The Effects of Environmental Policy on Endogenous Growth and Social Welfare

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This paper explores the effects of stricter domestic environmental policy on economic growth rate and social welfare by incorporating environmental pollution and expenditure for pollution abatement and control into an endogenous growth model (Barro 1990) that emphasises the role of government spending as the engine of economic growth. The analysis shows that on a balanced growth path, the general negative growth effect of the more stringent environmental policy does not hold true for some specific cases. For some other cases, both economic growth rate and social welfare level are found to increase or decrease together as a result of the stricter environmental policy; and the possibility of a trade-off between economic growth and social welfare is also found. (*JEL* Classifications: O41, Q20, H30)

I. Introduction

The effect of the more stringent environmental policy on economic growth and social welfare is now an important issue. It is because environmental factors are no longer free goods due to pollution, because the environmental policy influences economic activity in different ways both through restriction and abatement and through its impacts on production and consumer welfare, and because it is now a hot issue of Green Round in international trade. This paper presents an endogenous growth model, with a feature of environ-

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mental pollution and policy affecting production and social welfare. This model links between pollution level, environmental policy, economic growth and social welfare. By constructing this model, it is analysed how the more stringent environmental policy affects the long-run economic growth rate and then social welfare level.

Keeler *et al.* (1972), Foster (1973), Heal (1982), van der Ploeg and Withagen (1991), Tahvonen and Kuuluvainen (1993), and Hosoda (1994) show that optimal pollution control leads to lower steady state levels of capital stock and consumption than when pollution is ignored in the model. The results obtained from these studies are based on use of the neoclassical growth model. The main criticism of the neoclassical economic growth model is that it is not suitable as a model to explain economic growth, because it assumes exogenously determined population growth and technological innovation.

There are a few studies on endogenous growth models incorporating the issue of environmental pollution. Mohtadi and Roe (1992) employ the production function used by Rebelo's (1991) model,[1] and incorporate the environment as negative consumption as well as production externalities. Gradus and Smulders (1993) emphasise the importance of assumptions regarding production technology and the relation between pollution, production, and abatement. In case of constant returns to physical capital (i.e. Rebelo's (1991) model), they find that an increase in abatement activities crowds out investment and lowers the endogenous growth rate. In the case of human capital accumulation (i.e. Lucas' (1988) model), they find that physical capital intensity declines and the endogenous optimal growth rate is unaffected by increased environmental care, or is even higher, depending on whether or not pollution influences agents' ability to learn. More recently, Bovenberg and Smulders (1995) have explored the relationship between economic growth and environmental quality by using an endogenous growth model that incorporates pollution-augmenting technological change. According to Bovenberg and Smulders (1995), there are two opposite forces through which environmental policy affects the long-run economic growth rate. First, a decrease in pollution input and harvested

[1]The production function used by Rebelo (1991) is called AK function. The function is formulated as $Y=AK$, where Y is output, K is capital and A is constant. The marginal product of capital is equal to A, which subject to constant returns to scale.

resources directly reduces the productivity of reproducible inputs, and then adversely affects economic growth. Second, an improved quality of environment as a result of environmental policy positively affects productivity and growth. The second effect may dominate the first effect if environmental quality not only enters utility but also has an important productive role.

In applying an endogenous growth model, it is necessary to analyse the fundamental engine of long-run economic growth in the real economy. The fundamentals for rapid and sustained growth, which emerge from the growth experience of North-East Asian countries for the period 1965-90, are a stable macroeconomy, high human capital, an effective and secure financial system, limited price distortions, policies to develop agriculture and openness to foreign technology. Among the selective interventions by government, the export push strategy and public infrastructure are the most effective ones in terms of leading to structural change in the economy. In Korea, which has achieved sustained and accelerated economic growth rates in the last decades, the government has played an active and key role in accomplishing this. Along the same line as this argument, Barro's (1990) endogenous growth model, which emphasises the role of the government and tries to link growth to fiscal variables, is regarded as the most suitable model in explaining the sustainable high economic growth recorded in the North-East Asian countries like Korea.

Section II presents a version of the endogenous growth model developed originally by Barro (1990) but extended to include environmental pollution and policy. Section III and IV then uses the model to analyse the effects of environmental policy on economic growth and social welfare. Section V makes some concluding remarks.

II. The Model

This section further develops the model of Barro (1990), which considers the role of public services as an input to private production. It is this productive role that creates a potentially positive linkage between government and growth. Let G be the quantity of public services provided to each household-producer. We assume that these services are provided without user charges and

are not subject to congestion effects (which might arise for highways or some other public services). That is, the model abstracts from externalities associated with the use of public services (Barro 1990, p. S106). Following Barro (1990), the production function is assumed to exhibit constant returns to scale as follows:[2]

$$Y = \Phi(K, G) = K \cdot \phi\left(\frac{G}{K}\right), \tag{1}$$

where K denotes capital provided to private individuals. The production function exhibits constant returns to K and G together, but diminishing returns to K and G separately. That is, even with a broad concept of private capital, there are decreasing returns to private input (capital) if the (complementary) government input G does not expand in a parallel manner. The particular importance of the services from government infrastructure in this context is emphasised by some empirical studies such as Aschauer (1989a, b). The term Φ satisfies the usual conditions for positive and diminishing marginal products, so that $\phi' > 0$ and $\phi'' < 0$. The marginal product of capital is

$$\frac{\partial Y}{\partial K} = (1 - \phi' \cdot \frac{G}{Y}) \cdot \phi = (1 - \delta) \cdot \phi, \tag{2}$$

where δ is the elasticity of Y with respect to G (for a given value of K), so that $0 < \delta < 1$.

