By Force of Habit Formations and Exchange Rate Movements

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This paper sets up a habit persistence monetary model in line with Campbell and Cochrane (1995) with transaction costs in consumption. It discusses the behavior of exchange rates and consumption as well as other variables to various shocks. This paper shows that the habit persistence model fails to explain the exchange rate fluctuations and the movement of consumption due to a locally large value of relative risk aversion. (JEL Classifications: F30, F40)

I. Introduction

In monetary economics and finance, it is common practice to employ a representative agent model in order to derive some restrictions on consumption and asset returns and compare them with the actual data. Because the aggregate consumption data for the G-7 countries are smooth, it seems that the way in which the agent’s preference evaluate small gambles about certainty is critical for providing a good fit to the data. But the common constant-relative-risk-averse, expected utility function fails in this respect, as pointed out by many economists. Mehra and Prescott (1985) argue that the representative agent, expected additive utility model, sensibly restricted, cannot account for both the 0.8 percent average real return on debt and the nearly 7.0 percent average real return on equity that the U.S. data show for the 1889-1978 period. In related

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modelling exercises, Grossman, Melino and Shiller (1989), Cecchetti and Mark (1990), and Kandel and Stambaugh (1991) report that the representative agent model with a degree of relative risk aversion in the range of 20-30 perform fairly well. On the other hand, Kocheleakota (1990) shows that a representative model with a negative time preference can match the above historical average.

In recent years, models with habit formation have been quite successful in linking consumption and asset prices. Constantinides (1990) showed that once a habit formation is added to the standard model with power utility and lognormal distribution, the equity premium puzzles disappear. More recently, Boldrin, Christiano and Fisher (1995) replaced the power specification of utility with the habit persistence specifications proposed by Constantinides (1990) and investigated the implications of the model on the equity premium puzzle as well as the business cycles. Campbell and Cochrane (1995) present a model such that the habit formation is external,\(^1\) as in Abel's (1990) "keeping up with the Joneses" formulation and succeed in explaining many asset pricing puzzles including the risk free rate puzzle.

In international finance there is vast empirical literature that documents the excessive variation of exchange rates and the failure of the hypothesis that the forward exchange rate is an unbiased predictor of the future spot exchange rate. The effects of monetary shocks on exchange rates are documented to last long and persistent. While some economists explain this failure as the market inefficiency, others make efforts to set up a dynamic general equilibrium model and explain risk premiums. Among others Cardia (1991) set up a dynamic general equilibrium model and tried to explain the exchange rate volatility puzzles. The flexible price international finance model with various shocks could not generate the excessive volatility of exchange rates. Moreover, the flexible price small open economy models with a standard utility function fail to account for persistent effects of monetary shocks on exchange rates. As the habit formation approach is based on the analysis of the consumption path to some fundamental shock in the economy, it is natural that we are led to the question: Can the habit formation model explain the exchange rate puzzle which is

\(^1\)This says that an individual's habit level depends on the history of aggregate consumption rather than the individual's own past consumption.
closely related to consumption?
I begin by setting up a habit formation dynamic general equilibrium model with transaction costs in consumption in line with Campbell and Cochrane (1995). Then, using this model, I investigate the following questions. First, I explore whether this model can explain a long and persistent hump-shaped exchange rate effect of monetary shocks. Second, I discuss whether this model can give rise to volatile exchange rate movements. Third, I explore whether the comovements of exchange rates and other real variables are consistent in the data.

The main findings of this paper can be summarized as follows. First, when there exists a substantial degree of habit formation in consumption, an expansionary monetary shock to the home country does not lead to any noticeable change of consumption and exchange rates. This reflects the strong effect of the locally small intertemporal elasticity of substitution in consumption and leisure. Second, the introduction of habit formation reduces the close comovement of employment and output by making households desire very smooth consumption profile at the cost of too little variation of exchange rates. Finally, the relative variation of endogenous variables as well as the cross correlation between output and other endogenous variables nicely matches with the data.

The organization of the paper is as follows. In section II, I specify a habit formation models with a transaction cost in which the transaction cost is reduced when a representative household use money in purchasing goods. In section III, I discuss the equilibrium conditions and its implications. I discuss the state space analysis and quantitative implications in section IV and section V. Finally I give a concluding remarks in section VI.

