Incorporating the Effect of New and Disappeared Products into the Cost of Living: An Alternative Index Number Formula

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The current Consumer Price Index (CPI) is a biased estimator of the true cost of living (COL). Incorrect treatment of quality change in existing products and the introduction of new products is known to be a major source of the bias. One of the practical problems that make it hard to properly incorporate the effect of newly introduced or disappeared products into the CPI is that the price of a new product in the pre-introduction period and the price of a disappeared product in the period it disappears are unobservable. The present paper introduces an index number formula which overcomes that problem. The index number formula is *exact* for the constant-elasticity-of-substitution (CES) preference ordering if the elasticity of substitution is known. Unlike the formula introduced by Feenstra (1994), it separates a change in the COL into three parts, namely, new, disappeared and existing products. (*JEL* Classifications: C43, D11)

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I. Background

A bilateral cost-of-living (COL) index measures the ratio between the minimum expenditures required to attain an identical level of utility in two price situations. Thus, it takes account of the consumer's cost-minimizing behavior to achieve a certain level of utility, such as substitution between goods and services, and substitution between outlets. While the concept of the COL index is most appropriate for various purposes such as analysis of inflation, indexation, contract escalation, measurement of changes in the real values and international studies, the Consumer Price Index (CPI) is often used for those purposes. The CPI in the current form is a ratio of the costs of purchasing a basket of goods and services of constant quality and constant quantity in two different time periods. Hence, it takes no account of any possibility of reduction in the cost of achieving the same level of well-being through substitution between existing products and substitution between existing and new products (including existing products with different quality).[1]

Weaknesses of the CPI as a measure of the COL are well documented in the report by Boskin *et al.* (1996).[2] According to the report, the total upward bias, for the US, in the CPI-based rate of inflation as a measure of changes in the COL has been estimated at 1.1 percentage points per year. That is, if the annual percentage change in the CPI was measured as 5 percent, the true percentage change in the COL would be approximately 3.9 percent. When compounded over a long period, this has enormous ramification for the government budget, and for understanding and measuring economic progress. Since most social security benefits, individual income tax brackets and the personal exemption are indexed to the CPI, the report estimates that an upward bias of 1.1 percentage points per year compounded over the next 10-year period would contribute about $US 148 billion to the annual US federal deficit and $US 691 billion to the US national debt in the year 2006. The bias in the CPI also leads to misleading measures of real GDP growth, productivity growth, real wage and income growth. They report that when measured by deflating by an appropriate COL

[1] See the Bureau of Labor Statistics (1997) for the construction of the CPI.

[2] See Boskin *et al.* (1997), Gordon and Griliches (1997), Boskin and Jorgenson (1997) and subsequent discussion for its summary.

index, real average hourly earnings have risen about 13 percent since 1973 instead of falling by about 13 percent; real median family income has risen over 30 percent rather than about 4 percent; and the poverty rate is 9.0 percent instead of the official 13.8 percent.

II. New Goods and Quality Change Bias

Out of the total bias of 1.1 percentage points estimated by Boskin *et al.* (1996), 0.6 percentage points is due to the neglect or incorrect treatment in the CPI of quality change in existing products and the introduction of new products. Hausman (1997b) views that even this estimate is likely to be an underestimate, the reason being that "the consumer surplus from new goods has not been included in the report's estimate" (p. 96). While new products are continuously introduced into the consumer marketplace and a myriad of products disappear from shop shelves every month, the base period for the CPI is set only once every ten years for the US CPI, and every five years for the Australian CPI. The report's estimate is likely to be even more an underestimate. The reason is that the current CPI tries to measure the price changes of fixed-quality goods and services that are included in the basket, and hence neglects the benefit from having improved quality or new features of the existing items.

To devise a method to reduce such bias, it would be useful to investigate the current practice of the statistics agency in dealing with quality change and new products. The following explanation of how the US Bureau of Labor Statistics (BLS) responds, in computing the CPI, to the introduction of new products and quality change is based on BLS (1997) and Armknecht *et al.* (1997). The BLS classifies new products into three categories — namely, replacement items, supplemental items, and entirely new items. *Replacement items* include new versions or models of previously available items, such as new car models and apparel items made of a new fabric. The BLS uses three different methods to derive pure price changes for replacement items, eliminating the part of price changes attributable to quality change. When a new item that is directly comparable with the disappeared old item can be found, it is assumed that there is no change in the quality and the price of the new item is directly compared with that of the old item (*direct comparison*). When a replacement item is qualitatively different from the item it replaces, quality-induced price change is isolated by means of production-cost and mark-up or hedonic-regression techniques (*direct quality adjustment*). Price changes for replacement items that are neither directly comparable nor quality adjustable (noncomparable items) are imputed using the price changes of other similar constant-quality replacement items (*imputation*). The part of the price differential between the original and replacement items in excess of the imputed price change is assumed to be induced by quality change, and hence it is excluded in computing the CPI. Similarly, to the extent that it is different from the quality-adjusted price change, the price differential between the original and replacement item is excluded from the CPI.

Supplemental items are newly added brands of existing products. Thus, in this case, the new and old items are sold simultaneously. Examples for this group of new items include generic drugs, new brands of cereal and new types of discount airline fares. Such items have a chance to be included in the CPI through *sample rotation*. Each year, the BLS takes a new sample of *entry-level items* (ELI's) and outlets for about 20 percent of the geographic areas (*primary sampling units*: PSU's) on a rotating basis. To enter into the CPI during sample rotation, a new item should fall within the definition of an established ELI. That is the case because a new item has a positive possibility to be selected in a new sample only if it is priced in the Consumer Expenditure (CE) Survey and it has a non-zero market share. Only items that fall within the broad definition of an established ELI are priced in the CE Survey. When a new item replaces an existing item for pricing in a new sample, the two items are linked by *overlap pricing*. That is, in the period when re-sampling occurs, both the existing and new items are priced. For the corresponding ELI and PSU, the price change from the previous period to the current period is measured by the price change of the existing item, while the price change between the current and next period is measured by the new item. Hence, the price differential at the time of the introduction of the new item is regarded as the quality differential between the two items, and hence it is excluded from the CPI.

