Consumption Decisions with Stochastic Wage Income: Testing the Implications of an Approximate Solution

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In this paper, I use an approximate solution to model the optimal consumption when the representative consumer faces labor income uncertainty. This approximate consumption function is based on Zeldes' (1989) numerical solution to the optimal consumption problem with CRRA utility and stochastic labor income. Unlike the certainty equivalence solution, this model assumes that the consumer discounts expected future labor income at a rate higher than the real interest rate. It therefore takes into consideration the precautionary savings of the consumer. The first order implications of the approximate consumption function, with and without the liquidity constrained consumers, are tested using quarterly US data. The evidence lends support to the claims of the approximate consumption function, particularly when liquidity constrained consumers are included.

The empirical results of this paper imply that current consumption should be Granger caused by variables in the lagged information set. Meanwhile, consumption should be smoother than labor income, even when the latter follows an integrated process. Both implications have been documented in the literature. Based on this evidence, I conclude that the approximate model is a promising way of getting around the difficulties

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[**Seoul Journal of Economics** 1998, Vol. 11, No. 2]

involved in obtaining a closed form solution when utility is of the general decreasing absolute risk aversion type. (*JEL* Classification: E21)

I. Introduction

The simple life cycle/permanent income model described in most macroeconomic texts assumes that individuals determine their current consumption according to the sum of their financial assets and the present discounted value of their expected future labor income. This proposition is generally valid when labor income is nonstochastic. However, in the case of stochastic labor income, as being assumed in most circumstances, it is true only when some stringent assumptions on the momentary utility function and/or the labor income process of the consumers are maintained. This is because the underlying intertemporal optimization problem of a representative consumer cannot be explicitly solved with arbitrary uncertain labor income and concave utility.

A special case which validates the simple consumption decision rule is when the momentary utility is assumed to be quadratic. Quadratic utility is justified to some extent because it can be considered as a local approximation to the underlying utility of the consumer. The explicit consumption function obtained in this special case is often called the certainty equivalence solution, since *only the first moments* of the stochastic labor income appear in it. Indeed, since Hall (1978), most studies of the permanent income model have relied on the certainty equivalence solution, which has the important implication that consumption should be approximately a random walk. Rejections of the consumption random walk in a majority of the studies, with either aggregate or panel data, are often interpreted as evidence of the prevalence of liquidity constraints.

Although it is simple to handle for empirical studies, the quadratic utility function is implausible, because it implies that consumers have increasing absolute risk aversion. That is, consumers are supposed to be willing to pay more to avoid a given lottery as their consumptions increase. This implication is counter-intuitive. Therefore quadratic utility function does not lead to a realistic description of rational consumers' behavior under uncertainty.

There is thus a need to investigate the more realistic case of decreasing (or at least, nonincreasing) absolute risk aversion. In fact, recent studies of the effect of government budget deficit on consumption has proven that explicit modelling of labor income uncertainty is fruitful. It generates new insights when consumers are assumed to have decreasing absolute risk aversion utility function (Barsky, Mankiw and Zeldes 1986; Kimball and Mankiw 1989). These new developments also shed light on the prospect of studies of consumption with decreasing absolute risk aversion utility and stochastic labor income. A necessary condition for utility to exhibit nonincreasing absolute risk aversion is marginal utility being convex, or the third derivative of the utility being positive. As a result, optimal consumption should generally be less relative to the certainty equivalence level, reflecting their prudence or precautionary savings (Leland 1968; Sandmo 1970 and Dreze & Modigliani 1972).

The biggest disadvantage of considering this type of utility function is that in even very simple cases, a closed form solution does not exist. To allow for any empirical work, other restrictive assumptions would have to be imposed. An example of this is given by Caballero (1990, 1991). Caballero demonstrated that if utility is characterized by the constant absolute risk aversion, and both consumption and labor income innovations are i.i.d., then optimal consumption is the certainty equivalence level less a constant. The constant reflects the consumer's precautionary savings. Although his studies have emphasized precautionary savings, the explicit solution is reached at the sacrifice of realism because of his additional assumptions.

In this paper I study a compromise solution that is empirically tractable, yet consider the effects of labor income uncertainty. Based on a numerical simulation by Zeldes (1989), I investigate an approximate consumption function in which the consumer is assumed to discount the uncertain future labor income at a rate higher than the real interest rate. The implications of that consumption function is then tested with quarterly US data. Evidence suggests that this consumption function, particularly when we assume that a fraction of the consumers are liquidity constrained, is a good characterization of the aggregate US data. The claims of Zeldes's simulation are also supported by the data.

The primary purpose of this paper is to provide a starting point for probing the usefulness of this approximate consumption function in empirical studies. Because the assumptions are on the parameters of the consumption function, we do not have to assume whether the consumer's utility function is in one form or another. Different characteristics of the utility function are not our primary concern here, although there might be a correspondence between the parameters of the utility function and those of the approximate consumption function.

The rest of this paper is organized as follows: Section Ⅱ lays out the general intertemporal optimization problem that underlies the permanent income hypothesis. Two tractable cases of the general problem are when labor income is nonstochastic and when utility is quadratic. Both cases provide the bases for the usual claim that consumption should be proportional to financial assets and present value of expected future income. This section also presents several results of Zeldes' simulation. An approximate consumption function is formulated based on his simulation results. Section Ⅲ derives some implications of the approximate consumption function. Some relevant econometric issues and the data are also discussed there. Section Ⅳ presents and discusses the empirical results. In particular, the results are used to explain the two empirical anomalies in the consumption literature. The conclusion of the paper follows in Section V.

II. The Permanent Income Hypothesis with Stochastic Labor Income

A. *The General Intertemporal Consumption Problem of a Representative Consumer*

The infinitely-lived representative consumer is assumed to maximize the expected value of an intertemporally separable utility function, and to be subject to an intertemporal budget constraint. Specifically, at period t, the consumer solves the following problem:

$$\max \quad E_t = \sum_{i=0}^{\infty} \left(\frac{1}{1+\rho} \right)^i U(C_{t+i}) \tag{1}$$

$$\text{s.t. } C_{t+i} + A_{t+i} = W_{t+i} + (1+r)A_{t+i-1}, \quad i = 0,\ 1,\ 2,\ \cdots \tag{2}$$

and

$$\lim_{i \to \infty} \left(\frac{1}{1+r} \right)^{i} A_{t+i} = 0 \tag{3}$$

where

E_t = expectation conditional on all information available in period t;
ρ = rate of subjective time preference;
r = real rate of interest, assumed to be constant over time;
$U(\cdot)$ = momentary utility function, assumed to be monotonically increasing, concave and continuously differentiable to at least the third order;
C_t = consumption in period t;
W_t = labor income in period t;
A_t = nonhuman wealth in period t;

The intertemporal budget constraints in the model indicate that consumers can lend or borrow against future income flow at the market interest rate without quantitative limit, namely, capital markets are perfect. The transversality condition that the present value of the household's asset holding is zero in the infinite future guarantees insolvency will not arise for this individual.

The first order conditions for the above problem are

$$U'(C_t) = \left(\frac{1+r}{1+\rho} \right) E_t U'(C_{t+1}), \qquad t = 1, 2, 3, \cdots \tag{4}$$

These conditions reflect the fact that while being on the optimal consumption path, the consumer should not be able to increase expected utility by consuming one unit less today, increasing one unit of asset holding, and then increasing consumption tomorrow by $1+r$ units.

B. A Special Case: Nonstochastic Labor Income

Consider the simple case when labor income is variable over time but nonstochastic.[1] In addition, we assume that the time discount rate is equal to the real rate of return from nonhuman wealth. In this case, the first order conditions are

$$U'(C_t) = U'(C_{t+1}) \qquad t = 1, 2, 3, \cdots \tag{5}$$

[1]Alternatively, we can assume that the consumer has perfect foresight while letting labor income be stochastic.

Because U is concave, marginal utility is monotonically decreasing. The F.O.C.'s in (5) therefore hold if and only if

$$C_t = C_{t+1} \qquad t = 1, 2, 3, \cdots \tag{6}$$

Equation (2), the consumer's intertemporal budget constraints, can be combined to be written as

$$\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i C_{t+i} = (1+r)A_{t-1} + \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i W_{t+i}. \tag{7}$$

From (6) and (7), we get

$$C_t = \frac{r}{1+r}\{(1+r)A_{t-1} + \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i W_{t+i}\}. \tag{8}$$

We conclude, in this special case of nonstochastic labor income, that the optimal consumption decision of a representative consumer is to follow a flat consumption path. The consumer uses the capital market, which is assumed to be perfect, to shield consumption from changing over time in the presence of the ups and downs of the labor income stream. The optimal levels of consumption, from time t on, are equal to the annuity value of the sum of the t-th period nonhuman wealth, $(1+r)A_{t-1}$, and human wealth,

$$\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i W_{t+i}.$$

The latter is the present discounted value, as of period t, of all future (variable but certain) earnings.

A fact worth of noting here is that the consumption function derived in equation (8) is independent of whichever form of the momentary utility we assume it to be, as long as it is concave. In particular, it includes the special cases of quadratic, constant absolute risk aversion and constant relative risk aversion utilities that are frequently used in the literature.

