

Positional Advantage and Predatory Equilibria

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This paper considers, analysing a conflict between the prey and predator in a general equilibrium framework, how interactions among positional advantage, resource disparity, and the effectiveness of offense relative to defense affect the allocations of resources. Compared to the Nash equilibria, the prey, as a Stackelberg leader, tends to allocate more of resources to defense to utilize the deterrent effect of defense. The prey gains at the expense of the predator and the total consumable output of the economy. In contrast, as a Stackelberg leader, the predator, due to the provoking effect of offense, allocates less of resources to offense. This induces the prey to devote more of resources to production. The predator has an interest in enlarging the appropriable output even though he gets a smaller share. It is shown that both parties prefer the predator to be Stackelberg leader. (JEL Classifications: C70, D60, D74)

I. Introduction

Recent contributions to the analysis of conflict identified important factors, such as resource disparity among contenders and the forms of conflict technology, that are responsible for the allocation of resources between productive and unproductive or appropriative activities.¹ Hirshleifer (1991), for example, demonstrates that a

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¹This literature includes, among others, Becker (1983), Grossman and Kim (1991, 1995, 1996a, 1996b), Hirshleifer (1988, 1991, 1995), Neary (1996), Noh [*Seoul Journal of Economics* 1998, Vol. 11, No. 2]

relatively poorly endowed agent has a comparative advantage in appropriative activities relative to productive activities. This paper considers how positional difference between contenders affects resource allocations in a simple general equilibrium model of conflict.

Analysing a conflict between two contenders over the common pool of income, Hirshleifer (1995) shows that, when one agent acts as a Stackelberg leader, both sides' fighting or appropriative efforts become smaller and incomes are higher compared to the Nash case. In addition, the follower gains relative to the leader. However, these results are obtained without distinguishing offensive from defensive activities in appropriative activities. In their analysis of the security of property, Grossman and Kim (1995, 1996a) capture, with the defensive efforts being made before the offense, the deterrence effect of defense and obtain an equilibrium where no resources are allocated to the offense along with the conditions under which this equilibrium is likely to occur. But they did not consider how the deterrence effect changes the equilibrium allocation of resources compared to the Nash equilibrium. Nor did they consider the case where a potential offender acts as a Stackelberg leader.²

Considering a conflict between the prey and predator in a static environment, this paper investigates how positional difference interacts with resource disparity and the relative effectiveness of offense to affect the equilibrium resource allocation. Main results of the paper follows from the fact that, at the intersection of the reaction curves, the prey faces a downward sloping portion of the

(1995, 1996), Skaperdas (1992), Skogh and Stuart (1982), and Usher (1987).

²As the analysis of the paper shows, if the offender acts as a Stackelberg leader, he has an interest not to provoke the prey and the prey, on his part, has an interest to accept the restraining behavior of the offender. As historical examples, we find some barbarians in the era of the Roman Empire and some nations surrounding the ancient Chinese Empire sometimes take initiatives to reveal their pacifist intention, for example, by paying tribute to their potential prey. Also, we can think of the restraining behaviors of North Korea as the prudent action of the Stackelberg leader. Other examples of the prudent behavior of the offender could be found in the labor-management dispute or in the conflict between incumbent monopolist and new entrant. Finally, government is usually modeled as a leader when we analyse government as extracting rents from the people. In this case, the upward-sloping portion of the Laffer curve is an incidence of the restraining behavior of the government.

predator's reaction curve while the predator faces a upward sloping reaction curve of the prey. Therefore, given the benchmark of Nash equilibria, the prey as a Stackelberg leader has an incentive to allocate larger amount of resources to defense to take advantage of the deterrent effect while the predator as a leader restrains from allocating large amount of resources to offense in order not to provoke the prey.

In the interior solution, due to deterrence effect of defense, the prey devotes more than Nash level of resources to defense, which induces the predator to allocate less than Nash level of resources to offense. However, total amount of resources devoted to appropriative activities, the sum of offense and defense, becomes larger than the Nash level. It turns out that the prey gains at the expense of the predator and total consumable output in the economy. When the relative effectiveness of offense is sufficiently low and the ratio of the initial resource endowment of the prey to that of the predator is not too high, it takes only a small amount of defense for a complete deterrence. This case of nonaggression where no resources are allocated to offense is observed only when the prey acts as a Stackelberg leader.³

The predator as a Stackelberg leader allocates less than Nash level of resources to offense, which makes the prey devote larger than Nash level of resources to production. The predator has an interest in enlarging the appropriable output even though he gets a smaller share. The welfare of both prey and predator improves and total consumable output becomes larger.

Since positional asymmetry in either way results in a smaller amount of offense, the initial resource disparity should be more severe than Nash case against the predator in order for him to choose to become a pure predator. Finally, it is shown that both prey and predator prefer the predator to become Stackelberg leader.

³Grossman suggests that in actual conflicts nonaggression is a prevalent phenomenon. This does not imply, however, that the cases of Nash and the predator as a leader are empirically uninteresting. In actual conflicts, we also observe cases where the predator builds up the offensive weapons but does not actually attack the prey. Differentiating decisions on building up of weapons and on actual attack would be one possible extension of the model.