II. The Model with Habit Persistence

A. Household

a) Preferences
The economy consists of a continuum of identical infinite-lived households. Following Abel (1990), and Campbell and Cochrane (1995), suppose that a representative household derives utility from
the level of consumption relative to a time-varying subsistence or habit level and from the leisure. In particular, I assume that the habit is external in the sense that it is determined by the aggregate consumption of the nation as a whole, and not by the consumption of any individual household as in Campbell and Cochrane (1995). This simplifies greatly the analysis. The utility function of the representative household takes the form:

$$E_t \sum_{j=0}^{m} \beta^j \psi(C_{ej}; L_{ej}; H_{ej}) \psi(L_{ej}; H_{ej}),$$

(1)

where

$$\psi(C_{ej}; L_{ej}; H_{ej}) = \frac{\{C_{ej} - H_{ej}\}^{-\theta} \psi(L_{ej})^{-\tau}}{1 - \sigma}, \quad \sigma = 1,$$

(2)

and $H_t$ is the level of habit. $\theta$ is the utility curvature parameter about consumption and leisure. The utility of a representative household depends on a power utility of the difference between consumption and habit. The stochastic sequence of habits $H_{et}^{\infty}$ is regarded as exogenous by the household and tied to the stochastic sequence of aggregate consumption $C_{et}^{\infty}$ as follows. Define the surplus consumption ratio as

$$S_t = \frac{C_t - H_t}{C_t}.$$  

(3)

When habit $S_t$ is held constant as consumption $C_t$ varies, the local coefficient of relative risk aversion is

$$-\frac{C_t \psi(L_t)}{u_1} = \frac{\sigma}{S_t}.$$  

(4)

Here risk aversion rises as the surplus consumption ratio $S_t$ decreases. $\sigma$ is no longer the coefficient of relative risk aversion in this model. I need to specify how the habit $S_t$ evolves over time to aggregate consumption. As in Campbell and Cochrane (1995), suppose that the log surplus consumption ratio follows an AR(1) process:

$$s_{t+1} = (1 - \theta) S_t + \phi s_t + \mu (s_0)(c_{t+1} - c_t - \gamma_g),$$

(5)

where small letters represent the natural logarithms. $\gamma_g$ is the average detrended consumption growth rate. $\gamma_g = \mathbb{E}_t \Delta \bar{c}_{t+1}$. The parameter $\phi$ governs the persistence of the log surplus consumption

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2Abel (1990) calls this ‘Catching up with Joneses’.
ratio, and the sensitivity function $\mu(s)$ controls the sensitivity of $s_{t+1}$. That is, it governs the log habit $s_{t+1}$ to innovations in consumption growth $\epsilon_{t+1}$. $\mu(s)$ is defined as follows:

$$
\mu(s) = \begin{cases} 
\frac{1}{S} \sqrt{1 - 2(s - S)}, & s \leq S_{\max} \\
0, & s \geq S_{\max}
\end{cases}
$$

(6)

As in Campbell and Cochrane, let

$$
\bar{S} = S_{\max} \sqrt{\frac{\sigma}{1 - \alpha}}, \quad \bar{s} = \log \bar{S},
$$

and

$$
S_{\max} = \bar{S} + \frac{1}{2} (1 - \bar{S}^2).
$$

Rewriting equation (5)

$$
s_{t+1} = (1 - \alpha)\bar{S} + \phi s_t + \mu(s_t)(c_{t+1} - c_t - \gamma_d)
$$

(7)

$$
= (1 - \alpha)\bar{S} + q(t) + \mu(s_t)(c_{t+1} - \gamma_d).
$$

(8)

where $q(t) = \phi s_t - \mu(s_t)c_t$. The function $\mu(s)$ in equation (6) controls the sensitivity of the habit $s$ to changes in consumption. The price of risk depends on $\mu(s)$ as in Campbell and Cochrane (1995). It is desirable to have a non-constant $\mu(s)$ if one wants to generate state-dependent risk prices. Campbell and Cochrane (1995) choose $\mu(s)$ so that the risk free rate is constant for all $s$. However, there is no a priori reason to use the specification in equation (5) and not much is lost when the constant $\mu$-constant is chosen. As I use a log-approximation method to analyze the dynamics of the model in the next section, I assume a constant $\mu$-function as in Lettau and Uhlig (1995).

b) Budget Constraints and Transaction Costs

Assume that money reduces the costs of consumption transactions and the cost of time to shopping can be represented by a function of consumption levels and real balances, as in Feenstra (1986). That is, when a household has real balance holdings equal to $m$, it must expend additional $\Phi(C_t, m)$ units of goods to consume $C_t$ units as transaction costs. As in Feenstra (1986), I assume that each transaction cost function is homogeneous of degree one in both arguments with $\Phi_1 \geq 0$, $\Phi_2 < 0$, $\Phi_{11} \geq 0$, $\Phi_{22} \geq 0$, and $\Phi_{12} \leq 0$.

