Transitional Dynamics of Structural Changes

Sungik Park*

A simple three-factor, three-goods endogenous growth model with a Stone-Geary type utility function is utilized in order to analyze transitional dynamics of structural change. It is shown that sectoral contributions of respective industries to economic growth are variable and different, and that structural change in production and factor use favors the manufacturing industry as opposed to the agricultural industry during the transitional period. Theoretical results are calibrated, and compared with experiences of advanced countries in earlier periods. It is found that the simulation results are generally in accord with real economic data. In particular, the real interest rate and the ratio of investment to output are shown to be within reasonable ranges, which resolves the puzzles in neoclassical transitional dynamics raised by King and Rebelo (1993). (*JEL* Classifications: O11, O41)

I. Introduction

Modern endogenous growth literature has mainly dealt with how steady state or balanced growth rates are determined by economic fundamentals such as the potential of learning activity, investment in R&D, or education. Different growth performances are then attributed to differences in economic fundamentals across countries. The analyses in these papers usually assume all variables in the

* Department of International Trade, Kyungsung University, 110-1, Daeyeon-Dong, Nam-Gu, Pusan, 608-736, Korea. (Tel) +82-51-620-4421, (Fax) +82-51-625-4536, (E-mail) sipark@star.kyungsung.ac.kr. I am grateful to Ronald Jones and Sergio Rebelo for their comments and valuable suggestions. Comments from Hyun Park and two anonymous referees are also appreciated. All remaining errors are, of course, mine.
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economy grow at their long-run growth rates.[1] However, it is strongly believed that there is a transitional period, in which economic variables do not behave as predicted by balanced growth analyses.

There have been several studies about transitional dynamics, such as King and Rebelo (1993), Mulligan and Sala-i-Martin (1992), Barro and Sala-i-Martin (1995), and Rebelo (1992). Transitional dynamics may take place for several different reasons, as they pointed out. First, in neoclassical models such as Solow, they arise because the level of capital stock is different from that of the capital stock in a steady state. Using a neoclassical framework, King and Rebelo showed that the transitional dynamics of a neoclassical model were not appropriate in explaining the U.S. growth over the long-term. While some variation in parameter values could replicate some features of actual data, they invariably generated inconceivable implications for others. Therefore, their logical suggestion was to study transitional dynamics in the context of endogenous growth models.

In an economy endowed with several different kinds of capital stocks, transitional dynamics may arise due to an imbalance among these capital stocks. An initial imbalance results in higher marginal productivity of capital in short supply, and thus accumulation of capital takes place. Therefore, balance is restored in a steady-state equilibrium. Mulligan and Sala-i-Martin, and Barro and Sala-i-Martin worked on transitional dynamics in an endogenous growth model setting with two kinds of capital (i.e., human and physical capital).

Another mechanism for transitional dynamics is a time-varying intertemporal substitution. Suppose there is a subsistence level of consumption. If income in this economy is too low, the elasticity of intertemporal substitution is zero since all income is spent to maintain the subsistence level. However, as income grows, it converges to some constant ($1/\sigma$ in the case of CRRA utility) and so does the growth rate. In this case, transitional dynamics are built into the structure of preferences, unlike earlier cases. Rebelo

[1]For example, the AK model of Rebelo does not feature transitional dynamics. In general, the role of transitional dynamics in explaining economic growth is less emphasized in endogenous growth models than in neoclassical growth models.

studied transitional dynamics in order to explain different growth rates across countries in the presence of perfect capital mobility by incorporating Stone-Geary utility.[2] His focus was, nevertheless, on explaining different growth rates rather than transitional dynamics and its implications.

In these studies, it is notable that implications about change in the sectoral composition of production and factor use in the process of industrialization are missing. Mainly among development economists, economic growth is seen as closely interrelated to structural transformation from an agriculture-based to a manufacturing-based economy.[3] Modern economic growth, as Kuznets identified, is characterized by the change in the relative importance of sectors in terms of production and factor shares. There is solid evidence of universality of structural change in this respect. This paper begins with the perspective that a change in the relative importance of sectors may be viewed as one pattern of transitional dynamics.

The failure to explain the structural change partly comes from inappropriate industry classification. It is customary to have only one consumption goods, although other sectors (typically human capital, physical capital or R&D sector) are explicitly considered in the model. Therefore, these studies have not been able to explain structural change in the composition of consumption and corresponding structural change in production.

Another reason lies in that the standard model economy neglects a presence of subsistence level. In this regard, it has been noted that Stone-Geary utility is useful and convenient, since preference is non-homothetic and the income elasticity of demand for goods

[2]This utility function was also used by Matsuyama (1992) in order to explain how agricultural productivity was related to economic growth under closed and open economies respectively. Atkeson and Ogaki (1992) found economically significant differences in the behavior of poor and rich households in terms of the intertemporal elasticity of substitution by incorporating this utility function.

[3]The study of structural change in the process of economic growth has been a major theme in development economics (refer to works of Clark, Kuznets and Chenery). Disaggregation of industries are necessary in this field. However, their studies are mainly descriptive and lack formal models in terms of optimization behavior. Rather, they seem to support a disequilibrium approach. In these respects, their approaches are different from mine.

with positive subsistence level is less than unitary. Therefore, Stone-Geary utility has the ability to yield different growth rates during the transitional period.

The objective of this paper is to explain the transitional behaviors of economic variables, especially in terms of structural change during industrialization. In order to overcome the failure mentioned earlier, I incorporate Stone-Geary utility asymmetrically in a multi-sector economy model in the context of an endogenous growth model. Also, this model is used to fit the experiences of advanced countries in earlier periods when they were industrializing. Interesting findings are that the real interest rate and the ratio of investment to output are within reasonable ranges. These resolve the puzzles in neoclassical transitional dynamics raised by King and Rebelo (1993).

There has not been much work done concerning structural change in the process of industrialization in the formal context of endogenous growth models.[4] This paper attempts to fill this gap, utilizing a simple multi-sector endogenous growth model.

The structure of this paper is as follows. The next section briefly discusses the empirical regularity of structural change and economic growth. Then, in Section III, a simple three-factor, three-goods endogenous growth model with Stone-Geary utility is constructed. Transitional dynamics and the balanced growth paths of the closed economy are analyzed, and it is shown that structural change in production and factor use favors manufactured as opposed to agricultural goods. In Section IV, theoretical results are calibrated. In Section V, transitional paths for several economic variables are analyzed and compared with economic data from advanced countries in earlier periods. It is found that the simulation results are generally consistent with actual data. The final section contains concluding remarks.