II. The Model

Consider an economy where there exists a conflict between the prey and predator. From the exogenously given resources, R , of the economy, the prey and predator respectively have as their initial resource endowment R_1 and R_2 , $R=R_1+R_2$. We represent the disparity in resource endowment by the ratio of resource endowment of the prey to that of the predator, R_1/R_2 .

Each of the prey and predator can produce the consumable output by applying their respective resources to production. But, the predator has an option of taking away part or all of the output of the prey by using resources to offense while the output of the predator is assumed to be perfectly secure. Even though the output of the prey is vulnerable to attack, the prey can protect his output by devoting resources to defense.⁴

Accordingly, the resource constraints for the prey and predator are

$$R_1=D+P_1, \quad R_2=A+P_2. \quad (1)$$

D and A denote the amount of defense and offense respectively and P indicates the amount of resources used for production.

Output from productive efforts is represented by a simple linear technology

$$Y_i=kP_i, \quad i=1, 2, \quad k>0. \quad (2)$$

Y represents the output and k is the parameter of productive technology that is set to 1 for simplicity. The assumed productive technology indicates that the output of one agent is a function of only his own productive efforts so that outputs are produced independently of each other. Also, we assume that the productive technology is identical for two agents.⁵

⁴To save the space, this paper focuses on one-sided conflict. This situation could arise either because the output of the predator is intrinsically useless to the prey or the attack of the prey on the predator is not effective at all. However, in a two-sided conflict where the output of the predator is also appropriable, the main results of the paper would go through as long as defense and offense are distinguished from each other.

⁵Modeling differences in productive skills complicates the analysis greatly without affecting the main results. Possible existence of complementarity between productive efforts can be modeled as in Hirshleifer (1991). Also, see footnote 12.

How successfully the prey defends his output from the offense of predator is represented by the following appropriative technology

$$\pi = \frac{1}{1 + \theta \frac{A}{D}}, \quad \theta > 0, \quad (3)$$

where π indicates, given A , D , and θ , the fraction of prey's output that he retains. Note that $0 \leq \pi \leq 1$. Given D and A , as θ increases, π decreases. Therefore, θ can be interpreted as the parameter representing the effectiveness of offense relative to defense.⁶

The assumed technology indicates that, given the amount of offense, as the defensive efforts increase, the prey keeps a larger fraction of his produced output. Also, the technology shows diminishing returns both to the defense and offense. That is to say, we have $\partial \pi / \partial D > 0$, $\partial \pi / \partial A < 0$, $\partial^2 \pi / \partial D^2 < 0$, and $\partial^2 \pi / \partial A^2 > 0$.⁷

In this static model of conflict, we assume that the produced output of the prey is the only appropriable object.⁸ Finally, we assume that both agents try to maximize final consumptions that are given by $C_1 = \pi Y_1$ and $C_2 = Y_2 + (1 - \pi)Y_1$. C_1 indicates final consumption of the prey. With the substitutions of resource constraints and production functions, final consumptions of the prey and predator can be rewritten as functions of D and A

$$C_1 = \pi (R_1 - D), \quad C_2 = (R_2 - A) + (1 - \pi)(R_1 - D). \quad (4)$$

⁶Hirshleifer (1988) considers implications of ratio and difference forms of appropriative technologies for equilibrium resource allocations. See also Noh (1995). Dixit (1987) derives conditions on logistic form of appropriative technology that ensure advantages of strategic behavior in R & D and rent-seeking contests. However, these contests have a fixed prize while this paper takes a general equilibrium approach.

⁷We also obtain that $\partial^2 \pi / \partial D \partial A > (<) 0$ when $D > (<) \theta A$. Given that the amount of defense is greater than the effective amount of offense (θA), the larger is offense, the larger fraction of output is retained by the prey as he increases the defensive efforts. In Becker (1983), the appropriative technology, which is called the Political Influence Function, is assumed to have either positive or negative cross partials for all values of A , D , and θ .

⁸In Grossman and Kim (1995, 1996a), the resource base rather than the produced output is subject to appropriation. They also incorporate the destructive effect of offense.

III. Simultaneous Movement

Consider Nash equilibrium where two contenders move simultaneously. Since each agent makes decisions taking the other agent's choice as given, the first order condition for the interior solution of the prey is given by

$$\frac{\partial C_1}{\partial D} = \frac{\partial \pi}{\partial D} (R_1 - D) - \pi = \frac{\theta A}{(D + \theta A)^2} (R_1 - D) - \frac{D}{D + \theta A} = 0. \quad (5)$$

As the prey increases defensive efforts, the benefit is the larger fraction of his output that can be retained while the cost is the foregone production opportunity. Note that $\partial C_1 / \partial D > 0$ when $D = 0$ and $\partial C_1 / \partial D < 0$ when $D = R_1$, implying that the prey always allocates some positive amount of resources to defense and this amount is strictly less than his initial resource endowment.