When a new item cannot be classified into any of the established ELI's (*entirely new items*), it can only enter into the CPI through *major revisions* or creation of a new ELI and then inclusion during sample rotation between major revisions. In the case of the US CPI, a full implementation could take at least five years through rotation, or ten years through a major revision. Room air-conditioners, mobile phones, VCR's, and personal computers are good examples of this group of items.

Since the current CPI is designed to measure the price changes in a basket of goods and services that are of constant quality and constant quantity, what is attempted in the above methods is adjusting price changes so that the part of price changes caused by quality change is separated and eliminated. Thus, the above methods provide two sources of bias to the CPI as a measure of the COL. One is that the assumptions made for an adjustment method may be wrong. For example, in the case of *direct comparison*, the new replacement item may have different characteristics from the item it replaces, and hence the CPI includes a price change caused by quality change as well as a pure change in the price. If the quality of the new item is improved, then the CPI will overstate the increase in the price of the item which is supposed to be of constant quality. The other source is that the current CPI could miss out the cost savings in achieving a given level of utility by consuming better-quality substitutes. For example, suppose a *replacement item* where the *direct quality adjustment* method is applied. Assume that the weight of the item in the fixed CPI basket is 0.05, and its unit price increases by 3 percent of which 1 percent is due to quality change and the other 2 percent is a pure price change. Further assume that other prices are fixed and no substitution is made between items. The CPI will increase by 0.1 percent if a correct quality adjustment is made. However, it still is not a correct measure of the change in the COL because it does not take into account the contribution of quality change towards lowering the cost to achieve a given level of well-being. For example, if the same level of utility is achieved by consuming only 80 percent as many units of the new item as the old item, the COL will decrease by 0.88 percent.[3]

An appropriate method to deal with items of different quality would be treating them as an entirely new item. Under such an approach, quality-change bias becomes the same problem as the

[3] $(0.95 \times 1 + 0.05 \times 0.8 \times 1.03) - 1 = -0.0088.$

new/disappeared goods bias. It would be necessary for this approach to work that the basket of goods and services included in the CPI be revised each period, and an index number formula which can incorporate the changes in the COL, caused by a revised choice set, be employed for aggregation.

The present paper is concerned with the latter. One of important practical problems the above approach may face is that the price of a new good in the period preceding its appearance in the marketplace is not observed. Similarly, the price of a disappeared good is not observable in the period it disappears. If the implicit price change over the introduction or disappearing period is neglected, the change in the COL induced by an increased or reduced choice set will be missed out.[4] In the following sections, alternative approaches to overcoming the problem of unobservable prices of new and disappeared goods are examined. In the next section, an econometric approach is briefly described. Section IV explains an index number formula introduced by Feenstra (1994). Section V introduces a new index number formula to overcome the problem of unobservable prices. Finally, Section VI provides concluding remarks.

III. Econometric Approach

Hausman (1997a) attempts to measure the consumer's surplus arising from the introduction of a new brand of cereal. He estimates consumer's preferences and solves the estimated demand equations for the *reservation* (or *shadow*) *price* of the new good in the pre-introduction period. Then, he evaluates consumer's surplus based on the reservation price. This approach finds its theoretical grounds in Hicks (1940). According to Hicks, the price of a new good in the period preceding its introduction, namely reservation price, should be at a level which is just high enough to make its demand equal to zero.

Let $\hat{I}=\{1, 2, 3, \cdots, n\}$ be the consumer's choice set of n items,

[4]See Diewert (1987, Section 11) for a simple numerical illustration of the bias resulted from omitting an implicit price change in the initial period. See also Diewert (1996, p. 32) for a graphical illustration of the benefit from an increased choice set. Hausman (1997b, p. 97) also emphasizes the importance of the correct approach to a COL index when introducing new goods.

where item n is introduced in period 1 but unavailable in period 0. Define the *Marshallian* demand functions as

$$q_i^t = g_i(p_1^t, p_2^t, \cdots, p_{n-1}^t, p_n^t, X^t), \quad t=0,1 \text{ and } i \in \hat{I}, \tag{1}$$

where p_i^t are prices and X^t is the total expenditure in period t. Then, the *reservation price* of item n in period 0 (p_n^0) is implicitly given by

$$q_n^0 = 0 = g_n(p_1^0, p_2^0, \cdots, p_{n-1}^0, p_n^0, X^0). \tag{2}$$

The demand equations in (1) are econometrically estimated using a data set observed in the periods when p_n^t are observed, say for $t=1, \cdots, T$. Then, the reservation price for n can be obtained by solving (2) for p_n^0 after substituting p_i^0's, X^0 and the coefficient estimates into $g_n(\cdot)$.

To recover the cost (expenditure) function from the Marshallian demand functions, Hausman (1981) solves the following differential equation, which is obtained by equating the Marshallian demand to the Hicksian demand:

$$h_n(p, u) = \frac{\partial E(p, u)}{\partial p_n} = \frac{\partial X}{\partial p_n} = g_n(p, X), \tag{3}$$

where $h_n(\cdot)$ is the Hicksian demand function, $E(\cdot)$ is the cost function, p denotes the price vector $(p_1, p_2, \cdots, p_n)^T$, and u is the utility level. The constant of integration in the solution of the differential equation depends on the initial value of u. Hence, the constant can be equated to u^0 as cardinal utility index. Once the cost function is obtained, the true COL index of Konüs (1939) of period 1 in comparison with period 0 is computed by

$$P_K(p^0, p^1, u) \equiv \frac{E(p^1, u)}{E(p^0, u)}, \tag{4}$$

where u is a reference utility level. The natural utility levels u^t can be obtained through the indirect utility function:

$$u^t = v(p^t, X^t), \tag{5}$$

where the indirect utility function $v(p, X)$ is derived by solving $X = E(p, u)$ for u.

