital stocks and output of the economy.

VI. Concluding Remarks

This paper proposed an economic model to analyze the interdependence among capital accumulation, sexual division of labor and marriage rate in a perfectly competitive economic system. We built a dynamic model and provided the conditions for the existence of equilibria. We were concerned with the effects of amenity levels of various life styles, working efficiency and preference on the economic structure. The effects of changes in some parameters on the equilibrium economic structure were examined.

The study may be improved in multiple ways. For instance, one may simulate the model by computer, seeing actual dynamic processes over time. Our model simplifies many aspects of family life. We carried out the comparative static analysis with respect to z_{22} and A_{22} under very strict assumptions. We may directly analyze the effects of changes in some other parameters. It should be remarked that the system may contain more complicated behavior than we mentioned. For instance, if the amenity level that a man obtains from living with a woman is extremely higher than the amenity level that he lives singly and the supply of the female population is much lower than the male population, then the system may not have equilibrium or may have some kinds of corner solutions. We may extend the model in multiple ways. It is important to classify production into multiple sectors as male and female labor force may obtain different amenity levels from different professions. Issues related to the number of and bringing-ups of children are obviously one of the most important factors which affect marriage. It is also significant to introduce endogenous changes in preference parameters, η_{jm}, ξ_{jm}, λ_{jm}, amenity levels, A_{jm}, of life styles for different people and the parameter, θ (which is dependent on emotion and power structures of the female and male population).

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