Assume that government spending is financed by income tax revenue collected by a flat-rate τ:

$$G = \tau \cdot Y = \tau \cdot K \cdot \phi\left(\frac{G}{K}\right). \tag{3}$$

We assume that government has to balance its budget always. Government spending is not exogenously determined, but endogenously determined by total income. Assume that producers all have access to the same technology.

Pollution is assumed to be a by-product of capital accumulation (K), but it is reduced by private expenditure for pollution abatement (A) financed by some part of production:

$$P = P(K, A). \tag{4}$$

[2]All variables are implicit functions of time, t. This argument will be suppressed through the paper for notational convenience.

Pollution is assumed to be a flow variable, not a stock variable.[3] The pollution as a flow variable changes instantaneously if the capital stock changes or if the level of expenditure for abatement changes and this is particularly relevant for analysing pollution like noise. Additional unit of capital contributes at least as much pollution as preceding units ($P_{KK}>0$), and pollution control is efficient for low level of pollution abatement expenditure ($P_A<0$), but becomes less efficient as additional units of expenditures are applied ($P_{AA}>0$). Pollution can be reduced by natural assimilation, but in this model we ignore their probability, as the major results of the model are not changed.

Private expenditure for pollution abatement and control (A) is also financed by part of production by a flat-rate of income share (ε):

$$A(K,\ G)=\varepsilon\ \cdot Y=\varepsilon\ \cdot K\cdot\phi\left(\frac{G}{K}\right). \tag{5}$$

While the expenditure rate ε is constant, the abatement expenditure increases with production level. The more ambitious pollution abatement effort is associated with an increase of the national income share of expenditure for pollution abatement and control, following the more stringent environmental regulations and policies by government.

Any production that is not consumed or used for investment in government spending (G) and pollution abatement (A) leads to an increase in the stock of capital:

$$\dot{K}=(1-\tau-\varepsilon)\cdot Y(K,\ G)-C,\quad K(0)=K_0>0,\quad \lim_{t\to\infty}K(t)\geq 0, \tag{6}$$

where C denotes consumption and a dot denotes a time derivative. We abstract depreciation of the capital stock as the qualitative results remain unchanged.

The infinitely-lived representative individual seeks to maximise overall social welfare (W) as given by:

$$W=\int_0^\infty U(C,\ P)e^{-\vartheta t}dt, \tag{7}$$

where ϑ represents the positive and constant rate of time preference. U is an instantaneous utility function which is an increasing

[3]See Keeler *et al.* (1972), Becker (1982), Tahvonen and Kuuluvainen (1993) for the case of stock pollution.

and decreasing function of consumption (C) and environmental pollution (P), respectively (i.e. $U_C>0$ and $U_P<0$). The marginal utility of both consumption and pollution is diminishing ($U_{CC}<0$, $U_{PP}<0$). Assume that population, which corresponds to the number of workers and consumers, is normalised to one. We assume that $U_{CP}<0$: An increase in pollution level reduces the marginal utility from consumption.

Taking public spending (G) as *given*, an individual chooses a consumption path so as to maximise the utility function subject to the dynamic constraint. Individual choices, however, affect everybody's output. When each agent increases production, the national income increases. This increased income pushes up government expenditure through the increased tax revenue with a fixed ratio of government spending to output, and hence increases income again.

The representative individual chooses a consumption path so as to maximise the intertemporal welfare function (W) subject to the dynamic capital accumulation constraints in market economy in which the government levies an income tax and uses the proceeds to finance government spending and the private sector expenses for pollution abatement. The utility maximisation problem is solved by assuming that individuals take G as given. The current value Hamiltonian is

$$H=U[C,\ P(K,\ A)]+\theta\ \cdot\{(1-\tau-\varepsilon)\cdot K\cdot\phi\left(\frac{G}{K}\right)-C\},$$

where θ is the shadow price of capital. C and K are control and state variables, respectively. The current value Hamiltonian generates the following social optimum condition (See Appendix A for details):

$$g_d=\frac{\dot{C}}{C}=\eta\,[(1-\tau-\varepsilon)(1-\delta)\phi+\{U_P(P_K+\varepsilon(1-\delta)\phi P_A)+U_{CP}\dot{P}\}/U_C-\vartheta], \tag{8}$$

where $\delta\,(=(G/Y)\phi')$ is the elasticity of Y with respect to G (for a given value of K), so that $0<\delta<1$. And, $\eta=-U_C/(CU_{CC})$ is the elasticity of intertemporal substitution between current and future consumption. We assume that η is constant.