II. Brief Discussion on Empirical Regularity of Structural Change

Structural change may be defined in various ways. The concept

[4]There is one paper by Laitner (1994). However, his model is a variation of the neoclassical growth model with exogenous technology progress.

of structural change in this paper refers to the change in relative importance of sectors in terms of production and factor use.[5] More specifically, the structural change this paper focuses on is the structural transformation from an agriculture-based to a manufacturing-based economy. Industrialization or economic growth is then the central process of structural change. That is, structural change and economic growth are strongly interrelated.

There is solid evidence of universality of structural change in this respect. For example, Maddison (1980) collected data on the sectoral composition of employment in sixteen advanced countries in 1870 and recent years. Table 1 shows the shares of agriculture drastically fell to 8.3 percent in 1976 from 48.8 percent in 1870, while those of industry and services increased to 36.1 and 55.6 percent from 27.5 and 23.7 percent respectively. Table 3.4 in Maddison (1980) shows that the shares of agriculture fell to 5.6 percent in 1976 from 16.1 percent in 1950. However, those of industry and services increased to 41.5 and 52.9 percent in 1976 from 40.4 and 43.4 percent in 1950. These data confirm structural change favoring the manufacturing sector over the agricultural sector. This kind of structural change is observed by Kuznets (1960-1962, various issues), Syrquin (1988) and various other authors.

Structural change in expenditure shares also takes place. Engel's law, which refers to the decline in the share of food and the rise in the share of manufacturing in total consumption during industrialization, is cited as one typical phenomenon. Maddison also documented data on expenditure patterns in various times. According to his table 3.3, in England in 1688, food and drink absorbed about 41.9 percent of net expenditure, whereas these "necessities" accounted for only 12.5 percent of U.S. expenditure in 1976. Net expenditure in the U.S. in 1976 was about twenty times as high as that in England in 1688. Therefore, it is clear that economic growth brings major changes in expenditure patterns favoring the manufacturing industry as opposed to the agriculture industry.

[5]This definition is most commonly used one for structural changes in development and in economic history.

TABLE 1
STRUCTURE OF EMPLOYMENT
(Unit: Percentage share in total labor employment)

Year	1870	1950	1960	1970	1976
Agriculture	48.8	24.7	17.5	10.9	8.3
Industry	27.5	36.6	38.7	39.0	36.1
Services	23.7	38.7	43.8	50.1	55.6

Notes:
1) Figures are averages of the relevant values in 16 advanced countries.
2) This table is abstracted and reproduced from Table 3.2 in Maddison (1980).

III. The Model

A. Structure of the Closed Economy

The economy has three factors: land (N), unskilled labor (L) and the capital stock (K) which includes physical and human capital. Endowments of natural resources and labor are assumed to be fixed, while capital may be accumulated. There are three goods, agricultural goods (x_1), manufacturing goods (x_2) and new capital goods ($\dot{k}$),[6] in per-capita-terms notation. These are produced by different combinations of the above-mentioned factors as follows[7]:

$$x_{1t} = a_{1t}^{\alpha_1} n^{\alpha_2} (b_{1t} k_t)^{1-\alpha_1-\alpha_2}, \tag{1}$$

$$x_{2t} = a_{2t}^{\beta} (b_{2t} k_t)^{1-\beta}, \tag{2}$$

$$\dot{k}_t = \delta b_{3t} k_t, \tag{3}$$

where dot($\cdot$) denotes time derivative of the corresponding variable. The a_t's and b_t's are the shares of labor and capital across economic sectors,[8] that is, $0 \le a_{1t}, a_{2t}, b_{1t}, b_{2t}, b_{3t} \le 1$, $a_{1t} + a_{2t} = 1$

[6]Since depreciation is not considered in this model, new capital may be interpreted as the net capital increase rather than gross capital production.

[7]This model is a variation of Rebelo (1991). While he focused on the effects of taxation on the rate of growth, I concentrate on the structural changes during the transitional period.

[8]The service sector is not included in this paper. The share of the service sector has been one of the major concerns in development economics. However, the role of the service sector in consumption, production and

and $b_{1t}+b_{2t}+b_{3t}=1$, and the initial period capital stock, K_0, is given. Small case letters denote per capita quantities. Land is sector-specific but labor is mobile across two sectors. Capital is not directly consumable but enters as an argument into the production function. The capital-producing sector here may be broadly interpreted. It includes R&D investment and human capital accumulation, as well as physical capital accumulation. Note that the production function of this sector is assumed to be linear in capital in order to have a constant growth rate in the balanced growth path. Therefore, the capital-producing sector is the ultimate engine of growth. That is, the long-run growth rate of the economy is determined by the size of the capital-producing sector.[9]

A representative consumer maximizes lifetime utility

$$U=\int_0^\infty e^{-\rho t}u(c_{1t},\ c_{2t})dt. \tag{4}$$

Momentary utility is of the Stone-Geary type, that is, $u(c_{1t},\ c_{2t})=\phi\log(c_{1t}-\mu)+(1-\phi)\log c_{2t}$, $\mu\,(>0)$ is the subsistence level parameter for agricultural goods. Note that both agricultural output and manufacturing output are consumable and the subsistence level is assumed present only in the agricultural sector.

B. Equilibrium Path

The current value Hamiltonian, utilizing equilibrium conditions, $c_{1t}=x_{1t}$, and $c_{2t}=x_{2t}$, is

$$\begin{aligned}H_t=&\phi\log(a_{1t}^{\alpha_1}n^{\alpha_2}(b_{1t}k_t)^{1-\alpha_1-\alpha_2}-\mu)+(1-\phi)\log(a_{2t}^{\beta}(b_{2t}k_t)^{1-\beta})+\\&\lambda_t\delta b_{3t}k_t+\omega_{1t}(1-a_{1t}-a_{2t})+\omega_{2t}(1-b_{1t}-b_{2t}-b_{3t}),\end{aligned} \tag{5}$$

where λ is the shadow price of capital in terms of utility, and ω_1 and ω_2 are the Lagrangean multipliers associated with resource allocation. For notational convenience, I abbreviate the time dummy

growth is not clarified yet. Hence, for analytical simplicity, the service sector is abstracted.

