An Oligopoly Model of Commercial Fishing

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Population dynamics and oligopoly theory are combined to formulate international fishery under imperfect competition. It is assumed that fish harvesting countries behave as oligopolist, but their costs depend on their harvest rates as well as on the total fish stock, which is governed by the biological growth law. We show that the number of nonextinct equilibria is 0, 1, or 2, and characterize the dynamic behavior of the fish stock in terms of model parameters and initial fish stock level. Finally, we analyze how the steady state equilibrium fish stock is affected by entry of a new fishing country. (*JEL* Classifications: L13, L67)

I. Introduction

In his earlier papers Okuguchi (1996, 1998) has analyzed international commercial fishing under imperfect competition, where two countries harvest fish of a single species in an open access sea. It is assumed that the fish each country harvests is sold not only in its own country but also in the other country. Therefore the two countries form an international duopoly, where each country's harvesting cost is assumed to depend not only on its harvest rate but also on the total fish stock, whose intertemporal movement in the absence of fishing is assumed to be governed by the biological growth law (that is, logistic law).

In this paper we will extend the existence and stability analysis of Okuguchi's model to n-country case, namely we will analyze

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international oligopoly in fishery under imperfect competition. In the theory of oligopoly the qualitative properties, such as stability, of the dynamic processes often differ for duopoly and for oligopoly with more than two firms. We will show that for the model examined in this paper, most of his conclusion for the long-run dynamics will remain valid in the extended case, but the steady state equilibrium fish stock will be affected by the number of fishing countries.

II. Mathematical Model

Let x_{ki} denote the amount of fish harvested by country k and sold in country i ($k=1, 2, \cdots, n$; $i=1, 2, \cdots, n$). The inverse demand function in country i is assumed to be the following:

$$p_i=a_i-b_iY_i, \tag{1}$$

where $Y_i=x_{1i}+x_{2i}+\cdots+x_{ni}$. The fishing cost of country k is given by

$$C_k=c_k+\gamma_k X_k^2/X, \tag{2}$$

where $X_k=x_{k1}+x_{k2}+\cdots+x_{kn}$ and X is the total level of fish stock.[1] Therefore the profit of country k can be written as

[1]Suppose a fishing firm in a single country. Its harvest rate x of the fish is generally formulated as a function of the fish stock X and its fishing effort E,

$$x=h(X, E), \quad \frac{\partial h}{\partial X}>0, \quad \frac{\partial h}{\partial E}>0.$$

As a special case we can consider

$$x=eX^{\alpha}E^{\beta},$$

which leads to

$$E=e^{(-1/\beta)}x^{(1/\beta)}X^{(-\alpha/\beta)}.$$

Suppose the unit cost of fishing effort to be constant δ. Then the total cost C for harvesting x is

$$C\equiv\delta E.$$

If, in addition, $\alpha=\beta=(1/2)$ (technology of constant returns to scale in fish stock and fishing effort) and $\gamma\equiv\delta e^{(-1/\beta)}$, the total cost for harvesting x is given by

$$C=\frac{\gamma x^2}{X},$$

which is formally identical to (2) without opportunity cost, i.e. $c_k=0$. See Clark (1976).

$$\pi_k = \sum_{i=1}^{n} p_i x_{ki} - (c_k + \gamma_k X_k^2 / X), \tag{3}$$

which is a concave function of vector $(x_{k1}, x_{k2}, \cdots, x_{kn})$.

We assume that the n countries behave as Cournot oligopolists. Given X, this situation can be modeled as an n-person noncooperative game with the set of nonnegative vectors being the set of strategies for all players, and π_k being the payoff function of player k. This game is equivalent to a linear complementarity problem with positive definite coefficient matrix (see Okuguchi and Szidarovszky 1990, Section 3.2). Therefore there is a unique Cournot-Nash equilibrium. Excluding corner equilibrium, the first order conditions for country k's profit maximization are given by

$$\frac{\partial \pi_k}{\partial x_{ki}} = -b_i x_{ki} + a_i - b_i Y_i - 2\gamma_k X_k / X = 0.$$

Hence we have for all k and i,

$$x_{ki} = \frac{a_i}{b_i} - Y_i - \frac{2\gamma_k}{b_i X} X_k. \tag{5}$$

Adding these equations for all values of i leads to the following:

$$X_k = A - S - B\frac{2\gamma_k}{X} X_k, \tag{6}$$

where S is the total amount of fish harvested,

$$A = \sum_{i=1}^{n} \frac{a_i}{b_i}, \quad B = \sum_{i=1}^{n} \frac{1}{b_i}. \tag{7}$$