An advantage of this approach is that the precision of the estimate of the COL index can be measured in terms of standard errors. However, it has a disadvantage that a new good can be incorporated into the measure of the COL only after obtaining enough number of observations on the quantity and price of the new good. That is, new goods cannot be incorporated into the measure in a timely fashion. This is the case because the demand equation for the new good must be estimated first in order to derive the reservation price. The approach, of course, also suffers from the general disadvantages of an econometric approach to computing an index. That is, an econometric model has to be specified and estimated, and hence the resultant index could be sensitive to the specification and/or estimation technique used. Furthermore, the model may have to be re-estimated as more observations become available.

IV. Feenstra's Index

Feenstra (1994) employs an economic index number approach to measure the price index of imported goods to the US, incorporating new items and quality change in existing items into the index. Feenstra's index number formula is *exact* for a *constant-elasticity-of-substitution* (CES) aggregator function if the elasticity of substitution is known.[5] His result is an extension of the index number formula introduced by Vartia (1974) and Sato (1976), referred to as *Vartia index II*.[6] Their result is summarized in the following theorem. Throughout the discussions below, it is assumed that the observed values of quantities and prices are identical to their theoretical values. In other words, observed quantities, denoted q_i^t, are solutions to the consumer's problem with given prices p_i^t and total expenditure X^t.

***Theorem* 1 (Vartia 1974, 1976; Sato 1976)**
Let preferences be represented by the CES cost function

$$E(p,\ u,\ I) \equiv ue(p,\ I) \equiv u\left(\sum_{i\in I} \alpha_i p_i^{1-\sigma}\right)^{1/(1-\sigma)}, \tag{6}$$

where $u>0$ denotes utility level, $\sigma>0$ is the elasticity of substitution, $\alpha_i>0$ are constants, I denotes the set of items that are

[5] An index number formula $P(p^0,\ p^1,\ q^0,\ q^1)$ is said to be *exact* for an aggregator function $e(p)$ if $P(p^0,\ p^1,\ q^0,\ q^1)=e(p^1)/e(p^0)$; see Diewert (1976).
[6] See also Vartia (1976).

available in both period 0 and period 1, and $I \neq \phi$. Then, the true Konüs COL index can be exactly measured by the following index

$$P_K(p^0, p^1, u, I) = e(p^1, I)/e(p^0, I) = P_V(p^0, p^1, q^0, q^1, I), \tag{7}$$

where q^t are the quantity vectors corresponding to the price vectors p^t, and the *Vartia index II* is defined by

$$P_V(p^0, p^1, q^0, q^1, I) \equiv \prod_{i \in I} \left(\frac{p_i^1}{p_i^0} \right)^{w_i(I)}. \tag{8}$$

The weights, $w_i(I)$, are a function of the logarithmic means of budget shares. Namely,

$$w_i(I) \equiv \frac{L[s_i^0(I),\ s_i^1(I)]}{\sum_{j \in I} L[s_j^0(I),\ s_j^1(I)]}, \tag{9}$$

where the logarithmic mean function is defined by

$$L[s_i^0(I),\ s_i^1(I)] \equiv \frac{s_i^0(I) - s_i^1(I)}{\ln s_i^0(I) - \ln s_i^1(I)}, \tag{10}$$

and the budget shares are defined as

$$s_i^t(I) \equiv \frac{p_i^t q_i^t}{\sum_{j \in I} p_j^t q_j^t} \quad \text{for } i \in I, \text{ and } t = 0,\ 1. \tag{11}$$

Thus, the true COL index can be measured only by using observable values of prices and quantities if preferences are represented by a CES function. Note that, however, if the set of items available in period 1, denoted I^1, is different from the set of items available in period 0, denoted I^0, then the above index number formula is unworkable since the log of zero is undefined and the reservation prices are not observed. So, the above index is useful only when the sets of available items in both periods are identical.

Feenstra (1994) modifies this index number formula to allow for new goods and disappeared goods.

***Theorem* 2 (Feenstra 1994, Proposition 1)**

Let I^t be the set of items available in period t, and $I \subseteq (I^0 \cap I^1)$, $I \neq \phi$. Assume that preferences are represented by the CES unit cost function with $\sigma > 1$.[7] Then,

[7]When the choice set varies over time the elasticity of substitution must be greater than unity. The reason is that an elasticity of substitution that is less than unity implies that all items are essential for consumption and

$$e(p^1, I^1)/e(p^0, I^0) = P_F(p^0, p^1, q^0, q^1, I^0, I^1), \tag{12}$$

where

$$P_F(p^0, p^1, q^0, q^1, I^0, I^1) \equiv P_V(p^0, p^1, q^0, q^1, I)\left(\frac{\lambda^1}{\lambda^0}\right)^{1/(\sigma-1)} \tag{13}$$

$$\lambda^t \equiv \frac{\sum_{i\in I} p_i^t q_i^t}{\sum_{j\in I^t} p_j^t q_j^t} \quad \text{for } t=0,\ 1. \tag{14}$$

Thus, Feenstra's modified index is the Vartia index II multiplied by a function of the ratio of the proportion of the expenditure on a (sub)set of items available in both periods out of the total expenditure in period 1 to the proportion in period 0. New items are included in I^1, but not included in either I^0 or I, while disappeared items are included in I^0 but not in I^1 or I. So, all items included in I are available in both periods, and hence there is no problem in computing $P_V(p^0, p^1, q^0, q^1, I)$. Unobservable prices and quantities do not pose any problem in computing the ratio part of the index. Quality change can be viewed as a simultaneous occurrence of the introduction of a new item and the disappearance of an old item (in the case of replacement item) or simply as the introduction of a new good (in the case of supplemental item). That is, in the case of a replacement item, the old version of the item is included in I^0, but not in I or I^1, while the new version is included in I^1, but not in I or I^0. Hence, the above index properly incorporates quality change as well as new goods. Since the index consists of two parts, the Vartia index for price changes in existing items and the adjustment for new and disappeared items, the change in the COL due to the introduction of new items is not distinguishable from the change due to disappeared items.

