Taxation on Fixed Foreign Direct Investment and International Commodity Trade as a Potential Protector

Gangsun Rhee*

This paper presents a model to explain why foreign direct investment (FDI) is possible even in the closed loop game where no precommitment on future FDI tax rate is available. Using a simple two-period model, we explain how international commodity trade between the capital-exporting country and the capital-importing country can be a passive leverage to protect the fixed FDI from being confiscated. We derive the necessary and sufficient conditions to guarantee an interior FDI tax rate in the closed loop game. (JEL Classification: F40)

I. Introduction

One problem concerning foreign direct investment (FDI) is possible confiscation by the capital-importing country (CIC).1 Since many CIC’s can not make a long-term commitment on their policies, the possibility of confiscation after FDI has been made may be a major reason why the developing countries have difficulty attracting foreign capital.2 As Williams (1975) shows, confiscation

*Research Fellow Chungnam Development Institute 48-1 Eunhaeng-Dong, Jung-Gu Taegon, 301-060, KOREA, (Tel) +82-42-222-2162, (Fax) +82-42-222-2165. This paper is a revised version of a chapter of the author’s Ph.D. dissertation, Washington State University. I am indebted to Ray Batina, Fred Inaba, and Jeff Krautkraemer for helpful comments. Of course, the usual disclaimer applies.

1Eaton and Gersovitz (1983) and Dixit (1987) provide a non-technical description of the problems related with the possible confiscation of FDI.

by the poor CIC’s is not rare. Yet, we notice that FDI is positive, and in many cases confiscation does not occur.

In the literature, there are several approaches to explain how FDI’s can be positive in the real world, even if the FDI’s become a hostage to the hosting governments in the closed loop game. Eaton and Gersovitz (1984) explain why confiscation does not occur by introducing some managerial services that the foreign investor brings to the production process. The managerial services include such things as technical knowledge, organizational capabilities, and access to overseas markets. These services can not be confiscated. If the CIC tries to confiscate FDI, the investor can retaliate by withdrawing the managerial services. Eaton and Gersovitz study the effects of possible confiscation on international capital allocation, welfare level of the CIC, and factor prices. They also derive the conditions under which the confiscation does not occur.

Bond and Samuelson (1989) introduce a tax on FDI in each of two periods in a two period model to study the effects of the lack of long-term commitment on the choice of technique by firms. The investor and the government of the CIC negotiate the first period tax rate before FDI is made, and the government can commit to its tax rate for at least one period. Then, after the FDI has been made, the second period tax rate is negotiated. They allow the investor to change the production technique, such as the combination of capital and labor, depending on whether the policy commitment by the CIC is available or not. They conclude that commitment to future tax rates by the CIC is optimal as long as the distortions imposed by the tax treatment of capital and the CIC’s imperfect access to capital markets are not too large.

Cole and English (1991) present a model in which private agents in the CIC live forever and foreign investors make a FDI every period. In each period, FDI is made and then the CIC decides whether or not to confiscate. If confiscation occurs, investors retaliate by not investing in the future, which reduces consumption

---

2This is called the time inconsistency problem of government policy. Since Kydland and Prescott (1977) originally discussed this problem, it became a hot issue in the literature of economic policies. Some papers concerning this issue in the area of tax policies include Fischer (1980), Rogers (1986, 1987), Staiger and Tabellini (1987), Lapan (1988), Maskin and Newbery (1990), and Batina (1990, 1991, 1992a). Persson and Tabellini (1990) is a good text for this issue.
in the CIC in the future. They show that under reasonable assumptions, increased investment makes confiscation less likely to occur, and that the level of investment chosen by atomistic foreign investors may be non-optimal.

Finally, Batina (1992b) investigates a circumstance where FDI is positive and confiscation does not occur in a closed loop policy game. His model assumes that the government of the capital-exporting country (CEC) produces a public good from which the CIC also benefits. If confiscation of FDI occurs, the CEC government can refuse to provide the CIC with the public good. Using the public good as a protection-retaliation measure, necessary and sufficient conditions for the existence of an interior equilibrium in which FDI is positive and confiscation does not occur are derived.

This paper presents another theoretical framework to explain how FDI can be positive, even though the CIC government can not make a long term commitment on its policies. We specifically explain what conditions are necessary and sufficient to guarantee FDI is positive in a closed loop game. Our work is an extension of Batina (1992b). However, there is a major difference between this paper and Batina (1992b). While he assumed as protection-retaliation device a public good that is provided by the CEC and has a spillover effect to the CIC, we introduce international commodity trade between the CEC and the CIC as the device.