[9]In order to make endogenous growth feasible, the interest rate should not decline to zero in the model. That is, production function must be linear in producible factor (such as capital) as Rebelo's Ak model shows. In this model, the capital-producing sector has linear production function, which implies this sector is the only engine of the growth. This makes endogenous growth possible.

t in the following analysis. Therefore, first order conditions[10] for competitive equilibrium turn out to be

$$a_2 \phi \alpha_1 x_1 = (1-\phi)\alpha_1 \beta (x_1 - \mu), \tag{6}$$

$$(1-\alpha_1-\alpha_2)\phi x_1 = b_1 \lambda \delta k(x_1-\mu), \tag{7}$$

$$(1-\phi)(1-\beta) = b_2 \lambda \delta k, \tag{8}$$

$$\dot{\lambda} = \rho\lambda - \frac{x_1}{x_1-\mu}(1-\alpha_1-\alpha_2)\frac{\phi}{k} - \frac{(1-\phi)(1-\beta)}{k} - \lambda\delta(1-b_1-b_2), \tag{9}$$

and the transversality condition is

$$\lim_{t\to\infty} \lambda k e^{-\rho t} = 0. \tag{10}$$

Additional conditions from the Kuhn-Tucker theorem are

$$\omega_1 \geq 0, \qquad \omega_2 \geq 0,$$

$$\omega_1(1-a_1-a_2)=0, \qquad \omega_2(1-b_1-b_2-b_3)=0.$$

These result in

$$a = \frac{\phi\alpha_1}{\phi\alpha_1+(1-\phi)\beta\dfrac{x_1-\mu}{x_1}}, \tag{11}$$

$$-\frac{\dot{\lambda}}{\lambda} = \delta - \rho. \tag{12}$$

1) Dynamics of Output

During the transitional period, the a's and b's are not necessarily constant.[11] From (7) and (8) it is obvious $\dot{b}_1/b_1+\dot{\lambda}/\lambda+\dot{k}/k = -\{\mu/(x_1-\mu)\}\dot{x}_1/x_1$ and $\dot{b}_2/b_2+\dot{\lambda}/\lambda+\dot{k}/k=0$. Since $\dot{k}/k=\delta(1-b_1-b_2)$, we have

$$\frac{\dot{b}_1}{b_1} = \delta(b_1+b_2) - \rho - \frac{\mu}{x_1-\mu}\frac{\dot{x}_1}{x_1}, \tag{13}$$

$$\frac{\dot{b}_2}{b_2} = \delta(b_1+b_2) - \rho. \tag{14}$$

[10]Second order sufficiency conditions are easily verified. Since the objective function is concave, and the differential equation governing capital accumulation is linear in control and state variables, Mangasarian sufficiency conditions are satisfied.

[11]Dynamic paths of all the relevant variables which are discussed in this section will be calibrated in the next section and drawn in Figure 2.

The output growth rates of x_1 and x_2 are as follows from the above equations:

$$\frac{\dot{x}_1}{x_1}=(1-\alpha_1-\alpha_2)(\delta-\rho)\left\{\frac{x_1-\mu}{x_1-\mu(a_1\alpha_1+\alpha_2)}\right\}, \tag{15}$$

$$\frac{\dot{x}_2}{x_2}=(1-\beta)(\delta-\rho)-\frac{\beta a_1}{1-a_1}\frac{\dot{a}_1}{a_1}. \tag{16}$$

Then, using (11) and (15) we have[12]

$$\frac{\dot{a}_1}{a_1}=(1-\alpha_1-\alpha_2)(\delta-\rho)\left\{\frac{-(1-a_1)\mu}{x_1-\mu(\alpha_1 a_1+\alpha_2)}\right\}. \tag{17}$$

Now assume x_1 is greater than μ.[13] This implies that the growth rate of x_1 is positive from (15) and that of a_1 is negative from (17). Since x_1 is increasing, the growth rate of a_1 eventually becomes zero from negative values, and thus converges to $a_1^*=\phi\alpha_1/(\phi\alpha_1+(1-\alpha)\beta)$.[14] It is also obvious that the growth rate of x_2 is positive from (16). These imply that the growth rates of x_1 and x_2 are converging to $(1-\alpha_1-\alpha_2)(\delta-\rho)$ from below and $(1-\beta)(\delta-\rho)$ from above, respectively.

It remains to be seen how b_1 and b_2 behave during the transitional period. For analytical convenience, a phase diagram is drawn as in Figure 1. On the balanced growth path it is required that b_1 and b_2 be constant. Since $\dot{x}_1/x_1$ is always positive, the graph of $\dot{b}_1/b_1=0$ is located outside that of $\dot{b}_2/b_2=0$. Over time, the graph of $\dot{b}_1/b_1=0$ approaches that of $\dot{b}_2/b_2=0$, since $\mu/(x_1-\mu)$ is getting closer to zero. Arrows indicate how b_1 and b_2 move in each region, divided by these two graphs. Outside the graph of $\dot{b}_1/b_1=0$, b_1 and b_2 both increase, and inside the graph of $\dot{b}_2/b_2=0$, b_1 and b_2 both decrease. In the region between the two graphs, b_1 decreases and b_2 increases.

[12]Bang-bang solutions may show up in equation (17) under the assumption of a Stone-Geary type utility function. However, this asymptotic behavior is not a major concern here. Therefore, I rule out this possibility for analytical convenience.

[13]In fact, the following arguments fail to hold if x_1 is smaller than μ. In this case, utility is not well defined, and thus the maximization problem cannot be solved using this framework. Therefore, I did not consider this possibility.

[14]Asterisk(*) denotes the values of corresponding variables on the balanced growth path.

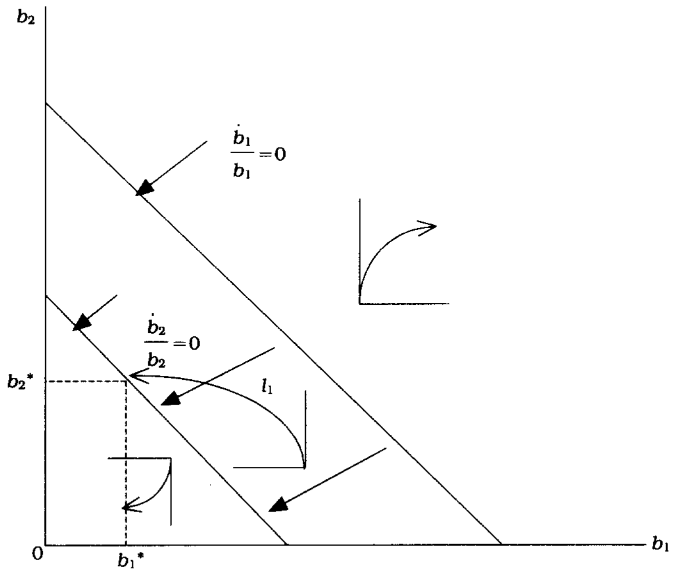


FIGURE 1

TRANSITIONAL DYNAMICS OF b_1 AND b_2

In the long run we must have $\delta(b_1+b_2)=\rho$ and $b_1/b_2=\phi(1-\alpha_1-\alpha_2)/(1-\phi)(1-\beta)$ from (7) and (8), which imply $b_1^*=\rho\phi(1-\alpha_1-\alpha_2)/\{\delta(\phi(1-\alpha_1-\alpha_2)+(1-\phi)(1-\beta))\}$ and $b_2^*=\rho(1-\phi)(1-\beta)/\{\delta(\phi(1-\alpha_1-\alpha_2)+(1-\phi)(1-\beta))\}$ on the balanced growth path. These imply that $\dot{b}_1/b_1$ should be negative and $\dot{b}_2/b_2$ should be positive during the transitional period in order to have a balanced growth path. It is also required that $b_1+b_2>\rho/\delta$, which suggests that $\dot{k}/k$ converges to $\delta-\rho$ from below. Therefore, the only viable transitional path must look like l_1. All other paths are not consistent with the values in the long run or do not lead to a balanced growth path.