Solving equation (6) for X_k we have

$$X_k = \frac{A - S}{1 + 2B\dfrac{\gamma_k}{X}}, \tag{8}$$

and by adding this equation for all values of k we get one equation for the single unknown S:

$$S = (A - S)\sum_{k=1}^{n} \frac{1}{1 + 2B\dfrac{\gamma_k}{X}}, \tag{9}$$

from which we see that

$$S = \frac{Af(X)}{1 + f(X)} \tag{10}$$

with

$$f(X) = \sum_{k=1}^{n} \frac{1}{1+2B\dfrac{\gamma_k}{X}}. \tag{11}$$

We assume that the fish stock changes according to $\dot{X}=X(\alpha - \beta X)$ in the absence of fishing.[2] Therefore it changes according to

$$\dot{X}=X\left(\alpha - \beta X - \frac{Af(X)}{(1+f(X))X}\right), \tag{12}$$

in the presence of international commercial fishing, where α and β are positive constants, α being the intrinsic growth rate. This time-invariant nonlinear differential equation will be examined in the next section.

III. Existence and Stability of the Steady State

Introduce the notation

$$g(X)=\frac{Af(X)}{(1+f(X))X}. \tag{13}$$

Simple differentiation shows that

$$f'(X) = \sum_{k=1}^{n} \frac{\dfrac{2B\gamma_k}{X^2}}{\left(1+2B\dfrac{\gamma_k}{X}\right)^2}, \tag{14}$$

which implies that

$$Xf'(X)-f(X) = \sum_{k=1}^{n} \frac{-1}{\left(1+2B\dfrac{\gamma_k}{X}\right)^2} <0. \tag{15}$$

Therefore

$$g'(X)=\frac{A(f'(X)X-f(X)-f(X)^2)}{X^2(1+f(X))^2}<0. \tag{16}$$

[2]This equation is known as the logistic law, which has been known to fit well with experimental data for many biological populations and widely used in the fishery literature, including Clark (1976). There are many alternative dynamic models of population. See May (1973) for some of alternative models.

Hence, g is strictly decreasing in X. Notice next that

$$f''(X)=\sum_{k=1}^{n}\frac{-\dfrac{4B\gamma_k}{X^3}}{\left(1+2B\dfrac{\gamma_k}{X}\right)^3}, \tag{17}$$

which implies inequality

$$-f''(X)\cdot\frac{X^2}{2}=\sum_{k=1}^{n}\frac{\dfrac{2B\gamma_k}{X}}{\left(1+2B\dfrac{\gamma_k}{X}\right)^3}<\sum_{k=1}^{n}\frac{1}{\left(1+2B\dfrac{\gamma_k}{X}\right)^2} \tag{18}$$
$$=f(X)-Xf'(X),$$

where we used relation (15) is the last step. Finally, simple differentiation and calculation show that the numerator of $g''(X)$ (which determines its sign) can be written as

$$A(f''X+f'-f'-2ff')X^2(1+f)^2-A(f'X-f-f^2)\{2X(1+f)^2+X^2 2(1+f)f'\}$$
$$=AX(1+f)\{(f''X-2ff')X(1+f)-(f'X-f-f^2)(2+2f+2Xf')\}. \tag{19}$$

The sign g'' coincides with the sign of the bracketed term, which can be simplified and bounded as follows:

$$f''X^2(1+f)-2f'X(1+f)+2f+4f^2+2f^3-2(Xf')^2$$
$$>-2(f-Xf')(1+f)-2f'X(1+f)+2f+4f^2+2f^3-2f^2 \tag{20}$$
$$=2f^3>0,$$

where we used relations (18) and (15). Hence g is strictly convex in X.

The nonextinct steady state or the bionomic equilibrium for the fish stock satisfies equation

$$\alpha-\beta X=g(X). \tag{21}$$

The left hand side is strictly decreasing and linear, the right hand side is strictly decreasing and strictly convex, therefore we have the following possibilities, as in Okuguchi (1996):

