I. DOCTORAL THESIS

DESIGNING RULE FOR THE CRITICAL PEAK PRICING PARAMETERS CONSIDERING PRICE-RESPONSIVENESS OF CUSTOMERS

소비자의 가격반응성을 고려한 피크요금제 파라미터 설계

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ABSTRACT

DESIGNING R CRITICAL PEAK PRICING PARAMETERS CONSIDERING PRICE-RESPONSIVENESS OF CUSTOMERS

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Recent introduction of deregulation in the power industries offers electricity retailers the chance to make profit through electricity retail business. Accordingly, the retailers need to establish proper strategies for designing and implementing their pricing schemes to grab the chance. Many other pricing schemes can be used for the purposes, however, critical peak pricing (CPP) is chosen in this thesis as a tool for the profit maximization for the following reasons. One, to our knowledge, it is less highlighted compared to other retail pricing schemes despite of its advantages. The other, it contains several designable components for the retailer perspective, for example, base and peak rates, number of events, and event duration.

In terms of the profit maximization, this thesis aims at presenting several analyses and intuitive guides for designing CPP scheme based on the price responsiveness of customers. The guides and designs include selection of the CPP parameters such as peak rate, number of events, and event duration. To achieve the goals, we first investigated how these parameters affect profit of the retailers. In this process, simple price-response model is adopted for the analyses. The results of the analyses allow us to provide the

following three important guidelines. First, the optimal peak rate is approximately inversely proportional to price responsiveness of customers. Second, the optimal peak rate does not change a lot as the number of events varies. Third, the optimal peak rate for single critical event case can be safely used even in case of the multi-events situations in terms of the profit. Fourth, we can ensure existence of a minimum number of events not to lose profit compared the uniform pricing or real-time pricing schemes. Finally, it is a reasonable choice to set event duration as an hour and number of the events as equal time period to total events hour, instead of evaluating each combination of two variables. They are verified by the illustrative examples.

In addition to examining the profit maximization of the retailers, we also explored the benefits that a CPP scheme would afford customers. The results showed that CPP could be advantageous to both customers and the retailer if the customers were sufficiently responsive to price fluctuations. It should be emphasized that such a win-win situation may be only achieved if the CPP scheme is properly designed; this, in turn, requires an analysis of the effects of the CPP parameters on profit and a methodology for selecting appropriate parameter values, which were presented in this thesis.

On the other hands, we pointed out that CPP is not free from the payback phenomenon: a rise in consumption after a critical event, like other time-varying retail pricing schemes. Also, we showed that this payback has a negative effect on profits and thus must be appropriately considered when designing a CPP scheme by the illustrative example which proves that CPP scheme which is designed without considering payback may not be an optimal anymore if the payback takes place. However, few studies have examined CPP scheme design considering payback. This thesis thus characterizes payback using three parameters (duration, amount, and pattern) and examines payback effects on the optimal schedule of critical events and on the optimal peak rate for two specific payback patterns. All the analysis is verified through numerical simulations. The results demonstrate the

need to properly consider payback parameters when designing a profit-maximizing CPP scheme.

KEYWORDS:

Critical Peak Pricing (CPP), Demand Response (DR), Deregulation, Demand elasticity, Price-responsive demand, Payback phenomenon

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CHAPTER 1. INTRODUCTION

1.1 MOTIVATIONS AND PURPOSES

In the electricity retail sector, entities such as utilities or load serving entities (LSEs) have provided electricity to their contracted customers. Traditionally, the operational goal of the retailers was to serve the customers without violating security criteria for the transmission network [1]. There are several available tools for the retailers for maintaining the security in the power system operation. Demand response (DR) program is one of them, which enhances operational efficiency of the power system through cutting customers' demand in peak time [2]. However, introduction of the deregulation in power industries gives another chance to retailers, participating in electricity wholesale market [3] with implementing time-varying retail rates to their customers [4]. In other words, the retailers can be a business entities in a deregulated market environment and pursue profit in return for reliable provision of the electricity to customers [5].

Concerning the deregulated electricity market circumstance that the retailers faced, wholesale real-time market prices are determined for each specified time interval [6]. The appellations for the prices are differ with countries or markets, however, real-time market clearing prices (RTMCPs) is widely used. RTMCP is a function of supply and demand which fluctuate every time and established under market mechanism. Retailers purchase electricity at this RTMCP to the wholesale market and resale it to customers at their own retail rate to make a profit.

In these process, inherent price volatility entailed in the RTMCP could induce a huge risk for the retailers because their retail rates are relatively frozen while RTMCPs are fluctuating consecutively [7]. Another literature in [8] also cautions retailers to consider these price risk for the similar reasons. In an extreme case, RTMCP can skyrocket that

possibly incurs huge loss of the retailers because of the large gap between their retail rate and the RTMCP. As one of the clear evidence for this, Pacific Gas and Electric Company (PG&E), electricity utility in the California, suffered from the gap during the California electricity crisis, eventually, declared bankruptcy which threated the secure operation of the regional power systems [9].

To deal with these price risk problem, several risk hedging strategies are addressed in previous literatures. In the financial engineering field, the risk is tried to be managed through derivatives, such as options and futures [10]. Nonetheless, such instruments have obvious limits, specifically, extreme variations and seasonal autocorrelation which can often be observed in the electricity wholesale market diminish their effects of the risk hedging compared to those for other commodity markets [11]. Direct contracts with generation companies (GenCo) can be another candidate for a remedy to deal with the price risk. Such contracts includes bilateral long-term supply agreements and forward contracts [12]. Study in [13] proposed a stochastic decision-making framework to determine optimal procurement level of the forward and pool market contract amounts for the retailer perspective, where its ultimate goal is to maximize profit while limiting degree of the risk. As a last candidate, dynamic pricing scheme, which belongs to DR program and the subject discussed in this thesis, can be used for the risk hedging purposes by reflecting the volatility of the RTMCP in retail rate [14].

Among many dynamic pricing schemes, three programs are often mentioned in the literatures, which are real-time pricing (RTP), in which reflecting volatility of the RTMCP into retail price using simultaneously fluctuating prices charged to customers; time-of-use (TOU), in which different rates are applied for different blocks of time; and critical peak pricing (CPP), which charges higher rates when a contingency situation occurs or the RTMCP is relatively high [2]. For all these programs, how to design

dynamic pricing schemes plays a crucial role for hedging price risk which is closely related to ability to maximize their profits.

In this thesis, we focused on CPP because it has distinguishable features and several advantages over other dynamic pricing schemes. While it cannot hedge price risk perfectly which can be done by RTP, it has less complexity compared to RTP which enables CPP scheme to be implemented more easily in real-world situation [15]. Note that customers who are participating in a RTP program should respond continuously to time-varying retail rate for every hour. Comparing to TOU, dispatchable CPP events can be a powerful operational tools for the retailers when they implement CPP scheme as a retail rate instead of the TOU. Although TOU has strong point in implementation issues because it is much simpler than CPP, it provides nothing to retailers for dealing with sudden increases of the RTMCP. Note that TOU has only a few block rates for specific time periods which are fixed in advance. As a result, CPP can be regarded as a reasonable alternative to RTP, especially for the residential customers who possibly have a difficulty in responding to time-varying retail rates. CPP can even be implemented in conjunction with TOU by adding dynamically administrable CPP events to TOU [15].

In previous literatures, two main problems are commonly addressed in terms of the profit-maximizing CPP scheme design. One is the events scheduling problem, determining when critical events should be triggered to maximize or minimize a certain target outcome. Profit also can be one of the objective and many previous studies have formulated and solved the events scheduling problem for the purposes. Literature in [16] set profit maximization as goal of the objective function for the retailer perspective and solved it through the dynamic programming framework under forecasted information on price and demand [16]. Similar problem is handled in a stochastic approach to reflect inherent uncertainties of some parameters, such as wholesale market price and temperature [17]. In literature [18], integer programming is used to solve the problem

while it ultimately aims at maximizing social welfare. The other problem in design of the CPP scheme is determination of the rate. In [19], a methodology is proposed for selecting proper level of the peak rate as well as the optimal events schedule under variable wind power generation.

Although many valuable findings on designing methods for CPP scheme to maximize profit of the retailers, there have been few studies examining how the CPP parameters other than the rates affect profit of the retailers, and even fewer that carried out an analytical approach to reflect responses of the customers in it [20]. Moreover, considering its clear relevancy and advantages over other dynamic pricing schemes, CPP scheme has received far less attention than RTP or TOU in state-of-the-art [20].

Concerning responses of the customers in more detail, the response tendencies of the customers to price change of the electricity has complex nature and hard to be analyzed. Payback phenomenon, which the curtailed or interrupted demand appears as delayed consumption after the restriction is finished [21]–[25], is one of them and actually observed in many DR programs. While literatures in [21]–[23] referred it as payback phenomenon, some other studies indicated it as load recovery [24],[25]. Under the situation that there is no official definitions on terms indicating that phenomenon, payback phenomenon will be used in rest of our discussion as a term referring it. The underlying cause of the payback phenomenon can be explained by several factors, however, basically due to the intrinsic attribute of the customers that they do not want to reduce overall consumption but shift in time because demand reduction induces decrease of their degree of satisfaction and convenience [26]. Among many other response pattern of the customers in DR program, payback becomes main concern of this thesis because of their economic effects on profit of the firms are analyzed in some previous studies. According to literature in [27], amount of the paid-back demand may exceed that of curtailed one because the energy conversion processes of customers' appliances requires

additional energy consumption, further, it lessens the market value of the demand-side resources due to cost incurred in serving the additionally consumed demand [28]. The economic effects are evaluated in [21] by analyzing the amount of peak reduction and the effect of payback on generation costs for air-conditioning loads. A mathematical payback model is provided in [24] to utilize it in analysis on payback effects on each market participant.

In a tale with other DR programs, the payback phenomenon can be observed in CPP scheme following a critical event [29]. Accordingly, it may affect the profits enjoyed by the retailers. In this situation, the designed optimal CPP scheme which maximizes the profit may not be an optimal anymore, thus, it must be appropriately considered and reflected when retailers design a profit-maximizing CPP scheme including above-mentioned two problem, event scheduling problem and determination of the rate [30].

1.2 LITERATURE SURVEY AND CONTRIBUTIONS

To fill the gaps in state-of-the-arts which discussed in the previous section, this thesis provides some analyses and try to show the value of it clearly using improvement over previous studies. For the purpose, in-depth literature review is carried out and our contribution is tried to be clarified as follows.

Concerning the literatures on design method of the dynamic pricing schemes, many studies focused on methodologies for the profit-maximization objective. A customer price response model is provided and an agent-based iterative learning method is used for determining the optimal real-time prices based on the proposed response model in [31]. Literature in [5] used nonlinear programming in the optimization process for determining optimal day-ahead real-time prices. Further in [5], various constraints are included in the process such as the operating constraints of a distribution network, the limitation for minimum and maximum demand along with model for customers' responses to prices. The interaction between the LSE and their customers, who are optimizing their own objectives, is explored in [32], where the real-time prices during a scheduling horizon are obtained by solving the profit maximization problem with a simulated-annealing-based price control algorithm. Another study, which assumes a deregulated market environment similar to that in Spain, optimally designs various types of TOU schemes with two, three, and six prices through quadratic nonlinear programming [33]. The study in [34] proposes a designing procedure for the rates and duration of TOU blocks and founds that a properly designed TOU scheme can improve both the profit of a distribution company and the saving of customers. Furthermore, in [35], it is shown that both the provider and the consumers may benefit from TOU pricing. Literature in [35] also details the conditions under which this win-win situation may occur and outlines how the optimal TOU rate may be determined. For the one design problem in the CPP, events scheduling problem, it

is formulated as optimization problem and solved by dynamic programming framework [16] and integer programming [19], respectively, as we already described in the previous section. For the other design problem, determination of the rate, we could only find a recent research which proposed a method to determine the optimal peak rate simultaneously with the optimal schedule of the critical events considering variable wind power generation [19]. Concerning the payback effects, we can find no previous studies which considers payback effects in the CPP design.

Through the careful analyses on previous literatures, we suggest following contributions in this thesis. First, effects of the CPP parameters including the peak rate, the number of critical events, and the duration of each critical event on profit are analyzed by considering the responses of customers to retail price change. Second, the profit index is introduced to examine the effects analytically. In the analyses, we provide some valuable findings such as the fact that optimal peak rate changes little with the change of the number of critical events or there exists the minimum number of critical events to avoid losing profit compared to uniform pricing scheme for the profit-maximizing retailers. Finally, intuitive guidelines and rules for selecting the CPP parameters such that the retailers may maximize its profit are suggested. In the development of analyses, a price response model in [36] is used to quantify the reduction in electricity consumption, and the optimal schedule of critical events is determined using the dynamic programming [37].

Considering payback effects in CPP scheme, we first build analytical model for characterizing payback phenomenon because we can access no previous studies on CPP which include the payback in their design or analyses. In the characterization, three parameters are used; duration, amount, and pattern. Further, a payback ratio and a payback function are introduced and defined for the payback amount and pattern, respectively. Then, the payback effects on the CPP design problems, critical event

scheduling problem are demonstrated based on the characteristics of customers' responses to a critical event. Finally, an analytical expression for the optimal peak rate considering payback is derived for a generalized payback pattern. Then, the payback effects on the optimal peak rate are further analyzed for two specific payback models: exponentially decreasing payback (EDP), to model an intense demand recovery a short time after a critical event, and uniformly distributed payback (UDP), to represent a redistribution of demand over a longer time period.

1.3 THESIS ORGANIZATION

The remainder of this thesis is organized as follows. In Chapter 2, we extract important information from previous studies and provide them as background knowledge for better understanding of our works and contributions. A model for response of the customers to price change, the events scheduling problem, and methods for forecasting prices and demand are included in this chapter. In Chapter 3, we describe and focus on how to design the CPP parameters and how these parameters affect profit of the retailers based on the price responsiveness model of customers which is provided in Chapter 2. In the discussion, we define and suggest profit index as a tool for the analyses. In Chapter 4, the effects of payback phenomenon on design of the CPP scheme are analyzed through the characterized payback parameters. In Chapter 5, the results of which are verified through numerical simulations, finally, Chapter 6 offers some concluding remarks.

CHAPTER 2. BACKGROUNDS

In this chapter, we provide some backgrounds which is helpful for better understanding of discussions in following chapters. First, structure of the electricity retail sector is provided in Section 2.1 together with simplified wholesale market model which is used in this thesis. Especially, the market model includes electricity prices and demands forecasting process. Then, well-known CPP design problem which is carried out in many previous literatures, the events scheduling problem, is provided in Section 2.2. In Section 2.3, customers' price responsiveness model which is suggested in [36] is introduced.

2.1 STRUCTURE OF THE ELECTRICITY WHOLESALE MARKET AND RETAIL SECTOR

The main objective of this thesis is profit maximization of the retailers who serve demand of the customers through well-designed retail pricing scheme. Thus, understanding structure of the electricity wholesale market and the retail sector that they faced is very important for the design. We provide it as a background knowledge in this section with several assumptions on the market and retail sector which are used in this thesis to simplify problem without losing generality.

2.1.1 ENTITIES IN THE ELECTRICITY RETAIL SECTOR

In the electricity retail sector assumed in this thesis, some large customers can participate in the wholesale market directly, while others are served by retailers. Here, we note that the terms LSE will be used in rest of this thesis to indicate each retailer. In real-world electricity markets, there are many kind of the retailers which have different

business models and appellations each other. To eliminate ambiguity in further discussion, we adopt the terms LSE for the retail service entity and assume that LSE buys electricity only from wholesale market. Bilateral contracts with GenCo or hedging risk through derivatives are not considered in this thesis. Concerning wholesale market which LSE participates in, we used simplified market model which is composed of single market, real-time market. In other words, there exist only single price for every specific time period, RTMCP. The structure described above is depicted in following Figure 2.1.

