Human Capital Formation and Saving-Investment Dynamics

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This paper explores the dynamics of saving, investment and the trade balance in a two-country version of human capital growth model. As observed in the post-war G7 data, the model economy generates a high saving-investment correlation, together with a countercyclical trade balance. The persistent movement of investment, sustained by the accumulation of human capital, turns out to play a key role in producing the realistic cyclical properties of the model economy. (*JEL* Classifications: C16, F32, F41)

I. Introduction

Among the empirical regularities of international data, the saving-investment correlation has been the focus of much attention in many studies. While a high saving-investment correlation could be interpreted as an evidence of capital immobility,[1] recent international real business cycle studies have been successful in producing such relations observed in the data, even assuming the perfect

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[1]Feldstein and Horioka (1980) test the hypothesis of perfect capital mobility by regressing investment rates on saving rates in a cross-section of 16 industrialized countries: the ratios of saving and investment to GDP are averaged over the period of 1960-74, or subperiods. They find that the coefficients on saving rates are in the range 0.87-0.94 and significantly different from zero. They conclude that most of any incremental saving tends to remain in the country in which the saving is done. Feldstein (1983) also finds no evidence that saving-investment correlation has weakened over time by extending the sample period to 1979.

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capital mobility. For example, Baxter and Crucini (1993) draw a high saving-investment correlation in a two-country version of one-good standard neoclassical growth model.[2] By adding adjustment costs to the capital accumulation process and by assuming that shocks are transmitted rapidly across countries, they get the reduced volume of international capital flows and thus bring the saving-investment correlation in line with the correlations found in the data. However, these studies fail to produce some relevant movements of economic variables. Besides the improper cross-country correlations,[3] the trade balance movement is out of the range found in the data. For example, though she finds a high saving-investment correlation, Baxter (1995) produces a procyclical traae balance: the balance of trade for the country in which the productivity shock originates records a surplus.[4] This fact is contradictive to the empirical findings, including Baxter and Crucini (1993) and Backus, Kehoe and Kydland (1992), which show that the trade balance is countercyclical, together with high saving-investment correlations in the industrialized countries. In addition, the implication that output and investment booms are associated with current account surpluses runs counter to the empirical findings of Sachs (1981) and more recently, Glick and Rogoff (1994).[5]

[2]In a similar two-country one-good setting, Backus, Kehoe and Kydland (1992) fail to produce a high saving-investment correlation. With complete market, agents are assumed to trade contingent claims on their income, thus saving being perfectly correlated across countries. At the same time, investment differs across countries, depending on the productivity shocks which each country is facing.

[3]Cross-country correlation puzzles have not yet solved in the international RBC literature: for example, cross-country consumption correlation is stronger than cross-country output correlation in the theoretical economy, but, the reverse is observed in the data.

[4]In her model, the correlation between the trade balance and output turns out to be positive (0.78 & 0.78) for both complete and incomplete market setting, with the shock process specified as Backus *et al.* (1992). The correlation remains positive (0.06) in the complete market setting even if no spillover of the shock is assumed. She attributes this feature to the reason that the home productivity shock makes foreigners want to decumulate assets to sustain consumption while waiting for the shock to arrive. Thus the foreign current account deficit must be offset by a current account surplus in the home country. (She interchanges the trade balance with the current account in the justification that the business cycle aspects of the current account is much attributable to that of the trade balance.)

[5]Sachs (1981) reports the negative coefficient on investment in the

Clearly, the movement of the trade balance, which measures the difference between saving and investment, is closely connected to the saving-investment correlation. In order to investigate the dynamics of these two economic variables, I develop a two-country version of a one-good stochastic growth model. The model economy produces a single final good by combining labor and two forms of capital physical and human capital, which are produced from different sectors: a market production sector and a non-market production sector.

I find that this model economy generates both a countercyclical trade balance and a high saving-investment correlation. Investment dynamics play a central role in generating these properties of the model economy. Fluctuations in the model economy arise from shocks to the total factor productivity. A favorable home productivity shock leads to an increase in the output level of the home country, which is associated with a rise in consumption and investment booms of the home country. While consumption changes smoothly, investment shifts to its most productive location. On impact, an increase of output is associated with an increase of investment and a decrease of the trade balance. In subsequent periods, sustained improvement in the trade balance is accompanied by the persistent movement of investment, sustained by the accumulation of human capital. As a result, the trade balance is countercyclical when investment sustains its procyclical movements.[6]

An increase in output causes an increase in consumption that is less than output as agents smooth their consumption over time and share their idiosyncratic shocks. Consumption is, thus, procyclical and less volatile, implying that saving is procyclical. This results in a high saving-investment correlation as is found in the data.

The paper proceeds as follows. The next section reviews empirical

regression of the current account on investment, using data from 14 OECD countries for the time period 1960-79. Glick and Rogoff (1992) also find the negative relationship between the current account and investment. They regress the change in the current account on the change in investment, using 1961-90 annual data for 8 industrialized countries. They find significantly negative coefficients on the change in investment ranging from -0.16 to -0.55, averaging -0.33.

[6]When the impact of technological shocks on investment is not persistent, as in the standard neoclassical model, the trade balance turns out to be procyclical.

properties of international business cycles from the industrialized countries. In Section III, the economic environment of the model is described and equilibrium is characterized. Parameter values of the model economy is calibrated and its cyclical properties are reported and their implications are discussed in Section IV. Section V concludes.

II. Empirical Regularities

In this section, the properties of the international business cycles are reported. These properties refer to moments of quarterly time series for the seven largest industrial countries (G7): Canada, France, Germany, Italy, Japan, the United Kingdom and the United States. Source of data is the IMF's International Financial Statistics (IFS). All data are detrended using the Hodrick-Prescott filter. A detailed description of the data appears in the Appendix.

Table 1 reports the correlation between saving and investment within countries. Many studies, including those of Obstfeld (1985) and Tesar (1991), have shown that saving and investment are highly correlated and the correlation is robust in the low frequencies as well as the high frequencies.[7]

In this paper, saving is defined as output minus private consumption minus government consumption. This definition is adopted because it is easily measured in both the data and the model economy and is compatible with the other studies mentioned above, which used a similar definition.[8] Table 1 shows that the correlation between saving and investment varies across countries but is large

[7]Obstfeld estimates correlations between quarterly changes in saving and investment rates, for seven OECD countries. He finds that correlations are positive and increasing as country size is larger. And Tesar replicates the Feldstein-Horioka regression for 23 OECD countries. In the regression for 17year (1960-86) averaged saving rates and investment rates, the coefficient on saving rates is not significantly different from unity. Even as the period over which the averages are calculated is shortened to 5 year, 3 year and 1 year, the strong coefficient on saving rates remains.