C. The General Case: Stochastic Labor Income – Certainty Equivalence and Precautionary Savings

When future labor incomes are uncertain, there is generally no closed form solution to the consumer's intertemporal optimization problem stated in equations (1), (2) and (3). An exception to that is when the momentary utility function is quadratic

$$U(C_t) = -\frac{1}{2}(C^* - C_t)^2,$$

where C^* is the bliss level of consumption. In this case, and with the additional assumption of $\rho = r$, equations (4) can be written as

$$C_t = E_t C_{t+1}, \qquad t = 1, 2, 3, \cdots$$

The consumer's optimal level of consumption in period t can hence be written as

$$C_t = \frac{r}{1+r}\{(1+r)A_{t-1} + \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t W_{t+i}\} \tag{9}$$

i.e., the optimal level of consumption in period t should be the annuity value of the sum of the t-th period nonhuman wealth, $(1+r)A_{t-1}$, and expected human wealth,

$$\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t W_{t+i},$$

where the latter is the present discounted value of expected future labor income.

The solution in equation (9) is usually called the "certainty equivalence" level of consumption when the labor income is stochastic. As long as utility is quadratic, this formulation of optimal consumption is independent of the stochastic properties of the labor income involved, aside from its first moments. In particular, it is independent of the second or higher moments of the labor income innovations, and of whether the labor income follows an integrated process or a stationary process.

An important implication of the certainty equivalence solution is that consumption in period t should not be forecasted by any variable, dated $t-1$ or earlier, other than the $(t-1)$-th period consumption itself, namely, consumption is a martingale with respect to the consumer's information set. This fact has been a fundamental theme in the literature on the studies of consumption. Since Hall (1978), numerous papers have appeared, either to test the validity of that implication, or to obtain estimates of important parameters, e. g., the consumer's attitude toward risk, by exploiting that implication.

An unattractive feature of the quadratic utility is that it implies increasing absolute risk aversion, that is, the consumer is assumed to be willing to pay more to avoid a given lottery as his or her consumption increases. This is clearly counter-intuitive. Therefore a utility exhibiting decreasing absolute risk aversion is a more realistic description of rational consumers' behavior under uncertainty.

When consumers have decreasing absolute risk aversion utility, their optimal consumption will be less than the certainty equivalence level, and their consumption profile over time will also be steeper (see, for example, Leland 1968; Sandmo 1970 and Dreze & Modigliani 1972). Caballero (1990) has shown that when the utility is characterized by the constant absolute risk aversion,[2] consumption should be the certainty equivalence level less a constant. The constant, which measures the precautionary savings, depends on the coefficient of absolute risk aversion and the riskiness of the labor income. For the aggregate labor income risks, it apparently depends on both the variance of the labor income shocks and the persistence of the labor income process.

Zeldes (1989) provides a numerical solution to the optimal consumption problem with momentary utility exhibiting constant relative risk aversion. The results of his simulations show that, when labor income is uncertain, optimal consumption deviates from the corresponding certainty equivalence level in, among others, the following ways:

(i) The marginal propensity to consume out of wealth, when labor income is uncertain, is consistently larger than that under the certainty equivalence. The difference between the two is especially significant when the amount of the (certain) nonhuman wealth is low relative to the expected future (uncertain) labor income.

(ii) The expected growth rate of consumption, when labor income is uncertain, is consistently higher than that under the certainty equivalence. The difference between the two also depends on the amount of the (certain) nonhuman wealth relative to the expected future (uncertain) labor income.

Based on these simulation results, Zeldes concluded that in determining optimal consumption levels, current assets play a much more important role than risky future labor incomes. Empirical studies, if based on the certainty equivalence, are con-

[2]And when certain restrictive assumptions are imposed on both the consumption and income processes. This point is not explicitly emphasized when he derived the simple result that the precautionary saving can be represented by a constant.

sequently inadequate. For example, "excess sensitivity" of consumption to transitory income and "excess growth" of consumption, which appear to be contradictory to the optimal consumption behavior under certainty equivalence, may actually be consistent with optimal consumption behavior which takes labor income uncertainties into account.

Zeldes also suggested a possible remedy of the consumption function for the purpose of empirical studies. That is, to put a weight $\chi(\cdot)$ less than one on human wealth before adding it to nonhuman wealth, and let α, the marginal consumption out of total wealth (which includes nonhuman wealth and present discounted and risk adjusted future labor incomes), to be a free parameter. α is possibly larger than its counterpart in the certainty equivalence case, reflecting a larger marginal propensity to consume out of wealth which is consistent with the result obtained in the simulation. After these adjustments, the approximate consumption function can be written as

$$C_t = \alpha\{(1+r)A_{t-1} + \chi(\cdot) \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i E_t W_{t+i}\}, \tag{10}$$

where $\chi(\cdot)$ is a function of, among other variables, the ratio of nonhuman wealth to expected future labor income. The higher the nonhuman wealth is relative to the expected future labor income, the less risky the consumer feels, and the higher $\chi(\cdot)$ should be.

D. Alternative Approximate Consumption Function with Stochastic Labor Income

For the purpose of empirical studies, the approximate consumption function (10) suggested by Zeldes is a valid compromise between tractability for practical purposes and unavailability of closed form solution when the utility is CRRA. It respects the optimization principle to the extent of discounting uncertain present value of expected future labor incomes more than it does with the current assets. An added justification is offered by the fact that any first order deviation of the choice variable from the optimality, as long as it is feasible in the sense of satisfying the life-time budget constraint, will result in a utility loss only of second order in terms of the deviation of the choice variable (Cochrane 1989).

Equation (10), however, is most appropriate in describing situations when the labor income risks in all future periods are the same, regardless how far in the future they are. For example, labor income is a constant (the unconditional mean) plus a random shock term which is independently and identically distributed, as was assumed by Zeldes in his simulations. In practice,[3] however, labor income is usually considered to be an integrated process (Mankiw and Shapiro 1985). Indeed, per capita real disposable labor income of US can well be characterized by the ARIMA (1, 1, 0) process[4]

$$\underset{(3.2)}{\Delta W_t = 8.2} + \underset{(5.5)}{0.442}\Delta W_{t-1} + \varepsilon_t \quad (t\text{-statistics are in parenthesis})$$

$$\sigma_\varepsilon = 25.2$$

When the labor income process has a unit root, shocks to the labor income will be permanent. Consequently, the further away in the future, the more risky is the expected future labor income perceived by the consumer. This suggests that a more realistic way to approximate the consumption function is to discount expected future labor income more when it is further away from the current period. I therefore take the following consumption function as the starting point of my empirical analysis,

$$C_t = \alpha\{(1+r)A_{t-1} + \sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^i E_t W_{t+i}\}. \tag{11}$$

The discount factor $\{1/(1+\beta)\}^i$ in the above expression can be thought of as being composed of two parts, i.e., $\{1/(1+\beta)\}^i = \{1/(1+r)\}^i \cdot (1-p)^i$, where $\{1/(1+r)\}^i$ is used to discount expected i-th period labor income to its present discounted value, while $(1-p)^i$ is a risk discount factor of the expected i-th period labor income because it is more risky than those of the earlier periods, given that labor income is an integrated process. When the consumer perceives substantial risk in his or her labor income, we may expect β to be significantly greater than r, while p, which equals $(\beta - r)/(1+\beta)$, to be significantly greater than zero. In particular, β should be a function of the ratio of nonhuman wealth to expected future labor income. The lower the nonhuman wealth is relative to the expected future labor income, the more risk the consumer perceives, and the larger β should be relative to r.

[3]Especially when aggregate data are used in the empirical analysis.
[4]See Campbell and Deaton (1989).

The formulation of consumption function as in equation (11) was also emphasized by Hayashi (1982) for the purpose of dealing with stochastic labor income. He motivate the formulation from a different perspective, because the simulations of Zeldes that I rely on did not appear until the rapid development of computing technology recently made them possible. Hayashi's empirical results are also different from what I get in this paper. I believe this is due to the short data set that he used in his empirical analysis.

III. Test Precautionary Saving in the Approximate Consumption Function

A. First Order Implication of the Approximate Consumption Function

Similar to the testing of permanent income hypothesis under certainty equivalence, we cannot estimate equation (11) directly because the expected values of future labor income are unobservable. In order to use the exiting techniques of estimating and testing rational expectation models, I derive first order implications of the approximate consumption function and test the time series restrictions imposed on the data by those conditions. Parameter estimates of α and β are also obtained from those conditions. Based on these estimates, the hypotheses of a modified version of the Zeldes' (and Hayashi's) model are subsequently tested against those of the certainty equivalence. Namely, the alternative hypotheses that $\beta > r$ and $\alpha > r/(1+r)$ are tested against the null hypotheses that $\beta = r$ and/or $\alpha = r/(1+r)$.

Lagging equation (11) one period, multiplying both sides of the lagged equation by $1+\beta$, subtracting the resulting equation from equation (11), and rearranging yields[5]

$$\begin{aligned} \Delta \ln C_t = (\beta - \alpha - \alpha\beta) + \left\{ \frac{\alpha(r-\beta)}{r} \right\} \left\{ \frac{Y_t - W_t}{C_{t-1}} \right\} \\ + \alpha \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) \left(\frac{W_{t+i}}{C_{t-1}} \right), \end{aligned} \tag{12}$$

where Y_t is the total disposable income in period t, which includes both labor income and asset income.

[5]See Appendix for detailed derivation.