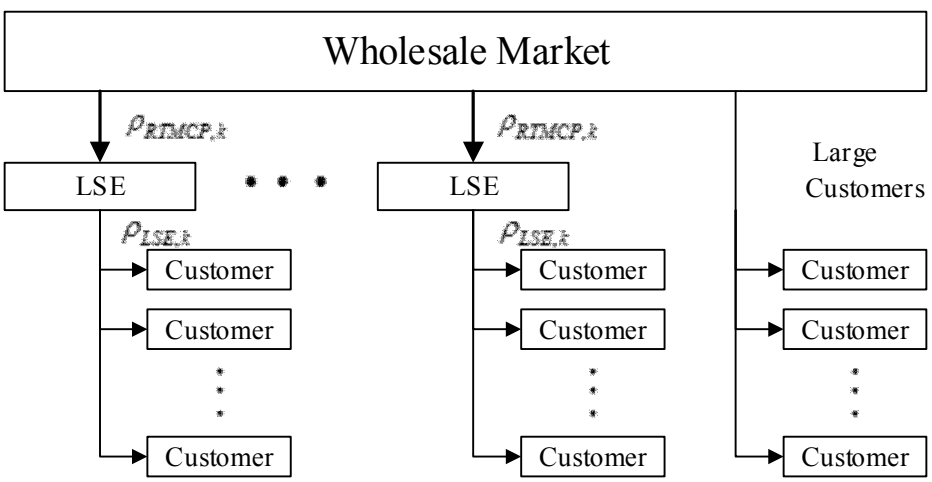

Figure 2. 1 Common structure of the electricity retail sector

Under the structure of the retail sector described in Figure 2.1, LSEs purchase electricity from the wholesale market at RTMCPs, which is denoted as $\rho_{RTMCP,k}$ and sell it to customers at individually designed rates, which is denoted as $\rho_{LSE,k}$. Subscript k means each specific time period.

For the LSE who implements CPP as a retail rate, $\rho_{LSE,k}$ becomes $\rho_{CPP,k}$ which implies the retail price under CPP in time period k. It can be expressed as

$$\rho_{CPP,k} = u_k \rho_{PEAK} + (1 - u_k) \rho_{BASE} \tag{2.1}$$

where u_k is the binary event decision variable that takes a value of one if the critical event is triggered in time period k, zero, otherwise. The variables ρ_{PEAK} and ρ_{BASE} are the peak and off-peak rates of the CPP scheme, respectively. The term of base rate is also widely used to refer off-peak rate. Normally, ρ_{BASE} is set as lower value compared to the uniform price of the LSE to attract customers in CPP plan, and ρ_{PEAK} is much higher than the one. However, given the high value of ρ_{PEAK}, there are some limitations on triggering in the critical events such as the total number of events during a specified time horizon and the minimum interval between successive triggering of critical events, which will be introduced in the following section.

2.1.2 ASSUMPTIONS ON ELECTRICITY MARKET

We already adopted an assumption that the sole electricity market price is RTMCP and it is determined by the real-time supply and demand status. The profit of the LSE is realized at the end of the scheduling time horizon for the actual RTMCP and demand. The design of a CPP scheme and calculation of the associated solution to the events scheduling problem, however, should be conducted before the information on the actual RTMCP and demand is available. Thus, in the design process of a CPP scheme, the RTMCP and demand need to be forecasted in advance for all time periods in the scheduling time horizon.

Various methods have been presented for using time series to forecast prices and demand [38]–[46]. Several types of autoregressive (AR) and autoregressive moving average (ARMA) models are investigated in terms of the performance of the models in

forecasting prices [38], and the autoregressive integrated moving average (ARIMA) model has been used to forecast next-day prices in [39]. It has been pointed out, though, that different time-series analyses, such as a dynamic regression model or a transfer function model, may be able to forecast next-day prices more accurately and efficiently by addressing the correlation problem [40]. The AR, ARMA, and ARIMA models have all been used to forecast demand as well [41]. Several studies have employed extended methods based on the basic structure of artificial neural networks (ANNs) to forecast prices and demand [42]–[46]. The study in [42] proposes implementing a similar days technique to select suitable input factors in order to more accurately forecast day-ahead prices with a simplified ANN model. Yet another study put forth a computationally efficient ANN-based method that used a decoupled extended Kalman filter for price forecasting [43]. Multiple ANNs, which together are called a committee machine, may be used to improve price predictions [44]. The ANN approach may also be used to forecast demand in relation to the temperature [45], though some argue that a weather compensation ANN model [46], could yield more accurate predictions.

More detailed and comprehensive descriptions of these methodologies are beyond the scope of this study. Instead, the autoregressive moving average (ARMA) model is briefly introduced because it is used in the numerical simulation presented later. The ARMA(p,q) model is expressed as [38]

$$x_k = ext + \sum_{i=1}^{p} \varphi_i x_{k-i} + \varepsilon_k + \sum_{j=1}^{q} \theta_j \varepsilon_{k-j} \tag{2.2}$$

where ext, φ_i, and θ_j are constant terms, x_k is the variable to be forecasted, ε_k is zero-mean white noise, and p and q are the orders of the ARMA model.

2.2 CRITICAL EVENTS SCHEDULING PROBLEM OF THE LSE

An events scheduling problem is an optimization problem that seeks to determine when critical events should be triggered in order to maximize or minimize a certain value. The events scheduling problem can be formulated in several different ways. For example, a profit-maximizing schedule of critical events can be developed by using dynamic programming deterministically based on the forecasted price and demand [16] or stochastically based on the probability distributions of the price and temperature [17]. The events scheduling problem can be solved by integer programming to maximize the total benefit of both LSEs and customers [18]. Alternatively, one study proposed using stochastic, nonlinear mixed-integer programming to solve the events scheduling problem in such a way that would minimize the total operational cost [19].

In order to develop a design methodology for the CPP in this thesis, the deterministic dynamic programming framework for maximizing profit of an LSE based on the forecasted information on price and demand in [16] is taken to solve the events scheduling problem. With some modification and rearrangement of the original objective function in [16] to reflect several assumptions in this thesis, the events scheduling problem can be reformulated as

$$\max_{u_k} \sum_{k=1}^{N} \{R_k - C_k\} \tag{2.3}$$

where N is the scheduling time horizon of the problem and R_k and C_k are the revenue and cost of the LSE in period k, respectively, and are defined as

$$R_k = u_k \rho_{PEAK} q_{CPP,k} + (1 - u_k) \rho_{BASE} q_{0,k} \tag{2.4}$$

$$C_k = u_k \rho_{RTMCP,k} q_{CR,k} + (1 - u_k) \rho_{RTMCP,k} q_{0,k} \tag{2.5}$$

The constraints on the events scheduling problem consist of conditions related to the maximum number of events, maximum event duration, maximum total event hours, and the minimum interval between successive events. These constraints are imposed in order to avoid inconveniencing customers and interrupting consumption by frequently issuing events. The specific descriptions of these constraints are presented next.

Maximum number of events (N_{CPP}): The number of critical events can be determined by counting the number of times u_k changes from zero to one. Then, by assuming that $u_k = 0$ for $k \leq 0$, this constraint can be presented as

$$\sum_{k=1}^{N} u_k \left(1 - u_{k-1}\right) \leq N_{CPP} \tag{2.6}$$

Maximum event duration (D_{CPP}): Once a critical event has been triggered, its duration should be no longer than a specified interval. Thus, the number of times $u_k = 1$ within the time window of $D_{CPP} + 1$ should be less than or equal to D_{CPP}. This condition has to hold for all moving time windows; this can be expressed as

$$\sum_{i=k}^{k+D_{CPP}} u_i \leq D_{CPP}, \qquad \forall k \in \{1, 2, \cdots, N - D_{CPP}\} \tag{2.7}$$

Maximum total event time (H_{CPP}): The maximum total event time during the scheduling horizon can be given regardless of N_{CPP} and D_{CPP}. In other words, if this constraint is not explicitly provided, H_{CPP} is naturally equal to N_{CPP} times D_{CPP}. If it is given, H_{CPP} should be less than or equal to N_{CPP} times D_{CPP}, and the total number of times $u_k = 1$ during the scheduling horizon should satisfy the following relationship:

$$\sum_{k=1}^{N} u_k \leq H_{CPP} \tag{2.8}$$

Minimum interval between successive events (Δk): In contrast to the maximum number of events constraint, the count of the interval should start just after u_k changes from one to zero. In addition, u_k should not change again during the subsequent interval of Δk. Thus, by assuming that $u_k = 0$ for $k \leq 0$, this constraint can be presented as

$$u_{k-1}\left(1-u_k\right) \sum_{i=k}^{k+\Delta k-1} \left|u_i - u_{i+1}\right| = 0, \qquad \forall k \in \{1, 2, \cdots, N - \Delta k + 1\} \tag{2.9}$$

Solving the events scheduling problem is identical to computing the optimal events schedule OS^*, set of optimal event time periods k^*s, such that u_{k^*}s are the solutions to Eq. (2.3) while satisfying all the constraints in Eqs. (2.6)–(2.9).

2.3 PRICE RESPONSIVENESS MODEL OF THE CUSTOMERS

If a group of customers consumes an amount of electricity, q_k, in period k, their net benefit at that time, denoted by S_k, can be presented as [36]

$$S_k = B(q_k) - \rho_k q_k \tag{2.10}$$

where $B(q_k)$ is the benefit derived from electricity consumption and ρ_k is the price of electricity in period k. If customers behave strategically in a competitive environment, they are likely to maximize their net benefits. Thus, $\partial S_k / \partial q_k$ should equal zero given ρ_k at the optimal point, and the following relation can thus be obtained:

$$\frac{\partial B(q_k)}{\partial q_k} = \rho_k \tag{2.11}$$

The relation in Eq. (2.11) suggests that reasonable customers tend to consume the amount of electricity for which the marginal benefit of consumption equals the electricity price. Such a benefit function is often assumed to take the following quadratic function form:

$$B(q_k) = B_{0,k} + \rho_{0,k}(q_k - q_{0,k})\left(1 + \frac{q_k - q_{0,k}}{2\beta_k q_{0,k}}\right) \tag{2.12}$$

where $B_{0,k}$, $q_{0,k}$, and $\rho_{0,k}$ are the nominal values of the benefit, demand, and price in period k, respectively. β_k is the elasticity constant of demand and is defined as [47]

$$\beta_k = \frac{\rho_k}{q_k}\frac{dq_k}{d\rho_k} \tag{2.13}$$

The variable β_k has a negative value because a price increase reduces demand. For example, β_k becomes –0.01 when demand decreases by 1% following a 100% increase in price. Indeed, the empirical results of the Statewide Pricing Pilot program showed that β_k ranges from –0.044 to –0.027 under CPP [48].

Then, after differentiating both sides of the Eq. (2.12) with respect to q_k and substituting Eq. (2.13) in, the resulting equation can be arranged for q_k as

$$q_k = q_{0,k}\left\{1 + \frac{\beta_k\left(\rho_k - \rho_{0,k}\right)}{\rho_{0,k}}\right\} \tag{2.14}$$

This equation implies that reasonable consumers who have a demand elasticity of β_k change consumption from $q_{0,k}$ to q_k when the price shifts from $\rho_{0,k}$ to ρ_k.

Under the CPP scheme, customers change consumption when a critical event is triggered or when the price moves from ρ_{BASE} to ρ_{PEAK}. Then, assuming demand elasticity is constant at β, the modified consumption for the critical event triggered in period k can be determined by replacing $\rho_{0,k}$ and ρ_k in Eq. (2.14) with ρ_{BASE} and ρ_{PEAK}, respectively,

$$q_{CR,k} = q_{0,k}\left\{1 + \beta\left(\frac{\rho_{PEAK}}{\rho_{BASE}} - 1\right)\right\} \tag{2.15}$$

Eq. (2.15) shows that customers with demand elasticity β reduce their consumption linearly to ρ_{PEAK}/ρ_{BASE}. In the real world, however, nominal demand $q_{0,k}$ does not occur when consumption changes to $q_{CR,k}$. Thus, $q_{0,k}$ should be interpreted as forecasted consumption, regardless of whether critical events have been triggered.

Using Eq. (2.15), we can define the cumulative curtailed demand $Q_{CUR,k}$ for a critical event starting in period k which is represented as:

$$\begin{aligned} Q_{CUR,k} &= \sum_{i=k}^{k+D_{CPP}-1} (q_{0,i} - q_{CR,i}) \\ &= \beta\left(1 - \frac{\rho_{PEAK}}{\rho_{BASE}}\right) \cdot \sum_{i=k}^{k+D_{CPP}-1} q_{0,i} \end{aligned} \tag{2.16}$$

CHAPTER 3. PROFIT-MAXIMIZING CPP SCHEME CONSIDERING CUSTOMERS' PRICE-RESPONSIVENESS

The main goal of this chapter is to provide a proper design strategy of the CPP scheme for the profit-maximizing LSE. Proper design includes determination of several parameters in the CPP scheme. Peak rates, number of the events, and duration of each event are rarely highlighted and assumed as a fixed constant in many previous literatures on CPP design. Thus, we provide designing rule for the above three CPP parameters in each Section 3.2, 3.3, and 3.4, respectively. Before the main design process, we define and suggest profit index in Section 3.1 as a tool for the design.

3.1 PROFIT INDEX

As a tool for the analysis, we first define a profit index for the LSE. Suppose that the maximum event duration is equal to one. Then, the profit index, PI_k, for each period k is defined as

$$PI_k = \sum_{i=k}^{k+D_{CPP}-1} q_{CR,i}(\rho_{PEAK} - \rho_{RTMCP,i}) - q_{0,i}(\rho_{BASE} - \rho_{RTMCP,i}) \quad (3.1)$$

The profit index in Eq. (3.1) represents the additional profit that the LSE will receive from the triggering of a critical event in period k. The objective function, or the profit of the LSE, in Eqs. (2.3)–(2.5) can be arranged by using the above definition of PI_k in Eq. (3.1) such that

$$\sum_{k=1}^{N}\{R_k - C_k\} = \sum_{k=1}^{N}\left\{u_k PI_k(\rho_{PEAK}) + q_{0,k}(\rho_{BASE} - \rho_{RTMCP,k})\right\}$$
$$= \sum_{k=1}^{N} u_k PI_k(\rho_{PEAK}) + \sum_{k=1}^{N} q_{0,k}(\rho_{BASE} - \rho_{RTMCP,k}) \tag{3.2}$$

The second term in Eq. (3.2) is constant given the forecasted price and demand. Additionally, in a period without critical event, there is no change in demand such that $q_{CPP,k}$ and $q_{0,k}$ in Eq. (3.1) are equal and PI_k become zero. Therefore, one may maximize the profit of the LSE by maximizing the summation of the profit indexes for the periods with critical events. Furthermore, it can be seen from Eq. (3.1) and Eq. (3.2) that this summation depends on the CPP parameters, such as the peak rate (ρ_{PEAK}), the number of critical events (N_{CPP}), and duration of the event (D_{CPP}). Consequently, by using the profit index, the effects of the parameters on the profit of the LSE can be analyzed and the method to select the suitable values of the parameters for the profit maximization can be devised, which are presented in the next sections.

Additionally, profit index can be used for solving events scheduling problem. Intuitively, one can find that critical event is triggered in time period that profit index has maximum value if $N_{CPP} = 1$. In case of $N_{CPP} \geq 2$, we cannot ensure that the time period which has bigger profit index value than other time periods is chosen as critical events period because of constraint in Eq. (2.9). Thus, we should solve the events scheduling problem using existing time consuming methodologies such as dynamic programming such as that in [16]. We developed novel algorithm which definitely reduces number of the calculation for the deterministic problem. Sometimes LSE could face large size of the problem due to too small time resolution or large number of the possible events. In this case, computational efficiency could appeal to LSE. We attached the algorithm and brief descriptions for it in Appendix A.