***Case* 1.** There is no real root of equation (21). This case is illustrated in Figure 1;

***Case* 2.** Equation (21) has only one root; and

$$g(0)=\frac{A}{2B}\sum_{k=1}^{n}\frac{1}{\gamma_k}>\alpha,$$

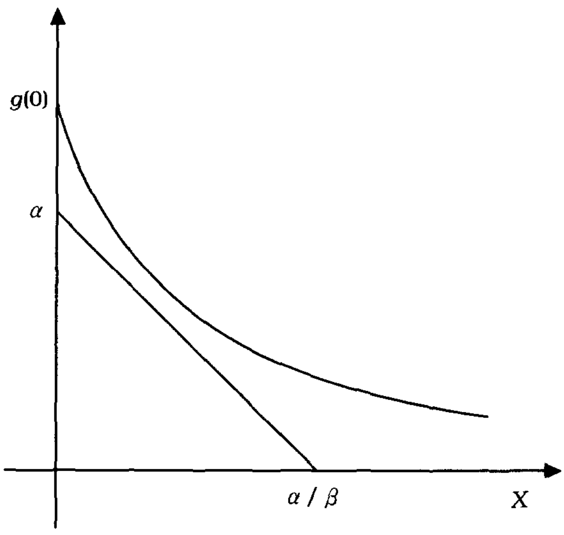

FIGURE 1
CASE 1 WITHOUT REAL ROOT

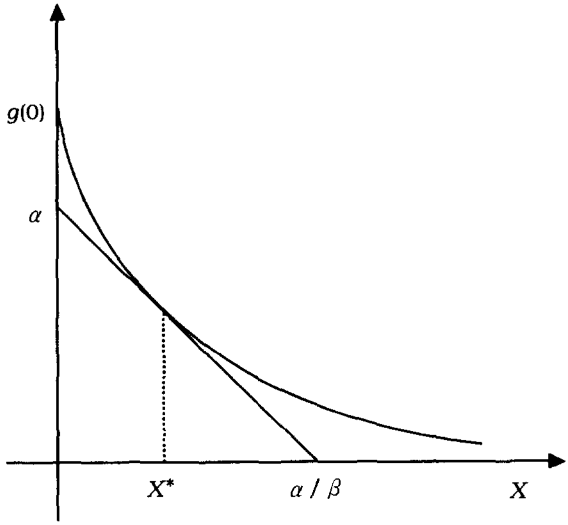

FIGURE 2
CASE 2 WITH A SINGLE ROOT

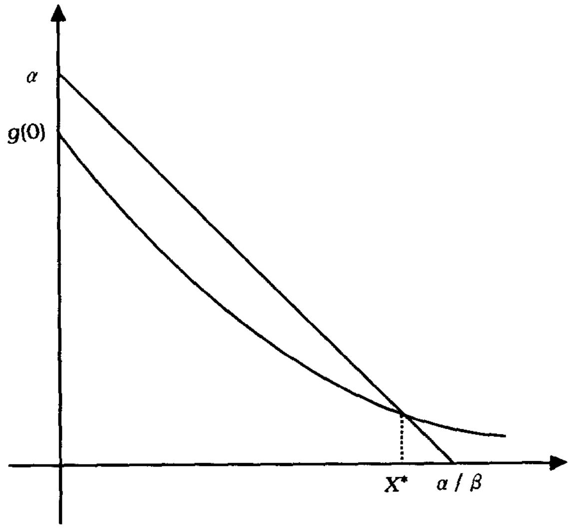

FIGURE 3
CASE 3 WITH A SINGLE ROOT

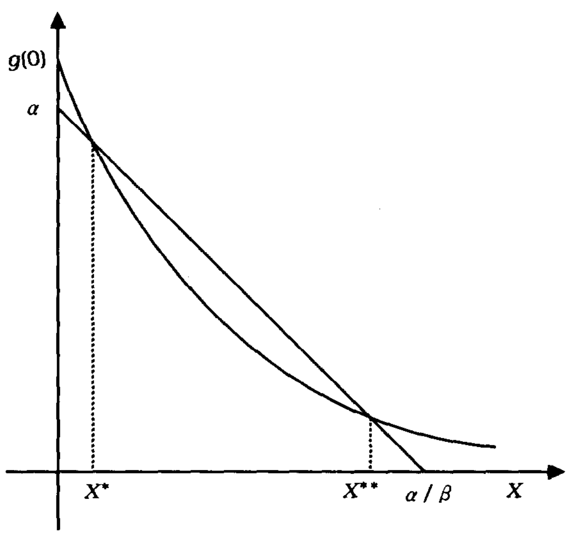

FIGURE 4
CASE 4 WITH TWO REAL ROOTS

which case is shown in Figure 2;

***Case* 3**. There is only one real root and $g(0) < \alpha$, which case is shown in Figure 3;

***Case* 4**. There are two real roots.

Notice that the facts $g(0) > 0$ and $\lim_{X\to\infty} g(X) = 0$ imply that these four cases contain all possibilities.