Drawbacks of the above index include the restrictiveness of the constant substitution-elasticity assumption and the unobservable value of σ in the formula. The value of σ, however, can be easily estimated via various methods. As in Feenstra (1994) and Feenstra and Shiells (1997), the σ could be estimated as a part of the coefficient of the price index of a subgroup of items, for which the above index is used for aggregation, in an upper-level demand equation. For example, suppose that preferences for various com-

hence the choice set cannot vary over time; see Feenstra and Shiells (1997, p. 254).

puter models (within ELI) are represented by the CES unit cost function while the upper-level preferences for different types of information-processing equipment (within *item stratum*) are represented by a translog unit cost function.[8] Then, the share equation for an ELI within the item stratum is given by

$$s_i^t(p) = \beta_0 + \beta_1 \ln \Phi_1^t + \frac{\beta_1}{\sigma_1 - 1} \ln \Lambda_1^t + \cdots + \beta_N \ln \Phi_N^t + \frac{\beta_N}{\sigma_N - 1} \ln \Lambda_N^t, \qquad (15)$$

where N is the total number of ELI's in the item stratum, β_i are the constants in the upper-level translog cost function, σ_i are the elasticity of substitution between varieties of ELI i, Φ_i^t are the multilateral Vartia index defined by

$$\Phi_i^t \equiv \prod_{\tau=1}^{t} P_V(p^{\tau-1}, p^{\tau}, q^{\tau-1}, q^{\tau}, \text{ELI } i), \qquad (16)$$

and

$$\Lambda_i^t \equiv \prod_{\tau=1}^{t} \left(\frac{\lambda_i^{\tau}}{\lambda_i^{\tau-1}} \right) = \left(\frac{\lambda_i^t}{\lambda_i^0} \right). \qquad (17)$$

If the fixed-base principle, rather than the chain principle, is used then Φ_i^t are simply the indices comparing period t with the base period 0.

V. An Alternative Index

There could be many different index number formulas that are exact for the CES aggregator function if we allowed the elasticity of substitution, denoted σ, to be included in the formula. If there was no change in the consumer's choice set over time, the Vartia index would be by far the most useful one since it does not include any unknown constants in its formula. However, when there exist new and/or disappeared goods, the Vartia index is unworkable because the unobservable reservation prices or zero budget shares cannot be handled by the formula.

A useful index number formula when the information on quan-

[8]In the US item-classification system, various models of computers are included in the ELI "Home Computers", and the ELI in turn belongs to the "Information-Processing Equipment" *item stratum* which includes such ELI's as Home Computers, Home Computer Software, Telephones, Typewriters and Calculators.

tities in the current period ($t=1$) is unavailable was introduced by Lloyd (1975):[9]

$$P_L(p^0,\ p^1,\ q^0,\ I) \equiv \left\{ \sum_{i \in I} s_i^0(I) \left(\frac{p_i^1}{p_i^0} \right)^{1-\sigma} \right\}^{1/(1-\sigma)}, \qquad (18)$$

where $s_i^0(I)$ are budget shares within the choice set I in period 0; see (11). This index is exact for the CES unit cost function when there is no change in the choice set. The usefulness of this formula is well demonstrated by Shapiro and Wilcox (1997), where they show how this formula can be used to produce timely price indices when the information on quantities is not immediately available. Note that q_i^1 are not included in the above formula.

For the purpose of incorporating new and disappeared items into the COL index, the above index formula cannot be used since the reservation prices are not available. We now introduce an alternative index number formula that overcomes the problem of the unobservable reservation prices, like Feenstra's index (13), and separates the whole change in the COL into the parts caused by price changes in existing items, the introduction of new items, and the disappearance of old items, unlike Feenstra's index.

Consider the distance function that is dual to the CES cost function (6)[10]

$$D(q,\ u,\ I) = \frac{1}{u} \left(\sum_{i \in I} \alpha_i^* q_i^{-\rho} \right)^{-1/\rho}, \qquad (19)$$

and the direct utility function

$$U(q,\ I) = \left(\sum_{i \in I} \alpha_i^* q_i^{-\rho} \right)^{-1/\rho}, \qquad (20)$$

where $\alpha_i^* \equiv \alpha_i^{1/\sigma} > 0$, and $0 > \rho \equiv (1-\sigma)/\sigma > -1$.[11] Due to the self-duality of the CES form and the exactness of the Lloyd's index (18), the following theorem can be readily derived.

Theorem 3

If preferences are represented by the CES functions, (6) and (19)–(20), then the true quantity index of Malmquist (1953) can be

[9]Shapiro and Wilcox (1997) credit this formula also to an unpublished 1996 manuscript by Brent R. Moulton of the BLS.

[10]See Woodland (1982, pp. 28-9) and Nahm (1997) for the self-duality between the CES unit cost function and utility function.