Obviously, the growth rate of each good is different on the balanced growth path as well as during the transitional period. We can also see that the share of manufacturing goods is increasing relative to agricultural goods, even after considering relative price changes during the transitional period. Looking at the relative share of px_2/x_1 where p is the relative price of x_2 in terms of x_1, reveals how sectoral proportions change over time. Since $\phi/(x_1-\mu)=(1-\phi)/px_2$ from utility, we have

$$\frac{\dot{p}}{p}+\frac{\dot{x}_2}{x_2}=\frac{x_1}{x_1-\mu}\frac{\dot{x}_1}{x_1}\geq\frac{\dot{x}_1}{x_1}. \tag{18}$$

This implies that the share of x_2 relative to that of x_1 is rapidly increasing in the initial stages of industrialization (since x_1 is close to μ) but approaches a constant, $(1-\phi)/\phi$, eventually (since μ will be negligible compared to x_1). Structural change from a primary-goods-based economy to an industrial economy like this is a typical phenomenon in the field of development economics, as surveyed in Syrquin (1988).

2) Dynamics of Relative Prices

The dynamics of price changes are easily determined along the equilibrium growth path. These are (12) and

$$\frac{\dot{p}}{p}=(\beta-\alpha_1-\alpha_2)(\delta-\rho)+\frac{a_1(\beta-a_1)-\alpha_2}{1-a_1}\frac{\dot{a}_1}{a_1} \tag{19}$$

from (15) - (18). The first term and the second term on the right hand side are negative and positive respectively by assuming $\beta<\alpha_1+\alpha_2$.[15] Therefore, the growth rate of p converges to $(\beta-\alpha_1-\alpha_2)(\delta-\rho)$ from above, while the growth rate of λ is fixed at all times. Note that the relative price moves in favor of agricultural goods as opposed to manufactured goods. On the balanced growth path, the share of agricultural goods relative to that of manufacturing goods is constant, since price changes exactly compensate differential growth rates in real terms between agricultural and manufactured goods. However, during the transitional period, the price change is not enough to compensate for this differential. Therefore, the consumption share of manufactured goods is becoming bigger than that of agricultural goods, as shown above.

The dynamic properties of prices can also be confirmed by utilizing marginal productivity conditions. To see these, consider the marginal products of labor and capital. Free mobility of labor and capital across sectors guarantees

$$\alpha_1 a_1^{\alpha_1-1}n^{\alpha_2}(b_1k)^{1-\alpha_1-\alpha_2}=p\beta a_2^{\beta-1}(b_2k)^{1-\beta}. \tag{20}$$

[15]This parameter restriction reflects the fact that the manufacturing growth rate is greater than the agricultural growth rate along the balanced growth path.

$$(1-\alpha_1-\alpha_2)a_1^{\alpha_1} n^{\alpha_2} (b_1k)^{-\alpha_1-\alpha_2} = p(1-\beta)a_2^{\beta} (b_2k)^{-\beta} = \lambda' \delta, \quad (21)$$

where λ' is the relative price of new capital in terms of x_1, so long as no complete specialization takes place. Therefore, implied dynamics for λ' will be

$$\frac{\dot{\lambda}'}{\lambda'} = -(\alpha_1+\alpha_2)(\delta-\rho) - \frac{a_1\alpha_1+\alpha_2}{1-a_1}\frac{\dot{a}_1}{a_1}, \quad (22)$$

which implies that the growth rate of λ' approaches $-(\alpha_1+\alpha_2)(\delta-\rho)$ from above. Note that λ and λ' are different in that the former is the marginal valuation of new capital in terms of utility and the latter is relative price of new capital in terms of x_1. However, these are clearly connected. If capital is employed in the production of new capital instead of natural resource-intensive goods, more capital is produced, which can be used in the production of natural resource-intensive goods in the following period. This results in higher utility in the following period. Since the increased production of natural resource-intensive goods will be $(1-\alpha_1-\alpha_2)/\dot{k}/k$, the relative price of new capital in terms of x_1 falls more slowly than the marginal valuation of new capital in terms of utility by this amount along the balanced growth path.

The real interest rate for loans denominated in capital goods (r_k) is different from that of loans denominated in agricultural goods (r_{x_1}). Since the marginal productivity of capital in the capital-producing sector is δ, equilibrium requires $r_k=\delta$. Then, an arbitrage argument implies $r_{x_1}=r_k+\dot{\lambda}'/\lambda'$ Therefore, r_{x_1} converges to $\delta-(\alpha_1+\alpha_2)(\delta-\rho)$ from above.

3) Real GDP Growth Rate and Sectoral Contribution

Real GDP in terms of x_1 is defined as $y=x_1+px_2+\lambda' i$, where i is $\dot{k}$. The real GDP growth rate is therefore

$$\frac{\dot{y}}{y} = \theta_1\frac{\dot{x}_1}{x_1} + \theta_2\frac{\dot{x}_2}{x_2} + \theta_2\frac{\dot{i}}{i} \rightarrow \{\theta_1^*(1-\alpha_1-\alpha_2)+\theta_2^*(1-\beta)+\theta_3^*\}(\delta-\rho), \quad (23)$$

where the θ_i's are shares of respective sectors(i), evaluated at base year prices. The aggregate growth rate is a weighted average of the different sectoral growth rates. During the transitional period, the exact path of the growth rate depends on the variable share and the variable growth rate of each sector. Because these also depend on endowments, we cannot predetermine how the growth rate approaches the rate on the balanced growth path. However, on the balanced growth path, since sectoral shares are constant,[16] and the real growth rates of respective sectors are $(1-\alpha_1-\alpha_2)(\delta-\rho)$, $(1-\beta)(\delta-\rho)$ and $(\delta-\rho)$, and their shares are θ_1^*, θ_2^* and θ_3^*, the growth rate of *GDP* in this economy approaches $\{\theta_1^*(1-\alpha_1-\alpha_2)+\theta_2^*(1-\beta)+\theta_3^*\}(\delta-\rho)$.