Let $X_0 = X(0)$ denote the initial total level of fish stock. In Case 1, $\dot{X}$ is always negative, therefore $X(t)$ is strictly decreasing. Since $X^* = 0$ is the only equilibrium, $X(t)$ converges to zero as $t \to \infty$, resulting in extinction of fish stock. In Case 2, $\dot{X}$ is always negative, unless $X(t) = X^*$. Therefore, if $X(0) > X^*$, then $X(t)$ converges to X^*, and if $X(0) < X^*$, then $X(t)$ converges to zero. In Case 3, $\dot{X} < 0$ for $X > X^*$, and $\dot{X} > 0$ for $X < X^*$. In the first case X decreases, and in the second case X increases. Therefore $X(t)$ converges to X^* regardless of the value of $X(0)$. In case 4, we have three subcases. If $X(0) < X^*$, then $X(t)$ decreases and converges to zero; if $X^* < X(0) < X^{**}$, then $X(t)$ increases, therefore $X(t) \to X^{**}$. If $X(0) > X^{**}$, then $X(t)$ decreases and $X(t) \to X^{**}$.

IV. Entry

In the preceding analysis we have assumed a fixed number of fishing countries. In this section we will examine the effects on the steady state equilibrium fish stock of entry of a new fishing country or the effects of the number of fishing countries. Let the number of fishing countries increase from n to $n+1$. The country $n+1$ is assumed to have the inverse demand function

$$p_{n+1} = a_{n+1} - b_{n+1} Y_{n+1} \tag{1'}$$

and the fishing cost

$$C_{n+1} = c_{n+1} + \frac{\gamma_{n+1} X^2_{n+1}}{X}. \tag{2'}$$

The steady state fish stocks can be compared on the basis of equation (21). Let $g(X)$ for n countries and that for $n+1$ countries be denoted by $g_n(X)$ and $g_{n+1}(X)$, respectively. We use similar notations for A, B and $f(X)$. To compare the steady states we have to examine the sign of

$$g_{n+1}(X)-g_n(X).$$

Since $A_{n+1}>A_n$, the above expression becomes positive if

$$\frac{f_{n+1}(X)}{1+f_{n+1}(X)}>\frac{f_n(X)}{1+f_n(X)}, \tag{22}$$

which holds if and only if

$$f_{n+1}(X)-f_n(X)>0. \tag{23}$$

It is in general impossible to know the validity of (23). In order to avoid this indeterminacy, we consider a simple case and assume symmetric countries, hence

$$a_k\equiv a,\ b_k\equiv b,\ c_k\equiv c,\ \gamma_k\equiv\gamma,\ k=1,\ 2,\ \cdots,\ n+1.$$

A simple calculation yields

$$f_{n+1}(X)-f_n(X)=\frac{n+1}{1+\dfrac{2(n+1)\gamma}{bX}}-\frac{n}{1+\dfrac{2n\gamma}{bX}}>0. \tag{24}$$

Hence, given X, the curve for $g_{n+1}(X)$ lies above that for $g_n(X)$.

Suppose $g_n(X)$ in Figures 1 and 2. In these cases the fish becomes extinct in the event of entry of a new country. In the case of Figure 3, $g_{n+1}(X)$ may have a unique intersection with the downward sloping line, touch it, or lie completely above it. Hence, the unique positive steady fish stock becomes smaller or the fish becomes extinct if the number of fishing countries increases. In the case of Figure 4 with two distinct steady state fish stocks, $g_{n+1}(X)$ may intersect twice with the downward sloping line, touch it or lie completely above it. If the first possibility occurs, the larger steady state fish stock decreases and the smaller one increases in the event of entry of a new fishing country, while in the case of the second possibility a unique steady state exists. Finally, if the third possibility arises, the fish becomes extinct.

V. Concluding Remarks

In an open access sea more than two countries, such as China, South Korea and Japan in the East Sea, engage in commercial fishing. In this paper we have formulated a dynamic model of international commercial fishing on the basis of Cournot oligopoly taking into consideration of the fish population dynamics. We have proved the number of nonextinct steady states to be 0, 1 or 2 and characterized the dynamic behavior of the fish stock. Finally, we have also analyzed how the steady states are affected by an increase in the number of fishing countries.

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References

Clark, C. W. *Mathematical Bioeconomics*. New York and London: John Wiley and Sons, 1976.

May, R. M. *Stability and Complexity in Model Ecosystems*. New Jersey: Princeton University Press, 1973.

Okuguchi, K. *Long-run Fish Stock and Imperfectly Competitive commercial Fishing*. Nagoya: Nanzan University, 1996.

_________. "Long-run Fish Stock and Imperfectly Competitive Commercial Fishing." *Keio Economic Studies* 35 (1998): 9-17.

_________, and Szidarovszky, F. *The Theory of Oligopoly with Multi-Product Firms*. Berlin/Heidelberg/New York: Springer-Verlag 1990, the revised version under preparation.