[11]Theorem 3 holds even if $\sigma > 0$, or equivalently $\rho > -1$, but Theorems 4 and 5 hold only if $\sigma > 1$, or equivalently $0 > \rho > -1$; see footnote 7.

exactly measured by

$$Q_{MI}(q^0,\ q^1,\ u,\ I) \equiv \frac{D(q^1,\ u,\ I)}{D(q^0,\ u,\ I)} = Q_{I1} = Q_{I0}, \tag{21}$$

where

$$Q_{I1} \equiv Q_{I1}(p^1,\ q^0,\ q^1,\ I) \equiv \left\{ \sum_{i \in I} s_i^1(I) \left(\frac{q_i^1}{q_i^0} \right)^{\rho} \right\}^{1/\rho}, \text{ and} \tag{22}$$

$$Q_{I0} \equiv Q_{I0}(p^0,\ q^0,\ q^1,\ I) \equiv \left\{ \sum_{i \in I} s_i^0(I) \left(\frac{q_i^1}{q_i^0} \right)^{-\rho} \right\}^{-1/\rho}. \tag{23}$$

See Appendix for a proof.

Now, to incorporate new and disappeared items into the index, recall the definitions of the choice sets in Theorem 2. Then, define the corresponding indices as

$$Q_{NEW} \equiv Q_{NEW}(p^1,\ q^0,\ q^1,\ I,\ I^1) \equiv \left\{ \sum_{i \in I} s_i^1(I^1) \left(\frac{q_i^1}{q_i^0} \right)^{\rho} \right\}^{1/\rho}, \text{ and} \tag{24}$$

$$Q_{OLD} \equiv Q_{OLD}(p^0,\ q^0,\ q^1,\ I,\ I^0) \equiv \left\{ \sum_{i \in I} s_i^0(I^0) \left(\frac{q_i^1}{q_i^0} \right)^{-\rho} \right\}^{-1/\rho}. \tag{25}$$

Note that $s_i^t(I^t)$ are budget shares within the set I^t, which are the naturally observed budget shares in each period, but the summation operators run only over the set I, which is the (sub)set of items available in both periods. So, if the σ is given, all of the above indices, Q_{I1}, Q_{I0}, Q_{NEW}, and Q_{OLD} are computable by using observable budget shares and quantity ratios in two periods.

Theorem 4

Suppose that preferences are represented by the CES functions. Then, the Malmquist quantity index incorporating new and disappeared items can be exactly measured by

$$Q_{MNEW}(q^0,\ q^1,\ u,\ I,\ I^1) \equiv \frac{D(q^1,\ u,\ I^1)}{D(q^0,\ u,\ I)} = Q_{NEW}; \tag{26}$$

$$Q_{MOLD}(q^0,\ q^1,\ u,\ I^0,\ I) \equiv \frac{D(q^1,\ u,\ I)}{D(q^0,\ u,\ I^0)} = Q_{OLD}; \text{ and} \tag{27}$$

$$\begin{aligned} Q_{MALL}(q^0,\ q^1,\ u,\ I^0,\ I^1) &\equiv \frac{D(q^1,\ u,\ I^1)}{D(q^0,\ u,\ I^0)} \\ = Q_{ALL} &\equiv \frac{Q_{NEW}\, Q_{OLD}}{Q_{I1}} \left(= \frac{Q_{NEW}\, Q_{OLD}}{Q_{I0}} \right). \end{aligned} \tag{28}$$

See Appendix for a proof.

Note that Q_{MNEW} measures the change in the real consumption quantity caused by the introduction of new items, and Q_{MOLD} the effect of the disappearance of old items, as well as the real quantity change in the existing items which is measured by Q_{MI} of (21). The result in (28) shows that the relationship between these parts is multiplicative. Thus, the real quantity index incorporating all three parts can be obtained by multiplying them by one another.[12]

To derive the respective COL indices, we utilize the well-known result that for a homothetic preference ordering the product of the Malmquist quantity index and its dual Konüs COL index equals the expenditure ratio.

Theorem 5

Let the preference ordering be represented by the CES functions, (6) and (19)–(20). Then, the Konüs COL index incorporating new and disappeared items can be exactly measured as follows:

$$P_{KI}(p^0, p^1, u, I) \equiv \frac{E(p^1, u, I)}{E(p^0, u, I)} = P_{I1} \equiv \frac{X^1(I)/X^0(I)}{Q_{I1}} = P_{I0} \equiv \frac{X^1(I)/X^0(I)}{Q_{I0}}; \tag{29}$$

$$P_{KNEW}(p^0, p^1, u, I, I^1) \equiv \frac{E(p^1, u, I^1)}{E(p^0, u, I)} = P_{NEW} \equiv \frac{X^1(I^1)/X^0(I)}{Q_{NEW}}; \tag{30}$$

$$P_{KOLD}(p^0, p^1, u, I^0, I) \equiv \frac{E(p^1, u, I)}{E(p^0, u, I^0)} = P_{OLD} \equiv \frac{X^1(I)/X^0(I^0)}{Q_{OLD}}; \text{ and} \tag{31}$$

[12] Q_{MNEW} is the product of the index for new items and the index for existing items, while Q_{MOLD} is the product of the index for disappeared items and the index for existing items. So, the real quantity index for all items can be obtained by multiplying Q_{MNEW} by Q_{MOLD}, and then dividing by the index for existing items, Q_{MI}.

$$
\begin{aligned}
P_{KALL}(p^0, p^1, u, I^0, I^1) &\equiv \frac{E(p^1, u, I^1)}{E(p^0, u, I^0)} \\
&= P_{ALL} \equiv \frac{P_{NEW}\,P_{OLD}}{P_{I1}}\left(=\frac{P_{NEW}\,P_{OLD}}{P_{I0}}\right) \\
&= \frac{X^1(I^1)/X^0(I^0)}{Q_{NEW}\,Q_{OLD}/Q_{I1}}\left(=\frac{X^1(I^1)/X^0(I^0)}{Q_{NEW}\,Q_{OLD}/Q_{I0}}\right) \\
&= \frac{X^1(I^1)/X^0(I^0)}{Q_{ALL}}
\end{aligned}
\tag{32}
$$

where $X^t(I) \equiv \sum_{i \in I} p_i^t q_i^t$; and $X^t(I^t) \equiv \sum_{i \in I^t} p_i^t q_i^t$ for $t=0, 1$. See Appendix for a proof.