The first order condition for the interior solution of the predator is

$$\frac{\partial C_2}{\partial A} = -1 - \frac{\partial \pi}{\partial A} (R_1 - D) = -1 + \frac{\theta D}{(D + \theta A)^2} (R_1 - D) = 0. \quad (6)$$

The predator also balances off, at the margin, the benefit and cost of an increased offensive efforts. The opportunity cost of foregone production is 1 for the predator while the corresponding opportunity cost for the prey is smaller and given by π . This is because the conflict is assumed to be one sided where the predator's output is perfectly secure while that of the prey is appropriable. This differential opportunity cost implies that, other things being equal, the prey has a comparative advantage in the unproductive activities of defense relative to production.

From the first order conditions, we derive the reaction functions of the prey and predator respectively as

$$D = -\theta A + \sqrt{\theta^2 A^2 + \theta A R_1}. \quad (7)$$

$$A = \frac{-D + \sqrt{\theta D(R_1 - D)}}{\theta}. \quad (8)$$

The reaction curve of the prey is upward sloping in the entire range. In contrast, the reaction curve of the predator slopes upward up to $D = (R_1/2)\{1 - (1/\sqrt{1+\theta})\}$ and then slopes downward until $D = \{\theta/(1+\theta)\}R_1$. When D becomes even larger, the best reaction of the predator is to set $A = 0$. This shape reflects the fact that as the defense of prey exceeds a certain amount, it becomes less profitable for the predator to engage in predation. The reaction curves are

shown to intersect at the downward sloping portion of the predator's reaction curve.⁹

A. Equilibrium Configuration

Following Grossman and Kim (1996a) we characterize the interior solution as part-time predation where the predator allocates only part of the resources to offense. The corner solutions where the predator allocates all or none of the resources to offense are characterized as pure predation and nonaggression respectively.

a) Part-time Predation

The interior Nash values of defense and offense are solved as¹⁰

$$D = \frac{2\theta}{1+4\theta+\sqrt{1+4\theta}} R_1, \quad (9)$$

$$A = \frac{4\theta}{(1+\sqrt{1+4\theta})(1+4\theta\sqrt{1+4\theta})} R_1.$$

Accordingly, the equilibrium value of π becomes

$$\pi = \frac{1+\sqrt{1+4\theta}}{1+2\theta+\sqrt{1+4\theta}}.$$

We note that

$$D = \frac{1+\sqrt{1+4\theta}}{2} A,$$

so that the amount of defense is greater than that of offense. This is due to the fact that the opportunity cost of the prey's defensive activities is smaller than that of the predator's offensive activities. In particular, when $\theta = 2$, then $D = 2A$, implying that $\pi = 1/2$. Since the equilibrium value of π is decreasing in θ , the prey keeps more than half of what he produces when $\theta < 2$.¹¹

⁹In a game between two identical players, it is shown by Gal-Or (1985) that when both reaction curves are upward sloping, then there is an advantage for the second mover. In Hirshleifer (1995), the reaction curves of both contenders slope upward because defense is not distinguished from offense. In this paper, however, defense is always a strategic complement to offense while offense is a strategic substitute when the amount of defense is large enough. See, for example, Bulow *et al.* (1985).

¹⁰Even though two reaction curves emanate from the origin, the origin is not the interior Nash solution because neither contender wants to stay at the origin. This is due to the ratio form of conflict technology.

¹¹Either in the conflict over the common pool of income as in Hirshleifer (1991) or in the two sided conflict as in Noh (1995), the amount of offense

Interestingly, we observe that the prey allocates constant fractions of his resources between defense and production regardless of the resource disparity while the predator allocates larger fraction of resources to offense as he becomes relatively poorer.

That a relatively poorly endowed agent allocates relatively more of resources to appropriative activities has been identified by Hirshleifer (1991). Distinguishing offense from defense, Noh (1995) notes that, an agent becomes more aggressive as his resource endowment becomes relatively smaller by allocating larger fraction of his resources to offense and smaller fractions to both defense and production. Since the offensive technology is not available to the prey in this paper, the prey allocates constant fractions of his resources to defense and production regardless of resource disparity. The predator, in contrast, becomes more aggressive as he becomes relatively poorer.

Note that $D < R_1$. A should be less than or equal to R_2 for the interior solution, which requires

$$\frac{R_1}{R_2} \leq \frac{1+4\theta+(1+2\theta)\sqrt{1+4\theta}}{2\theta} \equiv \lambda^*.$$

Since the right-hand side of the inequality is greater than 1, above inequality indicates that, given θ , either the resource base of the prey should be less than that of the predator or when the prey's resource base is larger, this disparity should not be too large. This condition makes sense because if the prey's resource base is considerably larger relative to the predator's resource base, it is better for the predator to specialize in predation.