First, a representative household faces the time constraint such that
where $N_t$ is the fraction of time devoted to work. Next, consider the household’s budget constraint. The household can lend or borrow an amount of $B_t^p$ in international financial markets and pays the lump-sum taxes of home currency, $T_t$ before the market opens. It also chooses the amount of home currency, $M_t$ to purchase the consumption goods at the goods market.

In addition to money and bonds, the household can invest in physical capital, $K_t$. For analytical simplicity, I assume that the household owns only its own country’s capital stock to rent to its country’s firm and there is no firm specific capital stock. Since we do not empirically observe large discrete capital stock adjustments, it is reasonable to introduce an adjustment cost in capital stock installments. If there are costs of installing capital, the capital stock will move more sluggishly. I assume that there are dead-weight costs of installing capital stock. To preserve the simple model structure as far as possible, I will adopt the Uzawa-Lucas-Prescott form of investment adjustment costs.

$$K_{t+1} = \psi \left( \frac{I_t}{K_t} \right) K_t + (1 - \delta_k) K_t,$$  \hspace{1cm} (10)

where $0 < \delta_k < 1$ is the rate of depreciation, and $\psi (l_t / K_t)$ is an increasing, concave, twice continuously differentiable function with $\psi(0) = \delta_k = 1$. $I_t$ is the composite investment of the home resident at period $t$, and $K_t$ is the composite capital stock of the home resident at period $t$. This formulation implies that Tobin’s $q$ is equal to $1 / \psi '(l_t / K_t)$ which equals one in the steady state. At the beginning of each period, the household receives wages, rents for capital. Thus its budget constraint and wealth at the beginning of the period $t$ are given by,

$$M_t + P_t B_t^p + P_t [C_t + \Phi (\theta)] + P_t I_t \leq \Theta_t - T_t,$$  \hspace{1cm} (11)$^3$

and

$$\Theta_t = M_{t-1} + (1 + r_{t-1}) B_{t-1}^p, P_t W_t H_t + D_t K_t.$$  \hspace{1cm} (12)

Here a money velocity $\nu_t$ at period $t$ is defined as

$^3$The function $\Phi (\theta)$ denotes transaction costs per unit of expenditure. Here it is assumed that total transaction costs are a convex function of expenditure and real balances.
$h_t = \frac{P_t C_t}{M_t}.$

$P_t$, $\Pi_t$, $W_t$, $D_t$, $r_t$ denote a domestic price level, domestic firm’s nominal profits, nominal wages, nominal rental rate for capital stock, and the real interest rate, respectively. The households are assumed to be subject to the borrowing constraint that prevents them from engaging in Ponzi game.

**B. Firm**

The representative firm maximizes profit

$$\Pi_t = \max_{[K^t, N^t]} P_t Y_t - D_t K_t^d - W_t N_t^d,$$  \hspace{1cm} (13)

where

$$Y_t = A_t F(K_t^d, z_t N_t^d)$$  \hspace{1cm} (14)

is output and $K_t^d$ and $N_t^d$ are demanded capital and labor at time $t$. $z_t$ and $A_t$ are the labor augmenting permanent technology progress, and transitory technology process at period $t$. I assume that the technology shock follows an $AR(1)$ process. The permanent changes in the total factor productivity, $z_t$ are taken as growing deterministically, i.e. $\gamma = z_t / z_{t-1}$ for all $t$ as in King, Plosser and Rebelo (1988, hereafter KPR (1988)).

$$\log A_t = \rho \log A_{t-1} + \frac{\xi_{t\rho}}{\xi_{\rho}}, \hspace{1cm} -1 < \rho < 1,$$  \hspace{1cm} (15)

where $E(\xi_{t\rho}) = 0$ and $\xi_{t\rho}$ is i.i.d. over time.

**C. Government**

Suppose that the government holds international bonds, $B_t^p$, levies lump-sum taxes, $T_t$, and supplies money, $M_t$ to finance its expenditures, $G_t$. Its period-by-period budget constraint is

$$\frac{\omega_t M_{t-1}}{P_t} + T_t + B_t^p - (1 + r_t) B_{t-1}^p = G_t.$$  \hspace{1cm} (16)

Here $\omega_t$ is the rate of growth of the nominal money supply.

$$\omega_t = \frac{M_t - M_{t-1}}{M_{t-1}}.$$  \hspace{1cm} (17)

To ensure that the size of the government debt does not grow without bound and to allow a convenient simulation of arbitrary changes in $G$, $T$, and $\omega_t$ without having to solve an optimizing
problem for government as in Cardia (1991), I assume that
\[ T_t = \gamma_t (1 + r_t) \frac{B_t^\theta}{P_t}, \quad \gamma_t > \sup r_t. \]
(18)