One important thing to note is the fact that sectoral contributions to economic growth differ. The marginal contribution of the agricultural sector is the smallest, reflecting the small value of $(1-\alpha_1-\alpha_2)$, while the total contribution may be biggest at the initial stages of industrialization because of its biggest share. The marginal contribution of the manufacturing sector is smaller than that of the capital-producing sector on the balanced growth path. However, this is not necessarily so during the transitional period. As shown earlier, the growth rate of manufacturing goods converges to $(1-\beta)(\delta-\rho)$ from above and that of capital goods converges to $(\delta-\rho)$ from below. Therefore, when a_1 is close to one (i.e. at the earlier stages of industrialization), the contribution of the manufacturing goods sector to growth is likely to be greater than that of the capital goods sector. This implies that the contribution to growth can be greater in the manufacturing goods sector than in the capital-producing sector during the transitional period. Although the capital-producing sector is assumed the ultimate engine of economic growth, the sectoral growth rate of this sector is not necessarily highest.

IV. Calibration

One strong implication of this model is that sectoral shares of labor and capital on the balanced growth path are independent of (initial) endowments. The transitional path is unique given (initial) endowments and parameter values. Therefore, multiple equilibria like Xie (1994) is ruled out. However, generally, the (initial) endowments affect the speed of convergence toward a balanced growth path.

[16]It was already shown that $\phi/(x_1-\mu)=(1-\phi)/(px_2)$, which implied $px_2/x_1=(1-\phi)/\phi$ on the balanced growth path. Therefore, if it is shown that $\lambda' i/x_1$ is constant, the shares of respective sectors are constant. This is obvious since $\dot{\lambda}'/\lambda'+\dot{i}/i=\dot{x}_1/x_1$ from (3), (15) and (22).

In order to find the dynamic paths of all relevant variables, the system of differential equations, which consists of equations, (13) - (17), needs to be solved. However, the differential equation system is not directly solvable. Therefore, numerical methods are applied. Since we know that there is only one stable path for given parameter values and endowments from Figure 1, we can trace an initial value of a_1, which leads to b_1^* and b_2^*. Because bigger (smaller) a_1 implies smaller (bigger) b_1, from the marginal productivity conditions, (20) and (21), if a_1 is bigger (smaller) than the value on the stable path, b_1 and/or b_2 will converge to zero (diverge to infinity). Therefore, by trial and error (simple shooting), I can determine the initial value of a_1, which ultimately leads to the balanced growth path.[17]

Once the initial value of a_1 is identified, the transitional dynamics of b_1 and b_2 are straight-forward, and those of other variables also can be determined, because all variables are directly connected with these factoral shares across industries. In order to identify the initial value of a_1, parameter values (such as δ, ρ, α_1, α_2, β, ϕ and μ), and endowments (n and k_0) need to be pre-determined. These parameter values determine long-run equilibrium in the economy and the paths of transitional dynamics.

To calibrate these values, I refer to Maddison's (1980) data. According to his Table 3.1, from 1870 to 1970, average per capita income (measured in 1970 U.S. dollars) of advanced countries rose about five-and-half-fold from U\$ 662 to U\$ 3651. Labor employment in agriculture was 0.64 in 1870, adjusting for the share of the service sector. Therefore, when there are no adequate parameter and endowment values, these values are used to fit the above data.

Cooley and Prescott (1994) assumed the time preference rate was 0.947, which implied $\rho \simeq 0.05$. Suppose the U.S. economy is on a balanced growth path. Since the aggregate real growth rate of the U.S. economy is about 3%, δ is set at 0.08 taking into account that $\dot{y}/y=\dot{k}/k=\delta-\rho$ on the balanced growth path. This implies $b_1^*+b_2^*=5/8$.

[17] There are other methods to trace transitional paths. The time elimination method is one of them, recommended by Mulligan and Sala-i-Martin (1992). However, state-like variables can not be found in an economy with multi-consumption goods. Therefore, in this paper, a simple shooting method is utilized.

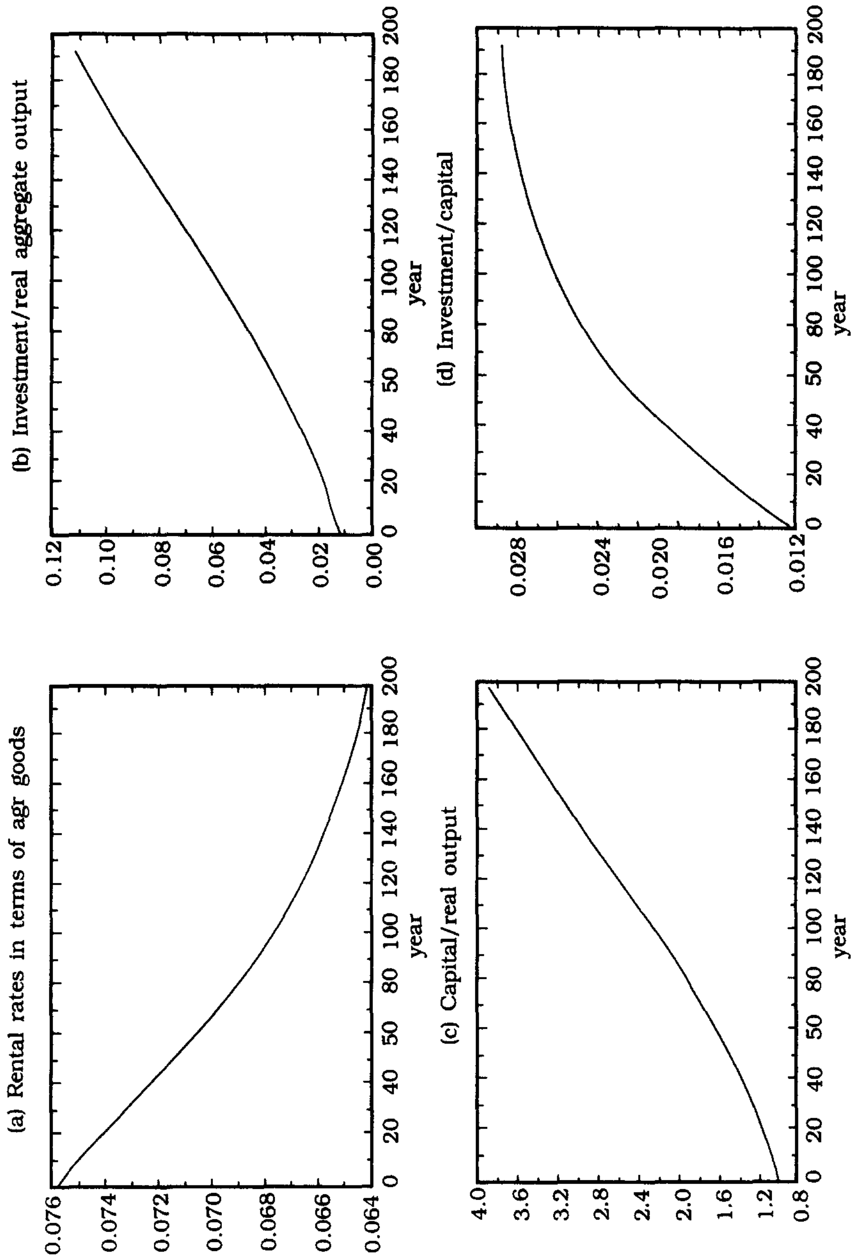
There are not many references about parameter values for factor intensities. Mankiw *et al.* (1992) estimated that physical capital's share was about one third and human capital's share was between one third and one half for the neoclassical production function. Since they did not disaggregate sectors, these values are not directly applicable. Grada (1981, Table 8.2) found factor shares for land, labor and capital of 0.18-0.27, 0.59-0.69 and 0.12-0.14 respectively in British agriculture in the late nineteenth century. However, other than land share, these are not usable here, since his measure of capital was based on a narrow concept of capital. Therefore, I let $\alpha_2=0.2$, $\alpha_1=0.4$ and $\beta=0.4$. These imply U.S. agriculture and manufacturing growth rates per capita are 1.2% and 1.8% respectively. Currently, the employment share of agriculture is about 13% out of total employment in agriculture, mining and manufacturing. Since $a_1^*=\phi\alpha_1/(\phi\alpha_1+(1-\phi)\beta)$, given parameter values, we can deduce $\phi=0.13$.