As θ increases, λ^* first decreases until it reaches the minimum value of approximately 7.24 when $\theta = (1+\sqrt{2})/2$ and increases thereafter. This shape is driven by the fact that, given the resource disparity, when θ increases, D/R_1 increases all along while A/R_2 increases up until $\theta < (1+\sqrt{2})/2$ but decrease thereafter. Consequently, as θ increases beyond this critical value, in order for the predator to become a pure predator, resource disparity should be larger.

is exactly matched by the same amount of defense or by the countervailing offense of the opposite party in the interior solution. That is to say, $A=D$. When the appropriative technology takes a form of $\pi = 1/[1+(A/D)^\theta]$, the equilibrium value of π becomes 1/2 for all values of θ . In this case only, we observe a strong form of the "paradox of power" in which final consumptions of contenders are equalized regardless of the initial disparities in resource endowment.

Final consumptions and resource wastage ratio, defined as the ratio of the sum of defense and offense to total resource, are derived as

$$\begin{aligned} C_1 &= \frac{1 + \sqrt{1 + 4\theta}}{1 + 4\theta + \sqrt{1 + 4\theta}} R_1, \\ C_2 &= R_2 + \frac{2\theta(\sqrt{1 + 4\theta} - 1)}{(1 + \sqrt{1 + 4\theta})(1 + 4\theta + \sqrt{1 + 4\theta})} R_1, \\ \frac{D+A}{R_1+R_2} &= \frac{\frac{2\theta(3 + \sqrt{1 + 4\theta})}{(1 + \sqrt{1 + 4\theta})(1 + 4\theta + \sqrt{1 + 4\theta})} R_1}{1 + \frac{R_1}{R_2}}. \end{aligned} \quad (10)$$

Since the output of prey is insecure, his final relative position becomes worse off compared to the initial relative position. The worsening of relative position is lessened as resource disparity becomes larger. That is, C_1/C_2 is increasing in R_1/R_2 . Resources wasted on unproductive appropriative activities increase as R_1/R_2 increases. This is because the prey maintains a constant fraction of his resources for defense while the predator becomes more aggressive as his resource endowment becomes relatively smaller. Therefore, consumable output of the economy would be larger as the predator is endowed with relatively larger resources. But, this would be resisted by the prey.

As offense becomes more effective, the predator becomes better off while the prey becomes worse off. Resource wastage ratio increases (decreases) as θ increases when $\theta < (>) 5 + 3\sqrt{3}$. This reflects the fact that when offense is sufficiently effective, the decrease in offense more than offsets the increase in defense.

b) Pure Predation

When $R_1/R_2 > \lambda^*$, the predator becomes a pure predator. In this case, $A = R_2$ and we derive $D = -\theta R_2 + \sqrt{\theta^2 R_2^2 + \theta R_1 R_2}$. This shows that the fraction of resources the prey allocates to defense decreases as resource disparity increases but this fraction increases as offense becomes more effective.

The equilibrium value of $\pi = 1 - [1/(1 + (1/\theta)(R_1/R_2))^{1/2}]$ shows that it increases as resource disparity increases. Final consumptions and resource wastage ratio are derived as

$$C_1 = \{(R_1 + \theta R_2)^{\frac{1}{2}} - (\theta R_2)^{\frac{1}{2}}\}^2.$$

$$C_2 = (R_1 + \theta R_2)^{\frac{1}{2}} \{ (R_1 + \theta R_2)^{\frac{1}{2}} - (\theta R_2)^{\frac{1}{2}} \} .$$

$$\frac{D+A}{R_1+R_2} = \frac{1 - \theta + \sqrt{\theta^2 + \theta \frac{R_1}{R_2}}}{1 + \frac{R_1}{R_2}} . \quad (11)$$

As resource disparity increases, even though the appropriable output becomes larger, the fraction that the predator takes away from the prey is sufficiently reduced that C_2 becomes smaller. Resource wastage ratio decreases as resource disparity increases. In contrast to the interior equilibrium, to maximize the consumable output of the economy, it is better to concentrate the economy's resources to the prey, which the predator would not agree voluntarily.¹² We also note that as offense becomes more effective, C_1 decreases while both C_2 and resource wastage ratio increase.¹³

IV. Prey Moves First

Suppose the prey can commit a certain amount of resources to defense before the predator chooses his actions. This structure captures the deterrence effect of defense. We investigate how the deterrence effect affects the allocation of resources.

A. Predator's Problem

The predator chooses A to maximize C_2 taking D as given. The first order condition for the predator is given by (6) and we note

¹²With linear productive technology as in this paper, Pareto improving redistribution of resources is not possible. When production function is concave, however, there exists a possibility of Pareto improving resource redistribution when resource redistribution is big enough. See Neary (1996) and Grossman and Kim (1996b).

¹³Consider North-South Korean relation and suppose that South Korea is the prey. Suppose θ becomes smaller, for example, with the military assistance by the United States to South Korea. Our analysis indicates that South Korea gains at the expense of North. If North had been already a pure predator, the increase in the consumable output of South outweighs the decrease in North's consumption. However, if the North were a part-time predator and offense of North were highly effective, then unless the assistance reduces the effectiveness of offense sufficiently, the decrease in North's consumption outweighs the increase in that of South. In this case, the assistance actually increases the North's offensive efforts.

that $\partial C_2 / \partial A$ is decreasing in A . Suppose that D is greater than $D_3 = \{\theta / (1 + \theta)\} R_1$. In this case, if we plug $A=0$ into equation (6), the sign becomes negative. When the prey allocates sufficiently large amount of resources to defense, it is not only difficult for the predator to appropriate but also it is not worth attacking. In other words, the amount of defense greater than or equal to D_3 deters predation completely.