B. First Order Implication: Nesting Liquidity Constraints

Previous studies of the permanent income hypothesis based on certainty equivalence indicate that the assumption of some consumers being bound by liquidity constraints is well supported by evidence in the US and most OECD countries (Campbell and Mankiw 1989, 1990, 1991). Following the hypothesis in that literature, I nest liquidity constraints by assuming a fraction λ of total disposable labor income in the economy accrues to the liquidity constrained consumers (type 1) who consume their current disposable labor income, while the remainder $1-\lambda$ accrues to individuals who are permanent-income consumers (type 2), setting their consumption level according to equation (11).[6] More specifically, let W_t be the total disposable labor income in period t. If W_{1t} represents the part of the W_t that accrues to the type 1 consumers, while the rest accrues to the type 2 consumers, then

$$C_{1t}=W_{1t}=\lambda W_t$$

$$C_{2t}=\alpha\{(1+r)A_{t-1}+\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^i E_tW_{2,\,t+i}\}$$

$$=\alpha\{(1+r)A_{t-1}+(1-\lambda)\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^i E_tW_{t+i}\},$$

where C_{1t} and C_{2t} are the consumption in period t of the two types of consumers respectively. Since the type 1 consumers are liquidity constrained, I assume they have no assets so that all assets in the economy are held by type 2 consumers. The type 2 consumers' t-th period budget constraint is

$$A_t+C_{2t}=(1+r)A_{t-1}+W_{2t}=(1+r)A_{t-1}+(1-\lambda)W_t,$$

hence the t-th period aggregate budget constraint is

$$A_t+C_t=(A_t+C_{2t})+C_{1t}$$

$$=(1+r)A_{t-1}+(1-\lambda)W_t+\lambda W_t=(1+r)A_{t-1}+W_t.$$

Total consumption is the sum of consumption by both types of consumers

[6]There are two types of liquidity constraints. One type is quantity constraints for credit, while the other is that different consumers face different interest rates. Here I model only the first type.

$$C_t=C_{1t}+C_{2t} = \lambda W_t + \alpha \{(1+r)A_{t-1} +(1-\lambda) \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta}\right)^i E_t W_{t+i}\}. \quad (13)$$

Lagging equation (13) on period, multiplying both sides of the lagged equation by $1+\beta$, subtracting the resulting equation from equation (13), and rearranging yields[7]

$$\begin{aligned} \Delta \ln C_t = & (\beta - \alpha - \alpha\beta) + \left\{ \frac{\alpha(r-\beta)}{r} \right\} \left(\frac{Y_t - W_t}{C_{t-1}} \right) \\ & + \lambda \left(\frac{W_t}{C_{t-1}} \right) - \lambda(1+\beta)(1-\alpha) \left(\frac{W_{t-1}}{C_{t-1}} \right) \\ & + \alpha(1-\lambda) \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) \left(\frac{W_{t+i}}{C_{t-1}} \right). \end{aligned} \quad (14)$$

C. Estimation and Testing: Some Econometric Issues

In order to estimate the parameters in equations (12) and (14), I rewrite them in the more succinct regression forms as

$$\Delta \ln C_t = (\beta - \alpha - \alpha\beta) + \left\{ \frac{\alpha(r-\beta)}{r} \right\} \left(\frac{Y_t - W_t}{C_{t-1}} \right) + e_t, \quad (12a)$$

and

$$\begin{aligned} \Delta \ln C_t = & (\beta - \alpha - \alpha\beta) + \left\{ \frac{\alpha(r-\beta)}{r} \right\} \left(\frac{Y_t - W_t}{C_{t-1}} \right) \\ & + \lambda \left(\frac{W_t}{C_{t-1}} \right) - \lambda(1+\beta)(1-\alpha) \left(\frac{W_{t-1}}{C_{t-1}} \right) + \varepsilon_t, \end{aligned} \quad (14a)$$

where

$$e_t = \alpha \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) \left(\frac{W_{t+i}}{C_{t-1}} \right),$$

and

$$\varepsilon_t = \alpha(1-\lambda) \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) \left(\frac{W_{t+i}}{C_{t-1}} \right).$$

Both regression equations are nonlinear, but only in their parameters. One way of proceeding is to estimate the unrestricted linear models

$$\Delta \ln C_t = A + B \cdot \left(\frac{Y_t - W_t}{C_{t-1}} \right) + e_t, \quad (12b)$$

and

$$\Delta \ln C_t = A + B \cdot \left(\frac{Y_t - W_t}{C_{t-1}} \right) + C \cdot \left(\frac{W_t}{C_{t-1}} \right) + D \cdot \left(\frac{W_{t-1}}{C_{t-1}} \right) + \varepsilon_t. \quad (14b)$$

[7]See Appendix for detailed derivation.

Because the parameters in equation (12a) are exactly identified, I could estimate A and B in equation (12b) first, and then solve for α and β from the estimates of A and B to obtain their point estimates. For equation (14a), because it is overidentified, the nonlinear overidentifying restrictions involved should be tested if I estimate the linear equation (14b). Considering the fact that equation (12a) and (14a) are well specified and are relatively simple nonlinear models, I therefore estimate the parameters directly by the nonlinear model (12a) and restricted nonlinear model (14a). An advantage of doing so is that I can perform statistical inferences directly on the parameters, rather than just obtain their point estimates, since I will also obtain the standard errors of the parameter estimates.

In estimating equation (12a) and (14a), several other issues need to be taken into consideration. First, the error terms are contemporaneous with the regressors, and consequently may be correlated with them. Moreover, because

$$\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i}(E_t-E_{t-1})\,W_{t+i},$$

is likely to be heteroskedastic, e_t and ε_t may be so too even though after being scaled by C_{t-1} the heteroskedasticity involved can be reduce to some extent.[8]

Because of these difficulties, the conventional method of non-linear least square cannot be used here because it will yield inconsistent estimates. The method I use in this paper is the generalized method of moments (GMM, see Hansen 1982; Hansen and Singleton 1982). It can yield parameter estimates and standard errors that are consistent even when the error term is correlated with the regressors and is heteroskedastic and serially correlated.

As in the estimation of many other rational expectation models, the appropriateness of GMM in this occasion is based on the fact that $E_{t-1}e_t=0$ and $E_{t-1}\varepsilon_t=0$, which follows from the law of the iterated projections. The orthogonality conditions

$$E(Z_{t-1}e_t)=0 \text{ and } E(Z_{t-1}\varepsilon_t)=0$$

[8]The error terms may also have a first order moving average structure. This is possibly due to the following facts: (i) there are measurement errors in C_t, Y_t, W_t, (ii) there are white noise transitory consumption caused by preference shocks, (iii) time aggregation or time averaging of data (Working 1960). These effects turn out to be unimportant for data used in this paper. See Section Ⅳ.A for more details.

should therefore hold on the data, where Z_{t-1} is a constant or any variable that belongs to the consumer's information set at $t-1$. When there are more instruments (hence more orthogonality conditions) than the estimated parameters, GMM also provides an asymptotic test of the overidentifying restrictions imposed on the model. The test statistic J has a χ^2 distribution with degrees of freedom equaling the number of moment conditions less the number of estimated parameters.

D. The Data

Seasonally adjusted quarterly US data are used in this paper to estimate (12a) and (14a). The data runs from 1953:2 to 1984:4 and are constructed by Blinder and Deaton (1985) from the US National Income and Product Account. They made several sensible adjustments to the data, which can be found in both their paper and that of Campbell (1987).[9] The series I use from their data set are real per capita total disposable income, real per capita disposable labor income, and real per capita consumption of non-durables and services. The consumption measure I use in estimation is real per capita consumption of nondurables and services divided by 0.7855. The scale factor 0.7855 is the sample mean of the ratio of total consumption spending to those on nondurables and services.

The real interest rate is constructed from the three month treasury bill rate in the secondary market and the consumer price index of all items by urban consumers. Both are obtained from the CITIBASE data tape (FYGM3 for treasury bill rate, PUNEW for CPI).

[9]They are, briefly, (i) removing the 1975 tax rebate from the disposable income series, (ii) subtracting consumer interest payments to business from NIPA disposable income series, (iii) adding personal nontax payments to state and local governments to both disposable income and the consumption series, (iv) treating clothing and shoes as durables, (v) dividing proprietors' income and personal income taxes, which are not done in NIPA, to capital income and labor income according the ratio of the two in the rest of the economy, (vi) deducting social insurance contributions from labor income, and (vii) consumer spending deflator used to construct real per capita data are adjusted in the same way as consumption.

IV. Empirical Results and Discussion

A. Empirical Results

With a constant and twice to fourth lagged $\Delta \ln C_t$ and $(Y_t - W_t)/C_{t-1}$ as instruments,[10] equation (12a) is estimated using generalized method of moments. When (12a) is estimated, the real rate of interest r in it is fixed as a constant. Eight different values are assigned to r, ranging from 0.10% per quarter to 1.25% per quarter.[11] This covers a wide range of possible values for real interest rate, including the sample mean of the real 3-month treasury bill rates for the 1953:2 – 1984:4 sample period, which is around 0.25% per quarter. The results of the estimation are listed in Table 1.

The claim that consumers discount expected future labor income at a higher rate than the real interest rate is supported by the results. β estimates are greater than r for all eight possible r values at less than 1% significance level. α, the marginal propensity to consume out of total wealth (nonhuman wealth plus present discounted and risk adjusted future labor incomes), is also estimated to be greater than $r/(1+r)$ for all eight values of r, with significance levels all below or around 5%. All these are consistent with the simulation results obtained by Zeldes (1989). The J-statistics in the table is asymptotically distributed as χ^2 with 5 degrees of freedom, because there are 7 moment conditions and 2 parameters to be estimated.[12] An unsatisfactory result in estimating equation (12a) is that the J-statistic is too large (significant at only slightly more than the 5% level) to accept the model, even though α and β estimates turn out to be quite reasonable.