3.2 SELECTION OF THE OPTIMAL PEAK RATE

In this section, design method for the optimal peak rate, which is the first parameter to be designed in a CPP scheme, is provided. We first get an optimal solution in an analytical manner for the simplest case of $N_{CPP} = 1$ and $D_{CPP} = 1$. Then, it will be expanded to more general cases without losing optimality. Further, the relationship between optimal peak rate and price-responsiveness of the customers which can be represented by demand elasticity is clearly investigated by analyzing resultant form of the optimal solution.

3.2.1 ANALYSIS FOR SIMPLIFIED CASE

For the most simple case of $N_{CPP} = 1$ and $D_{CPP} = 1$, we can get a quadratic function of ρ_{PEAK} by substituting Eq. (2.16) into Eq. (3.1). After some arrangement, resultant equation becomes

$$PI_k(\rho_{PEAK}) = a_{2,k}(\rho_{PEAK})^2 + a_{1,k}\rho_{PEAK} + a_{0,k} \tag{3.3}$$

where

$$\begin{aligned} a_{2,k} &= \frac{q_{0,k}\beta}{\rho_{BASE}}, \qquad a_{1,k} = q_{0,k}\left(1 - \beta - \beta\frac{\rho_{RTMCP,k}}{\rho_{BASE}}\right), \\ a_{0,k} &= q_{0,k}(\beta\rho_{RTMCP,k} - \rho_{BASE}) \end{aligned} \tag{3.4}$$

In the equation, $a_{2,k}$ has a negative value because it is reported that demand elasticity of the customers, β, is commonly has negative value [48]. Then, optimal value of ρ_{PEAK} which maximizes PI_k for time period k is always available because of well-known characteristics of the convex function. That is the optimal peak rate for the profit-maximizing LSE because maximizing the profit of the LSE is equivalent to maximizing value of the profit index. We will denote the value as $\rho^*_{PEAK,k}$. The optimal value of the quadratic function can be achieved by simple derivation, thus, the optimal peak rate, $\rho^*_{PEAK,k}$, should satisfies following condition

$$\frac{\partial PI_k\left(\rho_{PEAK}\right)}{\partial \rho_{PEAK}} = 2a_{2,k}\rho_{PEAK} + a_{1,k} = 0 \tag{3.5}$$

Solving Eq. (3.5) and substituting Eq. (3.4) gives following expression of $\rho^*_{PEAK,k}$ for $k \in \{1,2,\cdots,N\}$

$$\rho^*_{PEAK,k} = -\frac{a_{1,k}}{2a_{2,k}} = \frac{\rho_{BASE}}{2}\left(1-\frac{1}{\beta}\right) + \frac{\rho_{RTMCP,k}}{2} \tag{3.6}$$

3.2.2 Expansion for Generalized Case

For the extended case where $N_{CPP} = 1$ and $D_{CPP} > 1$ for $k \in \{1,2,\cdots,N - D_{CPP} + 1\}$, optimal peak rate can be achieved through similar procedure akin to the one for the simplest case of $N_{CPP} = 1$ and $D_{CPP} = 1$. Summing profit indexes associated to the time

periods in critical events take place, and following the procedures in Eqs. (3.3)–(3.6) result in

$$\rho^*_{PEAK,k} = \frac{\rho_{BASE}}{2}\left(1-\frac{1}{\beta}\right) + \frac{\sum_{i=k}^{k+D_{CPP}-1} \rho_{RTMCP,i} q_{0,i}}{2\sum_{i=k}^{k+D_{CPP}-1} q_{0,i}} \tag{3.7}$$

From the resultant Eq. (3.6) and (3.7), we can conclude that a critical event should be triggered when $PI_k(\rho^*_{PEAK,k})$ is greatest if $N_{CPP} = 1$ is assumed, regardless of whether is $D_{CPP} = 1$ or not. Consequently, the optimal peak rate $\rho^*_{PEAK,k}$ can be determined for a time period in which a critical event occurs. It should be noted that neither the second term in Eq. (3.6) nor that in Eq. (3.7) changes with respect to β. In addition, the magnitude of β is very small. As a result, it can be concluded that ρ^*_{PEAK} is approximately inversely proportional to $|\beta|$. In other words, when customers are less responsive to price changes (i.e., when $|\beta|$ is small), the optimal peak rate, $\rho^*_{PEAK,k}$, should be set at a high value in order to maximize the profit of the LSE.

When $N_{CPP} = 1$, ρ^*_{PEAK} can be analytically determined in a simple form of Eq. (3.6) or Eq. (3.7). However, in the general case where N_{CPP} and D_{CPP} are both greater than one, on the other hand, the summation of the profit indexes must be calculated for each combination of N_{CPP} numbers of elements among the set of time periods or $k \in \{1,2,\cdots,N\}$. For example, if $N = 10$ and $N_{CPP} = 2$, then we should choose two unordered outcomes from ten possibilities; thus, 45 combinatorial profit indexes should be computed without other constraints. In order to derive an analytical form of the optimal peak rate for $N_{CPP} > 1$, let us assume that the optimal events schedule is

determined as OS^* . Then, the summation of profit indexes for the determined N_{CPP} numbers of critical events can be expressed as

$$\sum_{k \in OS^*} PI_k(\rho_{PEAK}) = \sum_{k \in OS^*} \left\{ a_{2,k}(\rho_{PEAK})^2 + a_{1,k}\rho_{PEAK} + a_{0,k} \right\} \tag{3.8}$$

As in the case where $N_{CPP} = 1$, the optimal peak rate, ρ^*_{PEAK}, is determined by the following condition

$$\begin{aligned} \frac{\partial}{\partial \rho_{PEAK}} \sum_{k \in OS^*} PI_k(\rho_{PEAK}) &= \sum_{k \in OS^*} \frac{\partial PI_k(\rho_{PEAK})}{\partial \rho_{PEAK}} \\ &= \sum_{k \in OS^*} \left\{ 2a_{2,k}\rho_{PEAK} + a_{1,k} \right\} \\ &= 0 \end{aligned} \tag{3.9}$$

By using Eq. (3.4), the solution of Eq. (3.9) can be rearranged for the analytical form of the optimal peak rate as

$$\rho^*_{PEAK} = \frac{\rho_{BASE}}{2}\left(1 - \frac{1}{\beta}\right) + \frac{\sum_{k \in OS^*} \sum_{i=k}^{k+D_{CPP}-1} \rho_{RTMCP,i} q_{0,i}}{2 \sum_{k \in OS^*} \sum_{i=k}^{k+D_{CPP}-1} q_{0,i}} \tag{3.10}$$

Because the second term in Eq. (3.10) is not dependent on $|\beta|$, it can be also inferred that ρ^*_{PEAK} is approximately inversely proportional to $|\beta|$ in this case as well. Furthermore, the numerator and denominator of the second term both include $q_{0,i}$, and the numerator is the sum of demand weighted by the $\rho_{RTMCP,i}$s. Thus, if the $\rho_{RTMCP,i}$s in periods with critical events are not unreasonably different from each other, then ρ^*_{PEAK} will not significantly change as N_{CPP} increases as compared to the value when $N_{CPP} = 1$. When $|\beta|$ is small, the first term in Eq. (3.10) is large and, thus, the change in the second term with respect to N_{CPP} may be negligible. This, therefore, implies that the number of critical events has little impact on the optimal peak rate, and that the optimal peak rate for $N_{CPP} = 1$, which may be easily calculated, can be safely used even when $N_{CPP} > 1$.

For the case of $N_{CPP} \geq 2$, we remark that LSE could set different optimal peak rates for each events instead of applying unique ρ^*_{PEAK} calculated in Eq. (3.10). Theoretically, we can calculate $\rho^*_{PEAK,k}$s using profit indices for each time period that critical events take place, further, it gives more profit compared to the case that applying unique value of ρ^*_{PEAK}. However, it discolors the value of the CPP if we remind that simplicity of the CPP appeals to customers compared to RTP. Indeed, many utilities which implement CPP such as SCE (Southern California Edison) adopt single peak price scheme for this reason [49].

3.3 SELECTION OF THE NUMBER OF EVENTS

As a business entity, LSE aims at higher profit from implementing CPP scheme compared to legacy pricing scheme or other dynamic pricing schemes. One of the possible way to achieve it is manipulating the number of the events. In other words, the number of the events becomes controllable variable in the design process. In this section, we provide rules for selecting the number of the events to profit more than one of the popular legacy pricing scheme, uniform pricing scheme.

3.3.1 DETERMINING MINIMUM NUMBER OF THE EVENTS

3.3.1.1 MINIMUM NUMBER OF THE EVENTS TO PROFIT MORE THAN UNIFORM PRICING SCHEME

Under a CPP scheme, the off-peak rate, ρ_{BASE}, should be less than the uniform price, ρ_U, in order to attract customers because, otherwise, there would be no reason for them to sign a CPP contract. Therefore, in the absence of a critical event, an LSE should receive less profit under a CPP scheme than it would under a uniform pricing scheme. However, this reduction in profits during normal time periods may be offset by increased profit during the periods with critical events. It therefore follows that there is the minimum number of critical events that must be triggered for a CPP scheme to be more profitable than a uniform pricing scheme. The profit of LSEs when they sell electricity at ρ_U can be expressed as

$$\sum_{k=1}^{N}\left[q_{0,k}(\rho_U - \rho_{RTMCP,k})\right] \tag{3.11}$$

In order not to lose profit from the application of a CPP scheme, the profit of the LSE in Eq. (3.2) with the optimal peak rate should be greater than or equal to value of the Eq. (3.11). This condition can be represented after some arrangements as

$$\sum_{k \in OS^*} PI_k(\rho^*_{PEAK}) \geq \sum_{k=1}^{N} q_{0,k}(\rho_U - \rho_{BASE}) \tag{3.12}$$

The term on the right side is the loss in profit incurred by implementing CPP instead of uniform pricing during normal time periods. Thus, Eq. (3.12) explicitly shows that the sum of the profit indexes for critical events should be greater than or equal to loss of the profit in order to make CPP a reasonable option. The number of elements in OS^* is equal to the number of critical events, N_{CPP}, and the profit of the LSE always rises as N_{CPP} increases. As a result, the minimum number of events required to satisfy the minimum profit condition of the LSE, denoted as N_{CPP}^{min}, can be found. It can be formulated as

$$N_{CPP}^{\min} = \min_{u_k} \sum_{k=1}^{N} u_k \tag{3.13}$$

where its constraint is Eq. (3.12). Although it is difficult to represent N_{CPP}^{min} in an analytical form because N_{CPP} is a nonnegative integer, it can be numerically solved by mixed-integer nonlinear programming algorithm [50] using several commercial optimization software such as MOSEK [51] and CPLEX [52].

3.3.1.2 MINIMUM NUMBER OF THE EVENTS TO PROFIT MORE THAN OTHER DYNAMIC PRICING SCHEMES

The process for determination of the minimum number of events can be applied to the case that LSE intends to design CPP scheme which profits more than other dynamic pricing schemes, TOU and RTP, instead of uniform pricing scheme.

For the TOU case, the tariff can be composed of two, three, and six prices [33]. Among them, we adopt simple TOU pricing scheme which entails only two price level for the analyses purpose because designing an optimal TOU scheme is not a main concern of this thesis. The TOU rate for time period k, $\rho_{TOU,k}$, can be represented as follow.

$$\rho_{TOU,k} = \begin{cases} \rho_{TOU-HIGH} & k \in t_{TOU-HIGH} \\ \rho_{TOU-LOW} & k \in t_{TOU-LOW} \end{cases} \tag{3.14}$$

$\rho_{TOU-HIGH}$ and $\rho_{TOU-LOW}$ are the TOU rates in peak time and non-peak time, respectively. Peak and non-peak time of the TOU scheme are represented by $t_{TOU-HIGH}$ and $t_{TOU-LOW}$, respectively. Further, we set $\rho_{TOU-LOW}$ equal to ρ_{BASE}, then, demand of the customers becomes $q_{0,k}$ for $k \in t_{TOU-LOW}$. Using Eq. (2.14), the demand for $k \in t_{TOU-HIGH}$ which is denoted by $q_{TOU-HGIH,k}$ can be determined as

$$q_{TOU-HIGH,k} = q_{0,k}\left\{1 + \beta\left(\frac{\rho_{TOU-HIGH}}{\rho_{BASE}} - 1\right)\right\} \tag{3.15}$$

Then, profit of the LSE if it implements TOU can be calculated as

$$\sum_{k \in t_{TOU-HIGH}} \left[q_{TOU-HIGH,k} (\rho_{TOU-HIGH} - \rho_{RTMCP,k}) \right] + \sum_{k \in t_{TOU-LOW}} \left[q_{0,k} (\rho_{BASE} - \rho_{RTMCP,k}) \right] \tag{3.16}$$

By the condition that the value of Eq. (3.2) should be greater than or equal to value of the Eq. (3.15),

$$\sum_{k=1}^{N} u_k PI_k (\rho^*_{PEAK}) \geq \left[\begin{array}{l} \sum_{k \in t_{TOU-HIGH}} q_{TOU-HIGH,k} (\rho_{TOU-HIGH} - \rho_{RTMCP,k}) \\ - \sum_{k \in t_{TOU-HIGH}} q_{0,k} (\rho_{BASE} - \rho_{RTMCP,k}) \end{array} \right] \tag{3.17}$$

Similar to Eq. (3.12), the term in square bracket on the right side means profit loss due to implementing CPP instead of TOU scheme during the period of $k \in t_{TOU-HIGH}$. Same as case in the Subsection 3.3.1.1, LSE should find the solution for objective function in Eq. (3.13) while satisfying constraint in Eq. (3.17)

Same procedure is conducted for another dynamic pricing scheme, RTP. The price in the RTP scheme, denoted as $\rho_{RTP,k}$, can be expressed as [53]

$$\rho_{RTP,k} = \rho_{RTMCP,k} + m_k \tag{3.18}$$

where m_k is a design parameter that accounts for the objective, such as the margin of the LSE given various constraints. The profit of the LSE under the RTP scheme can be written as

$$\begin{aligned}\sum_{k=1}^{N}\{R_k - C_k\} &= \sum_{k=1}^{N} q_{RTP,k}\left(\rho_{RTP,k} - \rho_{RTMCP,k}\right) \\ &= \sum_{k=1}^{N} q_{RTP,k} m_k\end{aligned} \tag{3.19}$$

where $q_{RTP,k}$ is the demand, which is adjusted by $\rho_{RTP,k}$. Therefore, the RTP scheme completely hedges against price risk because $\rho_{RTMCP,k}$ is cancelled out and m_k is a parameter that determines the profit of the LSE. Different to TOU case, deriving analytical form of the objective function and constraint is difficult in this case because of complicated nature of the customers' response to continuously changing price level in RTP and flexibility in designing m_k. Thus, we will try to find the solution in numerical manner for the RTP case and compare it with that of CPP scheme in Chapter 5.