In the paper, we use a two-good model. In addition to the first good that is invested and traded between the CEC and the CIC, there is a second good that is owned by the CIC at the beginning of the second period and can be sold to the CEC agent. If the government of the CIC increases its tax rate on FDI in the second period after the FDI is fixed, the income of the CEC agent decreases, thus causing a decrease in the CEC’s demand for the CIC good. Hence, the sales revenue that the CIC agent can earn by selling the second good to the CEC will fall and this may deter the CIC from raising the tax on FDI. Trade in the second good passively protects FDI.

The sequence of the timing of the game is as follows. At the beginning of the first period, the CEC investor receives an endowment in units of the first good. The investor of the CEC distributes the endowment to investments at home and abroad in the first period. The second period has three stages. The firm produces its output in the first stage of the second period. After
output is fixed, the government of the CIC chooses its tax rate on FDI in the second stage of the second period, correctly anticipating the future responses of the private agents to the tax rate. After the tax rate is chosen, private agents choose consumption of the two goods in the last stage of the second period, and prices adjust in the international commodity markets so that exports equal imports.

The contents of the paper are as follows. Section II introduces the model; Section III summarizes the decision problems of the private and public sectors; and Section IV explains how the equilibrium is derived and under what conditions it exists. An example is presented in Section V. The conclusion is in the last section.

II. The Model

Consider a world economy that has one capital-exporting country (CEC) and one capital-importing country (CIC). An asterisk will be used to denote the CIC. Each country has the same fixed population that is normalized to one private agent. There are two goods that can be freely traded between the two countries. At the beginning of the first period, the private agent of the CEC receives a fixed amount of the first good as an endowment and invests it at home and in the CIC. In the second period, the agent receives investment income in units of the first good. The agent consumes a part of it as the second period consumption of the first good and the rest of the income is spent to finance second period consumption of the second good. The second good is imported from the CIC. On the other hand, the private agent of the CIC receives a fixed amount of the second good as an endowment at the beginning of the second period. The agent consumes a part of it and the rest is sold to the CEC agent to buy and consume the first good from the CEC. For simplicity, we assume that both goods are normal ones in each country and the agents do not consume in the first period.

The private agent of the CEC has the following preferences.

\[ U_{CEC} = u(c, y), \]

where \( c \) is the consumption of the first good and \( y \) is the consumption of the second good in the second period. It is assumed that \( u(c, y) \) is twice continuously differentiable, strictly quasi-concave,
strictly increasing, and indifference curves are asymptotic to both axes.

The budget constraints of the CEC private agent are

\[
\begin{align*}
X - x - x^* &= 0, \quad (2) \\
I - c - p^* y &= 0, \quad (3) \\
I &= (1+r)x + (1+r^* - \tau^*)x^*. \quad (4)
\end{align*}
\]

where \( X \) is the endowment of the first good received at the beginning of the first period, \( x \) and \( x^* \) are respectively the amounts of domestic and foreign investments, \( r \) and \( r^* \) are respectively the gross returns of the domestic and foreign investments, \( \tau^* \) is the tax rate on FDI imposed by the CIC government, \( c \) is the consumption of the first good, \( p^* \) is the relative price of the second good in the world market in the second period, and \( y \) is the consumption of the second good in the CEC. We set the price of the first good as the numeraire, \( p = 1 \). The agent of the CEC consumes a part of the investment income and uses the rest of it to buy \( y \) units of the second good. The second good is imported from the CIC and the CEC agent pays \( p^* y \) to buy \( y \) units of the second good from the CIC.

In the CIC, the private agent has the following utility function.

\[
U_{\text{CIC}} = u^*(c^*, y^*) + m^*(g^*),
\]

where \( c^* \) and \( y^* \) respectively represent the consumption of the first and second goods in the second period, and \( g^* \) is the amount of a public good. \( u^*(c^*, y^*) \) has the same properties as \( u(c, y) \) and \( m^*(g^*) \) is also twice continuously differentiable, strictly quasi-concave. \( \partial m^*(g^*)/\partial g^* > 0 \) for \( g^* \geq 0 \), and \( \partial m^*(g^*)/\partial g^* \to 0(0) \) as \( g^* \to \infty(0) \). There is no international spillover effect of \( g^* \).