The fact that equation (12a) does not provide a good fit to the data is understandable, because we have not taken the behavior of liquidity constrained consumers into account. By nesting the behavior of the liquidity constrained consumers, equation (14a) may conceivably increase its goodness of fit relative to that of equation (12a).

[10] I also experimented with other instruments, which yielded similar results. Results presented here are therefore robust to various choices of instruments.

[11] Or from 0.4% per annum to 5.0% per annum.

[12] Because (12a) is exactly identified, estimations with different r values all produce the same J-statistic, because the underlying linear model (12b) is the same for all estimations.

TABLE 1

ESTIMATION OF $\Delta \ln C_t = (\beta - \alpha - \alpha\beta) + \left\{\frac{\alpha(r-\beta)}{r}\right\}\left(\frac{Y_t - W_t}{C_{t-1}}\right) + e_t$:GMM

r (1)	α (2)	se(α) (3)	p-value $\left(\alpha > \frac{r}{1+r}\right)$ (4)	β (5)	se(β) (6)	p-value ($\beta > r$) (7)
0.10%	0.241%	0.085%	4.896%	1.059%	0.248%	0.006%
0.15%	0.343%	0.115%	4.720%	1.162%	0.278%	0.014%
0.20%	0.437%	0.141%	4.628%	1.258%	0.305%	0.026%
0.25%	0.526%	0.164%	4.584%	1.349%	0.328%	0.041%
0.50%	0.923%	0.253%	4.648%	1.754%	0.421%	0.144%
0.75%	1.272%	0.318%	4.857%	2.114%	0.490%	0.269%
1.00%	1.596%	0.370%	5.090%	2.450%	0.547%	0.400%
1.25%	1.902%	0.414%	5.321%	2.770%	0.595%	0.532%
	J-stat(5) = 10.622			p-value(J) = 5.941%		
0.4%	0.965%			4.237%		
0.6%	1.372%			4.649%		
0.8%	1.749%			5.033%		
1.0%	2.106%			5.396%		
2.0%	3.690%			7.017%		
3.0%	5.088%			8.457%		
4.0%	6.383%			9.801%		
5.0%	7.609%			11.082%		

Notes: 1. Quarterly real interest rate r is taken to be a fixed constant with different values listed in column 1.
2. The values of r, α and β in the first panel are measured in percentage per quarter, while those in the last panel are their corresponding values measured in percentage per annum.
3. Instruments used in estimation are C and twice to fourth lagged $\Delta \ln C_t$ and $(Y_t - W_t)/C_{t-1}$.
4. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).
5. The J-statistic is asymptotically distributed as χ^2 with 5 degrees of freedom, because there are 7 moment conditions and 2 parameters to be estimated.

The results of GMM estimation of equation (14a), using the same eight values of r as in Table 1, and a set of instruments including a constant and twice to fourth lagged $\Delta \ln C_t$, $(Y_t - W_t)/C_{t-1}$, W_t/C_{t-1} and W_t/C_t are listed in Table 2.

TABLE 2

ESTIMATION OF $\Delta \ln C_t = (\beta - \alpha - \alpha\beta) + \left\{\frac{\alpha(r-\beta)}{r}\right\}\left(\frac{Y_t - W_t}{C_{t-1}}\right) + \lambda\left(\frac{W_t}{C_{t-1}}\right) - \lambda(1+\beta)(1-\alpha)\left(\frac{W_{t-1}}{C_{t-1}}\right) + \varepsilon_t$: GMM

r	α	se(α)	p-value $\left(\alpha > \frac{r}{1+r}\right)$	β	se(β)	p-value ($\beta > r$)	λ	se(λ)	p-val (λ)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0.10%	0.180%	0.077%	14.978%	0.945%	0.243%	0.025%	0.317	0.0777	0.000
0.15%	0.261%	0.106%	14.757%	1.027%	0.271%	0.062%	0.317	0.0777	0.000
0.20%	0.337%	0.131%	14.630%	1.104%	0.296%	0.113%	0.317	0.0777	0.000
0.25%	0.410%	0.153%	14.563%	1.178%	0.318%	0.176%	0.317	0.0777	0.000
0.50%	0.746%	0.236%	14.615%	1.521%	0.404%	0.578%	0.317	0.0777	0.000
0.75%	1.053%	0.296%	14.877%	1.835%	0.467%	1.011%	0.317	0.0777	0.000
1.00%	1.343%	0.343%	15.169%	2.135%	0.518%	1.419%	0.317	0.0777	0.000
1.25%	1.622%	0.381%	15.452%	2.424%	0.560%	1.792%	0.317	0.0777	0.000
J-stat(10) = 12.243						p-value(J) = 26.913%			
0.4%	0.721%			3.780%					
0.6%	1.043%			4.106%					
0.8%	1.348%			4.416%					
1.0%	1.641%			4.713%					
2.0%	2.985%			6.083%					
3.0%	4.212%			7.341%					
4.0%	5.371%			8.539%					
5.0%	6.488%			9.697%					

Notes: 1. Quarterly real interest rate r is taken to be a fixed constant with different values listed in column 1.

2. The values of r, α and β in the first panel are measured in percentage per quarter, while those in the last panel are their corresponding values measured in percentage per annum.

3. Instruments used in estimation are C and twice to fourth lagged $\Delta \ln C_t$, $(Y_t - W_t)/C_{t-1}$, W_t/C_{t-1} and W_t/C_t.

4. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).

5. The J-statistic is asymptotically distributed as χ^2 with 10 degrees of freedom, because there are 13 moment conditions and 3 parameters to be estimated.

Estimates of α and β for various values of r are slightly lower than their counterparts in Table 1,[13] after the liquidity constrained consumers are taken into account. Nevertheless, β is still significantly greater than r for all values of r. This confirms the hypothesis that consumers, as long as they are not liquidity constrained, discount expected future labor income at a higher rate than the real interest rate. The marginal propensity to consume out of total wealth α is still estimated to be greater than r for all values of r, but not significantly so at conventional levels. The liquidity constrained consumers are estimated to consume about one third of the total disposable labor income in the economy. This result is consistent with the findings in the other relevant studies. Furthermore, the J-statistic, being insignificant in this case, indicates that modelling aggregate consumption behavior by equation (14a) cannot be rejected by the data.

Two problems that deserve some more scrutinies are the following: First, although I have been conservative in estimating equation (12a) and (14a) by treating the error terms e_t and ε_t as MA(1),[14] and using variables lagged at least twice as instruments, the fact that

$$e_t = \alpha \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) \left(\frac{W_{t+i}}{C_{t-1}} \right),$$

and

$$\varepsilon_t = \alpha (1 - \lambda) \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) \left(\frac{W_{t+i}}{C_{t-1}} \right),$$

does not strictly imply $E_{t-2}(e_t) = 0$ and $E_{t-2}(\varepsilon_t) = 0$, because C_{t-1} may be correlated with

$$\sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) W_{t+i},$$

from the perspective of period $t-2$.

To make sure that results in Table 1 and 2 are free of the above

[13] α and β are smaller in Table 2 than Table 1. This is understandable because results in Table 1 include those consumers whose consumption is determined by their current income. This tend to bias the estimation and make α and β in equation (12a) appear to be greater than those in equation (14a). The same principle applies in the magnitudes of α and β in Tables 3, 5, and 7 vs. in Tables 4, 6 and 8.

[14] See footnote 8.

problem, I divide the equations[15]

$$\Delta C_t=(\beta-\alpha-\alpha\beta)C_{t-1}+\left\{\frac{\alpha(r-\beta)}{r}\right\}(Y_t-W_t)$$
$$+\alpha\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i}(E_t-E_{t-1})W_{t+i},$$

and

$$\Delta C_t=(\beta-\alpha-\alpha\beta)C_{t-1}+\left\{\frac{\alpha(r-\beta)}{r}\right\}(Y_t-W_t)+\lambda W_t-\lambda(1+\beta)(1-\alpha)W_{t-1}$$
$$+\alpha(1-\lambda)\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i}(E_t-E_{t-1})W_{t+i},$$

by W_{t-2} instead of C_{t-1}. The resulting equations are therefore

$$\frac{\Delta C_t}{W_{t-2}}=(\beta-\alpha-\alpha\beta)\frac{C_{t-1}}{W_{t-2}}+\frac{\alpha(r-\beta)}{r}\left(\frac{Y_t-W_t}{W_{t-2}}\right)+e_t', \quad (12c)$$

and

$$\frac{\Delta C_t}{W_{t-2}}=(\beta-\alpha-\alpha\beta)\frac{C_{t-1}}{W_{t-2}}+\frac{\alpha(r-\beta)}{r}\left(\frac{Y_t-W_t}{W_{t-2}}\right)$$
$$+\lambda\frac{W_t}{W_{t-2}}-\lambda(1+\beta)(1-\alpha)\frac{W_{t-1}}{W_{t-2}}+\varepsilon_t', \quad (14c)$$

where

$$e_t'=\alpha\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i}(E_t-E_{t-1})\left(\frac{W_{t+i}}{W_{t-2}}\right)$$

and

$$\varepsilon_t'=\alpha(1-\lambda)\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i}(E_t-E_{t-1})\left(\frac{W_{t+i}}{W_{t-2}}\right).$$

Both e_t' and ε_t' are truly orthogonal to the consumer's information set as of period $t-2$.