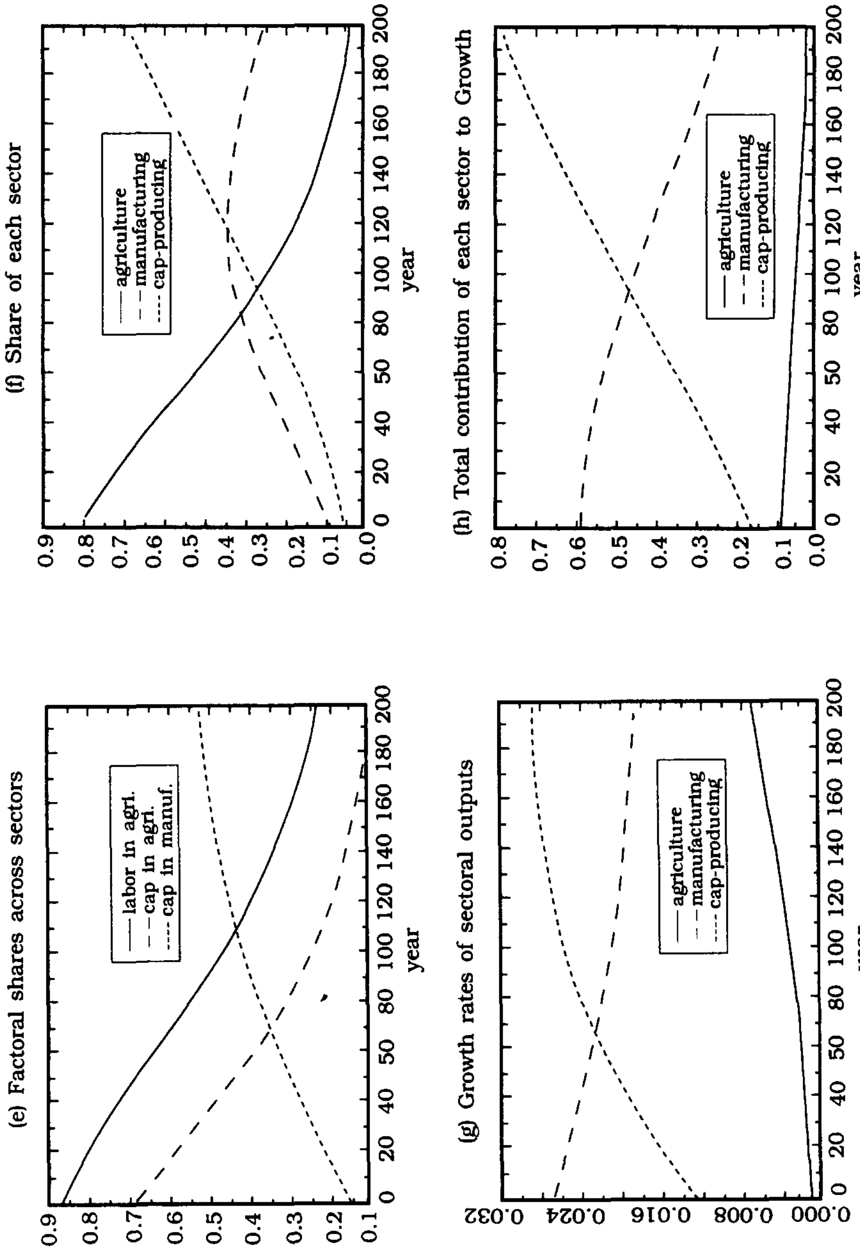
As for values of the initial endowments, I normalize $n=1$ and $k_0=1$. Given these values, the value of the subsistence level, μ, is not very important since it does not affect the path of transitional dynamics itself but determines the initial location on the transitional path. For convenience, I let $\mu=0.8$.[18] Then, it is feasible to nail down the actual time period from simulation results, using the data on the per capita income increase and labor employment share in agriculture as mentioned earlier. It turns out that year 60 in Figure 2 corresponds to 1870.

In summary, calibrated parameter values are as follows:

δ	ρ	α_1	α_2	β	ϕ	μ	n	k_0
0.08	0.05	0.4	0.2	0.4	0.13	0.8	1	1

[18]As μ becomes bigger, there is less room for production of manufacturing and capital goods. Therefore, given initial conditions, this negatively affects the manufacturing and the capital-producing sectors during the transitional period. For example, rental rate increases because of reduced investment in the transitional period. The share of the agricultural sector is higher, while those of the manufacturing and the capital-producing sectors are lower in terms of factoral shares and outputs. Reflecting these changes, growth rate is also lowered. However, these effects are transitory because the steady state values are not altered. I tried other values for μ. And it turned out that the transitional path itself was largely untouched while initial location of the transitional path was correspondingly affected. Therefore, the value of μ does not play a significant role here.