The budget constraint of the CIC private agent is

\[
p^*(Y^* - y^*) - c^* = 0. \quad (6)
\]

where \( Y^* \) is the endowment of the second good received at the beginning of the second period, \( y^* \) is the amount of the second good the agent consumes himself, and \( c^* \) is the amount of the first good the agent imports from the CEC and consumes in the second period. Notice that \( y = Y^* - y^* \). When the private agent of the CIC obtains \( Y^* \) units of the second good as the endowment, he uses \( y^* \) units of it for domestic consumption and the rest of it, \( y \), exports
to the CEC at the unit price $p^*$. The export revenue is $p^*y$, which is used to buy $c^*$ units of the first good from the CEC.

There is a large number of identical firms in each country. Again, we normalize them to one so that there is one firm in each country. Each firm of the CEC and the CIC uses capital to produce the first good. The technologies of the firms in the CEC and the CIC are simply given by $f(x)$ and $f^*(x^*)$. While the technology is generally different for each firm, i.e., $f \neq f^*$, it is assumed that $f(0) = f^*(0) = 0$. $f$ and $f^*$ are twice continuously differentiable, strictly concave, strictly increasing, and satisfy the Inada conditions. Notice that $f'(x) = r$ and $f^{''}(x^*) = r^*$ in equilibrium.

There are two world commodity markets. First, in the world commodity market of the first good, the aggregate supply is $(1+r)x + (1+r^*)x^*$ and the aggregate demand is $c + c^* + g^*$. Secondly, the aggregate supply of the second good is $Y^*$ and the aggregate demand is $y + y^*$. We concentrate on the second commodity market, while the first commodity market is automatically cleared by Walras’ law.

Our focus is to study how the existence of international commodity trade between the CEC and the CIC prevents the CIC from confiscating FDI. More specifically, we are interested in the conditions that result in $r^* < 1 + r^*$ so that confiscation does not occur. To focus on the determination of the FDI tax rate, we assume that all markets are competitive in the sense that all private agents regard the aggregate variables like price levels, tax policy, and the behavior of the other private agents as beyond their control. However, the government of the CIC may affect the variables by taking into account the behavior of the private agents and the demand and supply in the commodity markets when it chooses its tax policy on FDI.

Lastly, the government of the CIC is benevolent in the sense that it regards the utility function of its private agent as its objective function. The government of the CEC does not interfere in any transactions between the two countries at all. The government of the CIC has the following budget constraint,

$$r^*c^* = g^*. \quad (7)$$

\(^3\text{We may include labor in the production functions. However, it does not change our result, since labor and wage income are all fixed in the second period by assumption and are not subject to the FDI tax.}\)
We simply assume that a unit of the public good can be produced by a unit of the first good on a one-to-one basis.

III. The Decision Problems of the Private Agents and the CIC Government

It is convenient to summarize the decision problems of the private agents and the CIC government before we derive the equilibrium of the closed loop game between the private agents and the CIC government. We begin with the last stage of the second period since the equilibrium is calculated in a recursive way.

In the last stage of the second period, the private agent of the CEC chooses \( c \) and \( y \) to

\[
\max \ u(c, y), \tag{HP2}
\]

s.t. \( c = I - p^* y \),

\( (l, p^*) \) given,

where \( y \) represents the demand for the second good by the CEC private agent. \( I = (1+r)x + (1+r^* - \tau^*)x^* \) and \( p^* \) are given to the private agent, since the decisions on investments, production, and the tax rate were made before the decisions on \( c \) and \( y \), and the commodity market is assumed to be competitive. From (HP2), the optimal demand for the second good can be denoted as

\[
y = y(l, p^*). \tag{8}
\]

Substituting (8) into the budget constraint, we have the optimal \( c \).

In the last stage of the second period, the private agent of the CIC chooses \( c^* \) and \( y^* \) to

\[
\max \ u^*(c^*, y^*) + m^*(g^*), \tag{FP}
\]

s.t. \( c^* = p^*(Y^* - y^*) \),

\( (Y^*, p^*, g^*) \) given,

where \( y^* \) represents the demand for the second good by the private agent of the CIC. Again since the government decision is made before the decisions of the private agent on \( c^* \) and \( y^* \), and the commodity market is competitive, \( p^* \) and \( g^* \) are given to the private agent of the CIC. Due to the separable utility function, \( g^* \) drops
out, and the solution for the optimal demand for the second good is denoted as

$$y^* = y^*(Y^*, p^*).$$  \hspace{1cm} (9)

If (9) is substituted into the budget constraint, the optimal $e^*$ is determined.