As I will base the quantitative analysis on simulations of a log-linearized version of the equilibrium conditions, I must circumvent the unit root problem which is usually contained in the assumption of a constant international interest rate of a small open economy model. It is assumed that the bond holdings are a function of the interest rate to avoid the unit root in the model which invalidates the quantitative analysis. That is, the real interest rate decreases to the amount of net foreign assets.
\[ r_t = f(B_t), \quad f' < 0 \text{ for } B_t = B_t^P + B_t^\theta > 0. \]
(19)

III. Equilibrium

A. First Order Conditions

The first order necessary conditions for a solution to the household problem posed by equations (1) to (13) are the budget constraints with the following Euler equations:
\[ \theta \left(S_C(t)\right)^{\theta-\gamma-1}(1 - N_i)^{(1-\theta)(1-\gamma)} = \lambda_t P_t \left[ 1 + \Phi \left[v_t \right] + v_t \Phi' \left[v_t \right] \right], \]
(20)
\[ (1 - \theta)S_C(t)^{\theta-1}(1 - N_i)^{-\gamma-\theta+\gamma} = \lambda_t P_t \left[ 1 + \Phi \left[v_t \right] + v_t \Phi' \left[v_t \right] \right], \]
(21)
\[ \lambda_t \left[ 1 - \lambda_t P_t \Phi' \left[ u_t \right] \right] = \beta E \left[ \Lambda_{t+1} \right], \]
(22)
\[ \Lambda_t P_t = \beta E \left[ \Lambda_{t+1} R_{t+1} A_{t+1} \right], \]
(23)
\[ \Lambda_t \psi_{t} \left[ X_0 \right] = \beta E \left[ \Lambda_{t+1} \psi_{t} \left[ X_{t+1} \right] + D_{t+1} A_{t+1} \right], \]
(24)
\[ s_{t+1} = (1 - \phi s_t^0 + s_t^0 + \mu_t s_t^0) Y_t^0 - c_t^0 - d \frac{R_t}{K_t}, \]
(25)
\[ K_{t+1} = \psi \left( \frac{I_t}{K_t} \right) K_t \left( 1 - \delta_t \right) K_t, \]
(26)
\[ X_t = \frac{I_t}{K_t}, \]
(27)

where \( \Phi = \Phi \left[ z_t \right] \), \( v_t = c_t / m_t \), \( m_t = M_t / P_t \). Here \( \chi \left[ X_{t+1} \right] = \psi \left[ X_{t+1} \right] - \psi' \left[ X_{t+1} \right] X_{t+1} + 1 - \delta_k \) and \( \Lambda_0 \) is a Lagrange multiplier associated with the household’s budget constraint. In the later quantitative evaluations, I will assume that
\[ \Phi \left( \frac{c_t}{M_t/P_t} \right) = \gamma \left( \frac{c_t}{M_t/P_t} \right), \text{ where } \gamma > 0. \]

Equation (20), the first order condition for the consumption goods, says that the marginal utility of consumption goods equals the sum of the marginal utility of wealth and that of the liquidity service of currency, respectively. Equation (21) relates the marginal utility of leisure to the marginal utility of the wage rate. Equations (22) and (23) refer the intertemporal decision of the household, i.e. the decision of money holdings and net foreign asset holdings, respectively. In particular, these equations imply that the demand for the real balance is a decreasing function of the nominal interest rate. Equation (24) which is the first order condition with respect to the home representative household’s investment represents the evolution of Tobin’s q over time. Though I need not specify the functional form for adjustment cost function, \( \phi \), I should specify three parameters which describe the behavior around the steady state. First, I must specify the steady state value of Tobin’s q and the share of investment in national product. Since the steady state value of Tobin’s q is 1.0, I also set the value of this variable to 1.0 in steady state. And I will take the same investment share in steady state as in a model without adjustment cost. Next, I have to specify the parameter which determines the elasticity of marginal adjustment cost function. As there has been no study about this adjustment cost parameter value, I will present several results through sensitivity analysis in the next section.