[8]The exact definition of saving is the change in the market value of its wealth. But, when the market is assumed to be complete, the measure of saving is complicated since it is impossible to trace the change of ownership of asset. Moreover, the saving data available abstract from capital gains or losses on assets. For more discussions, see Backus *et al.* (1992).

and positive, with the mean of 0.66. Especially, the correlation is very high, over 0.80, for the United States and Japan.

The last column of Table 2 pertains to net exports. The trade balance is measured as the ratio of net exports-to-output.[9] For each of countries, the net exports-to-output ratio is countercyclical, in the sense that its contemporaneous correlation with output is negative. The negative correlation ranges from -0.20 for Germany to -0.61 for Italy, with the mean of -0.36. The countercyclical movement of the trade balance has also been documented by Backus, Kehoe and Kydland (1994) and Danthine and Donaldson (1993).[10]

Table 3 shows that the trade balance is highly persistent. The autocorrelation of net exports-to-output ratio extends from 0.56 in Canada to 0.82 in Japan. Compared to the other macroeconomic variables net exports are as persistent as consumption, but a little less persistent than output and investment.

Table 4 and 5 report the volatility of net exports. The standard deviation of net exports-to-output ratio ranges from a low of 0.43 percent for the United States to a high of 1.27 percent for Italy. These values mean high volatility, taking into account that its standard deviation is as high as that of other variables such as consumption-output ratio, investment-output ratio and saving-output ratio although net export is very small portion of the output relative to consumption, investment and saving.

Regarding cross-country regularities, Tables 6-8 present cross -country correlations. The correlation of output across countries is positive, with the average of 0.40, from Table 6. The correlation

[9]In the model economy, net exports are zero in the steady-state so that the percent deviation from its steady-state can not be computed. Thus, the ratio of net export to output is utilized here. Other studies avoid this problem by using different definitions. For example, Stockman and Tesar (1995) define detrended net exports as detrended exports minus detrended imports. Or Correia, Neves and Rebelo (1995) approximate log of net exports, keeping their signs. Quantitative results are not different between these definitions, except their standard deviations.

[10]Backus, Kehoe and Kydland (1994) document for the 11 OECD countries, using quarterly data over the post-war period. The correlation between the trade balance and output ranges from -0.17 to -0.68, with the mean of -0.37. Danthine and Donaldson (1993) also find negative correlation between the trade balance and output ranging from -0.14 to -0.67, with the mean of -0.37. They use the post-war quarterly data for the 11 industrialized countries.

TABLE 1
SAVING-INVESTMENT CORRELATIONS

	Saving-Investment Correlation
US	0.83
UK	0.45
Japan	0.80
Italy	0.45
Germany	0.72
France	0.64
Canada	0.74

Note: All variables are in the Hodrick-Prescott filtered logarithm.

TABLE 2
CORRELATIONS WITH OUTPUT

	y	c	i	s	NX/Y
US	1.00	0.86	0.90	0.87	-0.42
UK	1.00	0.83	0.74	0.71	-0.39
Japan	1.00	0.48	0.84	0.87	-0.35
Italy	1.00	0.67	0.77	0.78	-0.61
Germany	1.00	0.95	0.78	0.82	-0.20
France	1.00	0.48	0.82	0.84	-0.28
Canada	1.00	0.72	0.72	0.87	-0.24

TABLE 3
FIRST-ORDER AUTOCORRELATIONS

	y	c	i	s	NX/Y
US	0.84	0.83	0.90	0.74	0.79
UK	0.79	0.75	0.78	0.64	0.69
Japan	0.78	0.53	0.88	0.77	0.82
Italy	0.85	0.81	0.89	0.69	0.81
Germany	0.84	0.82	0.74	0.66	0.60
France	0.87	0.67	0.88	0.82	0.72
Canada	0.82	0.72	0.88	0.79	0.56

Note: All variables are the Hodrick-Prescott filtered and are in logarithm, except the net exports-to-output ratio (NX/Y) in level form.

TABLE 4
STANDARD DEVIATIONS

	y	c	i	s	NX/Y
US	1.66	1.20	5.34	7.00	0.43
UK	1.67	1.83	4.61	6.51	1.09
Japan	1.59	1.19	4.63	4.35	0.96
Italy	1.58	1.41	4.16	5.11	1.27
Germany	2.86	2.79	4.75	4.53	0.79
France	1.19	1.28	3.13	4.01	0.81
Canada	1.71	1.41	4.75	6.10	0.74

Note: All variables are the Hodrick-Prescott filtered and are in logarithm, except the net exports-to-output ratio (NX/Y) in level form.

TABLE 5
STANDARD DEVIATIONS

	C/Y	I/Y	S/Y	NX/Y
US	0.56 (64.88)	0.61 (15.52)	0.88 (15.57)	0.43 (0.91)
UK	0.65 (63.04)	0.65 (17.87)	0.96 (17.54)	1.09 (1.43)
Japan	0.84 (57.61)	1.04 (30.77)	1.03 (33.55)	0.96 (1.52)
Italy	0.75 (61.86)	0.71 (22.13)	0.94 (22.37)	1.27 (1.50)
Germany	0.50 (56.06)	0.64 (20.52)	0.66 (24.50)	0.79 (3.94)
France	0.76 (59.92)	0.47 (21.67)	0.70 (22.63)	0.81 (0.85)
Canada	0.71 (59.17)	0.82 (22.10)	1.06 (22.93)	0.74 (1.23)

Note: All variables are in level and the Hodrick-Prescott filtered. Figures in parentheses are the average of ratios. For example, the US consumption-output ratios average 64.88% over the time period. Absolute value of ratio is taken for NX/Y.