(a) Rental rates in terms of agr goods
0.076
0.074
0.072
0.070
0.068
0.066
0.064
0 20 40 50 80 100 120 140 160 180 200
year
(b) Investment/real aggregate output
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0 20 40 50 80 100 120 140 160 180 200
year
(c) Capital/real output
4.0
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
0 20 40 50 80 100 120 140 160 180 200
year
(d) Investment/capital
0.028
0.024
0.020
0.016
0.012
0 20 40 50 80 100 120 140 160 180 200
year


(f) Share of each sector
agriculture
manufacturing
cap-producing
year
(h) Total contribution of each sector to Growth
agriculture
manufacturing
cap-producing
year
(e) Factoral shares across sectors
labor in agri.
cap in agri.
cap in manuf.
year
(g) Growth rates of sectoral outputs
agriculture
manufacturing
cap-producing
year

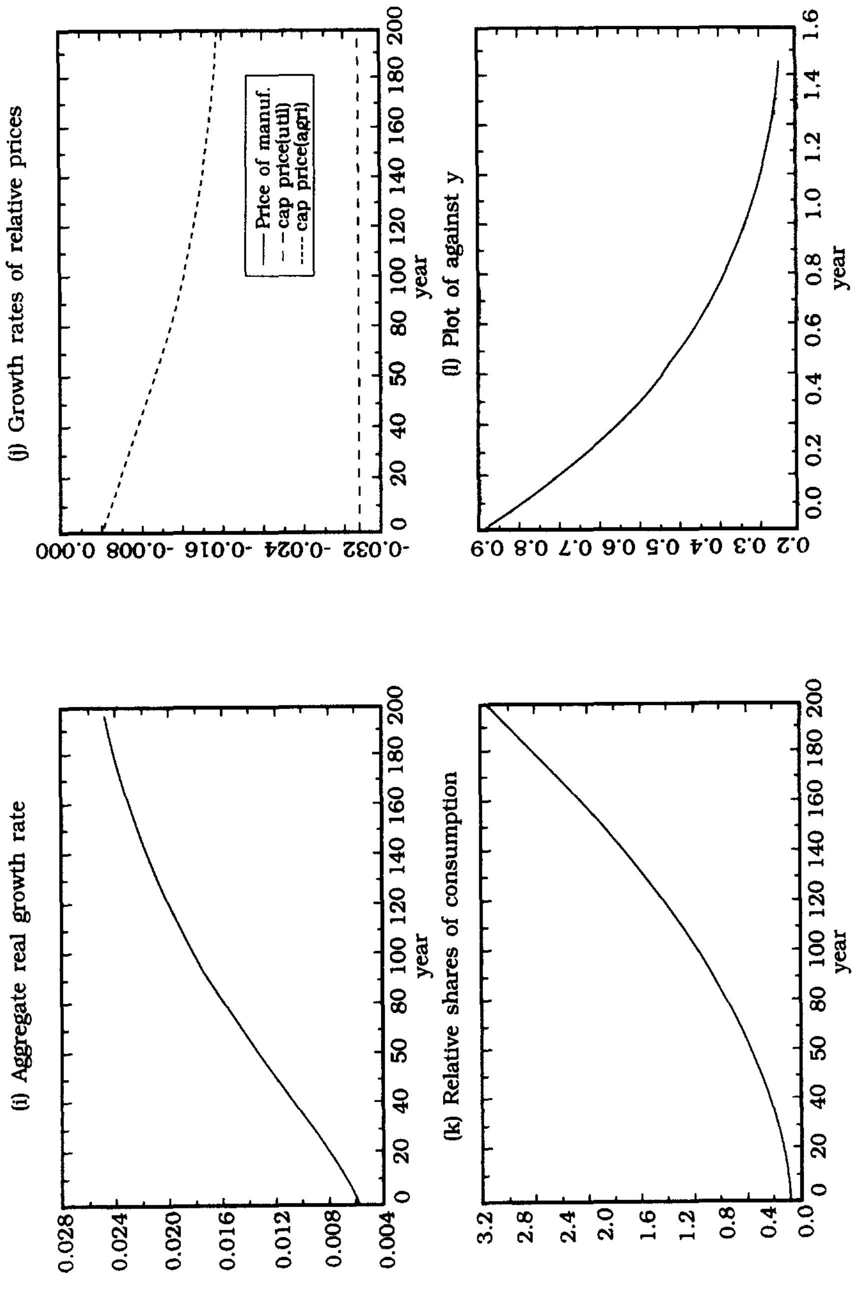

(j) Growth rates of relative prices
Price of manuf.
cap price(util)
cap price(agri)
year
(l) Plot of against y
year
(i) Aggregate real growth rate
year
(k) Relative shares of consumption
year

FIGURE 2

The corresponding values of economic variables on the balanced growth path are as follows:

a_1	b_1	b_2	$\frac{\dot{x}_1}{x_1}$	$\frac{\dot{x}_2}{x_2}$	$\frac{\dot{k}}{k}$	$\frac{\dot{y}}{y}$	$\frac{\dot{p}}{P}$	$\frac{\dot{\lambda}'}{\lambda'}$	$\frac{i}{y}$	$\frac{k}{y}$	$\frac{i}{k}$
0.13	0.06	0.57	0.012	0.018	0.03	0.03	-0.006	-0.018	0.15	5.0	0.03

V. Discussion of the Results

The results of simulations are drawn in Figure 2. The results are consistent with those in the previous section. These results are found to be broadly consistent with actual data arising from the experiences of advanced countries in earlier periods.

In order for transitional dynamics analysis to have empirical relevance, the speed of convergence must be reasonable. It is well known that the speed of convergence is too fast in neoclassical transitional dynamics when capital is narrowly interpreted. In this case, it is also shown by King and Rebelo (1993) that the real interest rate must have been as high as 105%. Increasing the capital share results in much more protracted transitional dynamics, and a much lower real interest rate. Therefore, by broadly interpreting capital which includes human and physical capital, neoclassical transitional dynamics may survive. However, King and Rebelo (1993) showed that the ratio of investment to output must have been unreasonably high in this case. In these respects, they concluded that neoclassical transitional dynamics had limited applicability in explaining actual data over long periods.

However, this model has not only a reasonable real interest rate but also a realistic ratio of investment to output. Real interest rates are found to be quite stable and within reasonable ranges. The real interest rate began at around 7.1% in 1870 and culminated at around 6.5% in 1970 as in (a) of Figure 2. Note that the interest rate on the balanced growth path is calibrated to be 6.2%, which fits with the long-term average of real rates of return on the stock market.

The investment share out of real GDP is increasing over time.

However, it is also within a realistic range, unlike neoclassical transitional dynamics. On the balanced growth path, the ratio is about 0.15, and the ratio of capital over real GDP is also increasing and reaches about 5.0 on the balanced growth path. These suggest the ratio of investment over capital is 0.03 on the balanced growth path, taking depreciation into account. Related to these, Kuznets found that there was a long-term rise in capital formation proportions in most countries. The average ratio of net domestic capital formation to NDP rose from 0.09 to 0.11 from the mid-nineteenth century to World War I and the average ratio of capital to GDP rose from 3.3 to 3.59 from the end of the nineteenth century through the twentieth century in ten advanced countries. These results are shown in (b)-(d) of Figure 2 and are consistent with the simulation results with some minor differences.