Once $y$ is determined by (9), the optimal supply of the second good is determined by

$$y = Y^* - y^*(Y^*, p^*).$$  \hspace{1cm} (10)

The price of the second good is determined by equating the demand equation (8) with the supply equation (10) as follows:

$$y(l, p^*) = Y^* - y^*(Y^*, p^*).$$  \hspace{1cm} (11)

Solving (11) for $p^*$, we have

$$p^* = p^*(Y^*, l).$$  \hspace{1cm} (12)

The government of the CIC takes action in the second stage of the second period. In the second stage of the second period, it chooses $r^*$ and $g^*$ to

$$\max \quad u^*(p^*y, Y^* - y) + m^*(g^*),$$

s.t. $$g^* = r^*x^*,$$

$$y = y(l, p^*),$$

$$p^* = p^*(Y^*, l),$$

$$l = (1 + r)x + (1 + r^* - r)x^*,$$

$$(Y^*, r, x, r^*, x^*) \text{ given},$$

where $y$ is the amount of the second good sold to the CEC at equilibrium. The decision on investments were made in the first period so that $r$, $x$, $r^*$, and $x^*$ are all given when the government of the CIC chooses $r^*$ and $g^*$.

Notice in (FG) that since the government has market power and makes its decisions before the private agents do, $p^*$ and $y$ respond to $r^*$. The government of the CIC takes into account the adjustment of $p^*$ and $y$ when it choose its tax rate on FDI. As we will show, $p^*$ and $y$ decrease when $r^*$ increases. If $r^*$ increases, $g^*$ also increases since $x^*$ is fixed. However, the increased $r^*$ reduces income in the CEC so that the market demand for the second good
shifts down as long as $y$ is a normal good. This lowers $p^*$ received by the CIC and thus lowers utility in the CIC. Hence, as we will show, confiscation may not be optimal.

The solution of (FG) is denoted as
\[ \tau^* = \tau^*(Y^*, r, x, r^*, x^*). \] (13)

Equation (13) and the government budget constraint determine $g^*$.

In the first stage of the second period, production occurs. $f(x)$ is produced in the CEC and $f^*(x^*)$ is produced in the CIC.

In the first period, the private agent of the CEC chooses $x$ and $x^*$ to
\[
\max \quad u(l-p^*y, y). \quad (\text{HP1}) \\
\text{s.t.} \quad X-x-x^* = 0, \\
I = (1+r)x + (1+r^*-\tau^*)x^*, \\
(X, r, r^*-\tau^*, p^*) \text{ given.}
\]

In equilibrium, expectations are realized. Under our assumption of perfect foresight, the CEC agent can solve the decision problems of the other agents in order to figure out what actions they will take.

IV. Equilibrium

An equilibrium in this closed loop game is defined as follows: the private agent of the CEC has an allocation of capital, $(x_e, x)$, the government of the CIC has a policy, $(\tau_e^*, g_e^*)$, and the private agents of the CEC and the CIC respectively have the demand and supply of the second good, $(y_e, Y^*-y_e^*)$, such that,

i. $(x_e, x_e^*)$ solves (HP1) for any $(\tau^*, g^*)$;
ii. $(\tau_e^*, g_e^*)$ solves (FG) given any $x^* \geq 0$;
iii. $(y_e, Y^*-y_e^*)$ solves (HP2) and (FP) for any $(\tau^*, g^*)$.

In this section, we calculate the equilibrium. Specific conditions to guarantee the existence and uniqueness of the equilibrium are also calculated.
A. The Optimal Demand for the Second Good

Recall (HP2) above. Substituting the budget constraint \( c = I - p^*y \) into the utility function, the decision problem of the CEC agent is choosing \( y \) to

\[
\max W = u(I - p^*y, y),
\]

where \( I \) and \( p^* \) are given.

The first order condition for the optimal \( y \) is

\[
\frac{\partial W}{\partial y} = -p^* \frac{\partial u}{\partial c} + \frac{\partial u}{\partial y} = 0,
\]

where the first term is the marginal loss of utility due to a decrease in the consumption of the first good, and the second term is the marginal gain of utility by an increase in the consumption of the second good. Solving the first order condition (14), we have the optimal demand for the second good (8).

The second order condition is

\[
\frac{\partial^2 W}{\partial y^2} = (p^*)^2 \frac{\partial^2 u}{\partial c^2} - 2p^* \frac{\partial^2 u}{\partial c \partial y} + \frac{\partial^2 u}{\partial y^2} < 0.
\]

Since we assume the utility function is strictly quasi-concave and the budget constraint is convex, (15) holds everywhere so that the optimal demand is unique if it exists.