TABLE 6

CROSS-COUNTRY OUTPUT CORRELATIONS

	US	UK	Japan	Italy	Germany	France	Canada
US	1.00	0.58	0.14	0.35	0.31	0.35	0.77
UK		1.00	0.31	0.35	0.41	0.52	0.50
Japan			1.00	0.34	0.33	0.54	0.03
Italy				1.00	0.40	0.60	0.43
Germany					1.00	0.38	0.50
France						1.00	0.30
Canada							1.00

TABLE 7

CROSS-COUNTRY CONSUMPTION CORRELATIONS

	US	UK	Japan	Italy	Germany	France	Canada
US	1.00	0.52	0.15	0.09	0.27	-0.20	0.69
UK		1.00	0.09	0.20	0.26	0.15	0.37
Japan			1.00	0.08	0.26	0.11	0.02
Italy				1.00	0.13	0.35	-0.02
Germany					1.00	0.27	0.33
France						1.00	-0.13
Canada							1.00

TABLE 8

CROSS-COUNTRY INVESTMENT CORRELATIONS

	US	UK	Japan	Italy	Germany	France	Canada
US	1.00	0.38	0.02	-0.07	-0.02	-0.01	0.32
UK		1.00	0.35	0.31	0.38	0.43	0.36
Japan			1.00	0.61	0.72	0.63	-0.07
Italy				1.00	0.64	0.61	0.44
Germany					1.00	0.61	0.33
France						1.00	0.43
Canada							1.00

Note: All variables are in the Hodrick-Prescott filtered logarithm. Statistics are computed for observations available for both series.

ranges from 0.30 between France and Canada to 0.77 between Canada and the United States, with the exceptional lows of 0.03 between Japan and Canada and 0.14 between the United States and Japan.

The cross-country consumption is in general positively correlated though less so than output. From Table 7, the average cross-country correlation of consumption is 0.19, compared to 0.40 for output. But the countries which have high output correlations between them tend to have high consumption correlations, too.

Table 8 reports cross-country investment correlations. The average of investment correlations across countries is 0.35, that is bigger than that of consumption. But, the cross-country investment correlations vary widely across countries from -0.07 to 0.72 and have little relations with the correlations of consumption or output.

III. The Model

A. Economic Environment

The model economy is a two-country version of Rebelo (1991)'s human capital growth model. There are two sectors in each country; a market production sector and a non-market production sector. A single final good is produced in the market production sector: this good is used for both consumption and investment: this single final good is traded through an international market to smooth consumption and to take advantage of favorable investment opportunities. A non-market production sector produces the good that is used for the accumulation of human capital.

Each country is inhabited by a large number of identical agents, who value leisure and consumption of the market good. Physical capital is mobile through international market while labor and human capital are immobile across national borders.

This model is described by the equations below. Since the countries are symmetric, only the equations representing the home country are presented. When it is necessary to distinguish the foreign country from the home country, a star is attached to all foreign country variables.

Representative agents in each country choose leisure L_t, and consumption, C_t, to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \ 0 < \beta < 1, \tag{1}$$

subject to their budget constraints listed below. The momentary utility function satisfies the usual properties: concavity and twice continuous differentiability.

With respect to the technology, each country produces a single good. The goods are produced using labor, physical capital and human capital with constant returns to scale technologies:

$$Y_t = F^M(\phi_t K_t, \ N_t H_t), \tag{2}$$

where ϕ_t is the fraction of physical capital (K_t) allocated to the market sector and N_t, the fraction of time devoted to the market sector and H_t, human capital. Here the production function combines a fraction ϕ_t of the stock of physical capital with $N_t H_t$ efficiency units of labor, which are the result of N_t hours of work undertaken by an individual with H_t units of human capital.

Each worker is endowed with one unit of time in each period and consumes L_t of leisure hours. N_t hours are devoted to the market sector, and the remaining $1-N_t-L_t$ hours to accumulation of human capital. Human capital is embodied in each worker and can be produced by combining physical capital, $(1-\phi_t)K_t$, with the human capital-augmented labor effort, $(1-N_t-L_t)H_t$. The evolution equation for human capital is given by

$$H_{t+1} = F^H[(1-\phi_t)K_t, \ (1-N_t-L_t)H_t] + (1-\delta_H)H_t, \tag{3}$$

where $F^H(\cdot)$ is homogeneous of degree one in physical capital and human capital-augmented labor effort, and δ_H is the depreciation rate of human capital.

Output can be used for consumption and investment (I_t). The difference between output and domestic absorption ($C_t + I_t$) is the trade balance (TB_t):[11]

$$Y_t = C_t + I_t + TB_t. \tag{4}$$

Domestic investment increases the stock of capital which evolves according to

$$K_{t+1} = I_t + (1-\delta_K)K_t, \tag{5}$$

[11]Here the trade balance is defined as net exports, which are exports minus imports.

where δ_K is the depreciation rate of physical capital.

Equations (2)−(5) imply that in the steady state, C_t, K_t, I_t and H_t, all grows at the same human capital growth rate, γ_H.[12] In Section Ⅲ−B, all variables in the transformed economy will be detrended by this growth factor.

While the growth of the economy is driven by the accumulation of human capital, fluctuations are driven by technology shocks, and domestic and foreign technology shocks interact in a manner described by the following bivariate autoregressions,

$$\begin{bmatrix} A_{m,t} \\ A^*_{m,t} \end{bmatrix} = \begin{bmatrix} \rho & \upsilon \\ \upsilon^* & \rho^* \end{bmatrix} \begin{bmatrix} A_{m,t-1} \\ A^*_{m,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix}, \tag{6}$$

where $[\varepsilon_t \ \varepsilon_t^*]'$ is distributed normally and independently over time. Here the persistence parameter, ρ, is important for the serial correlation of the technology variable within a country and diffusion parameter, υ, determines how much innovations to productivity which originate in one country are transmitted to the other country. The variance-covariance matrix for the innovations to the productivity shock process is

$$E(\varepsilon_t, \varepsilon_t^*)'(\varepsilon_t, \varepsilon_t^*) = \begin{bmatrix} \sigma_\varepsilon^2 & \psi \\ \psi & \sigma_{\varepsilon^*}^2 \end{bmatrix}. \tag{7}$$

Thus the contemporaneous correlation of the innovations to productivity is determined by the parameter, ψ.