Equation (8) is a modified version of the well-known Ramsey formula for the rate of economic growth and gives the optimum allocation between current and future consumption. The return to

capital adds to the current stock of capital and increases future output by the rate of $(1-\tau-\varepsilon)(1-\delta)\phi$. In the absence of a pollution problem, it is optimal to postpone consumption when the net marginal product of capital exceeds the rate of time preference, ϑ. With the pollution problem incorporated into the model, however, the future optimal consumption must be lowered when pollution grows and when capital accumulation increases the pollution level. Therefore, the social rate of return to capital (r_d) is equal to

$$(1-\tau-\varepsilon)(1-\delta)\phi+[U_P\{P_K+\varepsilon(1-\delta)\phi P_A\}+U_{CP}\dot{P}]/U_C.$$

It is optimal to postpone consumption ($\dot{C}/C>0$) when r_d exceeds the rate of time preference. Note that all partial derivatives are functions that vary over time as the point of evaluation changes over time. The necessary condition for positive economic growth is that the social rate of return to capital may not fall below the rate of time preference. In other words, the marginal product of capital after income tax (i.e. $(1-\tau)(1-\delta)\phi$) must be sufficiently larger than the rate of time preference to compensate for the marginal abatement costs associated with capital (i.e. $-\varepsilon(1-\delta)\phi$) and the utility losses due to pollution (i.e. $\{U_P(\cdot)+U_{CP}\dot{P}\}/U_C$). A fall of marginal product of capital would decrease r_d below time preference and economic growth would peter out.

If a larger and larger part of total output is used for pollution abatement and control (PAC) activities when the capital stock rises, then at some point in time it will be optimal to stop capital accumulation and economic growth. Therefore, pollution can remain constant when costs for PAC are kept in pace with capital accumulation. On the other hand, if increasing pollution as a result of capital accumulation decreases the utility of consumption, then society will wish to stabilise or decrease pollution levels by reducing capital accumulation and economic growth.

III. Effects of the Environmental Policies on Economic Growth Rate

By employing some explicit functions, this section theoretically analyses the effects of the stricter environmental policies on economic growth rate. First, we solve the model with explicit functions of production and social welfare. Next, we investigate the

effects of domestic environmental policy on the long-run economic growth rate.

We now choose specifications of the production, pollution and social welfare function to analyse the relationship between environment and economic growth.

$$Y(K,\ G) = \alpha K^{\beta} G^{1-\beta} = \alpha K \left(\frac{G}{K}\right)^{1-\beta}, \tag{9}$$

$$P(K,\ A) = \frac{K^{\gamma}}{A+\sigma}, \tag{10}$$

$$W = \int_0^{\infty} U(C,\ P) e^{-\vartheta t} dt = \int_0^{\infty} \frac{(C/P)^{1-1/\eta} - 1}{1-1/\eta} e^{-\vartheta t} dt, \tag{11}$$

where α, σ, β, and γ are parameters. The *Cobb-Douglas* production function (9) displays constant returns to scale in K and G together, but decreasing returns to K and G separately. Because the elasticity of substitution between K and G is unity, the elasticity of production with respect to government spending (δ) is equal to $(1-\beta)$: $0 < \delta < 1$. The pollution function (10) is an increasing and decreasing function of capital accumulation (K) and expenditure on pollution abatement (A), respectively. When $K=0$, the pollution level (P) is also equal to zero. The elasticity of substitution between K and A is unity. The social welfare function (11) implies constant intertemporal elasticity of substitution between current and future consumption ($\eta > 0$). The specified form of social welfare is customary in dynamic problems, with the simplifying constraint here that consumption and pollution have the same weight with opposite signs in the utility function. The instantaneous elasticity of substitution between consumption and pollution is unity.

The main obstacle in solving the model is that the derived results are too complicated (See Appendix B for details). To simplify our analysis, therefore, we assume that the constant σ in the pollution function (10) is equal to zero. This simplifies the pollution function to

$$P(K,\ A) = \frac{K^{\gamma}}{A}, \tag{12}$$

where this pollution function becomes homogeneous of degree $(\gamma - 1)$. This implies that a higher level of production increases the pollution level by the rate of $(\gamma - 1)$, provided the expenditure level

for pollution abatement (A) grows equally fast.