To see the properties of the demand function, totally differentiate (14) with respect to \( y, p^*, I, \) and \( \tau^* \) to obtain

\[
\frac{\partial y}{\partial p^*} = \frac{\frac{\partial u}{\partial c} - y \left( p^* \frac{\partial^2 u}{\partial c^2} - \frac{\partial^2 u}{\partial c \partial y} \right)}{\frac{\partial^2 W}{\partial y^2}} < 0,
\]

\[
\frac{\partial y}{\partial I} = \frac{p^* \frac{\partial^2 u}{\partial c^2} - \frac{\partial^2 u}{\partial c \partial y}}{\frac{\partial^2 W}{\partial y^2}} > 0,
\]

\[
\frac{\partial y}{\partial \tau^*} = -x^* \left( p^* \frac{\partial^2 u}{\partial c^2} - \frac{\partial^2 u}{\partial c \partial y} \right) \frac{\partial^2 W}{\partial y^2} < 0,
\]
where \( \frac{\partial^2 W}{\partial y^2} < 0 \) is given by (15). Equation (16) is the law of demand. Equation (17) represents the income effect. The last equation means that if \( \tau^* \) increases, the income of the CEC agent decreases so that demand also decreases. Since we assume that \( y \) is a normal good, we have \( \frac{\partial y}{\partial p^*} < 0, \frac{\partial y}{\partial l} > 0 \), and \( \frac{\partial y}{\partial \tau^*} < 0 \).

**B. The Optimal Supply of the Second Good**

Recall (FP). Substituting the budget constraint, \( c^* = p^*(Y^* - y^*) \), into the objective function, the decision problem of the CIC private agent becomes choosing \( y^* \) to

\[
\max \ W^* = u^*(p^*(Y^* - y^*), y^*) + m^*(g^*).
\]

where \( Y^* \), \( p^* \), and \( g^* \) are given.

The first order condition is

\[
\frac{\partial W^*}{\partial y^*} = -p^* \frac{\partial u^*}{\partial c^*} + \frac{\partial u^*}{\partial y^*} = 0. \tag{19}
\]

In (19), the first term represents the marginal loss of utility due to a decrease in the consumption of the first good. When \( y^* \) increases, the agent earns less sales revenue and has to reduce the consumption of the first good. The second term is the marginal gain of utility when \( y^* \) increases. Solving (19) for \( y^* \), we get the domestic demand function of the second good (9) above. Once \( y^* \) is determined, the optimal supply of the second good is immediately obtained by the constraint \( Y^* - y^* \), which is (10).

The second order condition is

\[
\frac{\partial^2 W^*}{\partial (y^*)^2} = (p^*)^2 \frac{\partial^2 u^*}{\partial (c^*)^2} - 2p^* \frac{\partial^2 u^*}{\partial c^* \partial y^*} + \frac{\partial^2 u^*}{\partial (y^*)^2} < 0. \tag{20}
\]

As in (15), the sign of \( \frac{\partial^2 W^*}{\partial (y^*)^2} \) is negative since \( u^* \) is assumed to be strictly quasi-concave and the budget constraint is convex. Hence, the optimal supply is unique if it exists.

Now differentiating (19) with respect to \( y^* \) and \( p^* \), we obtain

\[
\frac{\partial y^*}{\partial p^*} = \frac{\frac{\partial u^*}{\partial c^*} + (Y^* - y^*) \left( p^* \frac{\partial^2 u^*}{\partial (c^*)^2} - \frac{\partial^2 u^*}{\partial c^* \partial y^*} \right)}{\frac{\partial^2 W^*}{\partial (y^*)^2}}. \tag{21}
\]

where \( \frac{\partial^2 W^*}{\partial (y^*)^2} \) is given by (20). The sign of \( \frac{\partial y^*}{\partial p^*} \) is not clear immediately. Since \( y^* \) is a normal good and the CIC agent is
a net seller of the second good, a higher \( p^* \) makes the agent better off and the income effect increases \( y^* \). When \( p^* \) increases, on the other hand, the substitution effect also occurs, which decreases \( y^* \). Hence, the sign of \( \partial y^*/\partial p^* \) depends on which effect is stronger than the other. If the substitution effect is dominant, the sign of \( \partial y^*/\partial p^* \) is so negative that the supply, \( Y^*-y^* \), of the second good increases when \( p^* \) increases. If, however, the income effect is dominant, it is possible to have a negative-sloped supply curve of the second good.