The results of estimating equations (12c) and (14c) are listed in Tables 3 and 4, which lead to the similar conclusions we have obtained earlier from Tables 1 and 2. This tells us that either the error terms hardly have any first order moving average structure,[16] or the correlation between C_{t-1} and

$$\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i}(E_t-E_{t-1})W_{t+i},$$

from the perspective of period $t-2$, is too small to matter.

[15] They are the intermediate results (equations (A6) and (A11)) of the derivations contained in the appendix.

[16] Possibly because all the contributing effects approximately cancel with each other.

TABLE 3

ESTIMATION OF $\left(\frac{\Delta C_t}{W_{t-2}}\right)$

$=(\beta-\alpha-\alpha\beta)\left(\frac{C_{t-1}}{W_{t-2}}\right)+\left\{\frac{\alpha(r-\beta)}{r}\right\}\left(\frac{Y_t-W_t}{W_{t-2}}\right)+e_t'$:GMM

r	α	se(α)	p-value $\left(\alpha>\frac{r}{1+r}\right)$	β	se(β)	p-value ($\beta>r$)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.10%	0.205%	0.102%	15.209%	0.955%	0.266%	0.064%
0.15%	0.294%	0.138%	14.910%	1.045%	0.302%	0.151%
0.20%	0.377%	0.169%	14.735%	1.130%	0.333%	0.264%
0.25%	0.456%	0.197%	14.636%	1.210%	0.361%	0.393%
0.50%	0.815%	0.301%	14.618%	1.576%	0.470%	1.100%
0.75%	1.137%	0.376%	14.859%	1.907%	0.549%	1.762%
1.00%	1.439%	0.435%	15.147%	2.219%	0.613%	2.345%
1.25%	1.727%	0.484%	15.432%	2.519%	0.667%	2.854%
	J-stat(8) = 14.994			p-value(J) = 5.926%		
0.4%	0.820%			3.822%		
0.6%	1.175%			4.181%		
0.8%	1.509%			4.520%		
1.0%	1.826%			4.842%		
2.0%	3.259%			6.304%		
3.0%	4.547%			7.628%		
4.0%	5.754%			8.876%		
5.0%	6.909%			10.077%		

Notes: 1. Quarterly real interest rate r is taken to be a fixed constant with different values listed in column 1.

2. The values of r, α and β in the first panel are measured in percentage per quarter, while those in the last panel are their corresponding values measured in percentage per annum.
3. Instruments used in estimation are C and twice to fourth lagged $\Delta \ln C_t$ and $(Y_t - W_t)/C_{t-1}$.
4. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).
5. The J-statistic is asymptotically distributed as χ^2 with 8 degrees of freedom, because there are 10 moment conditions and 2 parameters to be estimated.

The second problem deserving some more discussions is that in all of the estimations so far, I have treated r as a fixed constant. Although various different values have been assigned to r, no explicit

TABLE 4

ESTIMATION OF $\left(\frac{\Delta C_t}{W_{t-2}}\right)=(\beta-\alpha-\alpha\beta)\left(\frac{C_{t-1}}{W_{t-2}}\right)+\left\{\frac{\alpha(r-\beta)}{r}\right\}\left(\frac{Y_t-W_t}{W_{t-2}}\right)$
$+\lambda\left(\frac{W_t}{W_{t-2}}\right)-\lambda(1+\beta)(1-\alpha)\left(\frac{W_{t-1}}{W_{t-2}}\right)+\varepsilon_t'$: GMM

r	α	se(α)	p-value $\left(\alpha>\frac{r}{1+r}\right)$	β	se(β)	p-value ($\beta>r$)	λ	se(λ)	p-val (λ)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0.10%	0.185%	0.079%	13.971%	0.959%	0.266%	0.062%	0.343	0.0647	0.000
0.15%	0.268%	0.108%	13.785%	1.043%	0.295%	0.124%	0.343	0.0647	0.000
0.20%	0.346%	0.133%	13.686%	1.122%	0.321%	0.202%	0.343	0.0647	0.000
0.25%	0.420%	0.156%	13.644%	1.197%	0.343%	0.289%	0.343	0.0647	0.000
0.50%	0.762%	0.242%	13.775%	1.546%	0.433%	0.782%	0.343	0.0647	0.000
0.75%	1.072%	0.305%	14.084%	1.864%	0.498%	1.267%	0.343	0.0647	0.000
1.00%	1.365%	0.353%	14.411%	2.167%	0.551%	1.709%	0.343	0.0647	0.000
1.25%	1.647%	0.393%	14.719%	2.459%	0.595%	2.105%	0.343	0.0647	0.000
	J-stat(13) = 13.642					p-value(J) = 39.954%			
0.4%	0.742%			3.837%					
0.6%	1.071%			4.170%					
0.8%	1.383%			4.486%					
1.0%	1.681%			4.790%					
2.0%	3.047%			6.182%					
3.0%	4.289%			7.457%					
4.0%	5.461%			8.667%					
5.0%	6.587%			9.837%					

Notes: 1. Quarterly real interest rate r is taken to be a fixed constant with different values listed in column 1.

2. The values of r, α and β in the first panel are measured in percentage per quarter, while those in the last panel are their corresponding values measured in percentage per annum.

3. Instruments used in estimation are C and twice to fourth lagged $\Delta \ln C_t$, $(Y_t-W_t)/C_{t-1}$, W_t/C_{t-1} and W_t/C_t.

4. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).

5. The J-statistic is asymptotically distributed as χ^2 with 13 degrees of freedom, because there are 16 moment conditions and 3 parameters to be estimated.

relationships have been established between the data on real interest rates and the r used in those previous estimations. This could be problematic when the hypotheses posed in this paper are tested. If there are large sample errors associated with the estimate of real interest rate, we may not be able to reject the null hypotheses of $\alpha = r/(1+r)$ and $\beta = r$ in favor of the alternative hypotheses of $\alpha > r/(1+r)$ and $\beta > r$.

To address and clarify this issue, I use the following two equations, (15) and (16), to estimate the values of r. Equation (15) is based on the assumption that the observed real interest rate is a constant (unconditional) mean plus a random disturbance term.[17] If we denote the before tax real three month treasury bill rate[18] as r_t, we then have

$$r_t = r + u_t, \tag{15}$$

where u_t is the error term associated with r_t.

The second equation that characterizes the real interest rate is based on the $(t-1)$-th period budget constraint

$$C_{t-1} + A_{t-1} = (1+r)A_{t-2} + W_{t-1}.$$

Substitute the A_{t-1} and A_{t-2} terms in the above equation by $(1/r)(Y_t - W_t)$ and $(1/r)(Y_{t-1} - W_{t-1})$ respectively, and rearranging yields

$$\Delta Y_t - \Delta W_t = r(Y_{t-1} - C_{t-1}).$$

If we assume that there are measurement errors in Y, W or C, we then have the regression equation

$$\Delta Y_t - \Delta W_t = r(Y_{t-1} - C_{t-1}) + v_t, \tag{16}$$

where v_t is the error term associated with the above equation. Equation (16) is a behavioral relationship which identifies the underlying real interest rate r on which the consumers' consumption decisions are based. Incidently, it is independent of whether some consumers are liquidity constrained, and of how many consumers are consuming their current labor income because of the liquidity constraints.

The results of jointly estimating equations (12a) and (15) by GMM, using a constant and twice to fourth lagged $\Delta \ln C_t$ and $(Y_t - W_t)/C_{t-1}$ as instruments, are listed in Table 5. Quarterly real

[17]This will be proven, by both the results of estimation and data plot, to be a poor approximation.

[18]I have also experimented with the real after tax three month treasury bill rate. The resulting estimate of r turned out to be a negative number.

interest rate r is estimated to be around 0.175%. The null hypotheses can be rejected at the conventional significance level. The J-statistic for the overidentifying restrictions is barely insignificant at the 5% level. The large J-statistic here is likely due to the fact that equation (15) is not an accurate approximation of the behavior of real interest rate.[19] This fact can be clearly seen from the time series plot of r_t in Figure 1, especially for the 1970's and 1980's.

Similar results are obtained when liquidity constrained consumers are included in the model, namely, when equations (14a) and (15) are jointly estimated by GMM. Quarterly real interest rate r is estimated to be around 0.256%. The fraction of the liquidity constrained consumers, λ, is estimated to be around 19%. The null hypotheses can also be rejected at the conventional significance level. The J-statistic for the overidentifying restrictions is significant at the 5% level, probably because of the same reason discussed above.

Joint estimation of equations (12a) and (16), and equations (14a) and (16), by GMM yields substantially higher estimate of r, as listed in Table 7 and Table 8 respectively. This is no doubt because the real rate of return on nonhuman wealth is risky. The hypotheses that $\alpha = r/(1+r)$ and $\beta = r$ can be rejected decisively by the results in Table 7, and less so for those in Table 8. The J-statistics for the overidentifying restrictions are insignificant at any conventional level for both estimations, reflecting an improved goodness of fit for equation (16) relative to that for equation (15).