3.3.2 Analysis on Minimum Number of the Events

When $N_{CPP} = 1$ and $D_{CPP} = 1$, the profit index with the optimal peak rate can be determined with Eqs. (3.3), (3.4), and (3.6) such that after some arrangements as

$$PI_k(\rho^*_{PEAK,k}) = -\frac{q_{0,k}\rho_{BASE}\left(1-\rho_{RTMCP,k}/\rho_{BASE}\right)^2}{4}\cdot\beta - \frac{q_{0,k}\rho_{BASE}}{4}\cdot\frac{1}{\beta} + \frac{q_{0,k}}{2}(\rho_{RTMCP,k}-\rho_{BASE}) \tag{3.20}$$

As $|\beta|$ is significantly smaller than one, changes in β dramatically impact the second term in Eq. (3.20) which includes $1/\beta$ unless $\rho_{RTMCP,k}$ is much greater than ρ_{BASE}. Additionally, because β is negative, it can be stated that the profit index decreases as $|\beta|$ increases, i.e., as the price responsiveness of customers becomes higher. It can be inferred from Eq. (3.20) that this trend in PI_k with respect to $|\beta|$ will be present in the general cases where $D_{CPP} > 1$ because the summation is a simple linear operation. The profit index means the compensation for the reduced profit with a critical event. Consequently, N_{CPP}^{min} increases as the price responsiveness of customers increases. Intuitively, if customers are less responsive to price changes, their demand will remain fairly constant when prices increase for the periods with critical events, thus resulting in a large profit index and a small N_{CPP}^{min}. In contrast, if they are highly responsive, demand will decrease significantly during the periods with critical event, which leads to a small profit index and a large N_{CPP}^{min}.

3.3.3 Remark on Maximum Number of the Events

The maximum number of critical events is limited not by the profit of the LSE but by the constraint on the maximum total event time, H_{CPP}. The value of H_{CPP} must be selected with care because it affects convenience of customers and the attractiveness of a CPP scheme in a competitive environment. Consequently, H_{CPP} places an upper bound on N_{CPP} due to the trade-off between the profit of the LSE and the convenience of customers.

3.4 SELECTION OF THE EVENTS DURATION

The number of events (N_{CPP}) and the event duration (D_{CPP}) are not independent parameters because of the constraint on maximum total event time (H_{CPP}) in Eq. (2.8). Moreover, it is obvious from Eq. (3.1) that the profit of the LSE increases as D_{CPP} increases until the constraint on H_{CPP} is satisfied. Therefore, for a given N_{CPP}, D_{CPP} is resultantly determined from Eq. (2.8) as follows

$$D_{CPP} = \lfloor H_{CPP} / N_{CPP} \rfloor \tag{3.21}$$

where $\lfloor x \rfloor$ is the floor function that gives the largest integer not greater than x.

On the other hand, a combinatorial selection of both N_{CPP} and D_{CPP} should be performed with the constraint on H_{CPP} when N_{CPP} is also a variable. This combinatorial selection problem can be qualitatively solved by the following reasoning. Suppose a simple example where $H_{CPP} = 3$, the demand is constant throughout the time periods, and the market prices are those given as in Figure. 3.1. When $D_{CPP} = 1$ and $N_{CPP} = 3$, profit is greatest when critical events are triggered in periods $k_{A,2}$, $k_{B,2}$, and $k_{C,2}$. In contrast, when $D_{CPP} = 3$ and $N_{CPP} = 1$, the profit is maximized if critical events are triggered in periods $k_{C,1}$, $k_{C,2}$, and $k_{C,3}$. The profit indexes for $k_{A,2}$ and $k_{B,2}$ are greater than that for $k_{C,2}$. However, the profit indexes around $k_{A,2}$ and $k_{B,2}$ are smaller than those for $k_{C,1}$ and $k_{C,3}$ around $k_{C,2}$. As a result, the constraint that $D_{CPP} = 3$ requires that $k_{C,1}$, $k_{C,2}$, and $k_{C,3}$ be the periods with critical events. This example clearly shows that D_{CPP} hinders the optimal selection of time periods for critical events, and becomes a factor reducing the degrees of freedom for the selection. Therefore, it is a reasonable choice to set $D_{CPP} = 1$ and $N_{CPP} = H_{CPP}$ in order to avoid a case-by-case evaluation of each combination of N_{CPP} and D_{CPP}.

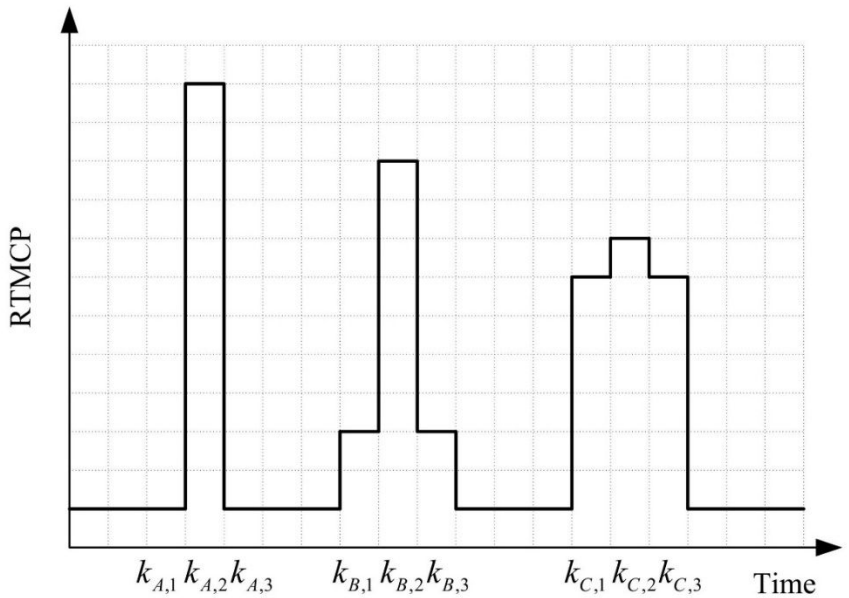

Figure 3. 1 Imaginary market prices used in the example

In addition, because of the constraint on the minimum interval between successive events (Δk), the largest profit indexes may not be chosen even with $D_{CPP} = 1$ if their time periods are too close together. As a result, there is no choice but to calculate the profit of the LSE for all combinations of N_{CPP} and D_{CPP} in order to find that which yields the greatest profit. While this initially appears to be a decidedly time-consuming task, N_{CPP} and D_{CPP} possess several useful characteristics that allow us to reduce the number of combinations. First, both N_{CPP} and D_{CPP} are positive integers, and profit is maximized not when N_{CPP} times D_{CPP} is less than H_{CPP}, but when N_{CPP} times D_{CPP} is exactly equal to H_{CPP}. This greatly limits the number of possible combinations because the combination of positive integers with the product of them being equal to H_{CPP} is limited. Second, N_{CPP} should fall within the range of $N_{CPP} \geq N_{CPP}^{min}$. This condition on N_{CPP} may achieve an additional reduction of the number of combinations.

CHAPTER 4. PAYBACK EFFECTS ON CPP IMPLEMENTATION

In CPP scheme, curtailed demand may appear as a form of delayed consumption after a triggering of a critical event, which is known as the payback phenomenon. This payback makes it complex and difficult to design and operate a suitable CPP scheme in terms of the profit maximization of the LSE. Therefore, several methods and analyses about the effects of the payback on the implementation of a CPP scheme are presented in this chapter.

In Section 4.1, we first need to develop analytical tools for characterizing payback because few studies have tried to analyze payback phenomenon in CPP scheme. For the purpose, two main features of the paid-back demand are captured, the amount and pattern, and expressed by analytical model.

In Section 4.2, two main problems in CPP design, events scheduling problem and determination of the optimal peak rate, are analyzed considering effects of the payback phenomenon. As a result of the analyses, an analytical expression for the optimal peak rate which is the modified form of that in Chapter 3 is derived for a generalized payback pattern. Further, the payback effects on the optimal peak rate are analyzed for two specific payback models, EDP and UDP.

4.1 CHARACTERIZING PAYBACK PHENOMENON

The ultimate goal of the characterization of the payback phenomenon which will be conducted in this section is to represent the payback into analytical expression. Some literatures such as [54] insist that payback can be represented concisely as the cross-elasticity in a mathematical form of the elasticity matrix, however, it is very difficult to

estimate accurate values for the elements and they vary with nominal level of the demand and price. Thus, developing analytical expressions which represent the payback phenomenon is essential for further analytical description. In this vein, we characterized payback by two main parameters, payback ratio and function in this section. Although it could not compose a complete set of bases for the features of the payback phenomenon perfectly, have meaning in categorizing the main properties of payback to design more suitable CPP scheme which considers payback effects in it.

4.1.1 PAYBACK RATIO

The curtailed demand, $Q_{CUR,k}$, may be under-, equally, or over-recovered following a critical event [21]–[23]. Thus, a payback ratio, denoted as $\alpha_{PB,k}$, is introduced as the ratio of recovered consumption to curtailed demand. The specific value of $\alpha_{PB,k}$ depends on the composition of customer demand. For example, one does not compensate for turning off lights during a critical event period by greater light use later on; such demand thus tends to be connected with under-payback, or $\alpha_{PB,k} < 1$. n contrast, a heating or air-conditioning system may require more post-event energy for transition to the target value from the decreased or increased room temperature arising during the critical event period; this tends to result in over-payback, or $\alpha_{PB,k} > 1$. Let $Q_{PB,k}$ be defined as the amount of paid-back demand for a critical event in period k. Then $Q_{PB,k}$ can be represented as:

$$Q_{PB,k} = \sum_{i=k+D_{CPP}}^{k+D_{CPP}+D_{PB,k}-1} q_{PB,i} = \sum_{n=1}^{D_{PB,k}} q_{PB,n+k+D_{CPP}-1} = \alpha_{PB,k} Q_{CUR,k} \tag{4.1}$$

where $q_{PB,i}$ is the recovered demand in period i and $\alpha_{PB,k}$ and $D_{PB,k}$ are the payback ratio and duration, respectively, for the critical event in period k.

4.1.2 PAYBACK FUNCTION

4.1.2.1 DEFINITION AND NORMALIZATION OF THE PAYBACK FUNCTION

The other key aspect of the payback phenomenon is its pattern. Let the payback function, $f_{PB,k}(n)$ for $n \in \{1,2,\cdots,D_{PB,k}\}$, be defined as the ratio of paid-back demand to $Q_{CUR,k}$ in the n-th time period from the end of the critical event. Then, $q_{PB,n+k+D_{CPP-1}}$ in Eq. (4.1) is expressed as:

$$q_{PB,n+k+D_{CPP}-1} = Q_{CUR,k} f_{PB,k}(n) \tag{4.2}$$

Comparing Eqs. (4.1) and (4.2), $f_{PB,k}(n)$ should satisfy the condition:

$$\sum_{n=1}^{D_{PB,k}} f_{PB,k}(n) = \alpha_{PB,k} \tag{4.3}$$

Let the normalized unit payback function, $f_{PB,k}^{U}(n)$, be defined as $f_{PB,k}^{U}(n) = f_{PB,k}(n)/\alpha_{PB,k}$, which satisfies the condition:

$$\sum_{n=1}^{D_{PB,k}} f_{PB,k}^{U}(n) = 1 \tag{4.4}$$

Then, $f_{PB,k}(n)$ is represented as $f_{PB,k}(n) = \alpha_{PB,k} f_{PB,k}^{U}(n)$ such that the payback function can be separately expressed by the payback ratio, $\alpha_{PB,k}$, and the payback pattern, $f_{PB,k}^{U}(n)$.

4.1.2.2 TWO SPECIFIC FEATURES OF THE PAYBACK

In real-world situations, it is difficult to specify a particular form for $f_{PB,k}^{U}(n)$. Nonetheless, some studies suggest a payback function model for analytic purposes particularly for water heating and air conditioning loads. Empirical results in [22] show that the payback pattern can be represented with an exponentially decreasing function for water heating loads. In [21] and [23], sets of decreasing values are specified as the payback function values for water heating loads and both water heating and air conditioning loads, respectively. This study thus adopts an EDP function as a specific payback pattern to model an intensive recovery of demand over a short time period immediately after a critical event. This takes the form of:

$$f_{PB,k}^{U}(n) = e^{-n\lambda}, \quad n \in \{1, 2, \cdots, D_{PB,k}\} \tag{4.5}$$

where λ is a constant, which is determined by solving the equation $\sum_{n=1}^{D_{PB,k}} e^{-n\lambda} = 1$ derived from Eq. (4.4). For example, $\lambda = \ln\{(1+\sqrt{5})/2\}$ can be obtained when

$D_{PB,k} = 2$. Figure 4.1 shows the shape of the EDP function for various values of $D_{PB,k}$ when $\alpha_{PB,k} = 1$. It should be noted from Figure 1 that the EDP functions are almost identical for $D_{PB,k}$ greater than five.

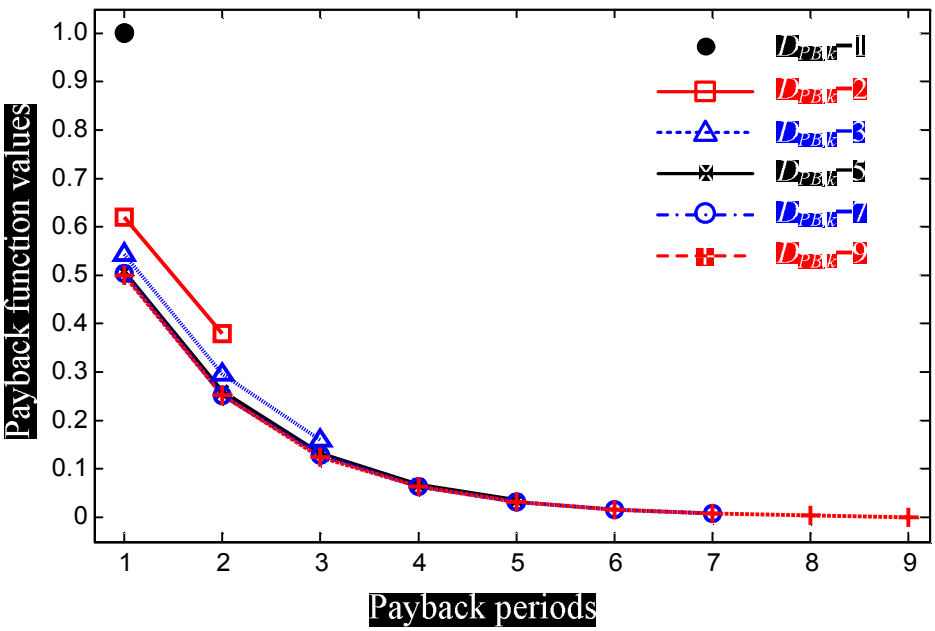

Figure 4. 1 The values of the payback function

Despite past studies [21]–[23] mentioning the EDP pattern, we cannot rule out the possibility that the curtailed demand is recovered fairly evenly during the payback period. Consequently, a constant function is used to model UDP and is analyzed as an additional specific payback pattern for comparison purposes. The UDP function is expressed as:

$$f^{U}_{PB,k}(n) = c, \quad n \in \{1, 2, \cdots, D_{PB,k}\} \tag{4.6}$$

where c is a constant that takes the value $c = 1/D_{PB,k}$ from Eq. (4.4).

4.2 PAYBACK EFFECTS ON CPP DESIGN

In this section, payback effects on two main problems in CPP design, events scheduling problem and determination of the optimal peak rate, will be presented.

4.2.1 PAYBACK EFFECTS ON EVENT SCHEDULING PROBLEM

The optimal schedule of critical events that was determined without considering payback may no longer be optimal once the payback phenomenon is taken into account. Further, the profit of the LSE may decrease if the additional costs arising from payback exceed the cost savings reaped through the critical event. A simple example in Figure 2 demonstrates such a scenario. Suppose that $N = 4$, $N_{CPP} = 1$, $D_{PB,2} = D_{PB,3} = 1$, $\alpha_{PB,2} = \alpha_{PB,3} = 1$, $\rho_{BASE} = 4$ cents/kWh, and $\rho_{PEAK} = 44$ cents/kWh. The nominal demand and RTMCPs are given in Figure 4.2. The customers' price responsiveness is assumed as $\beta = -0.05$.