In analogy to the quantity indices, the COL index P_{ALL} can be separated into the three indices P_{I0} $(=P_{I1})$, P_{NEW} and P_{OLD} which respectively represent the indices for existing items, new and existing items, and disappeared and existing items. Thus, the effect of the introduction of new items on the COL index can be measured by P_{NEW}/P_{I0}, while the effect of disappeared items by P_{OLD}/P_{I0}. Notice that P_{NEW}/P_{I0} equals $E(p^1, u, I^1)/E(p^1, u, I)$ and P_{OLD}/P_{I0} equals $E(p^0, u, I)/E(p^0, u, I^0)$. So, they measure the net effects on the COL of a changing choice set from I to I^1 or from I^0 to I. The relationship between the current index and the Feenstra index (13) can be easily seen by noting that the product of P_{NEW}/P_{I0} and P_{OLD}/P_{I0} corresponds to $(\lambda^1/\lambda^0)^{1/(\sigma-1)}$; see Appendix.

There is no guarantee that the new index, P_{ALL}, will always result in a lower inflation rate than does the CPI, but in general it will. To see why this is the case, notice that the CPI is in essence a Laspeyres price index. It is well known that when the preference ordering is homothetic the Laspeyres index is the upper limit of the true index.[13] So, if $I^0=I^1=I$ and the CPI is computed over the same set I, of which the prices and quantities are congruent with the CES preference ordering, it is guaranteed that the new index will not be greater than the CPI. Even if the true preference ordering is non-CES, it can still be shown that the new index is always smaller than CPI if $\sigma>1$ and p^1 is not vector-proportional to p^0.[14]

[13] See, for instance, Diewert(1993, p. 183).

[14] This can be proved by applying the inequality $(\sum s_i y_i^r)^{1/r} < (\sum s_i y_i^s)^{1/s}$ if $r < s$, $0 < s$, $y_i \neq 0$ for any i, and not all y_i are equal; see Hardy *et al.* (1964, p.

If $I^0 \neq I$, $I^1 \neq I$, or both, the set of the items included in the CPI would be different from that used for the new index, and hence there would be no guarantee that the above result will still hold. However, considering the fact that λ^1 is usually smaller than λ^0, that is, the proportion of the expenditure on new items to the total expenditure is usually larger than that of the expenditure on old and vanishing items, reveals that the new index will be smaller than the CPI in general; see (13).

The size of the bias in the CPI that can be corrected by using the new index would depend on the appropriate size of σ, the current level of CPI, and how the sets I, I^0 and I^1 are defined. In the case of the 1993 CPI, the US BLS found that about 1.5 percent (3.4%×0.43) of all price quotations involved items that had disappeared and no directly comparable items could be found for; see Nordhaus (1998, p. 60). If we assume that the value of expenditure is roughly proportional to the number of items, this implies that λ^0 is approximately 0.985. If we further assume that the expenditure on completely new or improved-quality items is worth 2 percent of the total expenditure, λ^1 would be 0.98. Most of the upper-level substitution bias of 0.15 percentage points, estimated by the Boskin report, could be eliminated if the new index number formula were used. So, as a whole, the new index could correct a bias of 0.7~1.2 percentage points if $\sigma = 1.5$~2.0 and CPI = 1.03~1.05 (compared with the previous year).[15]

As was the case for the Feenstra index, the current index also includes the unknown constant σ. A feasible solution to this problem could be the grid-search approach employed by Shapiro and Wilcox (1997). In that approach, the index number formula containing σ and a *superlative* index number formula, such as Fisher ideal or Törnqvist, are used to compute the COL indices, over a certain period of time, for a set of items that are available during the whole period.[16] Various values are tried for σ. Then,

26). Note that if $I^0 = I^1 = I$, then P_V (eq. 8) $= P_F$ (eq. 13) $= P_L$ (eq. 18) $= P_{ALL}$ (eq. 32). The P_L equals the left side of the above inequality if $y_i = p_i^1/p_i^0$ and $r = 1-\sigma$. If $s=1$ and y_i is defined as above, then the right side becomes the Laspeyres price index (CPI). Finally, note that $r<1$ when $1<\sigma$ as required in the text.

[15]The estimates are based on the formula, $\text{CPI} - (\text{CPI} - 0.0015) \times (0.98/0.985)^{1/(\sigma-1)}$; see (13).

[16]Diewert (1976) calls an index number formula *superlative* if it is *exact*

the value that produces the closest CES indices to the superlative indices could be selected as an appropriate estimate of σ. This estimate of σ, in turn, could be used to compute the indices incorporating new and disappeared items.

VI. Concluding Remarks

The index introduced in the present paper is exact for the CES preference ordering if the elasticity of substitution is given. Unlike the traditional indices, it is capable of incorporating the effect of new goods and disappeared goods on the COL. It is different from the one already introduced to the literature by Feenstra (1994) in the sense that the new index number can distinguish the effect of new goods from that of disappeared goods. This new feature could prove useful if one is interested in the separate effects of new goods and disappeared goods, on the COL. In the current literature, most researches are concerned with the bias in the CPI caused by incorrect treatment of the introduction of new goods. However, the effect of disappeared items on the COL would not be immaterial. For instance, the disappearance of cheap, simple old versions of some electronic equipment may contribute toward an increase in the COL. The unavailability of printer ribbon for some old dot-matrix printer could cost the owner as much as the price of a new-model printer.