We can show that $A=R_2$ is the best choice when $D_1 < D < D_2$, where

$$D_1, D_2 = \frac{1}{2(1+\theta)} \{ \theta (R_1 - 2R_2) \mp \sqrt{R_1^2 - 4R_1R_2 - 4\theta R_2^2} \}.$$

This range is possible only when

$$\frac{R_1}{R_2} > 2(1 + \sqrt{1 + \theta}).$$

The resource base of the prey must be at least four times as large as that of the predator for the predator to become a pure predator. We note that $D_1 > 0$ and $D_2 < D_3$. When the prey has an abundant resource base and at the same time he spends a relatively small amount of resources on defense, the predator devotes all of his resources to offense, rather than producing himself, hoping that he gets some fraction of huge output that the prey produces.

When $D > D_1$ or $D_2 < D < D_3$, the interior solution, denoted by A^* , is obtained in (8). Consequently, the behavior of predator is summarized as

$$A = \begin{cases} 0 & : D > D_3 \\ R_2 & : D_1 < D < D_2 \\ A^* & : 0 < D < D_1 \text{ or } D_2 < D < D_3. \end{cases}$$

B. Prey's Problem

The prey maximizes C_1 taking the reactions of the predator into considerations. The first order condition for the prey now becomes

$$\frac{dC_1}{dD} = \left(\frac{\partial \pi}{\partial D} + \frac{\partial \pi}{\partial A} \frac{dA}{dD} \right) (R_1 - D) - \pi. \quad (12)$$

The marginal benefit of increased defensive efforts includes the indirect effect of the induced change in A on π as well as the direct effect of increasing π .

When $A=0$, we have $dA/dD=0$, $\pi=1$, and $\partial \pi / \partial D=0$. Consequently, $dC_1/dD=-1$. But $A=0$ only when $D > D_3$. Since dC_1/dD

is decreasing in D , it follows that one possible equilibrium pair of strategies is given by

$$D = \frac{\theta}{1+\theta} R_1, \quad A=0. \quad (13)$$

Suppose $A=R_2$. Then, $dA/dD=0$. Setting the first order condition equal to 0 produces $D^2+2\theta R_2D-\theta R_1R_2=0$. Selecting the positive root, we get $D = -\theta R_2 + \sqrt{\theta^2 R_2^2 + \theta R_1 R_2}$. Since $A=R_2$ only when $D_1 < D < D_2$, the derived D should satisfy this condition. We can easily check that $D > D_1$ is always true. The condition $D < D_2$ is equivalent to $R_1/R_2 > \lambda^*$. Consequently, given this inequality, we have another possible equilibrium pair of strategies

$$D = -\theta R_2 + \sqrt{\theta^2 R_2^2 + \theta R_1 R_2}, \quad A=R_2. \quad (14)$$

Finally, consider the case when $A=A^*$. Substituting $\partial \pi / \partial D$, $\partial \pi / \partial A$, and dA/dD into the first order condition, we derive $D = (1/2)R_1$. Recall that $A=A^*$ only when either D falls between 0 and D_1 or between D_2 and D_3 . Note that $(1/2)R_1 > D_2$. The condition that $(1/2)R_1 < D_3$ implies $\theta > 1$. Therefore, given that $\theta > 1$, we have another possible equilibrium pair of strategies

$$D = \frac{1}{2}R_1, \quad A = \frac{\sqrt{\theta}-1}{2\theta}R_1. \quad (15)$$

Up to now, we found three possible equilibria each requiring certain conditions. When these conditions overlap with each other, we compare the values of C_1 in each equilibrium and choose the equilibrium along with the condition under which this equilibrium produces highest C_1 . These comparisons are made in the Appendix. Then the equilibrium behavior of prey is summarized as

$$D = \begin{cases} \frac{\theta}{1+\theta}R_1 : 0 < \theta < 0.654 \text{ and } \frac{R_1}{R_2} < \frac{4(1+\theta)}{\theta} \\ \quad \text{or } 0.654 < \theta < 1 \text{ and } \frac{R_1}{R_2} < \lambda^* \\ -\theta R_2 + \sqrt{\theta^2 R_2^2 + \theta R_1 R_2} : 0 < \theta < 0.654 \text{ and } \frac{R_1}{R_2} > \frac{4(1+\theta)}{\theta} \\ \quad \text{or } \theta > 0.654 \text{ and } \frac{R_1}{R_2} > \lambda^* \\ \frac{1}{2}R_1 : \theta > 1 \text{ and } \frac{R_1}{R_2} < \lambda^* \end{cases}$$

C. Equilibrium Configuration

a) Nonaggression

One distinctive feature when the prey utilizes the deterrent effect of defense is the existence of an equilibrium where the predator does not allocate any resources to offense. For this equilibrium, offense should be less effective relative to defense, $\theta < 1$, and resource disparity should not be too large.