A. Solution of the Model

As mentioned before, the utility maximisation problem is solved by assuming that the individual takes G as given. Given the functional forms specified in equations (9), (11) and (12), the current value Hamiltonian (H) reduces to the following:

$$H=\frac{(\varepsilon\alpha G^{1-\beta}CK^{\beta-\gamma})^{1-1/\eta}-1}{1-1/\eta}+\theta_2\cdot\{(1-\tau-\varepsilon)\alpha K^{\beta}G^{1-\beta}-C\}, \quad (13)$$

where θ_2 is the shadow price of capital. C and K are control and state variables, respectively. The current value Hamiltonian (13) generates the following economic growth rate (See Appendix C for details):

$$g_d=\frac{\dot{C}}{C}=\eta\cdot\left\{\frac{2\beta-\gamma}{1+\beta-\gamma}(1-\tau-\varepsilon)\alpha^{1/\beta}\tau^{(1-\beta)/\beta}-\frac{\vartheta}{1+\beta-\gamma}\right\}, \quad (14)$$

where $0<\beta<1$. Equation (14) shows that the economic growth rate depends on production technology, utility, pollution emissions and abatement. With constant η by assumption, the economy grows at the constant rate (g_d) in the steady-state.

It is optimal to postpone current consumption, if the net marginal product of capital, $\Psi(1-\tau-\varepsilon)\alpha^{1/\beta}\tau^{(1-\beta)/\beta}$, exceeds $\vartheta/(1+\beta-\gamma)$ where $\Psi=(2\beta-\gamma)/(1+\beta-\gamma)$. For example, when the pollution function is homogeneous of degree zero (i.e. $\gamma=1$), the current consumption must be postponed, if the net marginal product of capital exceeds (ϑ/β). When $\vartheta/(1+\beta-\gamma)<0$ (i.e. $\gamma>1+\beta$), the extent of postponement of consumption must be larger.

The marginal product of capital after income tax, $\Psi(1-\tau)\alpha^{1/\beta}\cdot\tau^{(1-\beta)/\beta}$, must be sufficiently larger than the rate of time preference, ϑ, to compensate for the marginal PAC costs associated with capital, $-\Psi\varepsilon\alpha^{1/\beta}\tau^{(1-\beta)/\beta}$, and the utility losses due to pollution, $-1/(1+\beta-\gamma)$. If a larger and larger part of total output is used for PAC when the capital stock rises, it is optimal to stop capital accumulation and economic growth at some moment in time.

B. Effects of the Stricter Environmental Policy on Economic Growth Rate

We now consider the effects of an increase in the expenditure for pollution control on the economic growth rate. Given the government spending (G), the total differentiation of equation (14) with respect to ε generates the effect of increased PAC expenditure on the long-run economic growth rate in a decentralised economy:

$$\frac{dg_d}{d\varepsilon} = -\Psi \cdot \eta \cdot \alpha^{1/\beta} \tau^{(1-\beta)/\beta}, \tag{15}$$

where $\Psi = (2\beta - \gamma)/(1+\beta-\gamma)$. According to equation (15), the sign of the effect of environmental policy depends on the relative values of β and γ. Considering the values of β and γ, we can obtain the following proposition on the effect of environmental policy on the economic growth rate:

***Proposition* 1**

A necessary and sufficient condition for $(dg_d/d\varepsilon)$ to be positive is that $(1+\beta) > \gamma$ and $2\beta < \gamma$.

Proof: The sign of $(dg_d/d\varepsilon)$ will be positive only when $\Psi < 0$. Because $\Psi = (2\beta - \gamma)/(1+\beta-\gamma)$, this will occur when either:

(i) $(1+\beta) > \gamma$ and $2\beta < \gamma$; or

(ii) $(1+\beta) < \gamma$ and $2\beta > \gamma$.

We can rule out case (ii) since it requires $\beta > 1$, as illustrated in Figure 1. Hence $(dg_d/d\varepsilon) > 0$ requires that (β, γ) lies in the shaded region in Figure 1. Only when (β, γ) lies in inside of the shaded triangle, the sign of $(dg_d/d\varepsilon)$ is positive.

Q.E.D.

It is generally argued that the more stringent environmental regulations and policies requiring an increase of expenditure for pollution abatement and control result in a decrease in the long-run rate of economic growth. The reason for the negative result of environmental policy on economic growth is the crowding out of investment caused by the increased pollution abatement activity following the more stringent environmental regulations and policies. This negative effect is qualitatively similar to that for the case of Rebelo's (1991) endogenous growth model in Gradus and Smulder (1993). Crowding out of investment arises in this model

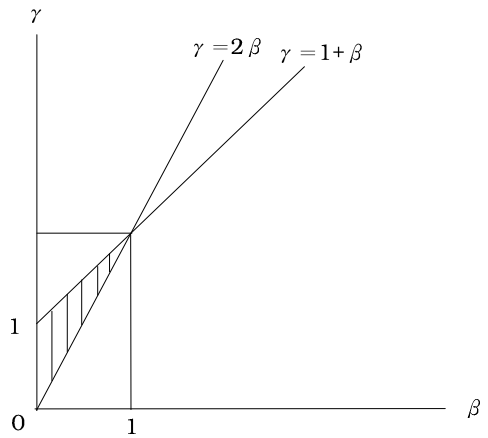

FIGURE 1
THE CONDITION FOR THE POSITIVE EFFECT OF DOMESTIC ENVIRONMENTAL POLICY ON THE ECONOMIC GROWTH RATE

because it does not allow for factor substitution[4] and variable marginal capital productivity.