The profit maximization conditions of the representative firm are given by

\[ \frac{D_t}{P_t} = \alpha \frac{Y_t}{K_t} \quad (28) \]

\[ \frac{W_t}{P_t} = (1 - \alpha) \frac{Y_t}{K_t} \quad (29) \]

The clearing conditions for the goods as well as the asset markets are given by

\[ Y_t = C_t [1 + \Phi (\phi)] + I_t + G_t + B^*_t - B^*_t - (1 + r_{-1}) \quad (30) \]

\[ M^*_t - M^*_t \quad (31) \]

\[ R^*_t - R^*_t \quad (32) \]

\[ r_t = f (B_t), \quad f' < 0 \text{ for } B_t > 0 \quad (33) \]
Finally the real and nominal interest rate are connected through the PPP rule.

\[(1 + \nu) = (1 + \nu_i)(1 + \varepsilon_i).\]  

(34)

The equilibrium conditions consist of the efficiency conditions of the home consumers and firms, as well as the clearing conditions of each goods market, capital rental market, labor market, money and bond market in each. Specifically, an equilibrium is an allocation of home agents \(C_h, S_h, K_h, X_h, l_h, H_h, B_h, M_h^{e_i} \), a sequence of prices and costate variables for the home country \(P_h, A_h, D_h, W_h, \iota, r_i^{0} \), and exchange rates \(x_t^{P} \) satisfying equilibrium conditions (20)–(33), and the uncovered interest parity condition, equation (34), given \(K_0, P_t^{i-1}, K_{00}, P_{N-1} \) and the exogenous stochastic processes \(\xi_t, \xi_t^{AA}, \xi_t^{\nu_{i-1}}\) satisfying equations (17)–(19).

B. State-Space Analysis

To analyze the dynamic implications of the model, I log-linearize the equilibrium conditions as in KPR (1988). Let’s represent the log-linearized version of the economy system in a state space.

\[\begin{align*}
N_{t+1} &= \Pi N_t + C_{t+1}, \\
X_t &= MN_t, 
\end{align*}\]  

(35) (36)

where \(N_t\) and \(\varepsilon_{t+1}\) are the vector of state variables, and vector of innovations at time \(t+1\) and \(X_t\) is the vector of control variables at time \(t\). In simulations, I assume that the exogenous stochastic processes \(Z_t = \{\omega_t, \xi_t, \xi_t^{\nu_{i-1}}\} \) are jointly covariance-stationary stochastic processes and have the following time series representation:

\[Z_t = \Phi Z_{t-1} + \xi_{t-1},\]  

(37)

with \(E_t \xi_t = 0, E_t \xi_t^{\nu_{i-1}} = \psi \) as in Cardia (1991).

With these apparatus, I analyze the response of the economy to shocks of technology and monetary policy using essentially the method of KPR (1988) in the next section. That is, I restrict my attention to the case of small fluctuations of the endogenous variables around a steady state growth path. Since most of the

\footnote{Here \(\xi_{\mu}\) is a logarithm of a detrended per capita government expenditure, i.e. \(\xi_{\mu} = \ln (x_t / x_t^0)\). This variable is assumed to follow an AR(1) process as in Rotemberg and Woodford (1992).}
following analysis will be done in stationary terms, it is more convenient to define a symmetric rational equilibrium in terms of a stationary one.

C. Stationary Transformed Economy

When there exists a growth trend in $z_t$, endogenous variables will exhibit trend in equilibrium, if it exists as well. Thus a stationary solution for the transformed variables will exist if the equilibrium conditions in terms of these variables do not contain $z_t$. A stationary equilibrium exists for the transformed (detrended) variables if the transformed variables do not involve $z_t$. The real variables of the economy are divided by $z_t$ and real balance for money is deflated by $z_t$ at each time.

As standard frequency domain techniques are used to compute the complex matrix spectral density function of all variables as in Lettau and Uhlig (1995), no simulations are necessary to obtain results for Hodrick-Prescott filtered series.