Under these parameter settings, customers eliminate half of their nominal demand when a critical event is triggered according to Eq. (2.16). Figure 4.3 and Table 4.1 present the modified consumption and profit levels under four different scenarios, respectively. For the case without payback, the profit is largest when a critical event is triggered in period $k = 2$; when payback is considered, however, the profit-maximizing critical event period shifts to $k = 3$. In other words, the optimal event schedule changes due to payback. In addition, profit decreases from \$30 in the case without payback to \$28 in the case of payback with optimal scheduling. This clearly indicates that payback may have a negative effect on an LSE's profits as literature in [28] insisted, simultaneously, suggesting that the event scheduling problem in the presence of payback must be solved as a separate optimization problem.

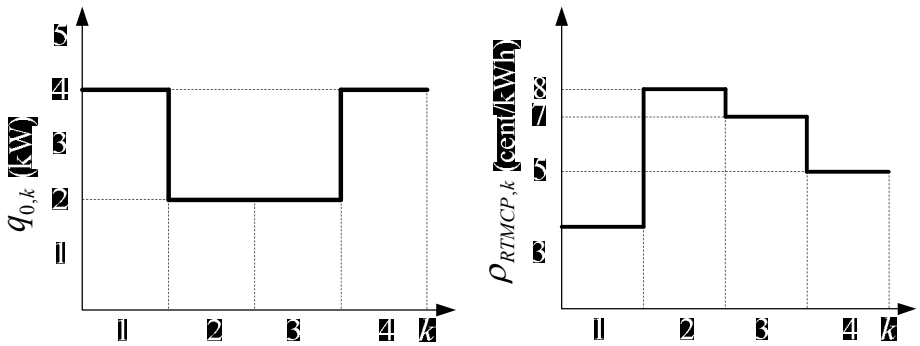

Figure 4. 2 Imaginary demands and market prices used in the example

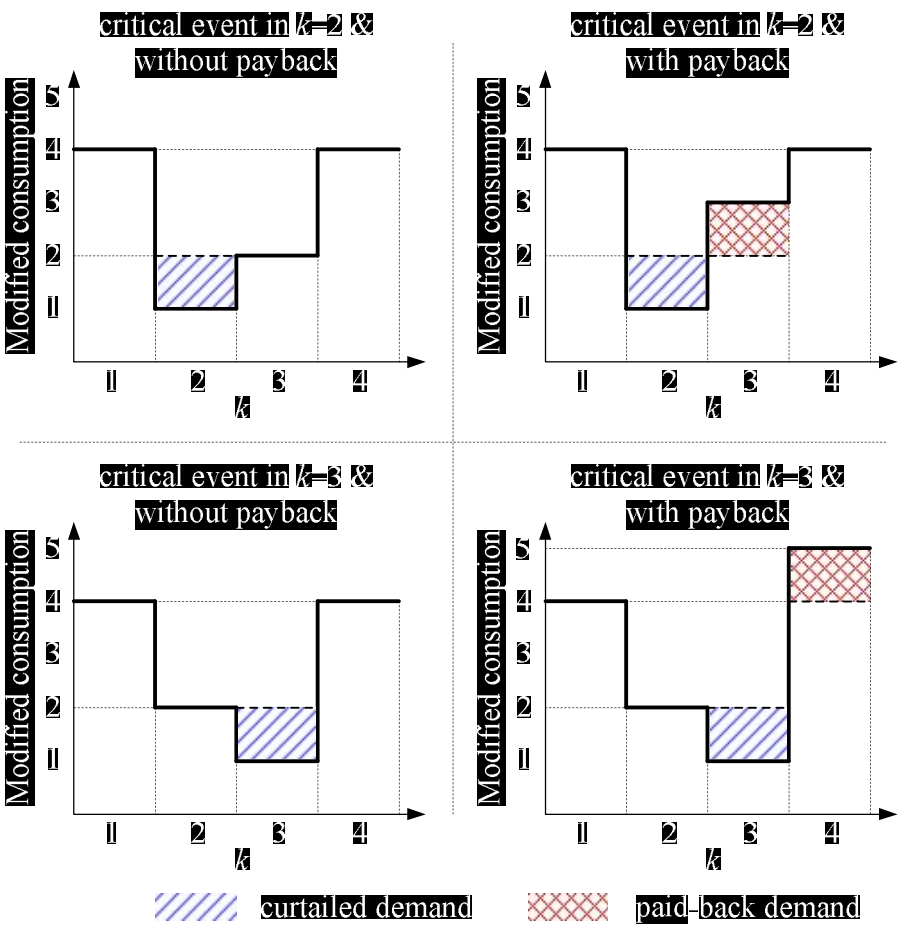

Figure 4. 3 A simple example to show how payback affects the optimal schedule of critical events

Table 4. 1 Profits of the LSE with and without payback for two event schedules in the example

Critical event period		Without payback ($)	With payback ($)
$k=2$	revenue	84	88
	cost	54	61
	profit	30	27
$k=3$	revenue	84	88
	cost	55	60
	profit	29	28

4.2.2 PAYBACK EFFECTS ON OPTIMAL PEAK RATE

4.2.2.1 ANALYSES ON GENERAL FORM OF THE PAYBACK FUNCTION

As we have already discussed in Chapter 3, defining profit index is the first step for deriving optimal peak rate. The profit index including the payback arising from a critical event in period k, $PI_{PB,k}(\rho_{PEAK})$, can be represented as:

$$\begin{aligned} & PI_{PB,k}(\rho_{PEAK}) \\ &= \sum_{i=k}^{k+D_{CPP}-1} \left\{ q_{CR,i}\left(\rho_{PEAK} - \rho_{RTMCP,i}\right) - q_{0,i}\left(\rho_{BASE} - \rho_{RTMCP,i}\right) \right\} \\ &+ \sum_{n=1}^{D_{PB,k}} \left\{ \alpha_{PB,k} Q_{CUR,k} f_{PB,k}^{U}(n) \left(\rho_{BASE} - \rho_{RTMCP,n+k+D_{CPP}-1}\right) \right\} \end{aligned} \tag{4.7}$$

As with the procedures for ρ^*_{PEAK} in Eq. (3.10), the optimal peak rate considering payback, $\rho^*_{PB,PEAK}$, can be obtained by substituting Eq. (2.17) into Eq. (4.7), differentiating with respect to ρ_{PEAK}, and solving the resulting equation for ρ_{PEAK}. After rearranging the terms, a specific form of $\rho^*_{PB,PEAK}$ is obtained as:

$$\rho^*_{PB,PEAK} = \left[\frac{\rho_{BASE}}{2}\left(1-\frac{1}{\beta}\right) + \frac{\sum_{k\in\mathbf{K}^*}\sum_{i=k}^{k+D_{CPP}-1} q_{0,i}\rho_{RTMCP,i}}{2\sum_{k\in\mathbf{K}^*} Q_{0,k}} \right] + \left[\frac{\rho_{BASE}\sum_{k\in\mathbf{K}^*}\alpha_{PB,k}Q_{0,k}}{2\sum_{k\in\mathbf{K}^*} Q_{0,k}} - \frac{\sum_{k\in\mathbf{K}^*}\left\{\alpha_{PB,k}Q_{0,k}\sum_{n=1}^{D_{PB,k}} f^U_{PB,k}(n)\rho_{RTMCP,n+k+D_{CPP}-1}\right\}}{2\sum_{k\in\mathbf{K}^*} Q_{0,k}} \right] \tag{4.8}$$

The terms in the first square bracket in Eq. (4.8) are equal to ρ^*_{PEAK} in Eq. (3.10). Thus, $\rho^*_{PB,PEAK}$ can be represented as:

$$\rho^*_{PB,PEAK} = \rho^*_{PEAK} + \Delta\rho_{PB,PEAK} \tag{4.9}$$

where $\Delta\rho_{PB,PEAK}$ indicates the payback effect on the optimal peak rate, the expression of which is as given in the second bracket in Eq. (4.8). The optimal event schedule, $\mathbf{K}^*$,

depends on $\alpha_{PB,k}$. Thus, $\alpha_{PB,k}$ should be included within the summation sign in Eq. (4.8). However, if $\mathbf{K}^*$ does not change and the payback parameters are the same for $k \in \mathbf{K}^*$, that is, $\alpha_{PB,k} = \alpha_{PB}$, $D_{PB,k} = D_{PB}$, and $f_{PB,k}^{U}(n) = f_{PB}^{U}(n)$ for $k \in \mathbf{K}^*$, then α_{PB} can be pulled out of the sum and $\Delta\rho_{PB,PEAK}$ can be represented as:

$$\Delta\rho_{PB,PEAK} = \frac{\alpha_{PB}}{2}\left\{\rho_{BASE} - \frac{\sum_{k\in\mathbf{K}^*}\left\{Q_{0,k}\sum_{n=1}^{D_{PB}} f_{PB}^{U}(n)\rho_{RTMCP,n+k+D_{CPP}-1}\right\}}{\sum_{k\in\mathbf{K}^*} Q_{0,k}}\right\} \tag{4.10}$$

Above Eq. (4.10) shows that, while maintaining the optimal schedule of critical events, the payback ratio, α_{PB}, has a linear relationship with the amount of change in the optimal peak rate. α_{PB} is not, however, related to whether the payback causes an increase or decrease in the optimal peak rate. On the other hand, the payback pattern, $f_{PB}^{U}(n)$, and duration, D_{PB}, affect both the amount and sign of the change in the optimal peak rate.

4.2.2.2 ANALYSES ON TWO PAYBACK FUNCTION

Despite the relationships between the payback parameters and the optimal peak rate, it is not evident whether payback causes an increase or decrease in the optimal peak rate based only on Eq. (4.10). This is because $\Delta\rho_{PB,PEAK}$ depends on the specific RTMCPs and nominal demand as well as the payback parameters; the interrelation among these factors is difficult to define conclusively, particularly in cases where $N_{CPP} \geq 2$. As a

result, the payback effects on the optimal peak rate will first be examined analytically for the simplest case ($N_{CPP} = 1$). These results will then be extrapolated to the general cases with $N_{CPP} \geq 2$.

Under the conditions that $N_{CPP} = 1$ and the optimal event schedule is determined as $\mathbf{K}^* = \{k^*\}$, $Q_{0,k}$ terms in the numerator and denominator of Eq. (4.10) cancel one another out. As a result, $\Delta\rho_{PB,PEAK}$ in Eq. (4.10) can be represented by more simplified form as follow

$$\Delta\rho_{PB,PEAK} = \frac{\alpha_{PB}}{2}\left\{\rho_{BASE} - \sum_{n=1}^{D_{PB}} f_{PB}^{U}(n)\,\rho_{RTMCP,n+k^*+D_{CPP}-1}\right\} \tag{4.11}$$

For EDP, most of the paid-back demand is concentrated in the initial time periods following the critical event. In addition, a critical event is usually triggered when the RTMCP (Please define) is high, such that the RTMCPs in the early time periods of the payback phase are likely to be higher than ρ_{BASE}. This implies that the second term inside the bracket in Eq. (4.11), which refers to the average RTMCP weighed by $f_{PB}^{U}(n)$ during the payback periods, is also likely to exceed ρ_{BASE} ; $\Delta\rho_{PB,PEAK}$ is thus negative. Consequently, for EDP, where the curtailed demand is recovered quickly, $\rho^*_{PB,PEAK}$, tends to be lower than ρ^*_{PEAK},. Additionally, as described in the previous section, the EDP functions hardly change for $D_{PB} \geq 5$. Therefore, for EDP, as D_{PB} increases, $\rho^*_{PB,PEAK}$, converges to a value, which is likely less than ρ^*_{PEAK}.

For UDP, $\Delta\rho_{PB,PEAK}$ in Eq. (4.11) can be simplified as:

$$\Delta\rho_{PB,PEAK} = \frac{\alpha_{PB}}{2}\left\{\rho_{BASE} - \frac{1}{D_{PB}}\sum_{n=1}^{D_{PB}} \rho_{RTMCP,n+k^*+D_{CPP}-1}\right\} \tag{4.12}$$

In Eq. (4.12), all relevant RTMCPs are equally weighted in the calculation of $\Delta\rho_{PB,PEAK}$. In addition, the RTMCPs are likely to be small, as the times in question are far from the critical event period. As a result, the absolute value of $\Delta\rho_{PB,PEAK}$ for UDP might be smaller than for EDP, suggesting that $\rho^*_{PB,PEAK}$ is greater for UDP than for EDP. In the extreme situation when the RTMCPs below ρ_{BASE} are dominant over a long payback duration, it is possible that $\Delta\rho_{PB,PEAK}$ becomes positive and thus $\rho^*_{PB,PEAK}$ exceeds ρ^*_{PEAK} Moreover, assuming that ρ_{BASE} is set close to the average of the RTMCPs over all periods, $\rho^*_{PB,PEAK}$ for UDP approaches ρ^*_{PEAK} as the payback duration increases. Nonetheless, in real situations, the payback duration is usually limited to a few time periods, and the RTMCPs around the critical event periods are likely to exceed ρ_{BASE}. Therefore, $\rho^*_{PB,PEAK}$ for UDP is still likely to be below ρ^*_{PEAK} even though it increases and approaches ρ^*_{PEAK} as the payback duration increases.

Until now, the analysis has considered payback effects on the optimal peak rate for the simplest case, $N_{CPP} = 1$. As indicated in Eq. (4.10), $\Delta\rho_{PB,PEAK}$ in the general case ($N_{CPP} \geq 2$) can be interpreted as a superposition of the effects for the $N_{CPP} = 1$ case, weighted by the $Q_{0,k}$ terms. In other words, $\Delta\rho_{PB,PEAK}$ for $N_{CPP} \geq 2$ can be determined as the weighted sum of N_{CPP} terms of $\Delta\rho_{PB,PEAK}$ for $N_{CPP} = 1$. Therefore, the above-presented analysis of payback effects on the optimal peak rate remains valid in cases with $N_{CPP} \geq 2$ unless $Q_{0,k}$ takes a very abnormal value for a certain critical event. Nonetheless, the payback effects on the optimal peak rate still depend strongly on the specific conditions of the RTMCPs and demand levels. As a result, the following section will perform numerical simulations for $N_{CPP} = 3$ given specific values of the RTMCP and

demand; this will allow verification of the payback effects for $N_{CPP} = 1$ and validate their application to cases with $N_{CPP} \geq 2$.

CHAPTER 5. ILLUSTRATIVE EXAMPLES

In order to demonstrate the performance and show effectiveness of the suggested CPP design methodologies suggested in this thesis, illustrative examples are provided in this chapter.

In Section 5.1, we try to show that design scheme of the profit-maximizing CPP provided in Chapter 3 is valid using forecasted demand and RTMCP data. Several findings in the Chapter 3 are also examined that; the optimal peak rate changes little with the number of critical events as long as the fluctuation of the RTMCP is not so severe; there exists the minimum number of critical events to avoid losing profit.

In Section 5.2, effects of the payback phenomenon on the implementation of a CPP are numerically examined. The object of examination includes the findings in Chapter 4 that change of the optimal event schedule of a CPP due to the payback and the optimal peak rate of a CPP considering the payback along with payback pattern and amount. Also, we calculated value of the CPP design considering payback for some case, the situation that market contains skyrocketing RTMCP. It suggested the design can enhance profit of the LSE up to 2.83% compared to the case applying optimal peak rate without considering payback.

5.1 DESIGNING PROFIT-MAXIMIZING CPP SCHEME

5.1.1 CONFIGURATIONS

The data on the RTMCP and demand are forecasted from actual historical data on the Pennsylvania-New Jersey-Maryland Interconnection (PJM), which is one of the regional

transmission organizations in the United States, for one month (May 2013) [55]. As a forecasting model, ARMA model in Eq. (2.2) is chosen. The orders and constant terms of the ARMA models are determined by using the SPSS tool [56]. Specifically, ARMA(3,0) and ARMA(5,9) are chosen as the most suitable models for the RTMCP and demand, respectively. The forecasted values of the RTMCP and demand in comparison with the actual values are shown in Figure 5.1 and 5.2, respectively.