Our study, however, shows that the negative growth effect of the more stringent environmental policy does not hold true for some cases. A stricter environmental policy increases the long-run rate of economic growth under some specific conditions on production and pollution functions (i.e. $(1+\beta) > \gamma$ *and* $2\beta < \gamma$). The increased pollution abatement activities crowd out investment. However, as soon as there are possibilities of substitution between K and G in our model, the crowding out effect will be dominated by substitution effects under the specific conditions on production and pollution functions (i.e. $(1+\beta) > \gamma$ *and* $2\beta < \gamma$). Less resources are available for capital accumulation (crowding out effect) as a result of the more stringent environmental policy, but under the condition $(1+\beta) > \gamma$ *and* $2\beta < \gamma$, this is dominated by the factor substitution towards government spending (G), the "engine of growth" in the

[4]Rebelo (1991) assumes the production function to be linear in the only input, capital, and Barro (1990) does not allow factor substitution between capital (K) and government spending (G). According to Barro, "The general idea of including G as a separate argument of the production function is that private inputs, represented by K, are not a close substitution for public inputs." (Barro 1990, p. 107)

context of our model.

IV. Economic Growth and Social Welfare

Because the economy is always in a position of steady-state growth (i.e. no transitional dynamics), it is straightforward to compute the attained social welfare, as long as the individual takes G as given. Given a constant economic growth rate in equation (14), the integral of social welfare function (11) can be simplified to yield (see Appendix D for derivatives):

$$W_d = \frac{\{C(0)/P(0)\}^{1-1/\eta}}{(1-1/\eta)\cdot\{\vartheta - g_d\cdot(1+\beta-\gamma)(1-1/\eta)\}}. \tag{16}$$

The condition $\vartheta > g_d \cdot (1+\beta-\gamma)(1-1/\eta)$ ensures that social welfare is *bounded*. Given an initial amount of capital, $K(0)$, the initial quantity of consumption, $C(0)$, is derived as:

$$C(0) = K(0)\cdot\{(1-\tau-\varepsilon)\,\alpha^{1/\beta}\,\tau^{(1-\beta)/\beta} - g_d\}. \tag{17}$$

Dividing both sides of equation (17) with $P(0)$ which is the initial level of pollution and then substituting it into equation (16) yield the social welfare function for the decentralised economy:

$$W_d = \left\{\frac{K(0)}{P(0)}\right\}^{1-1/\eta}\cdot\frac{\{(1-\tau-\varepsilon)\cdot\alpha^{1/\beta}\,\tau^{(1-\beta)/\beta} - g_d\}^{1-1/\eta}}{(1-1/\eta)\cdot\{\vartheta - g_d\cdot(1+\beta-\gamma)(1-1/\eta)\}}. \tag{18}$$

Equation (18) shows the relationship between social welfare (W) and economic growth (g_d) in the decentralised economy. As long as social welfare is *bounded*,

(i) in case $\eta > 1$, economic growth *decreases* social welfare; and

(ii) in case $\eta < 1$, economic growth *increases* social welfare.

Therefore, the relationship between economic growth and social welfare in the steady-state depends critically on the elasticity of intertemporal substitution of consumption (η) and the marginal efficiency of PAC expenditure (i.e. the value of γ) in the context of our model with Barro's (1990) Cobb-Douglas production function.

Barro (1990) shows that maximising the growth rate of the economy is equivalent to maximising social welfare (W). However, our study shows that in the presence of environmental pollution his argument does not hold true in some specific cases. The relationship between economic growth and social welfare is determined by the efficacy of PAC expenditure as well as the

production technology and the tastes in consumption and the environmental quality.

V. Concluding Remarks

The primary purpose of this theoretical study was to investigate the effects of environmental policy on economic growth and social welfare. Pollution and PAC expenditure are incorporated into Barro's (1990) endogenous growth model. Some of the conclusions we derive are as follows.

First, on a balanced growth path, the effects of increasing PAC expenditure on the economic growth rate and social welfare level depend critically on the efficacy of PAC expenditure on the abatement of pollution, the production technology and the tastes in consumption and the environmental quality.

Second, the generally accepted negative growth effect of the more stringent environmental policy does not hold true for some specific cases. A stricter environmental policy sometimes increases the long-run rate of economic growth under some specific conditions on both production and pollution functions (i.e. $(1+\beta) > \gamma$ and $2\beta < \gamma$). This implies that the growth effect of environmental policy of each country depends on the economic structure and the characteristics of pollution. In addition, the type of environmental policies used for pollution abatement and control is also one of the important factors on the growth effect of environmental policy, because different types of environmental policies have different impacts on economic structure, pollution emissions, and then economic growth.

Finally, under some specific assumption (i.e. $\eta < 1$), both the economic growth rate and social welfare level decline simultaneously as a result of increased PAC expenditure; and there is also the possibility of a trade-off between economic growth and social welfare.