C. The Optimal FDI Tax Rate

In order to characterize the decision problem of the CIC government, let us substitute the government budget constraint, \( g^* = \tau^* x^* \), into the utility function of (FG). Then, the decision problem becomes choosing \( \tau^* \) to

\[
\max W^* = u^*(p^*(I) y(I, p^*), Y^*-y(I, p^*)) + m^*(\tau^* x^*),
\]

where \( p^* \) and \( y \) are respectively the equilibrium price and the equilibrium level of the second good that are determined by the demand and supply functions, and \( Y^* \) and \( x^* \) are given when the CIC government chooses \( \tau^* \). Notice that even if the decision on \( \tau^* \) is made before the decisions of the private agents, the government of the CIC takes into consideration the effects of \( \tau^* \) on \( p^* \) and \( y \).

The first order condition of this decision problem is by the envelope theorem

\[
\frac{\partial W^*}{\partial \tau^*} = y \frac{\partial p^*}{\partial \tau^*} \frac{\partial u^*}{\partial c^*} + \frac{\partial m^*}{\partial g^*} = 0. \tag{22}
\]

The first term of (22) represents the marginal disutility due to the decrease of \( c^* \). Since \( \partial p^*/\partial \tau^* < 0 \) by (24) below, the sales revenue that the CIC agent receives by selling the second good decreases when \( \tau^* \) increases. Hence, \( c^* \) has to decrease, which is the first term. The second term is the marginal utility caused by the increase of the public good when \( \tau^* \) increases.

To derive \( \partial p^*/\partial \tau^* \), differentiate the market equilibrium condition (11) with respect to \( \tau^* \) and \( p^* \) to obtain

\[
\frac{\partial y}{\partial I} \frac{\partial I}{\partial \tau^*} d\tau^* + \frac{\partial y}{\partial p^*} dp^* = - \frac{\partial y^*}{\partial p^*} dp^*. \tag{23}
\]
Since \( \partial l / \partial \tau^* = -x^* \), (23) becomes

\[
\frac{\partial p^*}{\partial \tau^*} = \frac{x^* \frac{\partial y}{\partial I}}{\frac{\partial y}{\partial p^*} + \frac{\partial y^*}{\partial p^*}} < 0.
\]

(24)

Since the second good is a normal one, the numerator is positive. The sign of the denominator is not clear since the sign of \( \frac{\partial y^*/\partial p^*} \) can be both negative and positive. However, because of the Walrasian market stability condition\(^4\) that the slope of the supply curve is greater than the slope of the demand curve, the sign of the denominator should be negative, which means \( \frac{\partial p^*/\partial \tau^*} < 0 \).

If \(-y(\partial p^*/\partial \tau^*)(\partial u^*/\partial c^*)=x^*(\partial m^*/\partial g^*)\) for any \( \tau^* < 1+r^* \), we have an interior \( \tau^* \). Notice that if there is no international commodity trade, that is, \( \partial p^*/\partial \tau^* = 0 \), the optimal \( \tau^* \) is of course \( 1+r^* \) and confiscation occurs, since \( \partial W^*/\partial \tau^* = x^*(\partial m^*/\partial g^*) > 0 \).

The second order condition is

\[
\frac{\partial^2 W^*}{\partial (\tau^*)^2} = \frac{\partial y}{\partial \tau^*} \frac{\partial p^*}{\partial c^*} \frac{\partial u^*}{\partial \tau^*} + y \frac{\partial^2 p^*}{\partial (\tau^*)^2} \frac{\partial u^*}{\partial c^*} + y \frac{\partial p^*}{\partial \tau^*} \frac{\partial^2 u^*}{\partial (c^*)^2} - y \frac{\partial p^*}{\partial \tau^*} \frac{\partial^2 u^*}{\partial c^* \partial \tau^*} + (x^*)^2 \frac{\partial^2 m^*}{\partial (g^*)^2} < 0.
\]

(25)

If the second order condition is satisfied everywhere, the interior solution to (FG) is unique.

The graphical explanation is as follows. Define \( A = -y(\partial p^*/\partial \tau^*) \) \((\partial u^*/\partial c^*)\) and \( B = x^*(\partial m^*/\partial g^*) \), where \( A \) represents the marginal cost and \( B \) the marginal benefit. Notice that

\[
\frac{\partial A}{\partial \tau^*} = \frac{\partial y}{\partial \tau^*} \frac{\partial p^*}{\partial c^*} \frac{\partial u^*}{\partial \tau^*} - y \frac{\partial^2 p^*}{\partial (\tau^*)^2} \frac{\partial u^*}{\partial c^*} - y \frac{\partial p^*}{\partial \tau^*} \frac{\partial^2 u^*}{\partial (c^*)^2} + y \frac{\partial p^*}{\partial \tau^*} \frac{\partial^2 u^*}{\partial c^* \partial \tau^*} + \frac{\partial^2 u^*}{\partial (\tau^*)^2} \frac{\partial y}{\partial \tau^*} \frac{\partial y}{\partial c^*} \frac{\partial y}{\partial \tau^*}.
\]