A drawback of the new index number formula is that the elasticity of substitution must be estimated. A practical method to estimate the elasticity using historical data has been suggested. The restrictive feature of the assumption of a constant elasticity of substitution would not be as serious a problem as it usually is if the index number formula was applied to a group of items that belong to an ELI or item stratum where the substitution elasticities between its member items are close to one another. This reasoning is even more appealing considering that, as Gordon and Griliches (1997, p. 84) point out, the magnitudes of quality change bias are best assessed for each category since the size of bias differs so much across product categories.

for an aggregator function that can provide a second-order approximation to an arbitrary twice continuously differentiable linearly homogeneous function.

Appendix

***Proof of Theorem* 3:** Consider the well-known result

$$\frac{\partial \ln D(q,\ u,\ I)}{\partial \ln q_i} = s_i(I). \tag{A1}$$

So, from the CES distance function (19), we have

$$s_i^t(I) \equiv \frac{p_i^t q_i^t}{\sum_{j \in I} p_j^t q_j^t} = \frac{p_i^t q_i^t}{X^t(I)} = (\Psi^t(I))^{-1} \alpha_i^* q_i^{t-\rho},\ \text{and} \tag{A2}$$

$$s_i^t(I^t) \equiv \frac{p_i^t q_i^t}{\sum_{j \in I^t} p_j^t q_j^t} = \frac{p_i^t q_i^t}{X^t(I^t)} = (\Psi^t(I^t))^{-1} \alpha_i^* q_i^{t-\rho}\ \text{for}\ t=0,\ 1, \tag{A3}$$

where

$$\Psi^t(I) \equiv \left(\sum_{i \in I} \alpha_i^* q_i^{t-\rho} \right),\ \text{and}\ \Psi^t(I^t) \equiv \left(\sum_{i \in I^t} \alpha_i^* q_i^{t-\rho} \right). \tag{A4}$$

Substituting (A2) into (22) and (23), and noting that

$$\frac{D(q^1,\ u,\ I)}{D(q^0,\ u,\ I)} = \left(\frac{\Psi^1(I)}{\Psi^0(I)} \right)^{-1/\rho}, \tag{A5}$$

proves the theorem.

Q.E.D.

***Proof of Theorem* 4:** Substituting (A3) for $t=1$ into (24) and noting

$$\frac{D(q^1,\ u,\ I^1)}{D(q^0,\ u,\ I)} = \left(\frac{\Psi^1(I^1)}{\Psi^0(I)} \right)^{-1/\rho}, \tag{A6}$$

proves (26). Similarly, (27) can be proved by substituting (A3) for $t=0$ into (25) and noting

$$\frac{D(q^1,\ u,\ I)}{D(q^0,\ u,\ I^0)} = \left(\frac{\Psi^1(I)}{\Psi^0(I^0)} \right)^{-1/\rho}. \tag{A7}$$

Having proved (26) and (27), (28) is simply a restatement of the following relationship

$$Q_{MALL} = \frac{Q_{MNEW}\, Q_{MOLD}}{Q_{MI}}. \tag{A8}$$

Q.E.D.

***Proof of Theorem* 5:** Since the preference ordering is homothetic, (29) is obvious. To prove the other parts of the theorem, consider *Shephard's lemma.*

$$q_i^t = \frac{\partial E(p^t, u^t)}{\partial p_i} = u^t(\Omega^t(I^t))^{\sigma/1-\sigma} \alpha_i p_i^t \quad \text{(A9)}$$

where

$$\Omega^t(I^t) \equiv \left(\sum_{i \in I^t} \alpha_i p_i^{t\ 1-\sigma} \right) \text{ for } t=0,\ 1. \quad \text{(A10)}$$

Also note that

$$X^t(I^t) \equiv u^t \left(\sum_{i \in I^t} \alpha_i p_i^{t\ 1-\sigma} \right)^{1/(1-\sigma)} = u^t(\Omega^t(I^t))^{1/(1-\sigma)},\ \text{but} \quad \text{(A11)}$$

$$X^t(I) = \sum_{i \in I} p_i^t q_i^t = \sum_{i \in I} p_i^t \left\{ u^t \left(\Omega^t(I^t) \right)^{\sigma/(1-\sigma)} \alpha_i p_i^{t\ -\sigma} \right\} \quad \text{(A12)}$$

Substituting (A9) into (24), then into (30) together with (A11) and (A12) proves (30). Note that $E(p^1, u, I^1)/E(p^0, u, I) = [\Omega^1(I^1)/\Omega^0(I)]^{1/(1-\sigma)}$. The result in (31) can be similarly proved. In analogy to the proof of (28), the result (32) can be proved by noting that $P_{KALL} = (P_{KNEW}P_{KOLD})/P_{I1}$.

Q.E.D.