For the simulation, some CPP parameters other than variables which can be designed for profit maximization are fixed in advance. Especially, the values of ρ_{BASE} and H_{CPP} are set at 0.050 \$/kWh and 12 hours/month, respectively. Simultaneously, the interval of each time period is assumed to be one hour, meaning that the scheduling time horizon is $N = 744$.

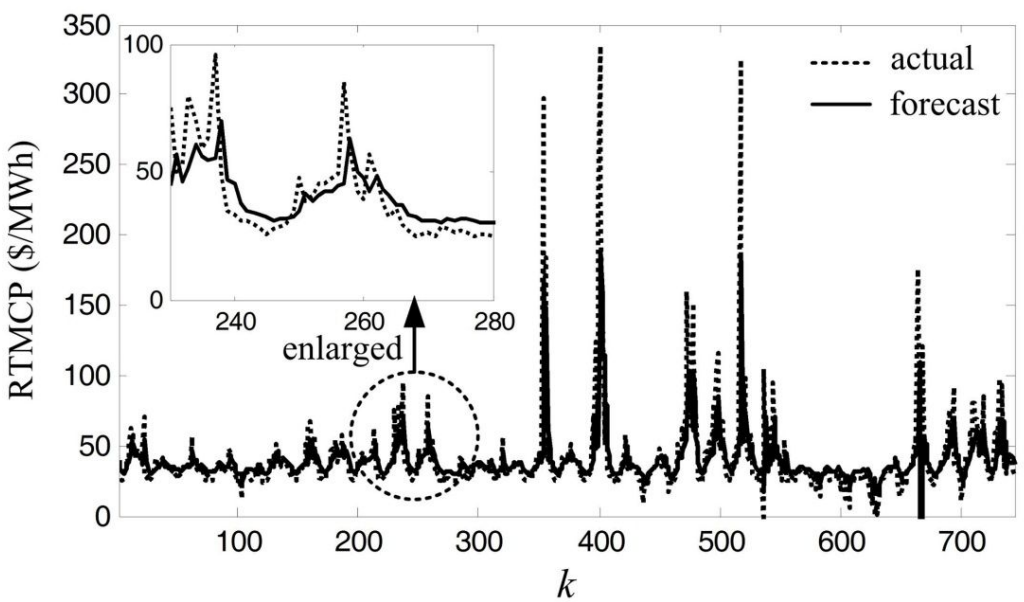

Figure 5. 1 Actual and forecasted data on the RTMCP used in Section 5.1

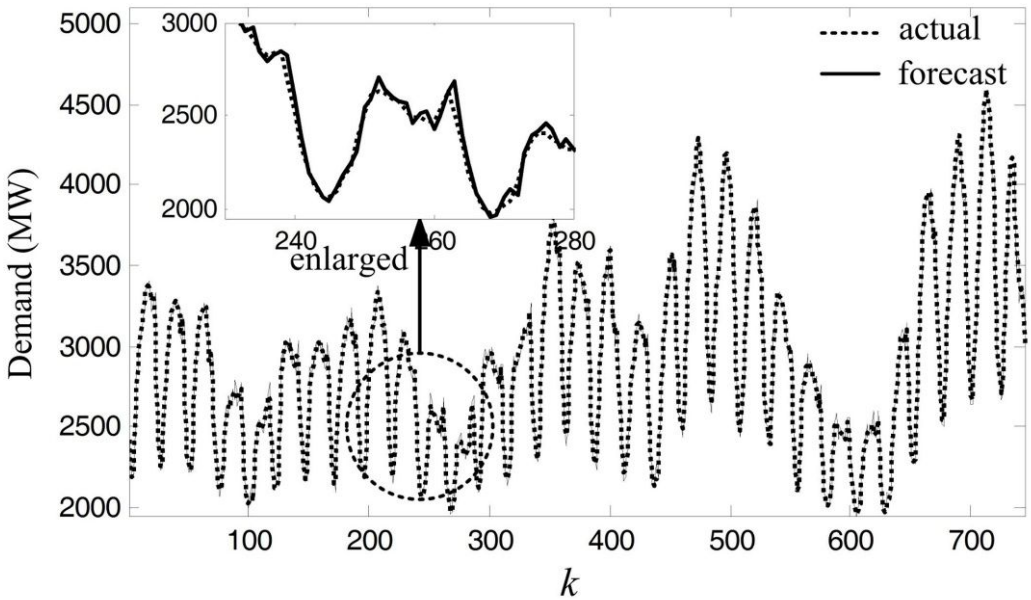

Figure 5. 2 Actual and forecasted data on the demand used in Section 5.1

5.1.2 EVENTS SCHEDULING PROBLEM AND PROFIT INDEX

In order to show the usefulness of the proposed profit index, the events scheduling problem is solved for $\beta = -0.02$, $\rho_{PEAK} = 1.900$ \$/kWh, $D_{CPP} = 1$, $N_{CPP} = 3$, and $\Delta k = 24$ hours. The solution is obtained as $OS^* = \{354,474,714\}$ with the increased profit of \$26.421 million from CPP. The periods with critical events are represented as stars in Figure 5.3 with the profit indexes over the scheduling time horizon. It seems that the highest three profit indexes provide the solution to the events scheduling problem. In this simulation, however, $PI_{712} = \$2.201$ million is greater than $PI_{353} = \$2.158$ million, but the constraint $\Delta k = 24$ results in the selection of $k = 354$ as the time period for a critical event. This result demonstrates that when the constraint on Δk is imposed, critical events cannot always occur in the periods with the highest profit indexes. Nonetheless, $k = 474$ and $k = 714$, the periods with the greatest and second greatest profit indexes,

are still selected. Consequently, it seems that the profit indexes, which can be calculated more simply and faster than the solution to the events scheduling problem, can offer an approximate picture of the operational strategy under a CPP scheme.

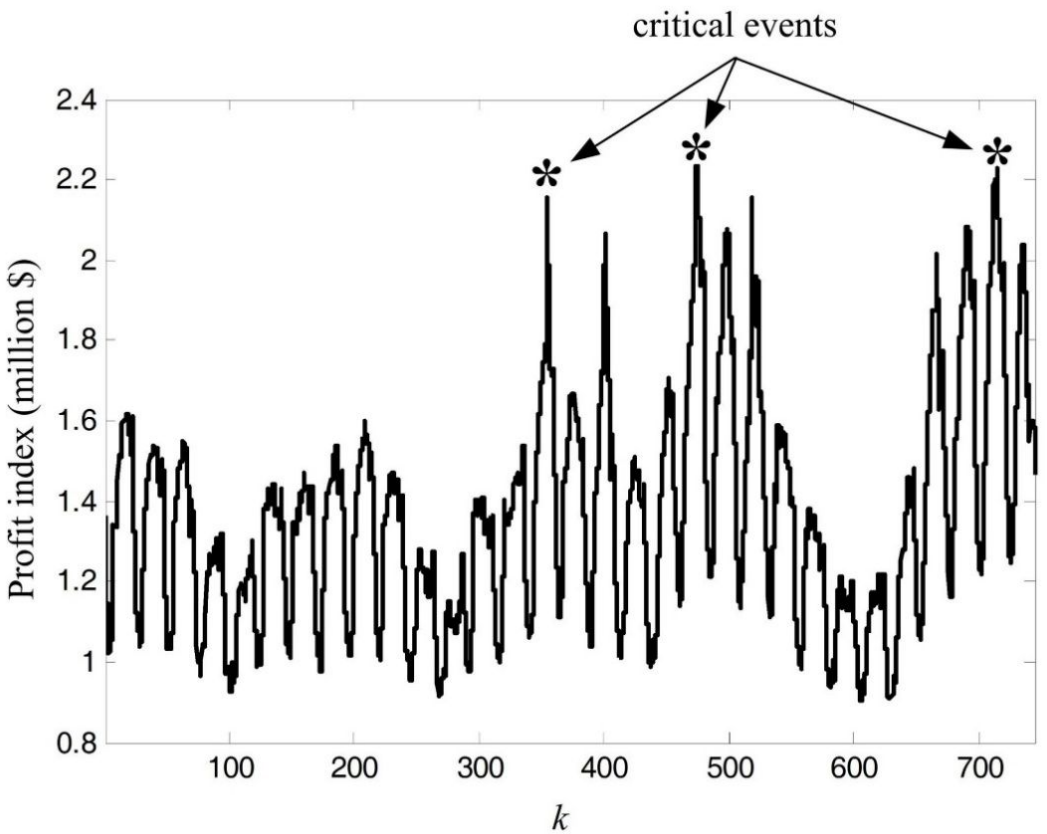

Figure 5. 3 Profit indexes for the scheduling time horizon

5.1.3 Optimal Peak Rate

For the CPP scheme with $N_{CPP} = 3$, $D_{CPP} = 1$, $\beta = -0.02$, and $\Delta k = 0$, the optimal peak rate can be determined from Eq. (3.10), which leads to $\rho^*_{PEAK} = 1.305$ \$/kWh. In

order to examine the improvement in profit under the optimal peak rate, the profits for $\rho^*_{PEAK} = 1.305$ \$/kWh and $\rho_{PEAK} = 1.900$ \$/kWh are compared. The later one, $\rho_{PEAK} = 1.900$ \$/kWh is arbitrarily chosen value. At a glance, $\rho_{PEAK} = 1.900$ \$/kWh seems to be better because a higher peak rate might be considered to lead to greater profit. However, the profit for $\rho_{PEAK} = 1.900$ \$/kWh is \$26.464 million, which is lower than that for $\rho^*_{PEAK} = 1.305$ \$/kWh (\$28.348 million) because a higher peak rate may result in an over-reduction of consumption by customers.

The optimal peak rate is calculated for various combinations of β and N_{CPP}, as shown in Figure 5.4 and 5.5. From Figure 5.4, it can be seen that ρ^*_{PEAK} is approximately inversely proportional to $|\beta|$. The result suggests that it is better to select a lower peak rate for customers that have high price responsiveness than for those that have low price responsiveness. Although Figure 5.4 shows a decreasing pattern with respect to $|\beta|$, these patterns are not monotonically decreasing functions of $|\beta|$. For example, when the section around $|\beta| = 0.05$ is enlarged, as shown as a subfigure in Figure 5.4, it can be seen that the optimal peak rate for $N_{CPP} = 1$ rises. This is because the optimal schedule of critical events may change as $|\beta|$ changes, and the values of $\rho_{RTMCP,k}$ for the modified event schedule may cause an increase in ρ^*_{PEAK}. After such an increase, however, ρ^*_{PEAK} decreases again with respect to increase of $|\beta|$, which shows that $|\beta|$ remains the dominant factor affecting ρ^*_{PEAK}.

Figure 5.5 shows the difference between each N_{CPP}'s ρ^*_{PEAK} value and that for the case where $N_{CPP} = 1$. From this figure we can see that N_{CPP} has little effect on ρ^*_{PEAK}, especially when $|\beta|$ is small. Specifically, the range of the difference in ρ^*_{PEAK}s for $|\beta| = 0.02$ is 0.004 \$/kWh, which is 0.31% of $\rho^*_{PEAK} = 1.305$ \$/kWh for $N_{CPP} = 1$. In contrast, the range in ρ^*_{PEAK}s for $|\beta| = 0.05$ is 0.037 \$/kWh, which is 6.00% of $\rho^*_{PEAK} = 0.617$ \$/kWh for $N_{CPP} = 1$. Therefore, it may be a practical choice to use ρ^*_{PEAK} for

$N_{CPP} = 1$ even in other cases when $N_{CPP} > 1$, particularly if customers are less responsive to price changes, that is, $|\beta|$ has a small value.

The effect of the minimum interval between successive events on profit of the LSE is now analyzed for various values of Δk when $\beta = -0.02$, $D_{CPP} = 1$, $N_{CPP} = 3$ and $\rho^*_{PEAK} = 1.305$ \$/kWh. The results are presented in Table 4.1. Profit of the LSE tends to decrease as Δk increases because a larger value of Δk prevents better time periods from being selected.

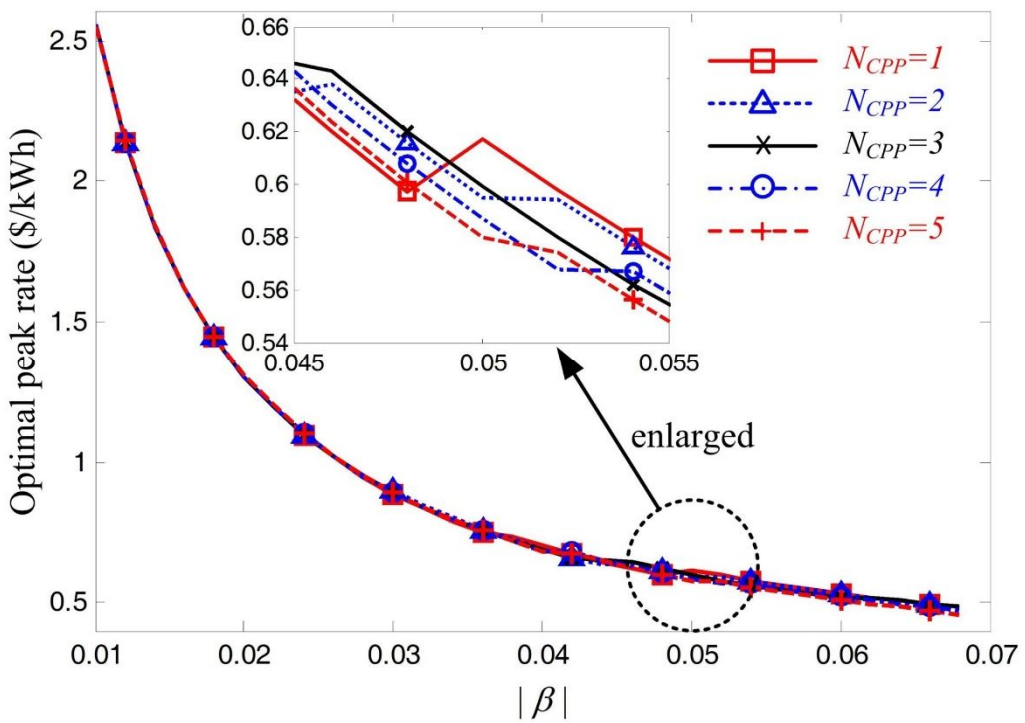

Figure 5. 4 Simulation results of the optimal peak rate for various values of the number of the events

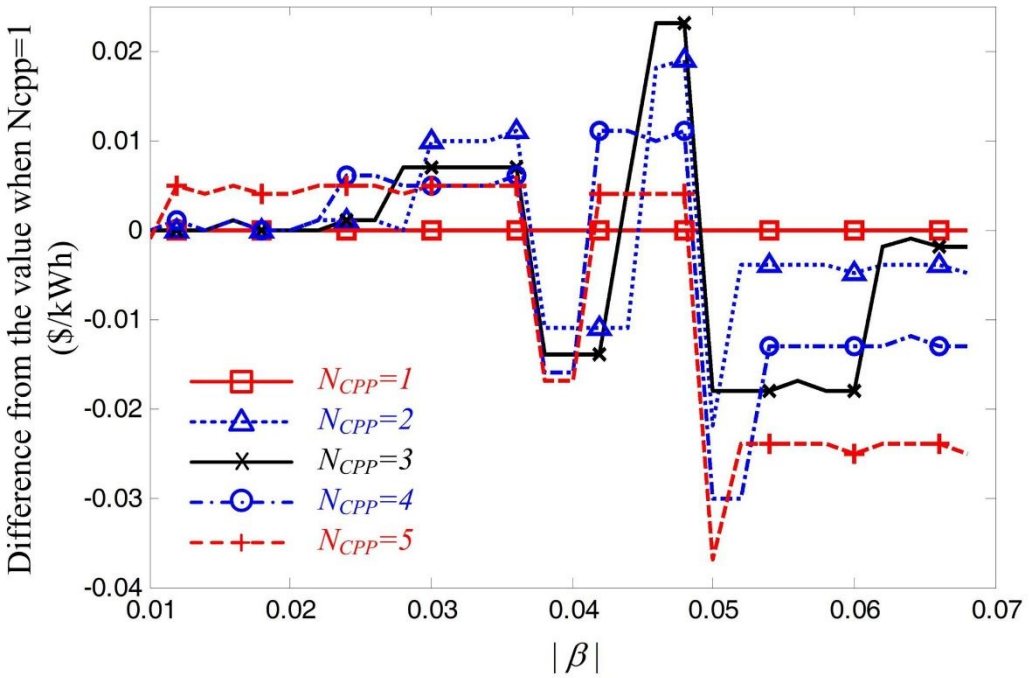

Figure 5. 5 Simulation results of the difference in the optimal peak rate for each $\boldsymbol{N_{CPP}}$ as compared to $\boldsymbol{N_{CPP} = 1}$

Table 5. 1 Effects of the optimal peak rate and minimum interval between successive events on forecasted profit of the LSE

Minimum event interval (hour)	**Forecasted profit (million dollar)**	**Profit change (%)**
0	28.348	0.000
24	28.075	0.963
48	28.075	–0.963
72	28.075	–0.963
96	28.075	–0.963
120	27.963	–1.358
144	27.891	–1.612

5.1.4 NUMBER OF THE EVENTS

In Section 3.3, we provided analytical method for selecting number of the events not to lose profit compared to the uniform pricing case. In this subsection, we proposed more efficient method for it using several numerical characteristics of the CPP parameters. Further, we also compare the outcome of the CPP with RTP, which is regarded as a pricing scheme that most dynamically administrates fluctuation of the RTMCP into retail rate. It gives several useful tips to LSE who considers implementing CPP instead of RTP in terms of the risk they should take.