(26)

\(^4\)When \( p^* \) increases, the substitution and income effects occur in both countries. The substitution effects in both countries and the income effect in the CEC are all negative, but the income effect in the CIC is positive. A stable market implies that the first three negative effects should dominate the positive income effect in the CIC.
\[
\frac{\partial B}{\partial \tau^*} = (x^*)^2 \frac{\partial^2 m^*}{\partial (g^*)^2} < 0. \tag{27}
\]

From the second order condition, we have
\[
\frac{\partial A}{\partial \tau^*} > \frac{\partial B}{\partial \tau^*}. \tag{28}
\]

Since \(\partial B/\partial \tau^* < 0\), there are two cases satisfying inequality (28). Figures 1 and 2 depict them. In both cases, the slope of the marginal cost curve exceeds the slope of the marginal benefit curve. As \(\tau^* \rightarrow 0\), \(g^* \rightarrow 0\) and \(\partial m^* / \partial g^* \rightarrow \infty\) so that the \(B\) curve is asymptotic to the vertical axis. If \(A = B\) for any \(\tau^* < 1 + r^*\) and \(\partial A/\partial \tau^* > \partial B/\partial \tau^*\), the optimal FDI tax rate is interior and unique.

D. The Optimal Allocation of Capital

Recall (HP1). Substituting the budget constraint \(x = X - x^*\) into the utility function, the decision problem of the CEC agent in the first period becomes choosing \(x^*\) to
\[
\max u(1+r)(x-x^*) + (1+r^* - \tau^*)x - p^*y, \quad y,
\]

where \(X, r, r^* - \tau^*, p^*\) are given to the agent.

The first order condition is
\[
\left( -p^* \frac{\partial u}{\partial c} + \frac{\partial u}{\partial y} \right) \frac{\partial y}{\partial x^*} - [(1+r) - (1+r^* - \tau^*)] \frac{\partial u}{\partial c} = 0. \tag{29}
\]

By (14), the first term of (29) is zero so that we have the following non-arbitrage condition,
\[
r = r^* - \tau^*, \tag{30}
\]

which is equivalent to
\[
f'(x) = f''(x^*) - \tau^*. \tag{31}
\]

Solving (31) and \(x = X - x^*\) for \(x\) and \(x^*\), we obtain
\[
x = x(X, p^*, \tau^*), \tag{32}
\]
\[
x^* = x^*(X, p^*, \tau^*). \tag{33}
\]

If we substitute (32) and (33) into (13), we have \(\tau^* = \tau^*(X, Y^*, p^*)\). Since \(p^*\) can be written as a function of \(\tau^*\) by (12), we can express the tax rate as a function of the endowments only.
V. Example

In this section, we present an example to illustrate how the model works. Let the utility functions of the CEC and the CIC be represented by

\[ U_{\text{CEC}} = a_1 \text{Inc} + a_2 \text{lny}. \]  
\[ U_{\text{CIC}} = a_1^* \text{Inc}^* + a_2^* \text{lny}^* + a_3^* \text{lng}^*. \]
where \(a_i\)'s and \(a_i^*\)'s are all positive constants.

It is straightforward to derive the demand and supply functions of the second good as follows,

\[
y = \frac{a_2 I}{(a_1 + a_2)p^*},
\]

\[
Y^* - y^* = \frac{a_1^* Y^*}{a_1^* + a_2^*}.
\]

By equating (36) with (37), we derive

\[
p^* = \frac{a_2(a_1^* + a_2^*) I}{(a_1 + a_2)a_1^* Y^*}.
\]

\[
\frac{\partial p^*}{\partial \tau^*} = -\frac{a_2(a_1^* + a_2^*) x^*}{(a_1 + a_2)a_1^* Y^*} < 0.
\]

The CIC government chooses \(\tau^*\) to

\[
\max W^* = a_1^* \ln(p^* y) + a_2^* \ln(y^* - y) + a_3^* \ln(\tau^* x^*).
\]

Using the envelope theorem, the first order condition is

\[
\frac{\partial W^*}{\partial \tau^*} = \frac{a_1^*}{p^*} \frac{\partial p^*}{\partial \tau^*} + \frac{a_3^*}{\tau^*} = 0.
\]

Substituting (38) and (39) into (40) and using the non-arbitrage condition \(r = r^* - \tau^*\), we obtain

\[
\tau^* = \frac{a_3^*(1+r)X}{a_1^* x^*}.
\]

Equation (41) implies that for an appropriate value of \(a_3^*/a_1^*\), it is possible to have an interior \(\tau^*\). By setting \(\tau^* < 1+r^*\) and using \(r = f'(x)\) and \(r^* = f'(x^*)\), we have

\[
\frac{a_3^*}{a_1^*} < \frac{x^*(1+f'(x^*))}{x(1+f'(x))}.
\]

As long as \(a_3^*/a_1^*\) has the range of (42), an interior \(\tau^*\) exists. If the CIC values the public good less, or values the first good more at the margin, the government of the CIC will choose a lower tax rate.