Relationship with Feenstra's Index: The part of the index for new and disappeared items can be obtained by

$$\frac{P_{NEW}P_{OLD}}{P_{I0}^2} = \frac{P_{NEW}P_{OLD}}{P_{I0}P_{I1}}$$

$$= \frac{\dfrac{(X^1(I^1)/X^0(I))}{Q_{NEW}} \times \dfrac{(X^1(I)/X^0(I^0))}{Q_{OLD}}}{\dfrac{(X^1(I)/X^0(I))}{Q_{I1}} \times \dfrac{(X^1(I)/X^0(I))}{Q_{I0}}} \quad \text{(A13)}$$

$$= \left(\frac{\lambda^0}{\lambda^1} \right) \left(\frac{Q_{I1}\, Q_{I0}}{Q_{NEW}\, Q_{OLD}} \right),$$

where

$$\lambda^t \equiv \frac{X^t(I)}{X^t(I^t)} \text{ for } t=0,\ 1. \quad \text{(A14)}$$

Substituting $s_i^t(I) \equiv \frac{p_i^t q_i^t}{\sum_{j \in I} p_j^t q_j^t} = \frac{p_i^t q_i^t}{X^t(I)}$ and $s_i^t(I^t) \equiv \frac{p_i^t q_i^t}{\sum_{j \in I^t} p_j^t q_j^t} = \frac{p_i^t q_i^t}{X^t(I^t)}$

for the corresponding terms in (22)−(25), then into (A13) yields

$$\left(\frac{\lambda^0}{\lambda^1}\right)\left(\frac{Q_{I1} Q_{I0}}{Q_{NEW} Q_{OLD}}\right) = \left(\frac{\lambda^0}{\lambda^1}\right)\left(\frac{\lambda^0}{\lambda^1}\right)^{1/\rho} = \left(\frac{\lambda^1}{\lambda^0}\right)^{1/(\sigma-1)}, \quad \text{(A15)}$$

since $\rho \equiv (1-\sigma)/\sigma$. The last term in (A15) is the adjustment term in Feenstra's index (13).

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References

Armknecht, Paul A., Lane, Walter F., and Stewart, Kenneth J. "New Products and the U.S. Consumer Price Index." In Timothy F. Bresnahan and Robert J. Gordon eds., *The Economics of New Goods – NBER Studies in Income and Wealth*, Vol. 58. Chicago: The University of Chicago Press, 1997.

Boskin, Michael J., Dulberger, Ellen R., Griliches, Z., Gordon, Robert J., and Jorgenson, Dale. "Toward a More Accurate Measure of the Cost of Living." Final Report to the Senate Finance Committee from the Advisory Commission to Study the Consumer Price Index, December 1996.

_________. "The CPI Commission: Findings and Recommendations." *American Economic Review* 87 (May 1997): 78-83.

Boskin, Michael J., and Jorgenson, Dale W. "Implications of Overstating Inflation for Indexing Government Programs and Understanding Economic Progress." *American Economic Review* 87 (May 1997): 89-93.

Bureau of Labor Statistics. "The Consumer Price Index." In *Handbook of Methods*, Chapter 17. April 1997.

Diewert, W. Erwin. "Exact and Superlative Index Numbers." *Journal of Econometrics* 4 (May 1976): 115-45.

_________. "Index Numbers." In J. Eatwell, M. Milgate and P. Newman, eds., *The New Palgrave: A Dictionary of Economics*, Vol. 2. London: The Macmillan Press (1987): 767-80.

_________. "The Economic Theory of Index Numbers: A Survey." In W.E. Diewert and A.O. Nakamura eds., *Essays in Index Number*

Theory, Vol. 1. Amsterdam: North-Holland, 1993.

________. "Comment on CPI Biases." *Business Economics* 31 (April 1996): 30-5.

Feenstra, Robert C. "New Product Varieties and the Measurement of International Prices." *American Economic Review* 84 (March 1994): 157-77.

________, and Shiells, Clinton R. "Bias in U.S. Import Prices and Demand." In Timothy F. Bresnahan and Robert J. Gordon eds., *The Economics of New Goods – NBER Studies in Income and Wealth*, Vol. 58. Chicago: The University of Chicago Press, 1997.

Gordon, Robert J., and Griliches, Zvi. "Quality Change and New Products." *American Economic Review* 87 (May 1997): 84-8.

Hardy, G. H., Littlewood, J. E., and Polya, G. *Inequalities*. Cambridge: Cambridge University Press, 1964.

Hausman, Jerry A. "Exact Consumer's Surplus and Deadweight Loss." *American Economic Review* 71 (September 1981): 662-76.

________. "Valuation of New Goods under Perfect and Imperfect Competition." In Timothy F. Bresnahan and Robert J. Gordon eds., *The Economics of New Goods – NBER Studies in Income and Wealth*, Vol. 58. Chicago: The University of Chicago Press, 1997a.

________. "The CPI Commission: Discussion." *American Economic Review* 87 (May 1997b): 96-7.

Hicks, J. R. "The Valuation of the Social Income." *Economica* 7 (1940): 105-24.

Konüs, A. A. "The Problem of the True Index of the Cost of Living." *Econometrica* 7 (1939): 10-29 (The original non-English version was published in 1924).

Lloyd, P. J. "Substitution Effects and Biases in Nontrue Price Indices." *American Economic Review* 65 (June 1975): 301-13.

Malmquist, S. "Index Numbers and Indifference Surfaces." *Trabajos de Estadistica* 4 (1953): 209-42.

Nahm, Daehoon. "A Duality Theorem for the Forms of Cost Function and Distance Function." Macquarie Economics Research Papers 3/97. Department of Economics, Macquarie University, February 1997.

Nordhaus, William D. "Quality Change in Price Indexes." *Journal of Economic Perspectives* 12 (Winter 1998): 59-68.

Sato, Kazuo. "The Ideal Log-Change Index Number." *Review of Economics and Statistics* 58 (May 1976): 223-8.

Shapiro, Matthew D., and Wilcox, David W. "Alternative Strategies for Aggregating Prices in the CPI." *Review-Federal Reserve Bank of St. Louis* 79 (May/June 1997): 113-25.

Vartia, Yrjö O. *Relative Changes and Economic Indices: A Licenciate Thesis in Statistics.* Helsinki: University of Helsinki, 1974.

_________. "Ideal Log-change Index Numbers." *Scandinavian Journal of Statistics* 3 (1976): 121-26.

Woodland, Alan D. *International Trade and Resource Allocation.* Amsterdam: North-Holland. 1982.