One more fact to note is that it is no longer appropriate to speak of convergence speed in terms of the GDP level. Since per capita GDP is growing on the balanced growth path, it is more relevant to mention it in terms of the growth rate. I found that the time needed for catching up half of the differential between capital growth rates in 1870 and the balanced growth path was forty-five years. It took eighty-nine years for three quarters of the growth rate differential to die out. This implies much more protracted transitional dynamics than those of neoclassical models.

Two major caveats are applied in this analysis. The first is the fact that the service sector is not included despite its large and increasing share in advanced countries. Therefore, in the following analysis, I implicitly assume that the share of the service sector is increasing as shown in the data, and appropriately adjust the shares of other sectors.

The second caveat is related to the empirical relevance of the capital-producing sector. Capital may be narrowly defined as investment in physical capital. Or, if broadly defined, capital includes investments in human capital and R&D as well as physical capital. However, data on these investments are not available. Therefore, I adopt the narrow definition when referring to actual data, where appropriate.

In the following sections, I will discuss the transitional dynamics in more detail in the order of shares of each sector, growth rates of (sectoral) outputs, relative prices, and relative shares of consumption.

A. Shares of Each Sector

As expected, the shares of labor and capital employment in agriculture are decreasing over time. The simulated share of agricultural employment was 0.30 in 1970, which was somewhat bigger than the actual value, 0.21 in terms of the average of advanced countries. However, total capital employment is increasing despite a shrinking capital share, while the decrease in labor share implies an absolute decrease in labor employment in agriculture. On the other hand, the factoral shares of the manufacturing and the capital-producing sectors are increasing. Eventually, all the factoral shares are converging to their steady-state values. This implies that per capita output of agriculture as well as the manufacturing and capital-producing sectors are increasing, which is consistent with Maddison (1980, Table 3.7). He showed that per capita labor productivity from 1870 to 1950 rose 0.7% and 1.4% annually in agriculture and manufacturing respectively.

The simulated agricultural output share is biggest in the initial stages, accounting for about 50% of GDP in 1870. However, it declines sharply as the economy grows, and thus is about 10% of total output on the balanced growth path. The shares of both the manufacturing and capital-producing sectors are increasing with that of manufacturing bigger than that of the capital-producing sector initially. However, as industrialization progresses, the share of manufacturing begins to fall, while that of the capital-producing sector continues to grow. Therefore, the time path of the manufacturing share is an inverted U-curve, while that of the capital-producing sector is upward sloping, as in (f) in Figure 2.

These facts are broadly consistent with the findings of Kuznets. He documented evidence that the agricultural share was negatively (and significantly) associated with per capita income in a study of the U.S. economy. It was also established that the reverse was true for the manufacturing sector.[19] This was also confirmed by Syrquin (1986, 1988).

[19]According to Kuznets, the agricultural shares were 0.35, 0.24, 0.17 and 0.12 in 1919-21, 1929, 1950 and 1955 respectively. Simulated results were 0.25, 0.21, 0.15 and 0.14 for these time periods.

B. Growth Rates of Outputs

The growth rate of the agricultural sector is the smallest throughout all transitional periods. However, the growth rate is increasing toward the balanced growth rate. The growth rate of the manufacturing sector is the biggest in earlier periods. This is because the initial output of manufacturing is too small and the preference structure requires a steady demand increase for manufacturing output. However, the growth rate of the manufacturing sector falls over time to 1.8%, while that of the capital-producing sector is continuously increasing. Along the balanced growth path, that of the capital-producing sector becomes the biggest and reaches 3%.

The total contribution of each sector to growth reveals similar pictures with one exception: that of agriculture is decreasing. This exception is due to a faster decrease in the share of agriculture than the increase in the growth rate of agriculture, which is consistent with real economic data.

The real GDP growth rate turns out to be increasing during the transitional period. This implies that, historically, the growth rate has been increasing.

C. Relative Prices

The relative prices of manufacturing and capital-producing sectoral outputs are decreasing during the transitional period. In terms of size, the fall in capital goods price in terms of utility is the biggest, and the next is that of capital goods in terms of agricultural goods, and the last is that of manufacturing goods in terms of agricultural goods. This suggests there have been systematic price changes during development, which favors agricultural goods against the others.

There is no consensus about how actual relative prices (the internal terms of trade) behave over the long-run. However, scant evidence suggests that the results are somewhat mixed.[20] It is said

[20]A closely related and hotly debated issue concerns the external terms of trade. Since Prebisch-Singer, it is argued that import-substitution industrialization is preferred due to the secular tendency for the external terms of trade to move against primary products. However, recent studies revealed that their argument was not supported empirically. For example, Balassa (1988) reported that the terms of trade of the developed countries in their

that studies on Soviet national income expected the prices of products based on natural resources (like agriculture) to rise relative to industrial products. Indeed, Maddison's data (1980, Table 3.5), spanning a quarter century, showed that the annual price growth rate of agricultural output was 4.5% as opposed to that of 4.9% in industry. This implies that the price of agricultural goods rose by 0.4% annually relative to manufacturing output over this period.

As for the relative price of capital goods, Kuznets found that relative prices of capital goods showed a clear upward trend contrary to simulation results. However, he cautioned that it was the price of construction, not of producers' durable equipment or inventories, that accounted for this increase in relative prices. Therefore, abstracting from rising construction costs, the evidence may not be firmly grounded.

D. Relative Shares of Consumption

As implied by the preference structure, the consumption share of manufacturing goods relative to that of agricultural goods is steadily increasing. However, it was found that it took a long time to reach the steady-state value. Even after one-hundred years passed, the relative shares of consumption rose only to around 2.3 in 1970 from 0.6 in 1870 compared to the steady-state value of 6.7.

VI. Concluding Remarks

In this paper, transitional dynamics were built into the structure of preference. The asymmetric presence of the subsistence level led to different transitional dynamic behavior across sectors. Therefore, this model was able to explain structural change in sectoral proportions, which were closely associated with economic growth. In particular, it was shown that structural change in production and factor share favors manufactured goods as opposed to agricultural goods during the transitional period. However, this was a transitional effect, since on the balanced growth path changes in

exchange of manufactured goods for primary products other than fuels with developing countries during the 1953-77 period declined by 10%. If adjustment is made for quality changes, the decline would be greater.

relative prices exactly counteracted differential growth rates between agriculture and manufacturing. Therefore, usual properties of balanced growth models were maintained.

Several dynamic paths of economic variables were calibrated and then compared with actual data from advanced countries in earlier periods. It was found that the simulation results were generally consistent with data. In particular, the real interest rate and the ratio of investment to output were shown to be within reasonable ranges. These resolve the puzzles in neoclassical transitional dynamics raised by King and Rebelo (1993). Therefore, it can be concluded that the transitional dynamics of the endogenous growth model with Stone-Geary utility better accommodate actual economic data over long time periods.

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