B. Discussion of the Empirical Results

The empirical finding that consumers, because of their prudence or decreasing absolute risk aversion, discount expected future labor income at a higher rate than the real interest rate has important implications in explaining the two most important empirical anomalies in the consumption literature, namely, the excess sensitivity and excess smoothness puzzles (Deaton 1987; Campbell and Deaton 1989 and West 1988).

First, because the approximate consumption function implies

[19]Of course, it may also result from the exclusion of liquidity constrained consumer in equation (12a). The results of Table 6 discussed in the next paragraph, however, does not support this conjecture.

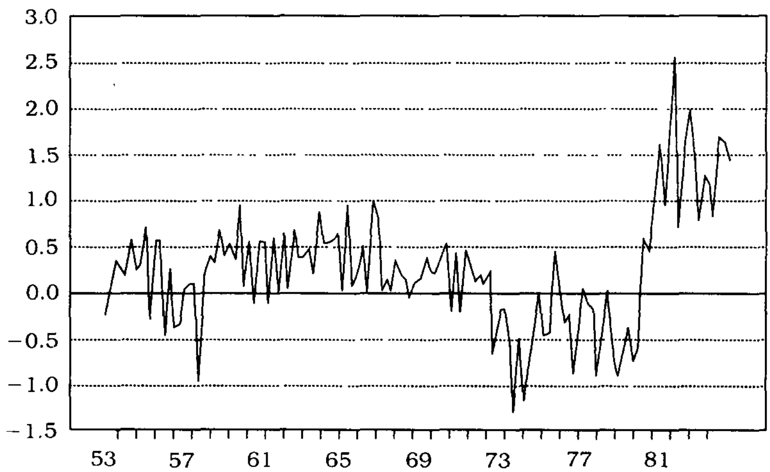

FIGURE 1

REAL INTEREST RATE

TABLE 5

SYSTEM ESTIMATION OF $\Delta \ln C_t = (\beta - \alpha - \alpha\beta) + \left\{\frac{\alpha(r-\beta)}{r}\right\}\left(\frac{Y_t - W_t}{C_{t-1}}\right) + e_t$ AND $r_t = r + u_t$: GMM

		s.e.	t-stat	significance level
r	0.175%	0.040%	4.374	0.001%
α	0.684%	0.134%	5.090	0.000%
β	1.997%	0.279%	7.154	0.000%
	J-statistic(11) = 18.325		p-value(J) = 7.434%	

Notes: 1. Initial values for the parameters in estimation are $r = 0.2\%$, $\alpha = 0.2\%$ and $\beta = 0.2\%$.

2. Instruments used in estimation are C and twice to fourth lagged $\Delta \ln C_t$ and $(Y_t - W_t)/C_{t-1}$.

3. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).

4. The J-statistic is asymptotically distributed as χ^2 with 11 degrees of freedom, because there are 14 moment conditions and 3 parameters to be estimated.

TABLE 6

SYSTEM ESTIMATION OF $\Delta \ln C_t = (\beta - \alpha - \alpha\beta) + \left\{ \frac{\alpha(r-\beta)}{r} \right\} \left(\frac{Y_t - W_t}{C_{t-1}} \right) + \lambda \left(\frac{W_t}{C_{t-1}} \right) - \lambda(1+\beta)(1-\alpha) \left(\frac{W_{t-1}}{C_{t-1}} \right) + \varepsilon_t$ AND $r_t = r + u_t$: GMM

		s.e.	t-stat	significance level
r	0.256%	0.035%	7.337	0.000%
α	0.870%	0.119%	7.324	0.000%
β	2.092%	0.293%	7.148	0.000%
λ	0.190	0.0669	2.846	0.442%
	J-statistic(22) = 49.628		p-value(J) = 0.066%	

Notes: 1. Initial values for the parameters in estimation are r=0.2%, α = 0.2%, β =0.2% and λ =0.3.

2. Instruments used in estimation are C and twice to fourth lagged $\Delta \ln C_t$, $(Y_t - W_t)/C_{t-1}$, W_t/C_{t-1} and W_t/C_t.

3. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).

4. The J-statistic is asymptotically distributed as χ^2 with 22 degrees of freedom, because there are 26 moment conditions and 4 parameters to be estimated.

TABLE 7

SYSTEM ESTIMATION OF $\Delta \ln C_t = (\beta - \alpha - \alpha\beta) + \left\{ \frac{\alpha(r-\beta)}{r} \right\} \left(\frac{Y_t - W_t}{C_{t-1}} \right) + e_t$ AND $\Delta Y_t - \Delta W_t = r(Y_{t-1} - C_{t-1}) + v_t$: GMM

		s.e.	t-stat	significance level
r	1.845%	0.259%	7.138	0.000%
α	4.897%	0.429%	11.402	0.000%
β	7.043%	0.684%	10.299	0.000%
	J-statistic(23) = 21.942		p-value(J) = 52.375%	

Notes: 1. Initial values for the parameters in estimation are r=0.2%, α = 0.2% and β =0.2%.

2. Instruments used in estimation are C and twice to fourth lagged $\Delta \ln C_t$, $(Y_t - W_t)/C_{t-1}$, $\Delta Y_t - \Delta W_t$ and $Y_t - C_t$.

3. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).

4. The J-statistic is asymptotically distributed as χ^2 with 23 degrees of freedom, because there are 26 moment conditions and 3 parameters to be estimated.

TABLE 8

SYSTEM ESTIMATION OF $\Delta \ln C_t = (\beta - \alpha - \alpha\beta) + \left\{\frac{\alpha(r-\beta)}{r}\right\}\left(\frac{Y_t - W_t}{C_{t-1}}\right)$
$+\lambda\left(\frac{W_t}{C_{t-1}}\right) - \lambda(1+\beta)(1-\alpha)\left(\frac{W_{t-1}}{C_{t-1}}\right) + \varepsilon_t$
AND $\Delta Y_t - \Delta W_t = r(Y_{t-1} - C_{t-1}) + v_t$: GMM

		s.e.	t-stat	significance level
r	1.695%	0.259%	6.541	0.000%
α	2.156%	0.469%	4.594	0.000%
β	2.886%	0.612%	4.712	0.000%
λ	0.241	0.0580	4.150	0.003%
	J-statistic(34)=27.691		p-value(J)=76.913%	

Notes: 1. Initial values for the parameters in estimation are r=0.2%, α= 0.2%, β=0.2% and λ=0.3.
2. Instruments used in estimation are C and twice to fourth lagged $\Delta \ln C_t$, $(Y_t - W_t)/C_{t-1}$, W_t/C_{t-1}, W_t/C_t, $\Delta Y_t - \Delta W_t$ and $Y_t - C_t$.
3. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).
4. The J-statistic is asymptotically distributed as χ^2 with 34 degrees of freedom, because there are 38 moment conditions and 4 parameters to be estimated.

$$C_t = (1-\alpha)(1+\beta)C_{t-1} + \alpha(r-\beta)A_{t-1} + \varepsilon_t'', \tag{17}$$

consumption in period t can be Granger caused by the consumer's asset holding in period $t-1$. This appears to be consistent with Hall's (1978) finding that lagged stock prices have predictive power for consumption.[20] Furthermore, because $rA_{t-1} = Y_t - W_t$, any variable that Granger causes Y_t and/or W_t may also Granger cause C_t. Adding the existence of liquidity constrained consumers only reinforces the arguments above.

Second, because the error term in equation (17) can be written as

$$\varepsilon_t'' = \alpha \sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^i (E_t - E_{t-1})\ W_{t+i},$$

the variance of the consumption innovation can be expressed in the

[20]The effects of stock prices on consumption change have different signs for the different lags, though. The coefficients on the first and third lags are positive, but those on the second and fourth lags are negative.

form of[21]

$$\left[\ \alpha \cdot \left\{P\left(\frac{1}{1+\beta}\right)\right\}^{-1}\right]^2 \cdot \mathrm{var}(\varepsilon_t),$$

where $P(L)W_t = \varepsilon_t$ is the autoregressive representation of the univariate labor income process. Under the hypothesis that real per capita disposable labor income follows the ARIMA(1, 1, 0) process of

$$\Delta W_t = 8.2 + 0.442\,\Delta W_{t-1} + \varepsilon_t,$$

so that,

$$P(L) = 1 - 1.442 \cdot L + 0.442 \cdot L^2,$$

the standard error of the consumption innovation for a unitary labor income innovation can be calculated from point estimates of α and β by $\alpha(1+\beta)^2/\{\beta(0.588+\beta)\}$. When the liquidity constrained consumers are included, the standard error of the consumption innovation should be $(1-\lambda)\alpha(1+\beta)^2/\{\beta(0.588+\beta)\}$.

For reference purpose, if labor income follows a random walk, namely,

$$W_t = W_{t-1} + \varepsilon_t \quad \text{or} \quad P(L) = 1 - L,$$

then the standard errors of the consumption innovation for a unitary labor income innovation will be $\alpha(1+\beta)/\beta$ for the case of no liquidity constraints, and $(1-\lambda)\alpha(1+\beta)/\beta$ for the case when some consumers are liquidity constrained.