With perfect international mobility of final goods, there is a single world resource constraint for the final good. In equilibrium, world output must equal world consumption and investment. Since the world comprises two countries, the world resource constraint is a size-weighted sum of resource constraint of each country:

$$\pi(Y_t - C_t - I_t - TB_t) + (1-\pi)(Y_t^* - C_t^* - I_t^* - TB_t^*) \geq 0, \tag{8}$$

where π denotes the size of home country, the fraction of the world population that lives in the home country.

When we consider adjustment costs in physical capital,[13] resource

[12]In the steady-state trade balance is zero in Equation (4).

[13]Since capital flow is highly volatile in the international RBC models, it is common to introduce a device to control the flow of physical capital. For example, Backus *et al.* (1992) use the multi-period construction for the technology of capital formation, say the Time-to-Build structure. Baxter and Crucini (1993) introduce convex costs of changing the physical capital stock. And Backus *et al.* (1994) assume the imperfect substitutability between foreign and domestic capitals.

constraint changes to

$$\pi\left(Y_t - \frac{\mu}{2}\frac{\left(K_{t+1} - \left(\frac{H_{t+1}}{H_t}\right)K_t\right)^2}{K_t} - C_t - I_t - TB_t\right) + \tag{9}$$
$$(1-\pi)\left(Y_t^* - \frac{\mu}{2}\frac{\left(K_{t+1}^* - \left(\frac{H_{t+1}^*}{H_t^*}\right)K_t^*\right)^2}{K_t^*} - C_t^* - I_t^* - TB_t^*\right) \geq 0.$$

Here, μ is the magnitude of adjustment costs in physical capital and the physical capital is penalized when its growth deviates from that of human capital. In the steady-state adjustment costs disappear as all variables grow at the same rate.[14]

In addition to the final good market, nations are linked by a financial market. The international financial market is assumed to be complete: individuals can smooth consumption as well as pool country-specific shocks. Most international RBC model follows this complete market assumption, as in Backus *et al.* (1992) and Baxter and Crucini (1993).[15]

B. Equilibrium

In the absence of distortions, pareto optimum and competitive equilibrium will coincide. Thus the equilibrium of the economy can be computed by solving the following Lagrangian problem:

$$\begin{aligned} L = \sum_{t=0}^{\infty} \beta^t [\pi U(C_t, L_t) + (1-\pi)U(C_t^*, L_t^*) \\ + \pi \Lambda_t\{(1-\delta_K)K_t + I_t - K_{t+1}\} + (1-\pi)\Lambda_t^*\{(1-\delta_K)K_t^* + I_t^* - K_{t+1}^*\} \\ + \pi Q_t\{F_t^H + (1-\delta_H)H_t - H_{t+1}\} + (1-\pi)Q_t^*\{F_t^{H^*} + (1-\delta_H)H_t^* - H_{t+1}^*\} \\ P_t\{\pi(F_t^M - C_t - I_t - TB_t) + (1-\pi)(F_t^{M^*} - C_t^* - I_t^* - TB_t^*)\}], \end{aligned} \tag{10}$$

where the momentary utility function $U(\cdot)$ is parameterized as

$$U(C_t, L_t) = \frac{(C_t^{\gamma} L_t^{1-\gamma})^{1-\sigma} - 1}{1-\sigma}.$$

Note that the Lagrangian problem mentioned above deals with no adjustment costs in physical capital. When adjustment costs in

[14] Mendoza (1991) and Gomme (1993) have also used this form of adjustment costs.

[15] Baxter and Crucini (1991) explore implications of the incomplete market. Especially, when we assume that there are no contingent claims on human capital, it is difficult to prove its equilibrium.

physical capital are considered, resource constraint changes to equation (9).

Defining $c_t = C_t / H_t$, $k_t = K_t / H_t$, $i_t = I_t / H_t$, $tb_t = TB_t / H_t$ and $\lambda_t = \Lambda_t / H_t^{\gamma(1-\sigma)-1}$, $q_t = Q_t / H_t^{\gamma(1-\sigma)-1}$, $p_t = P_t / H_t^{\gamma(1-\sigma)-1}$, the following first-order conditions implicity define c_t, l_t, n_t, i_t, tb_t, ϕ_t, k_{t+1}, H_{t+1}/H_t, q_t and p_t after λ_t is eliminated. Now, the conditions for the foreign country are omitted for compactness.

$$U_1(c_t, L_t) = p_t = U_1(c_t^*, L_t^*) \tag{11}$$

$$U_2(c_t, L_t) = U_1(c_t, L_t) F^M_{2,t}(\phi_t k_t, N_t) \tag{12}$$

$$\frac{q_t}{p_t} = \frac{F^M_{2,t}(\phi_t k_t, N_t)}{F^H_{2,t}\{(1-\phi_t)k_t, (1-N_t-L_t)\}} \tag{13}$$

$$p_t = \beta \left(\frac{H_{t+1}}{H_t}\right)^{\gamma(1-\sigma)-1} E_t[p_{t+1}\{F^M_{1t+1}(\phi_{t+1}k_{t+1}, N_{t+1}) + (1-\delta_K)\}] \tag{14}$$

$$q_t = \beta \left(\frac{H_{t+1}}{H_t}\right)^{\gamma(1-\sigma)-1} E_t[q_{t+1}\{(1-L_{t+1})F^H_{2t+1}\{(1-\phi_{t+1})k_{t+1}, (1-N_{t+1}-L_{t+1})\} + (1-\delta_H)\}] \tag{15}$$

$$\frac{F^M_{2,t}(\phi_t k_t, N_t)}{F^M_{1,t}(\phi_t k_t, N_t)} = \frac{F^H_{2,t}\{(1-\phi_t)k_t, (1-N_t-L_t)\}}{F^H_{1,t}\{(1-\phi_t)k_t, (1-N_t-L_t)\}} \tag{16}$$

$$\left(\frac{H_{t+1}}{H_t}\right) k_{t+1} = (1-\delta_K)k_t + i_t \tag{17}$$

$$\frac{H_{t+1}}{H_t} = F^H_t\{(1-\phi_t)k_t, (1-N_t-L_t)\} + (1-\delta_H) \tag{18}$$

$$\pi F^M_t(\phi_t k_t, N_t) + (1-\pi)F^{M^*}_t(\phi_t^* k_t^*, N_t^*) = \pi(c_t + i_t + tb_t) + (1-\pi)(c_t^* + i_t^* + tb_t^*), \tag{19}$$

where subscript denotes the derivative with respect to the corresponding argument.