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5.1.4.1 COMPARED TO THE UNIFORM PRICING SCHEME

In Section 3.3, we formulated analytical form of the objective function to find N_{CPP}^{min} and it was quite complicated and hard to solve. However, N_{CPP}^{min} can be found very easily in a numerical manner using the characteristic that N_{CPP} has an integer value. Iterative procedure in which N_{CPP} is incrementally increased and then checked to determine whether it satisfies the condition in Eq. (3.12) is key idea for it and will be conducted in this subsection.

Figure 5.6 shows the profit of the LSE for various values of N_{CPP} with respect to $|\beta|$ when ρ_{PEAK}^{*} is applied and $D_{CPP} = 1$. This figure clearly shows that, given β or price responsiveness of customers, profits rise as N_{CPP} increases. In addition, it can also be seen from Figure 5.6 that the interval between the profits for different N_{CPP}s is larger when $|\beta|$ is smaller. For example, when N_{CPP} increases from 2 to 3, $PI_{713} = \$2.835$ million is added to the profit level for $|\beta| = 0.02$, whereas $PI_{474} = \$1.009$ million is added to the profit level for $|\beta| = 0.06$. As described in Section 3.3, the demand remains

fairly constant during the periods with critical events if customers are less responsive to price changes. Thus, the profit index, or the additional profit from the increase in N_{CPP}, will be large. In contrast, demand will fall sharply if customers are sensitive to price increases; this will, in turn, yield a small profit index. Therefore, Figure 5.6 explicitly verifies this property of the profit index with respect to N_{CPP} and $|\beta|$.

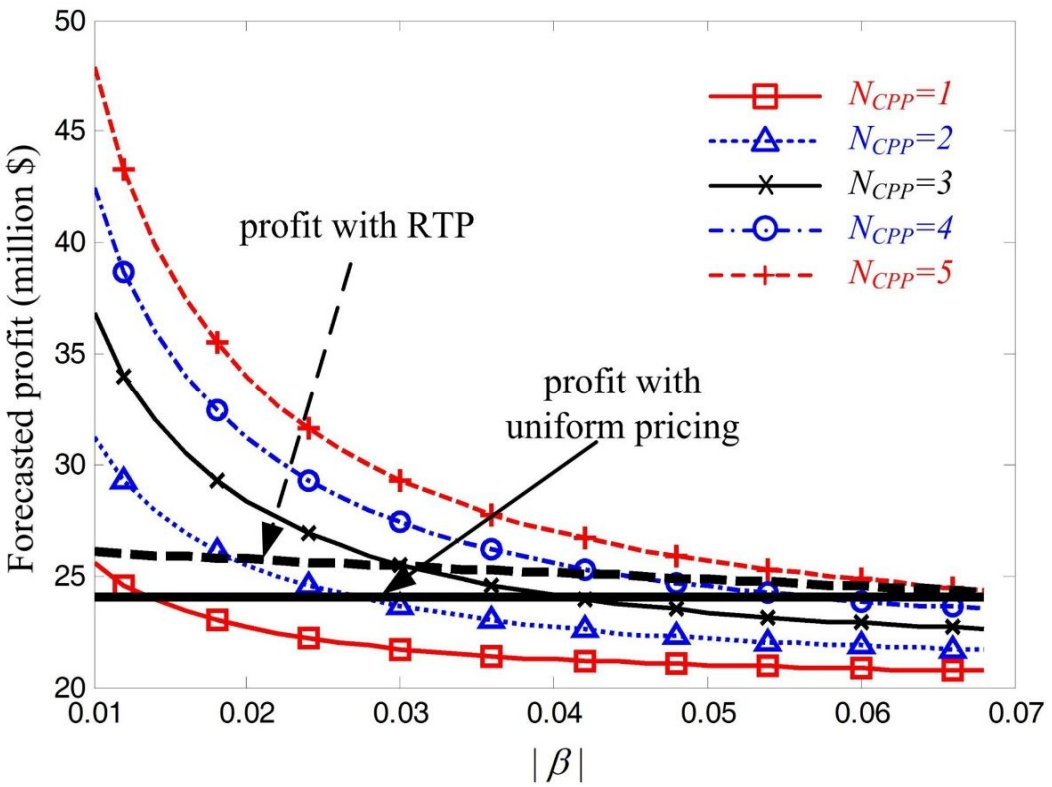

Figure 5. 6 Profit of the LSE for various values of $\boldsymbol{N_{CPP}}$ with respect to $|\boldsymbol{\beta}|$

The profit of the LSE for the CPP with various values of N_{CPP} is examined in comparison with the uniform pricing. Suppose that the uniform price, ρ_U, is 0.052 $/kWh, which is 4% higher than ρ_{BASE}; the profit of the LSE in this situation is represented by the solid line in Figure 5.6. Clearly, the profit under a uniform pricing scheme does not

change with respect to $|\beta|$. The minimum number of events below which the LSE loses profit, N_{CPP}^{min}, can also easily be checked with Figure 5.6. The results for N_{CPP}^{min} in a uniform pricing scheme are presented in Figure 5.7, which shows that N_{CPP}^{min} increases as $|\beta|$ increases. In a CPP scheme, the reduced profit that results from ρ_{BASE} being less than ρ_U should be offset by the profit indexes for critical events; thus, more critical events are necessary when the profit indexes are small. In other words, N_{CPP}^{min} is large when $|\beta|$ is high because the profit index for a high $|\beta|$ is small. Consequently, the results of N_{CPP}^{min} in Figure 5.7, represented by a solid line, agree with this reasoning of N_{CPP}^{min} with respect to $|\beta|$.

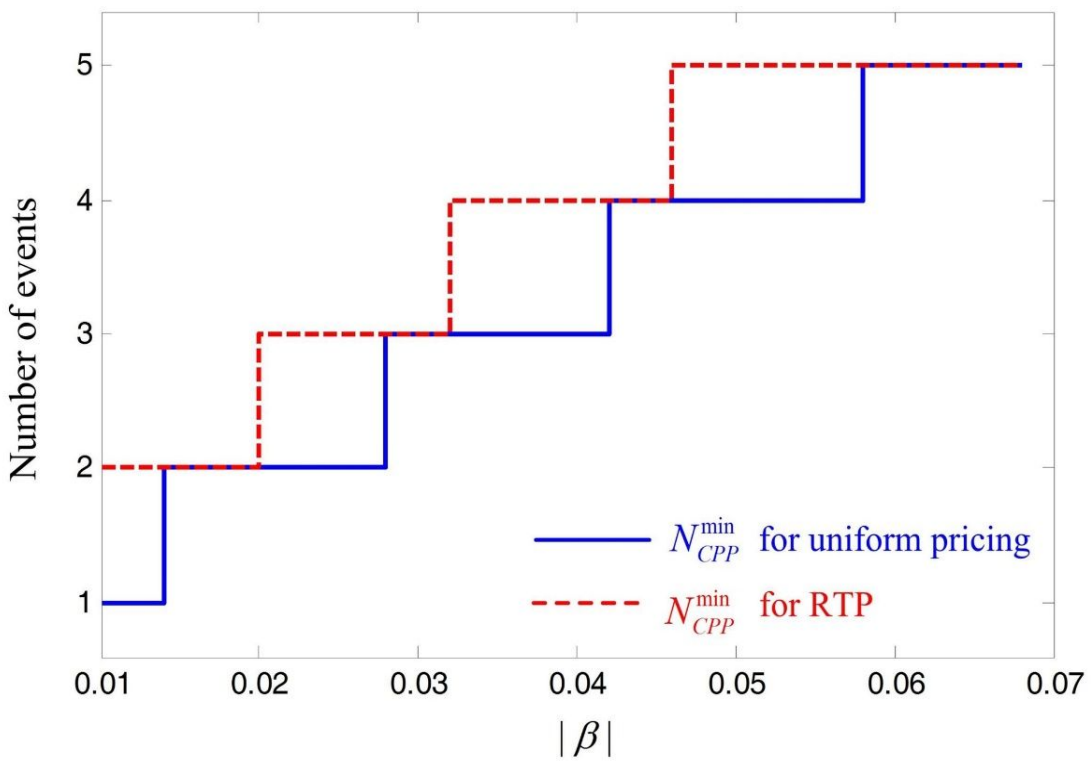

Figure 5. 7 Minimum number of events with respect to $|\boldsymbol{\beta}|$

5.1.4.2 COMPARED TO REAL-TIME PRICING

In Subsection 3.3.1.2, we introduced simple RTP design scheme for the analyses and represented it using designable component m_k. Though a more sophisticated RTP scheme design based on m_k would be of great interest and importance, it is beyond the scope of this thesis. Consequently, a simple but reasonable RTP scheme is composed for the comparison purposes in the simulation, which is given as

$$\rho_{RTP,k} = \rho_{RTMCP,k} + \begin{cases} 70 & \text{if } \rho_{RTMCP,k} \geq \rho_U \\ 5 & \text{otherwise} \end{cases} \tag{5.1}$$

where $m_k = 70$ and $m_k = 5$ are selected such that the profit under RTP is approximately equal to that under uniform pricing for customers with $|\beta| = 0.07$. The profit of the LSE with the RTP is also shown as a dashed line in Figure 5.6. It can be seen from Figure 5.6 that, as intended in the design in Eq. (5.1), the profit with the RTP is close to the profit with the uniform pricing when $|\beta| = 0.07$. For customers with the same $|\beta|$, the profit under CPP is not necessarily greater than that under RTP, and vice versa. Nevertheless, N_{CPP}^{min} associated with the RTP can be also determined, and the results are shown by a dashed line in Figure 5.7. As $|\beta|$ increases, both the profit under RTP and the profit index of the CPP decreases. Therefore, the N_{CPP}^{min} associated with RTP tends to increase as $|\beta|$ increases. In addition, the N_{CPP}^{min} for RTP is greater than that for uniform pricing because RTP is designed to yield greater profit than uniform pricing within the range of $|\beta|$ in the simulation. However, Figure 5.7 demonstrates that there are some regions of $|\beta|$ in which the N_{CPP}^{min}s associated with RTP and uniform pricing are equal to each other. This occurs because N_{CPP}^{min} is a positive integer determined by a ceiling function.

In order to better understand the characteristics of CPP, the conditions under which CPP converges to uniform pricing or RTP are investigated. Without binding on the constraints, such as the maximum total event time (H_{CPP}) or the minimum interval between successive events (Δk), critical events are triggered in every time period in order to maximize the profit of the LSE; thus, the CPP scheme becomes equivalent to the uniform pricing scheme such that $\rho_U = \rho^*_{PEAK}$. When the constraints are imposed for the convenience of the customers, however, the CPP ceases to resemble the uniform pricing scheme. On the other hand, the CPP scheme may be equivalent to the RTP scheme regardless of the constraints. In other words, CPP is the same as the RTP when m_k is designed as follows

$$m_k = \begin{cases} \rho^*_{PEAK} - \rho_{RTMCP,k} & \text{if } k \in OS^* \\ \rho_{BASE} - \rho_{RTMCP,k} & \text{otherwise} \end{cases} \tag{5.2}$$

However, the RTP scheme with m_k from Eq. (5.2) no longer completely hedges against price risk because $\rho_{RTMCP,k}$ is not cancelled out when we substitute it to profit function of the LSE, Eq. (3.19). Consequently, when the RTP scheme is designed to once again hedge against price risk, its prices vary continuously with $\rho_{RTMCP,k}$, thus CPP is differentiated from RTP.

5.1.5 EVENT DURATION

Event duration, D_{CPP}, should be selected by considering total event hours, H_{CPP}, which reflects the degree of inconvenience that customers are willing to accept. Because

D_{CPP} cannot be determined independently of and is bound with N_{CPP}, the profit of the LSE should be evaluated for the combinations of D_{CPP} and N_{CPP} that satisfy the constraints on H_{CPP} in Eq. (2.8).

In this simulation, two scenarios are composed. The first one is to change D_{CPP} for a constant $N_{CPP} = 3$, while the other is to change both D_{CPP} and N_{CPP} for $H_{CPP} = 12$ hours. The results are presented in Table 5.2 for $\beta = -0.02$, confirming that profit increases as D_{CPP} rises for the constant N_{CPP}. On the contrary, profit rather tends to decrease as D_{CPP} increases even though the differences in profit level are not prominent. In other words, $(D_{CPP}, N_{CPP}) = (1,12)$ is selected as the best parameter because, under the constraint on H_{CPP}, an increase in D_{CPP} reduces the freedom to choose better parameters related to N_{CPP}. As a result, without other reasons for setting $D_{CPP} > 1$, it is desirable to set $D_{CPP} = 1$ for a specified value of H_{CPP}.

Table 5. 2 Forecasted profit for each cases of event duration and number of the events

	Event duration (hour)	**Number of the events**	**Optimal peak rate ($/kWh)**	**Forecasted profit (million dollar)**
	1	3	1.305	28.348
Constant	2	3	1.305	31.164
N_{CPP}	3	3	1.303	36.570
	4	3	1.302	41.806
	1	12	1.307	52.771
Constant	2	6	1.306	47.439
H_{CPP}	3	4	1.307	44.734
	4	3	1.302	41.806

6	2	1.302	36.071

5.1.6 Discussion on the Benefit of Customers

This thesis focuses on maximizing profit of the LSE; however, it is important to examine the benefits of customers by the designed CPP. These benefits are directly tied to the costs that the customers face. The costs of customers with the CPP for various values of N_{CPP} are shown in Figure 5.8 along with the cost under a uniform pricing scheme.