The results derived from this paper imply that there exists the possibility of positive economic growth and social welfare effects of the more stringent environmental policies. However, these positive effects can be achieved by the use of appropriate environmental policies that correspond to the economic structure including consumer preference on environment and the characteristics of pollution in each country.

Appendix

Appendix A

The current value Hamiltonian generates the following first order conditions:

$$\frac{\partial H}{\partial C} = U_C - \theta = 0, \tag{A1}$$

$$\begin{aligned}\dot{\theta} &= \vartheta\theta - \frac{\partial H}{\partial K} \\ &= \theta\vartheta - U_P P_K - U_P P_A A_K - \theta(1-\tau-\varepsilon)\phi + \theta(1-\tau-\varepsilon)\frac{G}{K}\phi' \\ &= \theta\{\vartheta - (1-\tau-\varepsilon)(\phi - \frac{G}{K}\phi')\} - U_P P_K - U_P P_A A_K \\ &= \theta\{\vartheta - (1-\tau-\varepsilon)(1 - \frac{G}{Y}\phi')\phi\} - U_P P_K - U_P P_A A_K,\end{aligned} \tag{A2}$$

where $\frac{G}{K} = \phi \cdot \frac{G}{Y}$.

$$\dot{K} = (1-\tau-\varepsilon)\cdot K \cdot \phi \cdot \frac{G}{K} - C, \tag{A3}$$

$$\lim_{t\to\infty} e^{-\vartheta t}\theta K = 0. \tag{A4}$$

Appendix B

Given (9)-(11), the current value Hamiltonian is

$$H = \frac{(\varepsilon\alpha K^{\beta-\gamma}G^{1-\beta} + \sigma K^{-\gamma})^{1-1/\eta}\cdot C^{1-1/\eta} - 1}{1-1/\eta} + \lambda \cdot \{(1-\tau-\varepsilon)\alpha K^{\beta}G^{1-\beta} - C\},$$

where λ is the shadow price of capital. The first-order conditions are

$$\frac{\partial H}{\partial C} = (\varepsilon\alpha K^{\beta-\gamma}G^{1-\beta} + \sigma K^{-\gamma})^{1-1/\eta}\cdot C^{-1/\eta} - \lambda = 0, \tag{B1}$$

$$\begin{aligned}\dot{\lambda} &= \vartheta\lambda - \frac{\partial H}{\partial K} \\ &= \vartheta\lambda - \lambda(1-\tau-\varepsilon)\alpha\beta K^{\beta-1}G^{1-\beta} - C^{1-1/\eta}\cdot \\ &\quad (\varepsilon\alpha K^{\beta-\gamma}G^{1-\beta} + \sigma K^{-\gamma})^{-1/\eta}\cdot\{\alpha(\beta-\gamma)K^{(\beta-\gamma-1)}G^{1-\beta} - \sigma\gamma K^{-(1+\gamma)}\},\end{aligned} \tag{B2}$$

$$\dot{K} = (1-\tau-\varepsilon)\alpha K^{\beta}G^{1-\beta} - C, \tag{B3}$$

$$\lim_{t\to\infty} e^{-\vartheta t}\lambda K=0. \tag{B4}$$

Total differentiation of (B1) leads to

$$\begin{aligned}\frac{1}{\eta}\cdot(\varepsilon\alpha K^{(\beta-\gamma)}G^{(1-\beta)}+\sigma K^{-\gamma})^{(1-1/\eta)}C^{-1/\eta}\cdot\frac{\dot{C}}{C}\\ =(1-\frac{1}{\eta})(\varepsilon\alpha K^{(\beta-\gamma)}G^{(1-\beta)}+\sigma K^{-\gamma})^{-1/\eta}\cdot\\ \{(\beta-\gamma)\varepsilon\alpha K^{(\beta-\gamma)}G^{(1-\beta)}-\sigma\gamma K^{-\gamma}\}\cdot\frac{\dot{K}}{K}-\dot{\lambda}.\end{aligned} \tag{B5}$$

By using (B3), we get

$$\begin{aligned}\frac{1}{\eta}\cdot\frac{\dot{C}}{C}=\alpha\cdot\{\beta+(1-\frac{1}{\eta})\cdot\Delta\}(1-\tau-\varepsilon)\cdot\left(\frac{G}{K}\right)^{(1-\beta)}\\ +\frac{1}{\eta}\cdot\Delta\cdot\frac{C}{K}-\vartheta,\end{aligned} \tag{B6}$$

where

$$\Delta=\frac{(\beta-\gamma)\varepsilon\alpha K^{\beta-\gamma}G^{1-\beta}-\sigma\gamma K^{-\gamma}}{\varepsilon\alpha K^{\beta-\gamma}G^{1-\beta}+\sigma K^{-\gamma}}.$$

From (B3),

$$\frac{C}{K}=\alpha\cdot(1-\tau-\varepsilon)\cdot\left(\frac{G}{K}\right)^{(1-\beta)}-\frac{\dot{K}}{K}. \tag{B7}$$