From $D = \{\theta / (1 + \theta)\}R_1$, we note that, with smaller θ , the amount of defense needed for complete deterrence becomes smaller. With a complete deterrence, the prey retains all that he produces and the predator specializes in production. Final consumptions and resource wastage ratio are given by

$$\begin{aligned} C_1 &= \frac{R_1}{1 + \theta}, \\ C_2 &= R_2, \end{aligned} \tag{16}$$

$$\frac{D+A}{R_1+R_2} = \frac{\frac{\theta}{1+\theta} \frac{R_1}{R_2}}{1 + \frac{R_1}{R_2}}.$$

More resources are wasted on defense either when offense is more effective or when resource disparity is more severe.

b) Part-Time Predation

The predator occupies himself both in production and offense only when $\theta > 1$ and $R_1/R_2 < \lambda^*$. To prevent the predator from becoming a pure predator, resource disparity should be within a certain limit. Even if this condition is satisfied, the predator is completely deterred by the prey when θ is less than 1.

The prey allocates exactly half of his resources to defense regardless of the values of θ . From (15), we derive that $D = \{\theta / (\sqrt{\theta} - 1)\}A$ in part-time predation. Consequently, once it is determined that a complete deterrence is not the best policy, it is better to fix defense at the amount that is greater than that of offense for all values of θ .

In this region, the prey still takes advantage of the deterrent effect of defense because the prey's reaction curve cuts through the downward sloping section of the predator's reaction curve. That is to say, the prey allocates more than Nash amount of resources to defense. Consequently, the equilibrium value of π , given by $1/\sqrt{\theta}$

is greater than the Nash case. In particular, when $\theta = 4$, π becomes $1/2$ which is obtained when $\theta = 2$ in Nash case.

Interestingly, however, total amount of resources wasted on appropriative activities, $D+A$, becomes larger when the prey utilizes the deterrent effect. Final consumptions and resource wastage ratio are given by

$$\begin{aligned} C_1 &= \frac{1}{2} \theta^{-\frac{1}{2}} R_1, \\ C_2 &= R_2 + \frac{1}{2} (1 - \theta^{-\frac{1}{2}})^2 R_1, \\ \frac{D+A}{R_1+R_2} &= \frac{\frac{\theta + \sqrt{\theta} - 1}{2\theta} \frac{R_1}{R_2}}{1 + \frac{R_1}{R_2}}. \end{aligned} \quad (17)$$

Compared to the Nash case, the prey produces a smaller amount of consumable output even though he retains a larger fraction. It turns out that the absolute amount of prey's final consumption becomes greater. In contrast, since the increased output that the predator produces himself is more than offset by the reduced output that he takes from the prey, predator's welfare deteriorates. When the prey utilizes the deterrent effect, he gains at the expense of both the predator and total consumable output. Clearly we have a first mover advantage and this result contrasts to the second mover advantage in Hirshleifer (1995) where offense is not distinguished from defense in appropriative activities.

c) Pure Predation

When the relative resource base of prey is sufficiently large, the predator allocates all of his resources to offense. More specifically, the predator becomes a pure predator either when $0 < \theta < 0.654$ and $R_1/R_2 > 4(1+\theta)/\theta$ or when $\theta > 0.654$ and $R_1/R_2 > \lambda^*$.

When $\theta > 0.654$, the required resource disparity for the interior solution is identical to the Nash case. Otherwise, since the prey is likely to completely deter the predator, resource disparity should be more severe than the Nash case for pure predation to occur. As shown in the Appendix, $4(1+\theta)/\theta$ is greater than λ^* when $\theta < 0.654$. The equilibrium pair of strategies, π , final consumptions, and resource wastage ratio are identical to those in Nash pure predation.

V. Predator Move First

Suppose the predator moves first and sets the amount of offense before the prey chooses his actions.

A. Prey's Problem

The prey chooses D to maximize C_1 taking A as given. From the first order condition in (5), the prey always allocates some positive amount of resources to defense as given in (7).

B. Predator's Problem

Taking D in (7) as given, the predator chooses A . Differentiation of C_2 with respect to A gives

$$\frac{dC_2}{dA} = -1 - \left(\frac{\partial \pi}{\partial A} + \frac{\partial \pi}{\partial D} \frac{dD}{dA} \right) (R_1 - D) - \frac{dD}{dA} (1 - \pi). \quad (18)$$

Substitutions of $\partial \pi / \partial A$, $\partial \pi / \partial D$, and dD/dA into above equation produce

$$\frac{dC_2}{dA} = -(1 + \theta) + \frac{(\theta R_1 + 2\theta^2 A)}{2(D + \theta A)}.$$