Equation (11) states that the price of final goods is equated to its marginal utility across countries. This condition implies that non-separability in utility could lead to non-perfectly correlated cross-country consumption correlation.

Equation (12) shows that marginal utility of consumption is equated to the marginal disutility of working as the standard neoclassical model.

The shadow price of human capital in terms of consumption is determined by equation (13), where shadow price is equated to the ratio of marginal products of human capital between two sectors.

The accumulation of physical capital is governed by equation

(14). The discounted utility of capital invested this period rather than consumed is evaluated and compared to the utility of final good consumed this period. Notice that the future utility is discounted by the discount factor as well as by human capital accumulation, which is endogenous in the model. Thus, when the return of physical capital is equated across countries, the accumulation of human capital matters here.

Equation (15) governs the accumulation of human capital. An additional unit of human capital is contributed to both sectors through the $(1-L)$ fraction of time allocated to working in a period. This is shown by the R.H.S. of Equation (15) in the discounted form, and is equated to its shadow price, q_t.

Equation (16) says that marginal rate of technical substitution is equal across two sectors. The laws of motion for physical and human capital are represented by equation (17) and equation (18), respectively. Equation (19) represents the world resource constraint for the final goods.

A stationary competitive equilibrium for this economy consists of a set of policy functions, $c=C(s)$, $c^*=C^*(s)$, $L=L(s)$, $L^*=L^*(s)$, $N=N(s)$, $N^*=N^*(s)$, $i=I(s)$, $i^*=I^*(s)$, $tb=TB(s)$, $tb^*=TB^*(s)$, $\phi=\Phi(s)$, $\phi^*=\Phi^*(s)$, $k'=K(s)$, $k^{*\prime}=K^*(s)$, $H'/H=H(s)$, and $H^{*\prime}/H^*=H^*(s)$, where the state of the world, s, represents $\{A_m, A_m^*, k, k^*\}$[16]; and pricing functions, $p=P(s)$, $q=Q(s)$ and $q^*=Q^*(s)$, that satisfy the household's efficiency conditions (11)−(16) and their foreign counterparts; the firm's efficiency conditions, which are implicitly expressed in conditions (12)−(16) and their foreign counterparts; the laws of motion, (17), (18) and their foreign counterparts; and world market clearing condition for goods, (19).

The system of equations above characterizes the competitive equilibrium, but does not have an analytic solution. I will solve the model by log-linearizing the equations around the steady state, thus imposing certainty equivalence. The resulting linear system is solved by the method of undetermined coefficients (see Christiano 1991).

[16]Obviously, the state of the world includes human capital in the original problem before the transformation such that $s=\{A_m, A_m^*, K, K^*, H, H^*\}$.

IV. Calibration

A. Model Parameterization

The production technologies are specified as

$$F_t^M(\phi_t K_t,\ N_t H_t) = A_{m,t}\ (\phi_t K_t)^{\alpha}(N_t H_t)^{1-\alpha},$$

and

$$F_t^H\{(1-\phi_t)K_t,\ (1-N_t-L_t)H_t\} = A_{h,t}\{(1-\phi_t)K_t\}^{\theta}\{(1-N_t-L_t)H_t\}^{1-\theta}.$$

Table 9 summarizes the parameter values used in the benchmark economy. To be compatible with the literature, values of many parameters are adopted from the previous works. As in Baxter and Crucini (1993), the capital share in the market sector, α, is set equal to 0.42. Regarding the capital share in the non-market sector, θ, there is little a priori information. I choose $\theta = 0.05$ as the benchmark case and will conduct a sensitivity analysis.[17] The discount factor, β, is set at 0.9875. The magnitude of adjustment costs (μ) in the physical capital is set at 1 in the benchmark economy. The growth rate of the economy, γ_H, is set at 1.004 of the quarterly rate. I choose $\pi = 1/2$ in order to investigate the symmetric case.

The steady-state labor hours in the market sector, N, is set at 0.20. The depreciation rates, δ_K and δ_H, are restricted to have a common value, 10% of annual rate for the benchmark economy. In the sensitivity analysis, the depreciation rate of human capital down to 0.012 is also used in the experiments.[18] The intertemporal elasticity of substitution, $1/\sigma$, is set at 1/2. The scale parameter for the market production function, A_m, is normalized to unity, which represents a choice of units. As a result, the scale parameter for the non-market production function, A_h, is computed from the set of steady-state version of Equation (11)−(19), which determines the consumption expenditure share parameter, γ.

Regarding technology shock process, a symmetric AR coefficient matrix is chosen, following Baxter and Crucini, such that[19, 20]

[17]When Uzawa-Lucas specification, say $\theta = 0$, is adopted, the quantative properties of the model economy do not much change.

[18]King and Rebelo (1990) note that $\delta_H = 0.012$ from Mincer is the lowest value they found.

[19]Backus *et al.* (1992) estimate the technology shock process from the industrialized countries. But, their shock process needs some modifications

TABLE 9

BENCHMARK PARAMETER VALUES

	Parameter	Value
α	(Capital share in the market sector)	0.42
θ	(Capital share in the non-market sector)	0.05
N	(Labor hours in the market sector)	0.20
δ_K	(Depreciation rate of physical capital, quarterly)	0.10/4
δ_H	(Depreciation rate of human capital, quarterly)	0.10/4
γ_H	(Growth rate of human capital, quarterly)	1.004
π	(Size of the home country)	1/2
β	(Discount factor)	0.9875
$1/\sigma$	(Intertemporal elasticity of substitution)	1/2
μ	(Magnitude parameter of adjustment costs)	1
A_m	(Scale parameter for the market production function)	1
ρ	(Persistence coefficient of AR matrix)	0.93
υ	(Diffusion coefficient of AR matrix)	0.05
σ_ε^2	(Variance of innovations to technology shock)	0.0087^2
ψ	(Covariance of innovations to technology shock)	0.4×0.0087^2

$$\begin{bmatrix} 0.93 & 0.05 \\ 0.05 & 0.93 \end{bmatrix}.$$

The variance-covariance matrix of innovations is chosen as

$$\begin{bmatrix} 0.0087^2 & 0.4\times 0.0087^2 \\ 0.4\times 0.087^2 & 0.0087^2 \end{bmatrix},$$

where the standard deviation, 0.0087, is chosen such that the output volatility of the model economy mimics that of the data. This method has also been used in Cooley and Hansen (1989).