Substitution of (B7) into (B6), therefore, yields

$$\frac{1}{\eta}\cdot\left(\frac{\dot{C}}{C}+\Delta\cdot\frac{\dot{K}}{K}\right)=(\beta+\Delta)\cdot(1-\tau-\varepsilon)\cdot\alpha^{1/\beta}\tau^{(1-\beta)/\beta}-\vartheta. \tag{B8}$$

Note that $G=\tau\alpha K^{\beta}G^{(1-\beta)}$, and then $(G/K)=(\alpha\tau)^{1/\beta}$. In the steady-state, all quantitative variables grow at a same rate. Define $\dot{K}/K=g_{DK}$. Take logarithms of (B7) and derivatives of both sides and get

$$\frac{\dot{C}}{C}=\frac{\dot{K}}{K}=g_{DK}. \tag{B9}$$

(B9) means that in the steady-state the growth rate of consumption is equal to that of capital stock. Therefore, (B8) becomes

$$\frac{\dot{C}}{C}=\eta\cdot\left\{\frac{\beta+\Delta}{1+\Delta}\cdot(1-\tau-\varepsilon)\cdot\alpha^{1/\beta}\tau^{(1-\beta)/\beta}-\frac{\vartheta}{1+\Delta}\right\}. \tag{B10}$$

(B10) is the social optimum condition for the decentralised economy.

Appendix C

The first-order conditions generated from (13) are

$$\frac{\partial H}{\partial C}=(\varepsilon\alpha G^{(1-\beta)}CK^{(\beta-\gamma)})^{(1-1/\eta)}\cdot\frac{1}{C}-\theta_2=0, \tag{C1}$$

$$\begin{aligned}\dot{\theta}_2&=\vartheta\theta_2-\frac{\partial H}{\partial K}\\&=\{\vartheta-\alpha\beta(1-\tau-\varepsilon)\left(\frac{G}{K}\right)^{(1-\beta)}\}\theta_2\\&\qquad-(\beta-\gamma)\frac{1}{K}(\varepsilon\alpha G^{(1-\beta)}CK^{(\beta-\gamma)})^{(1-1/\eta)},\end{aligned} \tag{C2}$$

$$\dot{K}=(1-\tau-\varepsilon)\alpha K^{\beta}G^{1-\beta}-C, \tag{C3}$$

$$\lim_{t\to\infty}e^{-\vartheta t}\theta_2K=0. \tag{C4}$$

Total differentiation of (C1) leads to

$$\begin{aligned}&\frac{1}{\eta}\cdot(\varepsilon\alpha G^{(1-\beta)}K^{(\beta-\gamma)})^{(1-1/\eta)}C^{-1/\eta}\cdot\frac{\dot{C}}{C}\\&\quad=(\beta-\gamma)(1-\frac{1}{\eta})(\varepsilon\alpha G^{(1-\beta)}K^{(\beta-\gamma)})^{(1-1/\eta)}C^{-1/\eta}\cdot\frac{\dot{K}}{K}-\dot{\theta}_2.\end{aligned} \tag{C5}$$

By using (C2), we get

$$\begin{aligned}\frac{1}{\eta}\cdot\frac{\dot{C}}{C}&=(\beta-\gamma)(1-\frac{1}{\eta})\cdot\frac{\dot{K}}{K}+\alpha\beta(1-\tau-\varepsilon)\cdot\left(\frac{G}{K}\right)^{(1-\beta)}\\&\qquad+(\beta-\gamma)\cdot\frac{C}{K}-\vartheta.\end{aligned} \tag{C6}$$

From (C4),

$$\frac{\dot{K}}{K}=\alpha(1-\tau-\varepsilon)\cdot\left(\frac{G}{K}\right)^{(1-1/\eta)}-\frac{C}{K}. \tag{C7}$$

Note that $G=\tau\alpha K^{\beta}G^{(1-\beta)}$, and then $(G/K)=(\alpha\tau)^{1/\beta}$. Substitution of (C7) into (C6), therefore, yields

$$\begin{aligned}\frac{1}{\eta}\cdot\frac{\dot{C}}{C}&=(1-\tau-\varepsilon)\cdot\{\beta+(\beta-\gamma)(1-\frac{1}{\eta})\}\cdot\alpha^{1/\beta}\tau^{(1-\beta)/\beta}\\&\qquad+(\beta-\gamma)\cdot\frac{1}{\eta}\cdot\frac{C}{K}-\vartheta.\end{aligned} \tag{C8}$$

Substitution of C/K in (C7) into (C8) yields

$$\frac{1}{\eta}\cdot\frac{\dot{C}}{C}=(1-\tau-\varepsilon)\cdot\{\beta+(\beta-\gamma)(1-\frac{1}{\eta})\}\cdot\alpha^{1/\beta}\tau^{(1-\beta)/\beta}$$
$$+(\beta-\gamma)\cdot\frac{1}{\eta}\cdot\{(1-\tau-\varepsilon)\alpha^{1/\beta}\tau^{(1-\beta)/\beta}-\frac{\dot{K}}{K}\}-\vartheta. \quad \text{(C9)}$$