Values of these expressions for various point estimates of α, β and λ obtained in the previous regressions are listed in Table 9. Whenever liquidity constrained consumers are included, values of $(1-\lambda)\alpha(1+\beta)^2/\{\beta(0.588+\beta)\}$ and $(1-\lambda)\alpha(1+\beta)/\beta$ (in columns 4, 5, 8 and 9) never exceed 1 for all values of r listed in the table (including those estimated in Tables 5-8) and are rarely more than 0.5 for realistic values of r. For models that do not include liquidity constrained consumers, values of $\alpha(1+\beta)^2/\{\beta(0.588+\beta)\}$ (in columns 2 and 3) exceed 1.0 only when r is higher than 0.75% per quarter (or 3% per annum). For the more realistic values of r (less than 2% per annum), it is always less than 1.0. For the case when labor income is a random walk, $\alpha(1+\beta)/\beta$ (in columns 6 and 7) is again always smaller than 1, and mostly around or below 0.5 for

[21]See the derivation in Flavin (1981) for the certainty equivalence case. The similar conclusion of $\alpha \cdot \{P(1/1+\beta)\}^{-1}$ can be derived for the alternative solution presented here.

TABLE 9

THE STANDARD ERROR OF CONSUMPTION INNOVATION RESULTED FROM A UNITARY LABOR INCOME INNOVATION

$$\alpha \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta}\right)^{i} (E_t - E_{t-1})\ W_{t+i} = \alpha \cdot \left\{ P\left(\frac{1}{1+\beta}\right) \right\}^{-1}$$

$$\text{AND } (1-\lambda)\alpha \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta}\right)^{i} (E_t - E_{t-1})\ W_{t+i} = (1-\lambda)\cdot\alpha\cdot \left\{ P\left(\frac{1}{1+\beta}\right) \right\}^{-1}$$

r	$\frac{\alpha(1+\beta)^2}{\beta(0.588+\beta)}$		$\frac{(1-\lambda)\alpha(1+\beta)^2}{\beta(0.588+\beta)}$		$\frac{\alpha(1+\beta)}{\beta}$		$\frac{(1-\lambda)\alpha(1+\beta)}{\beta}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0.10%	0.389	0.366	0.222	0.216	0.230	0.217	0.132	0.128
0.15%	0.504	0.480	0.296	0.288	0.298	0.284	0.175	0.170
0.20%	0.593	0.570	0.356	0.345	0.352	0.338	0.211	0.205
0.25%	0.666	0.644	0.406	0.393	0.396	0.382	0.241	0.233
0.50%	0.899	0.883	0.573	0.553	0.535	0.525	0.340	0.329
0.75%	1.030	1.020	0.670	0.646	0.614	0.608	0.399	0.385
1.00%	1.116	1.110	0.736	0.708	0.667	0.663	0.439	0.423
1.25%	1.178	1.175	0.783	0.753	0.706	0.703	0.468	0.450
0.175%(From Table 5)	0.586				0.349			
0.256%(From Table 6)			0.576				0.344	
1.845%(From Table 7)	1.210				0.744			
1.695%(From Table 8)			0.974				0.584	

Notes: 1. The values of $\alpha(1+\beta)^2/\{\beta(0.588+\beta)\}$ in columns (2) and (3) are calculated according to the point estimates of α and β listed in Tables 1 and 3.

2. The values of $\alpha(1+\beta)/\beta$ in columns (6) and (7) are calculated according to the point estimates of α and β listed in Tables 1 and 3.

3. The values of $(1-\lambda)\alpha(1+\beta)^2/\{\beta(0.588+\beta)\}$ in columns (4) and (5) are calculated according to the point estimates of α, β and λ listed in Tables 2 and 4.

4. The values of $(1-\lambda)\alpha(1+\beta)/\beta$ in columns (8) and (9) are calculated according to the point estimates of α, β and λ listed in Tables 2 and 4.

the most plausible values of r.

The outcomes of these calculations reveal that, contrary to the case of certainty equivalence solution, the approximate consumption decision rule proposed in this paper predicts that consumption *should be smoother* than the disposable labor income even when the latter can be characterized as an integrated process. This prediction is confirmed by the evidence in the aggregate consumption and disposable labor income data of the US, as long as the real interest rate is not unusually high. In other words, after I modify the optimal consumption so that consumers will, because of their prudence, discount expected future uncertain labor income at a higher rate than the real interest rate, the "excess smoothness" paradox suggested by Deaton (1987) will no longer exist. Adding the existence of the liquidity constrained consumers reduces the standard error of the consumption innovation to an even smaller magnitude of $\alpha \cdot (1-\lambda) \cdot [P\{1/(1+\beta)\}]^{-1}$.

The point here is clearly different from that of Quah (1990). His paper shows that an integrated labor income process can be decomposed into various combinations of permanent and transitory parts which preserve the dynamic property of the univariate labor income process. At least one of those decompositions will give rise to a smooth consumption process if the consumer can recognize the two parts of the labor income. His arguments rationalize the smoothness of aggregate consumption even when the hypothesis that consumption follows the certainty equivalence solution is maintained. My point, however, simply argues that Deaton's paradox may not arise in the scenario of this paper.

V. Concluding Remarks

In this paper, I used an approximation to model the optimal consumption of a representative consumer when he or she faces uncertain labor income. This approximate consumption function is based on the numerical solution (of Zeldes) to the optimal consumption problem with CRRA utility and stochastic labor income, which takes into consideration the precautionary savings of the consumer. It assumes that the consumer discounts expected future labor income at a rate higher than the real interest rate. The further in the future labor income is, the more it should be discounted. The first order implications of the approximate consumption function, with and without the liquidity constrained consumers, are tested using quarterly US data. The evidence lends support to the claims of the approximate consumption function, particularly when liquidity constrained consumers are included.

Even though the model is not an exact solution to the intertemporal optimization problem, it appears to be a promising way of getting around the difficulties involved in obtaining a closed form solution when utility is of the general decreasing absolute risk aversion type. For example, the "excess sensitivity" and "excess smoothness" puzzles are easily understandable in this model, even without resorting to the assumption that the income process is composed of permanent and transitory parts which are observed only by the consumer but not the econometrician. An important purpose of this paper is to provide a starting point for probing the usefulness of this model for empirical purposes.

Many other issues concerning both the validity of the model and its application, if it is valid, are not addressed in this paper. First, the cointegration implication of the model described in Section III can be tested both on US and international (say, OECD countries) data. Second, β should be a function of the ratio of current assets to expected future labor income. Moreover, it may depend on the risk involved in the labor income, which in turn depends on the variance of the labor income and the persistence of the labor income process.[22] As for the idiosyncratic risks, it may depend on, among others, the marginal tax rate on labor income and the profession the consumer is engaged in.

These implications can, in principle, be tested both on international data and panel data. Difficulties may arise, nevertheless, when one uses disaggregated data. For example, some characteristics of consumers are not controllable in constructing a sample. More specifically, a consumer may appear to be in a more risky situation than others, but he may not discount future labor income more simply because he is less prudent and less risk averse than the others. The fact that he is in that situation could be the result of self selection. The selection bias of this kind needs careful treatment should disaggregate data be used in testing theories involving precautionary savings etc.

If the theory proves to be a good description of consumption, an good application of it is to calculate aggregate wealth accumulation in a life-cycle version of the model. A calculation of asset holdings, as the difference between the certainty equivalence consumption and that predicted by the approximate model, by different age

[22]This can be measured, say, by the sum of the coefficients of the moving average representation for ΔW_t as used in Campbell and Mankiw (1987), or by the variance ratio proposed by Cochrane (1988).

groups will give us an idea how much wealth is accumulated due to the prudence or the precautionary saving of the consumers.

Appendix

A. First Order Implication with No Liquidity Constrained Consumers

Consumption in period t is

$$C_t = \alpha\{(1+r)A_{t-1} + \sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i} E_t W_{t+i}\}. \tag{A1}$$

Lag the above equation one period, we get consumption in period $t-1$ as

$$C_{t-1} = \alpha\{(1+r)A_{t-2} + \sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i} E_{t-1} W_{t-1+i}\}. \tag{A2}$$

Multiply both sides of the C_{t-1} expression by $1+\beta$, we get

$$\begin{aligned}(1+\beta)C_{t-1} &= \alpha\{(1+\beta)(1+r)A_{t-2} + \sum_{i=1}^{\infty}\left(\frac{1}{1+\beta}\right)^{i-1} E_{t-1} W_{t+i-1} + (1+\beta)W_{t-1}\} \\ &= \alpha\{(1+\beta)(1+r)A_{t-2} + \sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i} E_{t-1} W_{t+i} + (1+\beta)W_{t-1}\}\end{aligned} \tag{A3}$$

Subtract (3) from (1)

$$\begin{aligned}C_t - (1+\beta)C_{t-1} &= \alpha\{(1+r)A_{t-1} - (1+\beta)(1+r)A_{t-2} - (1+\beta)W_{t-1}\} \\ &\quad + \alpha\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i}(E_t - E_{t-1})W_{t+i} \\ &= \alpha\{(r-\beta)A_{t-1} + (1+\beta)A_{t-1} - (1+\beta)(1+r)A_{t-2} - (1+\beta)W_{t-1}\} \\ &\quad + \alpha\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i}(E_t - E_{t-1})W_{t+i}\end{aligned}$$

$$= \alpha(r-\beta)A_{t-1} - \alpha(1+\beta)C_{t-1} + \alpha\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i}(E_t - E_{t-1})W_{t+i}, \tag{A4}$$

which is

$$C_t = (1-\alpha)(1+\beta)C_{t-1} + \alpha(r-\beta)A_{t-1} + \alpha\sum_{i=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{i}(E_t - E_{t-1})W_{t+i}, \tag{A5}$$

or

$$\Delta C_t = (\beta - \alpha - \alpha\beta) C_{t-1} + \alpha (r - \beta) A_{t-1} + \alpha \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) W_{t+i}, \quad \text{(A6)}$$

so

$$\Delta \ln C_t \simeq \frac{\Delta C_t}{C_{t-1}} = (\beta - \alpha - \alpha\beta) + \alpha (r - \beta) \frac{A_{t-1}}{C_{t-1}}$$
$$+ \alpha \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) \left(\frac{W_{t+i}}{C_{t-1}} \right)$$
$$= (\beta - \alpha - \alpha\beta) + \left| \frac{\alpha (r - \beta)}{r} \right| \left(\frac{r A_{t-1}}{C_{t-1}} \right) + \alpha \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) \left(\frac{W_{t+i}}{C_{t-1}} \right)$$
$$= (\beta - \alpha - \alpha\beta) + \left| \frac{\alpha (r - \beta)}{r} \right| \left(\frac{Y_t - W_t}{C_{t-1}} \right) + \alpha \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) \left(\frac{W_{t+i}}{C_{t-1}} \right). \quad \text{(A7)}$$