B. Simulation Results

From now, the quantitative properties of the theoretical economy are discussed. First, we look at the dynamic response of economic variables in order to understand how the model economy behaves in a response to a productivity shock. Figure 1−3 plot the impulse responses to an one standard-deviation innovation in the home country's technology shock. Figure 1 shows the benchmark economy, in which the magnitude of adjustment costs in physical capital, μ,

because solow residuals are computed without physical capital series.

[20]When I adopt the Backus *et al.* (1992)'s AR coefficient matrix, quantative properties of theoretical economies changes little.

is set at unity, Figure 2, in which μ is set at zero, that is, there is no adjustment costs in physical capital, Figure 3, the standard neoclassical model.

Throughout Figure 1 – 3, we see an increase in domestic output on impact in response to an one-time positive shock to domestic productivity. This shock also raises consumption, but less than the increase in output. Investment, however, grows more than output. The ability to trade internationally permits a country to have a greater response of investment to changes in expected rates of return. The trade balance moves initially into deficit. As time passes, the investment boom dissipates, and the trade deficit turns to a surplus.

This pattern of dynamic responses is found to be similar for all the models considered. There are, however, some differences. First, immobile human capital accumulation produces persistence of economic variables. Since the production of the final good uses physical capital as well as human capital, output does not response promptly as in the standard neoclassical model. As the human capital accumulates in response to the inflow of physical capital, the output begins to increase with a lag. But the impulse response of the output persists for the same reason that the accumulated human capital affects production for a longer period. This characteristic produces a humpshaped response of output found in Figure 2. Human capital also reduces the volatility of the investment response.

Second, the introduction of adjustment costs in the physical capital significantly reduces the volatility of investment and the trade balance as plotted in Figure 1. The movement of persistent and less volatile investment leads to gradual improvement in the trade balance, that is accompanied by the output slowing down back to its steady-state. This impulse-response pattern gives rise to a negative contemporaneous correlation between net exports and output.

Next, we turn to moments generated from simulations. These moments are computed from 50 simulations, each of 148 periods. This number of time period corresponds to the sample length for Canada, Japan, UK and US: 1957:1 – 1993:4. Table 10 – 11 report results.

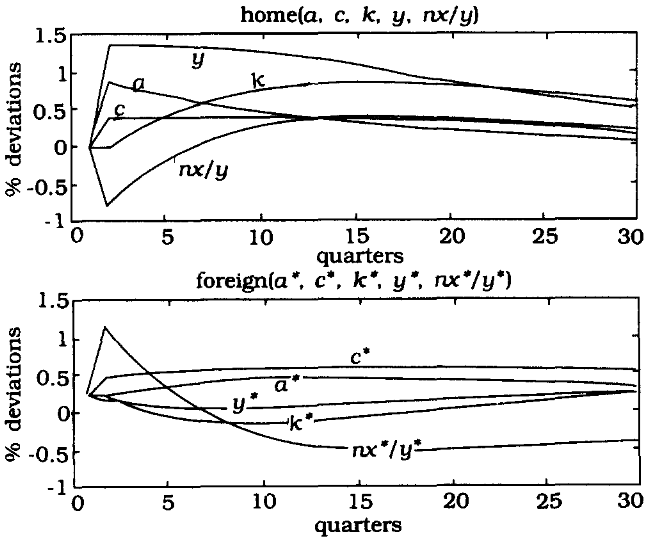

FIGURE 1 − 1

BENCHMARK ECONOMY ($H-K$ WITH ADJUSTMENT COSTS)

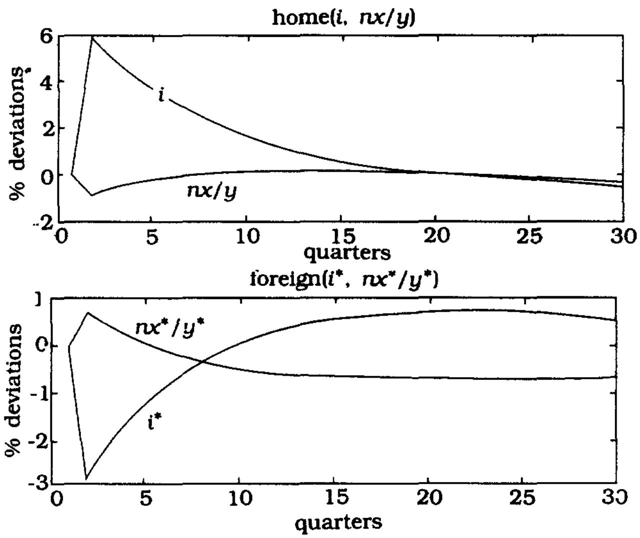

FIGURE 1 − 2

BENCHMARK ECONOMY ($H-K$ WITH ADJUSTMENT COSTS)