Now to derive specifically the equilibrium values of \(x\), \(x^*\), and \(a_3^*/a_1^*\) for which \(\tau^* < 1+r^*\), let \(f(x) = Ax^\alpha\) and \(f'(x^*) = A^*(x^*)^{\alpha - 1}\) where \(A\) and \(A^*\) are positive constants and \(\alpha\) is a positive fraction. By (31), we have

\[
a A(X-x^*)^{\alpha - 1} = a A^*(x^*)^{\alpha - 1} - \tau^*.
\]
TABLE 1

<table>
<thead>
<tr>
<th>(\tau^*)</th>
<th>(x^*)</th>
<th>(a_3^<em>/a_1^</em>)</th>
<th>(1+r^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.7690</td>
<td>0.0377</td>
<td>2.1403</td>
</tr>
<tr>
<td>0.2</td>
<td>0.7331</td>
<td>0.0745</td>
<td>2.1679</td>
</tr>
<tr>
<td>0.3</td>
<td>0.6925</td>
<td>0.1092</td>
<td>2.2017</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6478</td>
<td>0.1406</td>
<td>2.2425</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6002</td>
<td>0.1676</td>
<td>2.2908</td>
</tr>
<tr>
<td>0.6</td>
<td>0.5515</td>
<td>0.1894</td>
<td>2.3466</td>
</tr>
<tr>
<td>0.7</td>
<td>0.5034</td>
<td>0.2061</td>
<td>2.4095</td>
</tr>
<tr>
<td>0.8</td>
<td>0.4573</td>
<td>0.2179</td>
<td>2.4787</td>
</tr>
<tr>
<td>0.9</td>
<td>0.4144</td>
<td>0.2256</td>
<td>2.5534</td>
</tr>
<tr>
<td>1.0</td>
<td>0.3752</td>
<td>0.2298</td>
<td>2.6326</td>
</tr>
</tbody>
</table>

Assuming that \(X=1\), \(A=1\), \(A^*=2\), and \(\sigma=0.5\), we can solve (41) and (43) for the equilibrium values of \(x^*\) and \(a_3^*/a_1^*\) with simulating values of \(\tau^*\). The results are summarized in Table 1.

In Table 1, we notice that the FDI tax rate is interior; i.e., \(\tau^*<1+r^*\). For a given \(a_3^*/a_1^*\), the CEC agent correctly anticipates the future \(\tau^*\) and decides \(x^*\) by arbitrage condition (31). In the second period, the CIC government chooses an interior \(\tau^*\) for the fixed \(x^*\) and the given \(a_3^*/a_1^*\). Table 1 shows that for an appropriate value of \(a_3^*/a_1^*\), it is possible to have an interior \(\tau^*\) even with a fixed \(x^*\). We also notice that when the tax rate increases, more capital moves from FDI to domestic investment; i.e., a higher \(\tau^*\) makes \(x^*\) decrease and \(x=1-x^*\) increase.

VI. Conclusion

This paper analyzed a framework which demonstrates that international commodity trade between the CEC and the CIC can serve as a protection device for FDI. Necessary and sufficient conditions for an interior FDI tax rate were provided in a closed loop game between the private agents and the CIC government. We emphasized that when the CIC government increases its tax rate on fixed FDI, it causes an income effect for the CEC agent, and the demand for the good that the CIC agent sells to the CEC decreases. The decrease in demand generally reduces the sales
revenue that the CIC agent earns by exporting the good to the CEC and forces the CIC agent to consume less of the imported good. Hence, increasing the FDI tax rate has both positive and negative effects on CIC welfare. Therefore, it is possible to have an interior tax rate. We derived specific conditions to ensure an interior tax rate.

A contribution of this paper is to present a model that can explain why positive FDI occurs even in a closed loop game. Ex ante, the CIC's offer very favorable policies toward FDI's, but they have an incentive to change their policies once firms have invested in their countries. This paper presented one reason why the closed loop policy of the CIC's may not be as radical as confiscation.

(Received August, 1997)

References