B. First Order Implication with Liquidity Constrained Consumers

$$C_t = \lambda W_t + \alpha \{ (1+r) A_{t-1} + (1 - \lambda) \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i E_t W_{t+i} \} \quad \text{(A8)}$$

Lag (11) one period and multiply $1+\beta$ on both sides, we get

$$(1+\beta) C_{t-1} = \lambda (1+\beta) W_{t-1} + \alpha \{ (1+\beta)(1+r) A_{t-2}$$
$$+ (1 - \lambda) \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i E_{t-1} W_{t+i} + (1 - \lambda) W_{t-1} \}. \quad \text{(A9)}$$

Subtract (12) from (11), we have

$$C_t - (1+\beta) C_{t-1} = \lambda W_t - \lambda (1+\beta) W_{t-1} + \alpha (1 - \lambda) \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) W_{t+i}$$
$$+ \alpha \{ (1+r) A_{t-1} - (1+\beta)(1+r) A_{t-2} - (1+\beta)(1 - \lambda) W_{t-1} \}$$
$$= \alpha \{ (r - \beta) A_{t-1} + (1+\beta) A_{t-1} - (1+\beta)(1+r) A_{t-2} - (1+\beta) W_{t-1} + (1+\beta) \lambda W_{t-1} \}$$
$$+ \lambda W_t - \lambda (1+\beta) W_{t-1} + \alpha (1 - \lambda) \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) W_{t+i}$$
$$= \alpha (r - \beta) A_{t-1} - \alpha (1+\beta) C_{t-1} + \alpha (1+\beta) \lambda W_{t-1} + \lambda W_t - \lambda (1+\beta) W_{t-1}$$
$$+ \alpha (1 - \lambda) \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) W_{t+i}, \quad \text{(A10)}$$

or

$$\Delta C_t = (\beta - \alpha - \alpha\beta) C_{t-1} + \alpha (r - \beta) A_{t-1} + \lambda W_t - \lambda (1+\beta)(1 - \alpha) W_{t-1}$$
$$+ \alpha (1 - \lambda) \sum_{i=0}^{\infty} \left(\frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) W_{t+i}, \quad \text{(A11)}$$

and therefore

$$\Delta \ln C_t \simeq \frac{\Delta C_t}{C_{t-1}} = (\beta - \alpha - \alpha\beta) + \left\{\frac{\alpha(r-\beta)}{r}\right\}\left(\frac{Y_t - W_t}{C_{t-1}}\right)$$
$$+ \lambda\left(\frac{W_t}{C_{t-1}}\right) - \lambda(1+\beta)(1-\alpha)\left(\frac{W_{t-1}}{C_{t-1}}\right) \tag{A12}$$
$$+ \alpha(1-\lambda)\sum_{\iota=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{\iota}(E_t - E_{t-1})\left(\frac{W_{t+\iota}}{C_{t-1}}\right).$$

(Received February, 1998; Revised May, 1998)

References

Barsky, Robert B., Mankiw, N. Gregory and Zeldes, Stephen P. "Ricardian Consumers with Keynesian Propensities." *American Economic Review* 76 (1986): 676-91.

Blanchard, Olivier J. "Debt, Deficit, and Finite Horizons." *Journal of Political Economy* 84 (1985): 1161-76.

________, and Mankiw, N. Gregory. "Consumption: Beyond Certainty Equivalence." *American Economic Review* 78 (No. 2 1988): 173-77.

Blinder, Alan S. and Deaton, Angus S. "The Time Series Consumption Function Revisited." *Brookings Papers on Economic Activities* Vol. 2 (1985): 465-511.

Caballero, Ricardo J. "Consumption Puzzles and Precautionary Savings." *Journal of Monetary Economics* 25 (1990): 113-36.

________. "Earnings Uncertainty and Aggregate Wealth Accumulation." *American Economic Review* 81 (No. 4 1991): 859-71.

Campbell, John Y. "Does Saving Anticipate Declining Labor Income." *Econometrica* 55 (No. 6 1987): 1249-73.

________, and Deaton, Angus. "Why Is Consumption So Smooth?" *Review of Economic Studies* 56 (1989): 357-74.

________, Mankiw, N. Gregory "Are Output Fluctuations Transitory?" *Quarterly Journal of Economics* 102 (November 1987): 857-80.

________. "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence." *NBER Macroeconomic Annual, 1989.*

________. "Permanent Income, Current Income, and Consumption." *Journal of Business and Economic Statistics* 8 (July 1990): 265-79.

________. "The Response of Consumption to Income: A Cross-Country Investigation." *European Economic Review* 35 (1991): 723-67.

Cochrane, John H. "How Big Is the Random Walk in GNP?" *Journal of Political Economy* 96 (October 1988): 893-920.

________. "The Sensitivity of Tests of the Intertemporal Allocation of Consumption to Near-Rational Alternative." *American Economic Review* 79 (No. 3 1989): 319-37.

Deaton, Angus. "Life-Cycle Models of Consumption: Is the Evidence Consistent with the Theory?" In Truman Bewley ed. *Advances in Econometrics, Fifth World Congress* Vol. 2. Cambridge University Press, 1987.

Dreze, Jacques H. and Modigliani, Franco. "Consumption Decisions under Uncertainty." *Journal of Economic Theory* 5 (1972): 308-35.

Evans, Paul. "Are Consumers Ricardian? Evidence for the United States." *Journal of Political Economy* 96 (1988): 983-1004.

________. "Are Consumers Ricardian? Evidence for Nineteen Countries." Mimeograph, Department of Economics, The Ohio State University, 1991a.

________. *Lecture Notes for ECON 811: Applied Macroeconomics*. The Ohio State University, 1991b.

________, and Hasan, Iftekhar. "Are Consumers Ricardian? Evidence for Canada." Mimeograph, Department of Economics, The Ohio State University, 1991.

Flavin, Marjorie A. "The Adjustment of Consumption to Changing Expectations about Future Income." *Journal of Political Economy* 89 (1981): 974-1009.

Hall, Robert. "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence." *Journal of Political Economy* 86 (1978): 971-87.

Hayashi, Fumio. "The Permanent Income Hypothesis: Estimation and Testing by Instrumental Variables." *Journal of Political Economy* 90 (1982): 895-916.

Hansen, Lars Peter, "Large Sample Properties of the Generalized Method of Moments Estimators." *Econometrica* 50 (1982): 1029-54.

________, and Singleton, Kenneth. "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." *Econometrica* 50 (No. 5 1982): 1269-86.

Kimball, Miles S. "Precautionary Saving in the Small and in the

Large." *Econometrica* 58 (No. 1 1990): 53-73.

_______, and Mankiw, N. Gregory. "Precautionary Saving and the Timing of Taxes." *Journal of Political Economy* 97 (No. 4 1989): 863-79.

Leland, Hayne E. "Saving and Uncertainty: The Precautionary Demand for Saving." *Quarterly Journal of Economics* 82 (1968): 465-73.

Mankiw, N. Gregory, and Shapiro, Matthew D. "Trends, Random Walks and Tests of the Permanent Income Hypothesis." *Journal of Monetary Economics* 16 (1985): 165-74.

Nagatani, Keizo. "Life Cycle Saving: Theory and Fact." *American Economic Review* 62 (1972): 344-53.

Newey, Whitney K., and West, Kenneth D. "A Simple, Positive Definite, Heteroskedasticity and Autocorrelation Consistent Matrix." *Econometrica* 55 (1985): 703-08.

Quah, Danny. "Permanent and Transitory Movements in Labor Income: An Explanation for 'Excess Smoothness' in Consumption." *Journal of Political Economy* 98 (No. 3 1990): 449-75.

Sandmo, Agnar. "The Effect of Uncertainty on Saving Decisions." *Review of Economic Studies* 37 (1970): 353-60.

Skinner, Jonathan. "Risky Income, Life Cycle Consumption, and Precautionary Savings." *Journal of Monetary Economics* 22 (1988): 237-55.

West, Kenneth D. "The Insensitivity of Consumption to News about Income." *Journal of Monetary Economics* 21 (1988): 17-33.

Working, Holbrook. "Notes on the Correlation of First Differences of Averages in a Random Chain." *Econometrica* 28 (1960): 916-18.

Zeldes, Stephen P. "Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence." *Quarterly Journal of Economics* 104 (1989): 275-98.