IV. Quantitative Implications

A. Parameter Values

All parameter values used in this paper are reported in Table 1. Most of them are taken from Cardia (1991), and Campbell and Cochrane (1995). In particular, one needs to note the intertemporal elasticities of consumption and labor supply because these parameter values are important in the quantitative implications of the model. Even though many RBC models assume unit elasticity of intertemporal substitution ($\varepsilon_C = \sigma_C^{-1} = 1$) which is taken from Hansen and Singleton (1982), many empirical studies on consumption tell us to be more cautious and conservative in choosing the value. Thus the baseline model of this paper takes lower values of intertemporal elasticity of consumption, $\sigma_C = 2$, i.e. $\varepsilon_C = 1/2$. I also choose a conservative intertemporal elasticity of labor supply, $\varepsilon_w(= H_w)$ equal to 1. These intertemporal elasticities of consumption and labor supply are much smaller than those in Rotemberg and Woodford (1992). The value of elasticity of $i/k$ with respect to Tobin’s $q$, $q_a$ is the cost adjustment elasticity which reflects the volatility of investment. Since previous studies have not estimated
Table 1
THE CALIBRATED PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description of Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>1.004</td>
<td>steady state quarterly growth rate of technology</td>
</tr>
<tr>
<td>( s_H )</td>
<td>0.58</td>
<td>steady state labor share</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.025</td>
<td>rate of depreciation of capital stock</td>
</tr>
<tr>
<td>( r_h )</td>
<td>0.011</td>
<td>steady state rate of return</td>
</tr>
<tr>
<td>( \varepsilon_c(\sigma, \lambda^{-1}) )</td>
<td>( \frac{1}{2} )</td>
<td>intertemporal elasticity of consumption</td>
</tr>
<tr>
<td>( \varepsilon_w )</td>
<td>1</td>
<td>intertemporal elasticity of labor supply</td>
</tr>
<tr>
<td>( -\frac{1}{1+\xi} )</td>
<td>.5</td>
<td>the log-log interest rate elasticity of money demand</td>
</tr>
<tr>
<td>( \varepsilon_{nk} )</td>
<td>1</td>
<td>elasticity of substitution between capital and labor</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.97</td>
<td>the serial correlation of the habit formation</td>
</tr>
<tr>
<td>( G )</td>
<td>0.10</td>
<td>steady state government spending share</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>5, 15</td>
<td>elasticity of ( \frac{l}{k} ) to Tobin’s q</td>
</tr>
</tbody>
</table>

\[
V = \begin{bmatrix}
0.129E−02 & 0.131E−04 & -0.103E−03 \\
0.375E−03 & 0.200E−04 & 0.369E−03 \\
\end{bmatrix}, \quad \Phi = \begin{bmatrix}
0.5 & 0 & 0 \\
0 & 0.95 & 0 \\
0 & 0 & 0.95 \\
\end{bmatrix}
\]

this cost adjustment parameter, I will choose 5 and 15 as the value in the baseline model.\(^5\)

With the transaction cost function of \( \Phi\left(\frac{C_t}{M_t/P_t}\right) = \gamma\left(\frac{C_t}{M_t/P_t}\right)^{\zeta} \), with \( \zeta > 0 \), the log-log money demand elasticity to the interest rate is \(-\frac{1}{1+\zeta}\). I will take Lucas’ (1993) estimate for this elasticity which equals -0.5 for the U.S. and determine the parameter values. Though it is desirable to estimate the corresponding monetary base measure of major industrial countries, some countries do not have the comparable measure for the US \( M_t \).\(^6\) Therefore, I will use the estimate for the monetary growth rates for the German economy in Cardia (1991). The parameter values related to the habit formation are taken from Campbell and Cochrane (1995). In particular, the values of the serial correlation parameter \( \phi \) of the

\(^5\)Baxter and Crucini (1993) used the elasticity of 15 as a benchmark parameter value.

\(^6\)Since money is defined as \( M_t \), I also have \( [M_t/(PC)] = 0.34 \) for the US.
habit formation is set to 0.97. The steady state surplus consumption ratio $\overline{S}$ and the maximum surplus consumption ratio $S_{\text{max}}$ are set to 0.049 and 0.081. Finally, the steady-state wealth elasticity of real interest rate is set at a low value, $10^{-4}$, so that in the short run the model behaves as if the real interest rate is constant as in Uribe (1997).

B. Implications of the Model

In this subsection I review the main goal of this paper and see whether the habit formation model with transaction cost can explain the business cycle properties of the small open economy, in particular the behavior of the exchange rate. I compare the moments of the model with properties of data drawn from major industrial economies. I also compare a habit formation model with a model without habit formation.

a) The Impulse Responses of Positive Real and Monetary Shocks

The first issue that I address is if actual data impulses correspond to the dynamic responses of exchange rates and real activities to monetary shocks implied by this habit persistence model.

First, let’s compare the impulse response of the habit formation model with that of the model without habit formation. Consider the response of the exchange rate to real shocks. In the flexible price international monetary model without habit persistence in consumption, a positive real shock in the home country leads to a substantial increase of consumption and investment as well as a substantial increase of labor as in Figure 1. As a positive real shock improves the productivity of domestic production sector, the marginal cost of production decreases. As a result, domestic price goes down and a nominal exchange rate appreciates via PPP.