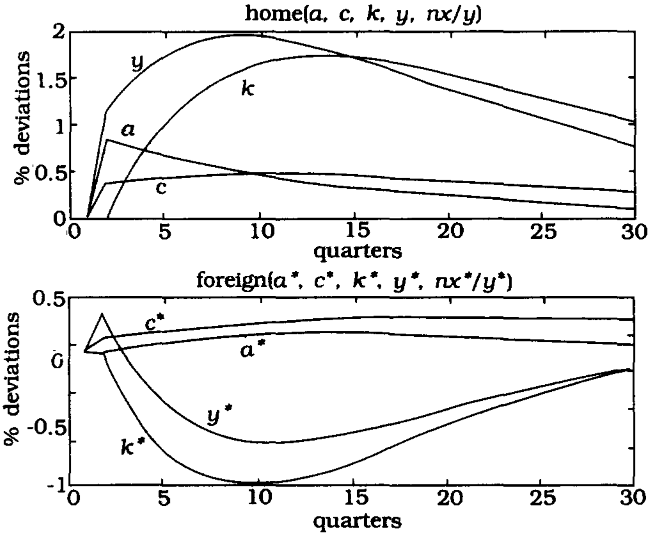

FIGURE 2 – 1
HUMAN CAPITAL MODEL WITHOUT ADJUSTMENT COSTS

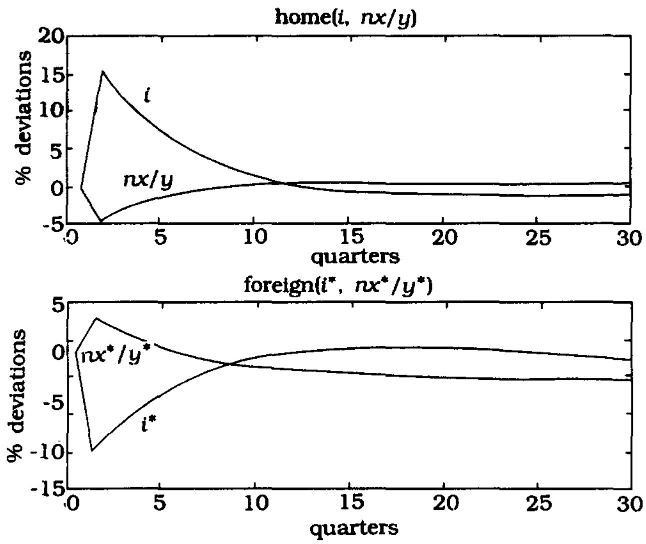

FIGURE 2 – 2
HUMAN CAPITAL MODEL WITHOUT ADJUSTMENT COSTS

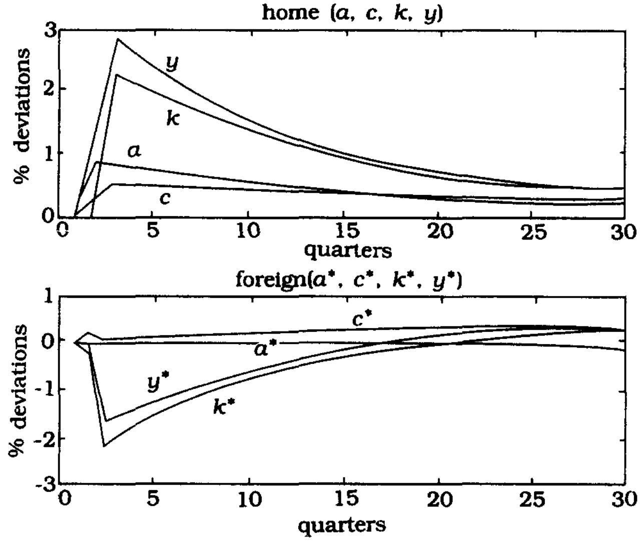

FIGURE 3 − 1
STANDARD NEOCLASSICAL MODEL

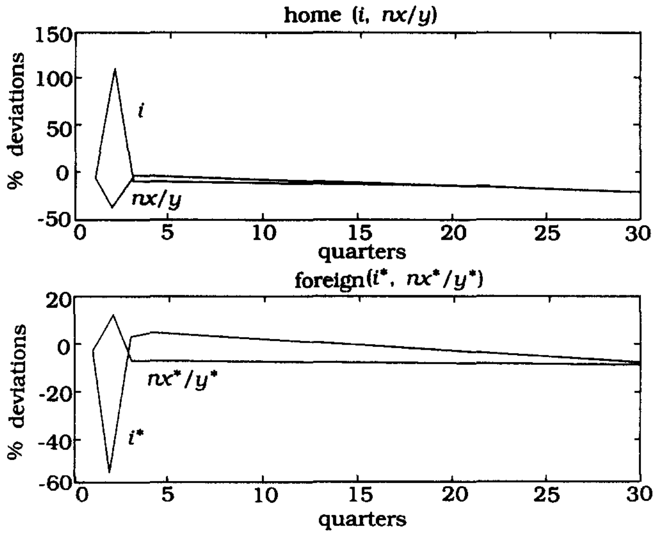

FIGURE 3 − 2
STANDARD NEOCLASSICAL MODEL

TABLE 10

PROPERTIES OF THEORETICAL ECONOMIES

	Benchmark	Human Capital without adjustment costs	Standard Neoclassical
Correlations			
Corr(i, s)	0.87	0.30	-0.11
Corr(nx, y)	-0.34	-0.04	0.17
Corr(c, y)	0.88	0.79	0.78
Corr(i, y)	0.84	0.35	-0.10
Standard Deviations			
Std(nx)	1.14	4.95	41.35
Std(y)	1.77	1.86	3.17
Std(c)	0.65	0.56	0.69
Std(i)	6.63	15.64	83.55
Auto Correlations			
AutoCorr(nx)	0.68	0.66	-0.08
AutoCorr(y)	0.72	0.80	0.78
AutoCorr(c)	0.75	0.77	0.76
AutoCorr(i)	0.65	0.65	-0.08
Correlations			
Corr(y, y^*)	0.29	0.06	-0.68
Corr(c, c^*)	0.92	0.77	0.56
Corr(i, i^*)	-0.54	-0.91	-0.99

Saving and Investment are positively correlated in the model, and stronger than in the data. Their correlation is 0.87 in the benchmark economy from Table 10, compared to 0.66, the average of the data. For the standard neoclassical model, the correlation is found to be negative (-0.11) since savings are perfectly correlated across countries with complete market while investments are quite different across countries facing different technological opportunities. For the human capital model without adjustment costs, the correlation turns positive, but is not strong: the correlation becomes 0.30. In addition to immobile labor hours, human capital accumulation makes it more difficult for the return of physical capital to be equated across countries. Further more, by adding adjustment costs, which reduce the volume of capital flow, domestic investment is more likely to be connected to the domestic factor such as immobile labor and human capital, and is less likely to be related to the technological shocks.

The trade balance is found to be countercyclical as observed in the data. The correlation between net exports-to-output ratio and output is -0.34, which is close to the average of the data, -0.36. The movement of the trade balance is closely connected to that of investment. Since investment is a component of net export by definition and is more volatile than consumption, the highly positive investment-output correlation (0.84) leads to the negative trade balance-output correlation. When investment and output is less correlated (0.35) in the case of the human capital model without adjustment costs, the trade balance turns out to be slightly negatively correlated (-0.04) with output. The trade balance is even positively correlated (0.17) with output when investment is negatively correlated (-0.10) with output in the case of the standard neoclassical model.