When $A=0$, above expression becomes infinite, implying that the predator always devotes a positive amount of resources to offense. If we evaluate the expression at $A=R_2$, we find that $dC_2/dA > 0$ when $R_1/R_2 > 2[(1+2\theta)/(1+\theta)\sqrt{1+2\theta}]/\theta \equiv \lambda^{**}$. Consequently, we have $A=R_2$ when $R_1/R_2 > \lambda^{**}$. Otherwise we obtain the interior solution from (18) as

$$A = \frac{1 + \theta - \sqrt{1 + 2\theta}}{2\theta\sqrt{1 + 2\theta}} R_1.$$

C. Equilibrium Configuration

The upward sloping reaction curve of the prey implies that the predator can induce the prey to allocate a smaller amount of resources to defense when the predator himself allocates a smaller amount of resources to offense. In contrast to the deterrent effect of defense, we have the provoking effect of offense when the predator acts as a Stackelberg leader.¹⁴

a) Part-Time Predation

The predator allocates some resources to production only when $R_1/R_2 < \lambda^{**}$. The upper bound of resource disparity for the interior solution decreases monotonically and reaches the minimum value of approximately 10.77 when $\theta = 1 + \sqrt{2}$ and then it increases monotonically as θ increases. We also find that $\lambda^{**} > \lambda^*$ for all values of θ . This implies that there exists a range of resource disparities within which the predator engages in some productive activities when he moves first while the predator becomes a pure predator when the predator and prey move simultaneously. (Or when the prey moves first with a large value of θ .) Since the predator restrains from allocating a large amount of resources to offense, in order for the predator to become a pure predator, the resource disparity should be greater than the Nash case.

The equilibrium strategies are given by

$$A = \frac{1 + \theta - \sqrt{1 + 2\theta}}{2\theta\sqrt{1 + 2\theta}} R_1, \quad D = \frac{1}{2} \left(1 - \frac{1}{\sqrt{1 + 2\theta}} \right) R_1.$$

Note that $D = [(\sqrt{1 + 2\theta} - 1)/(1 + \theta - \sqrt{1 + 2\theta})]A$. This shows that if $\theta > 4$, then $A > D$ and vice versa. The instance where the amount of offense is greater than that of defense is possible only when the predator is a Stackelberg leader with a highly effective offense technology.

Relative to the Nash case, defense as well as offense become smaller, which implies that resources wasted on unproductive activities become smaller. Since, in terms of total consumable output, the case of the prey as a leader is worse than Nash, it follows that the case of the predator as a leader is most productive.

Interestingly, the prey ends up retaining a larger fraction of his production. That is to say, the equilibrium value of $\pi = (\sqrt{1 + 2\theta} - 1)/\theta$ is greater than that obtained in Nash interior solution. This suggests that the predator as a Stackelberg leader has an interest in letting the prey devote more of resources to production so that, although he takes a smaller fraction of prey's output, the absolute

¹⁴It can be shown that as the predator reduces offense, the prey reduces defense at a faster rate. The provoking effect is also influenced by the effectiveness of offense. Calculations show that $d(dD/dA)/d\theta > 0$ when $R_1 > 2(1 + \sqrt{2})\theta A$. Therefore, when the resource base of the prey is larger than some multiple of effective amount of offense, the provoking effect is larger as θ increases.

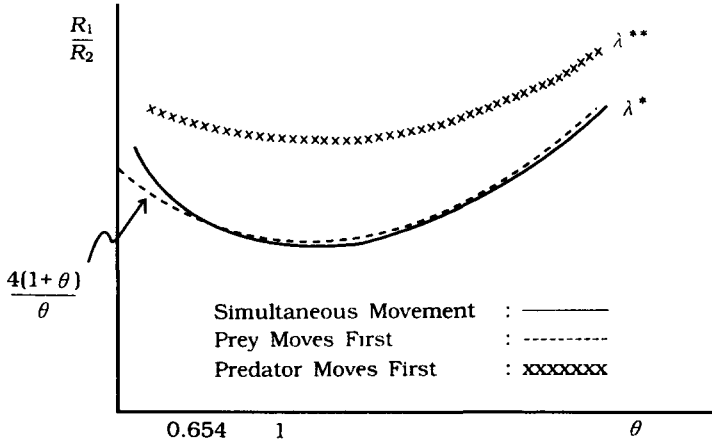


FIGURE
BOUNDARIES OF EQUILIBRIUM CONFIGURATION

amount of prey's output becomes larger. Note that this result of farming the prey is obtained in a static setting.

Final consumptions and resource wastage ratio are given by

$$\begin{aligned}
 C_1 &= \frac{R_1}{\sqrt{1+2\theta}}, \\
 C_2 &= R_2 + \frac{1+\theta - \sqrt{1+2\theta}}{2\theta} R_1, \\
 \frac{D+A}{R_1+R_2} &= \frac{\frac{1+(\theta-1)\sqrt{1+2\theta}}{2\theta} \frac{R_1}{R_2}}{1+\frac{R_1}{R_2}}.
 \end{aligned} \tag{19}$$

Compared to the Nash, the welfare of both predator and prey improves. It is interesting to observe that the prey becomes better off because that is in the interest of the predator. The restraint of the predator in appropriative activities causes himself and the prey to devote more resources to production.

b) Pure Predation

The equilibrium π , final consumptions, and resource wastage ratio are the same as in the Nash case. However, recall that, due to the provoking effect of offense, the predator acts as if he is not too aggressive. Therefore, resource disparity should be greater than

the Nash case to force the predator to become a pure predator.