Therefore, (C9) can be simplified to

$$\frac{1}{\eta}\cdot\left\{\frac{\dot{C}}{C}+(\beta-\gamma)\cdot\frac{\dot{K}}{K}\right\}=(1-\tau-\varepsilon)\cdot(2\beta-\gamma)\alpha^{1/\beta}\tau^{(1-\beta)/\beta}-\vartheta. \quad \text{(C10)}$$

In the steady-state, all quantitative variables grow at the same rate. Define $\dot{K}/K=g_{DK}$. Rearrange (C3) to get

$$\frac{C}{K}=(1-\tau-\varepsilon)\cdot\alpha^{1/\beta}\tau^{(1-\beta)/\beta}-g_{DK}. \quad \text{(C11)}$$

Take logarithms and derivatives of both sides and get

$$\frac{\dot{C}}{C}=\frac{\dot{K}}{K}=g_{DK}. \quad \text{(C12)}$$

(C12) means that in the steady-state the growth rate of consumption is equal to that of capital stock. Therefore, (C10) becomes

$$\frac{\dot{C}}{C}=\eta\cdot\left\{\frac{2\beta-\gamma}{1+\beta-\gamma}\cdot(1-\tau-\varepsilon)\cdot\alpha^{1/\beta}\tau^{(1-\beta)/\beta}-\frac{\vartheta}{1+\beta-\gamma}\right\}. \quad \text{(C13)}$$

Appendix D

The social welfare function was defined as follows:

$$W=\int_0^\infty U(C,P)e^{-\vartheta t}dt=\int_0^\infty\frac{(C/P)^{1-1/\eta}-1}{1-1/\eta}e^{-\vartheta t}dt. \quad (11)$$

Take limits of the term inside the integral in (11) when t tends to infinity and let them go to zero:

$$\frac{1}{1-(1/\eta)}\cdot\lim_{t\to\infty}\{(C_t^{1-1/\eta}\cdot P_t^{-(1-1/\eta)}-1)\cdot e^{-\vartheta t}\}$$
$$=\frac{1}{1-(1/\eta)}\cdot\lim_{t\to\infty}(C_t^{1-1/\eta}\cdot P_t^{-(1-1/\eta)}\cdot e^{-\vartheta t}-e^{-\vartheta t})$$
$$=\frac{1}{1-(1/\eta)}\cdot\lim_{t\to\infty}(C_0^{1-1/\eta}\cdot P_0^{-(1-1/\eta)}\cdot e^{[(gd-ge)(1-1/\eta)-\vartheta]t}-e^{-\vartheta t})$$

$$= \frac{1}{1-(1/\eta)} \cdot C_0^{1-1/\eta} \cdot P_0^{-(1-1/\eta)} \cdot e^{\{(g_d-g_e)(1-1/\eta)-\vartheta\}t},$$

where $C_t^{1-1/\eta} = C_0^{1-1/\eta} \cdot e^{g_d(1-1/\eta)t}$;

$P_t^{1-1/\eta} = C_0^{1-1/\eta} \cdot e^{g_e(1-1/\eta)t}$; and

$g_e = \dot{P}/P$.

Therefore,

$$W = \int_0^\infty \frac{(C/P)^{1-1/\eta}-1}{1-1/\eta} \cdot e^{-\vartheta t}$$

$$= \frac{1}{1-(1/\eta)} \int_0^\infty C_0^{1-1/\eta} \cdot P_0^{-(1-1/\eta)} \cdot e^{\{(g_d-g_e)(1-1/\eta)-\vartheta\}t} \qquad \text{(D1)}$$

$$= \frac{\{C(0)/P(0)\}^{1-1/\eta}}{(1-1/\eta) \cdot \{\vartheta-(g_d-g_e)(1-1/\eta)\}}.$$

By totally differentiating (C1) with $(C/P) = \varepsilon\,\alpha\,G^{1-\beta}CK^{(\beta-\gamma)}$ and then by using (C2), we derive the following formula:

$$g_e = \frac{\dot{P}}{P} = (\beta-\gamma) \cdot \left\{ \frac{C}{K} - (1-\tau-\varepsilon)\,\alpha^{1/\beta}\,\tau^{(1-\beta)/\beta} \right\} = (\gamma-\beta) \cdot g_d, \qquad \text{(D2)}$$

where $(C/K) = \{(1-\tau-\varepsilon)\,\alpha^{1/\beta}\,\tau^{(1-\beta)/\beta} - g_d\}$. From (D2), we get $(g_d - g_e) = (1+\beta-\gamma) \cdot g_d$. Therefore, the integral of social welfare function can be obtained by substituting (D2) into (D1) as follows:

$$W_d = \frac{\{C(0)/P(0)\}^{1-1/\eta}}{(1-1/\eta) \cdot \{\vartheta - g_d \cdot (1+\beta-\gamma)(1-1/\eta)\}}. \qquad \text{(D3)}$$

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