The impulse response of the habit formation model shows that there is little response of consumption and exchange rates. This is due to a habit formation in consumption. While the household in the model without habit formation uses the opportunity of increased productivity to work harder to build up capital, the household in the habit formation model will not have an incentive to do so. As the household with habit formation does not want and expect to change its consumption in the future, it does not work hard even if a favorable environment to work harder forms. The response of consumption to a positive real shock is very small and
investment output labor consumption exchange rate
Quarters after shock
Percent deviation from steady state
0 1 2 3 4 5 6 7 8 -0.5 0 0.5 1 1.5 2 2.5 3 3.5 4

**Figure 1**
**Impulse Responses to a Shock in Technology** ($\varphi = 5$, $\bar{S} = 1.00$)

investment output labor consumption exchange rate
Quarters after shock
Percent deviation from steady state
0 1 2 3 4 5 6 7 8 -0.5 0 0.5 1 1.5 2 2.5

**Figure 2**
**Impulse Responses to a Shock in Technology** ($\varphi = 5$, $\bar{S} = 0.05$)
as a result, the response of the exchange rate to the shock is tiny as in Figure 2. The impulse responses of the other endogenous variables to exogenous shocks in the habit formation model are very similar to those of no habit formation model.

Second, the impulse response of the endogenous variables to a monetary shock is similar to that to a real shock. Not only in the model without habit formation but also in the habit formation model the exchange rate appreciates and price decreases to a positive domestic monetary shock. The paradoxical phenomena of the so-called 'an exchange rate puzzle' and 'a price puzzle' in the international finance literature result from this small open economy model as in Figure 3. This result reflects partly the assumption of the perfect capital mobility. With perfect capital mobility, the impact of a domestic money supply on the interest rate is almost nil. Therefore the velocity of money changes too little to the shock. With a little increase of consumption and a extremely high elasticity of a net foreign asset to the interest rate, the money market equilibrium condition implies that the price decreases and the exchange rate appreciates to a positive monetary shock. In the habit formation model, the effects of a positive monetary shock on consumption and exchange rates are very small because the habit formation makes the household locally very risk averse, which implies a very low local elasticity of substitution. That is, the household wants to smooth consumption extremely, making consumption very unresponsive to shocks. Moreover, the labor supply of the household decreases after a positive monetary shock because of the low intertemporal elasticity of substitution of labor. Though the price and the exchange rate puzzles still appear in the habit formation model, it is not severe as in Figure 4.

Third, consider the effect of government spending shock on the economy. With money supply given, the domestic government can finance its extra expenditures by borrowing from abroad. As the demand for foreign currency increases, the exchange rate appreciates and interest rates increase. The increase of a government spending leads to an increase of aggregate demand. As the aggregate

\[ \pi \cdot c_t = \psi(k) \]

where \( \psi(\cdot) \) is the velocity of money.

As the relative risk aversion in this habit formation model is given by equation (4), the parameterized value of the local risk aversion is 40.
Figure 3
Impulse Responses to a Shock in Money ($\varphi = -5, \bar{\mu} = 1.00$)

--- : consumption
* : output
--- : investment
•••• : labor
••• : exchange rate

Investment output labor consumption exchange rate
Quarters after shock
Percent deviation from steady state
0 1 2 3 4 5 6 7 -1 0 1 2 3 4 5 6

Figure 4
Impulse Responses to a Shock in Money ($\varphi = -5, \bar{\mu} = 0.05$)

--- : consumption
* : output
--- : investment
•••• : labor
••• : exchange rate

Investment output labor consumption exchange rate
Quarters after shock
Percent deviation from steady state
0 1 2 3 4 5 6 7 -2 -1 0 1 2 3 4
investment output labor consumption exchange rate
Quarters after shock
Percent deviation from steady state
0 1 2 3 4 5 6 7 8 -0.05 0 0.05 0.1 0.15 0.2 0.25 0.3

FIGURE 5
IMPULSE RESPONSES TO A SHOCK IN GOVERNMENT ($\varphi = 5, \bar{S} = 1.00$)

investment output labor consumption exchange rate
Quarters after shock
Percent deviation from steady state
0 1 2 3 4 5 6 7 8 -0.1 -0.05 0 0.05 0.1 0.15 0.2

FIGURE 6
IMPULSE RESPONSES TO A SHOCK IN GOVERNMENT ($\varphi = 5, \bar{S} = 0.05$)
investment output labor consumption exchange rate
Quarters after shock
Percent deviation from steady state
0 1 2 3 4 5 6 7 8 -1 0 1 2 3 4 5 6

**Figure 7**

**Impulse Responses to a Shock in Technology** ($\phi = 15, \bar{S} = 0.05$)

investment output labor consumption exchange rate
Quarters after shock
Percent deviation from steady state
0 1 2 3 4 5 6 7 8 -2 -1 0 1 2 3 4 5 6

**Figure 8**

**Impulse Responses to a Shock in Money** ($\phi = 15, \bar{S} = 0.05$)
investment output labor consumption exchange rate
Quarters after shock
Percent deviation from steady state
0 1 2 3 4 5 6 7 8 -0.1 -0.05 0 0.05 0.1 0.15 0.2 0.25 0.3

**Figure 9**

**Impulse Responses to a Shock in Government** ($\varphi = 15, \bar{S} = 0.05$)

demand increases, the demand for import as well as domestic production for goods increases. As a result trade balance deteriorates and investment and labor supply increase. The impact of a government spending to the endogenous variables, on the whole, is tiny as in Figures 5 and 6.