The trade balance is more volatile than in the data; the standard deviation of the net exports-to-output ratio is 1.14, compared to 0.87, the average of the data. This high volatility of trade balance is due to the high volatility of investment. The standard deviation of investment is 6.63, that is out of the range observed in the data (3.13-5.34).

The trade balance is as persistent as in the data. The autocorrelation coefficient of net exports-to-output ratio is 0.68, which is close to the average of the data, 0.71. Its persistence is largely due to persistent investment. In the standard neoclassical model, investment moves erratically; the autocorrelation of investment is -0.08. There, investment promptly responses to the innovations to the technological shocks by moving national borders. But investment also dissipates as soon as possible. In the human capital model, however, investment does not response as quickly and strongly as in the standard neoclassical model, but its effect persists as the human capital gradually responds.

Regarding the international comovements, output and consumption are positively correlated across countries, respectively. But, as found in the international RBC literature, cross-country consumption correlation is stronger than cross-country output correlation. In the data, the reverse is generally observed. Furthermore, the cross-

TABLE 11
SENSITIVITY ANALYSIS

	Benchmark	$\theta = 0.42$	$\mu = 0.5$	$\delta_H = 0.012/4$
Correlations				
Corr(*i*, *s*)	0.87	0.87	0.71	0.87
Corr(*nx*, *y*)	-0.34	-0.23	-0.25	-0.28
Corr(*c*, *y*)	0.88	0.86	0.88	0.86
Corr(*i*, *y*)	0.84	0.86	0.68	0.83
Standard Deviations				
Std(*nx*)	1.14	1.64	1.92	0.94
Std(*y*)	1.77	2.75	1.64	1.55
Std(*c*)	0.65	0.78	0.56	0.56
Std(*i*)	6.63	6.83	8.12	6.02
Auto Correlations				
AutoCorr(*nx*)	0.68	0.66	0.67	0.71
AutoCorr(*y*)	0.72	0.69	0.75	0.73
AutoCorr(*c*)	0.75	0.77	0.76	0.75
AutoCorr(*i*)	0.65	0.62	0.66	0.66
Correlations				
Corr(y, y^*)	0.29	0.34	0.27	0.21
Corr(c, c^*)	0.92	0.90	0.89	0.92
Corr(i, i^*)	-0.54	-0.32	-0.73	-0.56

country investment correlation is negative, contrary to the positive correlations found in the data. In order to solve the consumption/output puzzle, some studies succeed in producing less correlated cross-country consumption. For example, Devereux *et al.* (1992) study a one-good two-country model in which independent, country-specific productivity shocks reduce the interdependence of the world economy, which, in turn, reduces the cross-country consumption correlation. Stockman and Tesar (1995) explore whether nontraded goods and taste shocks can reduce cross-country consumption correlation by enhancing non-separabilities in utility. In this respect, the model economy does not much enhance non-separability in utility because in response to technological shocks some part of hours are devoted to leisure (*L*) and the other, to human capital formation $(1-N-L)$. Thus, the response of leisure is weak and non-separability does not break the close link between foreign and domestic consumption. The consumption/output puzzle in the international RBC literature has not been fully resolved and is still a clear challenge to future research.

Table 11 reports a sensitivity analysis. Experiments for various parameter values basically do not change the characteristic of the model. Third column pertains to the moments generated when the capital share of non-market production sector is equal to that of market production sector, say $\theta=0.42$. The difference from the benchmark economy is that the trade balance is less countercyclical (the correlation with output is -0.23), and variables are more volatile (e. g., the standard deviation of the trade balance is 1.64). Fourth column shows the moments generated when the magnitude of adjustment costs in physical capital is reduced, say $\mu=1/2$. Compared to the benchmark economy, investment and saving are less correlated (the correlation is 0.71), and the trade balance is less countercyclical (the correlation with output is -0.25), and variables are more volatile as the capital flow increases. When the annual depreciation rate of human capital is reduced to down to 0.012, moments are reported in the fifth column. Cyclical properties do not change, except the less volatility of variables (e. g., the standard deviation of the trade balance is 0.94).

V. Conclusions

The purpose of this paper is to investigate the dynamics of saving, investment and the trade balance. The trade balance is obviously connected to the movement of saving and investment since the trade balance is the difference between saving and investment. Empirical regularities show that investment and saving are highly positively correlated, together with countercyclical trade balance.

In order to investigate the dynamics of these two variables, I develop a two-country version of human capital growth model. Human capital and adjustment costs incorporated into the model economy turn out to play a key role in reducing the volume of capital flow. Accordingly, domestic investment is more likely to be connected to the domestic factors and to be correlated with national savings.

The trade balance also turns out to be countercyclical as observed in the data. In response to technological shocks, an increase of output is, on impact, associated with an increase of investment and a decrease of the trade balance. In subsequent periods, sustained improvement in the trade balance is accompanied by the persistent movement of investment and output slowing down to their steady states. As a result, the trade balance turns out to be countercyclical when investment sustains its procyclical movements.

The model economy also produces other cyclical properties, which is comparable to those observed in the post-war data for G7 countries: The trade balance is volatile and persistent.

Data Appendix

The business cycle statistics documented in tables are computed from the International Monetary Fund's International Financial Statistics (IFS).

The data from IFS are as follows. The real output series are GDP (99b.r) except for Japan, real output of which is measured in GNP (99a.r). The remaining series are computed by deflating with the output deflator, computed as the ratio of nominal to real output. The nominal output series are nominal GDP (99b.c) or GNP (99a.c). The remaining series are private consumption (96f.c, divided by the output deflator), investment (gross fixed capital formation, 93e.c, divided by the output deflator), savings (nominal output-private consumption-government consumption, 99b.c or 99a.c-96f.c-91f.c, divided by the output deflator) and net exports-to-output ratio (ratio of exports of goods and services less imports of goods and services to output, (90c.c-98c.c)/(99b.c or 99a.c)).

All data are seasonally adjusted and quarterly data except population. The population data are log-linearly interpolated from the annual frequency and are used to convert all variables to per capita quantities. All series are detrended by the Hodrick-Prescott filter after logging, except net exports-to-output ratio that is measured in level form.

The sample periods for US, UK, Japan and Canada are 1957:1-1993:4; France, 1970:1-1993:4; Germany, 1978:3-1993:4; Italy, 1970:1-1993:3.

Cross-country statistics are computed for observations available for both series.

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