VI. Who Moves First?

Recall that the prey, as a Stackelberg leader, consumes more than he does in the Nash equilibrium. Also, the prey's consumption is increased from the Nash level when he is a follower. Comparison of welfare levels of the prey in these cases shows that the prey enjoys higher level of consumptions as a follower. This is true regardless of the effectiveness of offense.¹⁵ We know that the predator gains as a leader but loses as a follower. This suggests that both predator and prey prefer the predator to become Stackelberg leader.¹⁶

VII. Concluding Remarks

Investigating a conflict between the prey and predator in a general equilibrium framework, this paper shows how positional asymmetry interacts with resource disparity and the relative effectiveness of offense to affect the equilibrium allocation of resources.

In Nash equilibrium, as the ratio of resource endowment of the prey to that of the predator increases, the amount of resources wasted on unproductive activities increases as long as the predator allocates some resources to production. This amount, however, decreases once the predator becomes a pure predator. As offense

¹⁵To compare the welfare levels of the prey, first note that $\lambda^{**} > 4(1+\theta)/\theta$ when $\theta < 0.654$. Therefore, we have to make welfare comparisons in four regions. First, C_1 in (19) is greater than C_1 in (16). Second, C_1 in (19) is greater than C_1 in (17) only when $\theta > 1/2$. However, this region is possible only when $\theta \geq 1$. Third, C_1 in (19) is greater than C_1 in (11) only when $R_1/R_2 < 4\theta\sqrt{1+2\theta}/(\sqrt{1+2\theta}-1)^2$. It can be easily shown that the right hand side of the inequality coincides with λ^{**} . Finally, the prey consumes the same level consumption when the predator is a pure predator in both cases.

¹⁶If the destructiveness of offense is explicitly modeled, the prey can be better off when he is a leader rather than a follower. In this event, there exists a struggle to become the first mover, which may result in either simultaneous movement or some negotiations that are not considered in the paper. Also, if we consider a repeated conflict, it is more likely that the predator becomes the leader due to the costlines of deterrence.

becomes more effective, resource wastage in the economy becomes larger when the predator is a pure predator. In part-time predation, however, wastage of resources begin to decrease as offense becomes more effective beyond a certain point.

Compared to the Nash equilibrium, the deterrent effect of defense permits the prey to improve his welfare but only at the expense of the predator and total consumable output of the economy. The provoking effect of offense allows the predator to improve his welfare as well as that of the prey. Also, it is shown that it is in the interest of both prey and predator for the predator to become Stackelberg leader.

Appendix

When $D = \{\theta / (1 + \theta)\} R_1$ and $A = 0$, the consumption level of the prey, denoted by V_1 , is given by

$$V_1 = \frac{1}{1 + \theta} R_1.$$

When $D = -\theta R_2 + \sqrt{\theta^2 R_2^2 + \theta R_1 R_2}$ and $A = R_2$, V_2 , the consumption level of the prey in this case becomes

$$V_2 = \{(R_1 + \theta R_2)^{\frac{1}{2}} - (\theta R_2)^{\frac{1}{2}}\}^2.$$

When $D = (1/2)R_1$ and $A = \{[1 - (1/\sqrt{\theta})]/(2\sqrt{\theta})\}R_1$, V_3 , the consumption level of the prey when the predator is a part-time predator, is derived as

$$V_3 = \frac{1}{2} \theta^{-\frac{1}{2}} R_1.$$

First, we have $V_3 - V_1 > 0$. Therefore, when $\theta > 1$, $D = \{\theta / (1 + \theta)\} R_1$ will not be chosen. Next, it can be shown that

$$V_2 > V_1 \text{ only if } \frac{R_1}{R_2} > \frac{4(1 + \theta)}{\theta}.$$

Comparison between λ^* and $4(1 + \theta)/\theta$ shows that the former is greater than the latter when θ is larger than approximately 0.654. Therefore, when $\theta < 0.654$ and $R_1/R_2 < 4(1 + \theta)/\theta$, $D = -\theta R_2 + \sqrt{\theta^2 R_2^2 + \theta R_1 R_2}$ can not be the equilibrium strategy for the prey.

Finally, we derive that

$$V_2 > V_3 \text{ only if } \frac{R_1}{R_2} > \sqrt{2} - \frac{1}{2} \theta^{-\frac{1}{2}}.$$

Since $\sqrt{2} - (1/2)\theta^{-1/2}$ is less than λ^* , it follows that when $R_1/R_2 > \lambda^*$, $D = (1/2)R_1$ can not be the equilibrium strategy. These findings are summarized in the text.

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