Finally, I perform the sensitivity analysis by increasing the elasticity of capital stock to Tobin’s $q$ to 15 as in Baxter and Crucini (1993). As firms can adjust their capital stock more flexibly, the impulse responses of capital stock and output to the shocks become larger as in Figures 7, 8, and 9. The impulse responses of consumption and the exchange rate are almost the same as before.

b) Variabilities and Serial Correlations

In this subsection, I compare volatilities and serial correlations of the real variables of baseline model with those of data to see the overall performance of the model. The column labelled ‘Data’ in Tables 2 and 3 is reproduced from Fiorito and Kollintzas (1994) where moments are calculated for actual time series that have been Hodrick-Prescott filtered. This column reports composite data moments of six countries (Australia, Canada, Finland, Germany, Japan, and United Kingdom).
First, consider the standard deviation of the variables in model and data. A prominent feature about the exchange rate movement is its (excessive) volatility relative to other real variables as can be seen in Table 2 where some selected moments of data are presented. The standard deviation of exchange rates in data is between 5 and 4. However, the standard deviation of exchange rates in the baseline model is too small compared to it. This is due to the fact that consumers respond very little to monetary shocks and to real shocks in the habit formation model. These distorting features of the model are reflected in the small standard deviation of consumption. The standard deviation of the exchange rates in the habit formation model is just one-tenth of that in the no-habit formation model. When I decrease the capital adjustment cost, the standard deviation of investment increases while that of the exchange rates decreases. This is because households do not adjust their consumption profile, but adjust capital stock to the shock. Note that the standard deviations of output, consumption, and investment match well with those of data.

Next, consider the contemporaneous correlation between output and other endogenous variables. In the flexible price model without habit formation, consumption, and investment comove very closely as in data. But the correlation between output and employment in the no-habit formation model is almost perfect (0.99), while the correlation in data is modest. In the habit formation model, the correlation between output and employment decreases because households which wish to maintain their consumption profile do not change their labor supply to the shock. The price or the nominal exchange rate moves overly countercyclically in both habit formation model and no-habit formation model compared to the data.

Finally, note that the cross correlations between output and other endogenous variables match well with that of data. The simulation result shows that the properties of the data which a habit formation model generate are very similar to those of no-habit formation model except the variation of exchange rates and consumption.
### Table 2

#### Moments of Data

<table>
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<th>Variable</th>
<th>S.D.</th>
<th>Cross Corr.</th>
<th>with Output</th>
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<td>P^c</td>
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</table>

Note: Y, C, I \_f, I \_e, N, e, P^g, P^c denote output, consumption, fixed investment, equipment investment, employment, nominal exchange rate, GNP deflator, consumption price index.

Source: Fiorito and Kollintzas (1994) and IMF.
### Table 3

<table>
<thead>
<tr>
<th>Variable S.D.</th>
<th>Cross Corr. with Output</th>
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<td>$X_{t-4}$</td>
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### V. Concluding Remarks

This paper specifies a small open economy such that a risk premium varies over time as in Campbell (1996), Campbell and Cochrane (1995), Abel (1990), and Constantinides (1990) who have recently proposed a simple asset pricing model with habit formation. It investigates the impacts of various shocks in a habit formation general equilibrium model with a transaction cost, and compares the habit formation model with a no-habit formation model in many aspects. It shows that the habit formation model generates too small variations of consumption as well as that of the exchange rate compared to data, while it reduces the comovement of output and employment. This reflects the strong effect of the locally small intertemporal elasticity of substitution in consumption and leisure. The habit formation introduced by Campbell and Cochrane (1995) to explain the equity premium puzzle failed to generate the volatile movements of exchange rates.

In the future, it is desirable to incorporate the sticky price
property with the habit formation and look at the effect of monetary and real shocks on exchange rates as well as consumption.

(Received September, 1998; Revised November, 1998)

References


Rotemberg, J. J., and Woodford, M. "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity." *Journal of