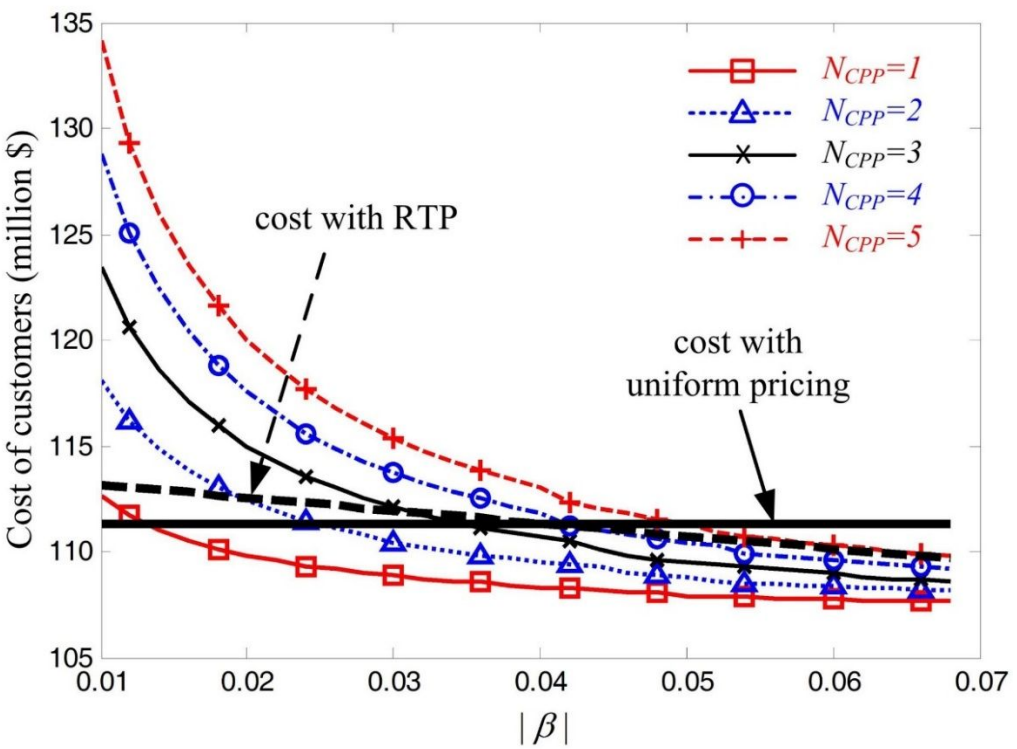

Figure 5. 8 Cost of the customers for various values of $\boldsymbol{N_{CPP}}$ with respect to $|\boldsymbol{\beta}|$

It shows that customers who are less responsive to price changes face greater costs under a CPP scheme for all the cases where $N_{CPP} \geq N_{CPP}^{min}$, that is, where the LSE profits more from a CPP scheme than from a uniform pricing scheme. Therefore, when customers have a small $|\beta|$, only the LSE reaps the benefits from a CPP scheme as their profits are mostly generated from increased consumer costs.

In contrast, when customers are highly responsive to price changes, the LSE may profit in two ways: one is from a decrease in demand that reduces the purchase cost of electricity and the other is from the cost that customers pay. Additionally, it is possible that customers may reduce their demand enough to actually decrease their costs even under CPP. For example, Figure 5.6 and Figure 5.8 show that when $|\beta| = 0.06$ and $N_{CPP} = 5$, the LSE receives \$0.8 million more in profit (CPP: \$24.85 million, uniform pricing: \$24.05 million), and customers pay \$1 million less (CPP: \$110.29 million, uniform pricing: \$111.29 million) under a CPP scheme as compared to a uniform pricing scheme. Consequently, if $|\beta|$ is sufficiently large, both the LSE and the customers may benefit from CPP.

5.2 Results in Payback Phenomenon

5.2.1 Configurations

For the simulations in Section 5.2, forecasted data on the RTMCP and demand based on actual data from PJM are used akin to those in the Section 5.1 [55]. However, to show the result more illustratively, we choose different actual data with those in the previous section as an input data for the forecasting, those for January 2014. Then, we carried out same forecasting procedure and resultant forecasted data on RTMCP and demand are depicted in Figure 5.9.

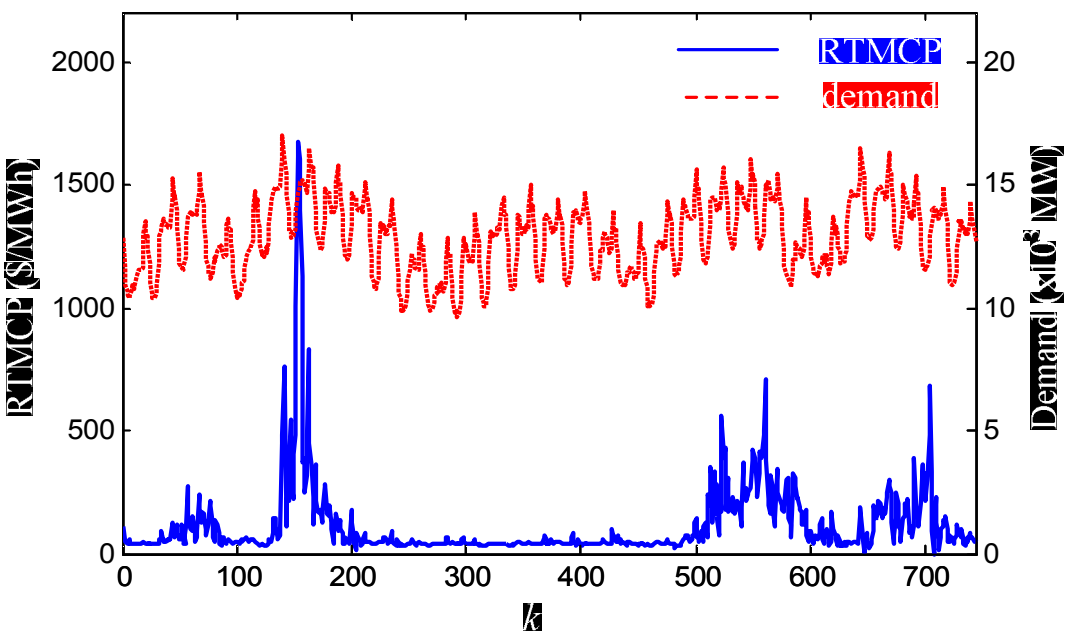

Figure 5. 9 Forecasted data for RTMCP and demand used in the Section 5.2

Under the data, we set $N_{CPP} = 3$, ρ_{BASE} =0.120 \$/kWh, and $N = 744$. Additionally, minimum interval between successive events which is already denoted as Δk is set at 48 hours. The reason for this is to give enough times to customers for recovering their curtailed consumption after critical event period, which was not fixed for the simulations carried out in the previous section. A customer price responsiveness of $\beta = -0.05$ is adopted and it is assumed that the payback parameters are equal for all critical event time period, that is, $\alpha_{PB,k} = \alpha_{PB}$, $D_{PB,k} = D_{PB}$, and $f_{PB,k}^{U}(n) = f_{PB}^{U}(n)$.

5.2.2 Payback Effects on the Optimal Event Schedule

The effects of payback on the event scheduling problem are examined under the conditions that $\alpha_{PB} = 1$ and $D_{PB} = 3$ hour. For set-ups both with and without payback, the optimal peak rate is arbitrarily selected as $\rho_{PEAK} = \rho_{PB,PEAK} = 120$ cents/kWh. Simulations of three different scenarios were undertaken, as listed in Table 4.3, including without payback, with EDP, and with UDP. The specific values of the payback functions are also given in Table 4.3 (first column).

The results for the simulated optimal schedule and corresponding profits are also listed in Table 4.3. The optimal schedules differ from one another according to not only whether or not payback occurs but also the payback pattern as we can see in the second column of the table. This suggests that the payback phenomenon must be considered to properly solve the event scheduling problem. Furthermore, the profit is larger in the non-payback case than in either case with payback as we can see in the last column of the table, verifying that the payback phenomenon has a negative effect on the profit of the LSE due to the additional cost of the paid-back demand.

Table 5. 3 Simulation results for the optimal schedule and the corresponding profit of the LSE

$f_{PB}(n)=\{f_{PB}(1), f_{PB}(2), f_{PB}(3)\}$	**Optimal schedule of critical events**	**Profit (million dollars)**
Without payback	{154, 561, 668}	46.476
Exponentially decreasing payback {0.54, 0.30, 0.16}	{157, 561, 644}	40.265
Uniformly distributed payback {1/3, 1/3, 1/3}	{157, 561, 704}	40.843

5.2.3 PAYBACK EFFECTS ON THE OPTIMAL PEAK RATE

The effects of payback on the optimal peak rate are simulated by changing the payback duration and ratio. The payback duration is set to change from one to ten ($D_{PB} \in \{1,2,\cdots,10\}$), and the range of the payback ratio is taken from [21] as $0.80 \leq \alpha_{PB} \leq 1.06$, with 271 equidistant values of α_{PB} selected within this range. For each combination of D_{PB} and α_{PB}, the optimal peak rate is determined from Eq. (3.10) and the corresponding profit for the LSE is calculated. For the case without payback, we find that $\rho^*_{PEAK} = 177.68$ (cents/kWh), which yields a profit of \$52.355 million.

The simulation results for the optimal peak rate, $\rho^*_{PB,PEAK}$, for three values of α_{PB} are presented with respect to α_{PB} in Figure 5.10 for EDP and Figure 5.11 for UDP, where $D_{PB} = 0$ indicates the case without payback. As described in Section 4.2, the values of $\rho^*_{PB,PEAK}$ are below ρ^*_{PEAK} in all simulations because the RTMCP values around critical

event periods are larger than ρ_{BASE}. Therefore, $\rho^*_{PB,PEAK}$ should be set below the usual level if the payback phenomenon is expected. In addition, Figure 5.10 and 5.11 shows that $\rho^*_{PB,PEAK}$ tends to be smaller for a short payback duration than for a long one, regardless of the payback pattern. This suggests that the LSE should select a lower optimal peak rate if the payback period is expected to be short.

Figure 5.10 and 5.11 also demonstrate how the payback pattern affects the optimal peak rate. The value of $\rho^*_{PB,PEAK}$ is smaller for EDP than for UDP, particularly for a long payback duration. This is because the RTMCPs in the late time periods of the payback duration are smaller than those in the early time periods, but they are all equally weighted when determining $\rho^*_{PB,PEAK}$ for UDP. Moreover, in contrast with the fact that $\rho^*_{PB,PEAK}$ for UDP increases as D_{PB} increases, we can see that $\rho^*_{PB,PEAK}$ for EDP hardly changes for $D_{PB} \geq 5$.

As shown in Figure 5.10 and 5.11, the effect of α_{PB} on the changing shape of $\rho^*_{PB,PEAK}$ is insignificant when compared with the effect of D_{PB}. Nevertheless, to examine the effects of α_{PB} on $\rho^*_{PB,PEAK}$ in greater detail, the change in $\rho^*_{PB,PEAK}$ with respect to α_{PB} is represented in Figure 5.12 for a few values of D_{PB}. As Eq. (4.10) and the associated analyses indicate, $\rho^*_{PB,PEAK}$ decreases linearly as α_{PB} increases, irrespective of the payback duration and pattern. This linear relationship between α_{PB} and $\rho^*_{PB,PEAK}$ holds only as long as the optimal event schedule, $\mathbf{K}^*$, does not change. When the optimal schedule changes, a step change of $\rho^*_{PB,PEAK}$ occurs, as can be clearly observed from the cases $D_{PB} = 1$ for EDP and UDP and $D_{PB} = 5$ for UDP in Figure 5.12. However, Figure 5.12 shows that $\rho^*_{PB,PEAK}$ decreases linearly after the step change as long as the modified optimal schedule is maintained.

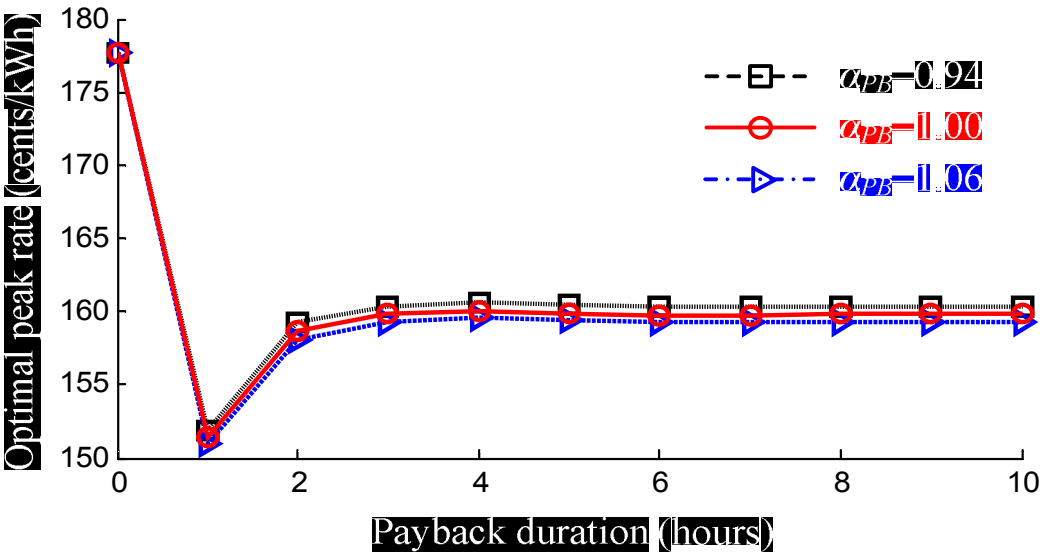

Figure 5. 10 Simulation results of the optimal peak rate with respect to the payback duration for the EDP

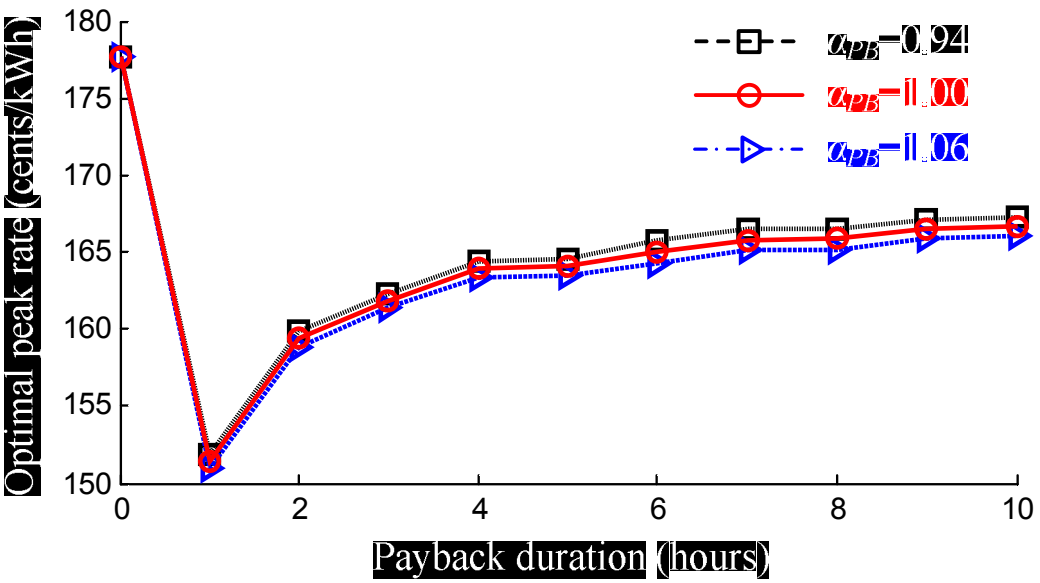

Figure 5. 11 Simulation results of the optimal peak rate with respect to the payback duration for the UDP

Finally, it is necessary to check whether the values of $\rho^*_{PB,PEAK}$ determined in the simulations are optimal. Figure 6 illustrates the profits with payback for $D_{PB} \in \{1,3,5\}$ and $\alpha_{PB} = 1$ with respect to the peak rate for the two payback patterns.

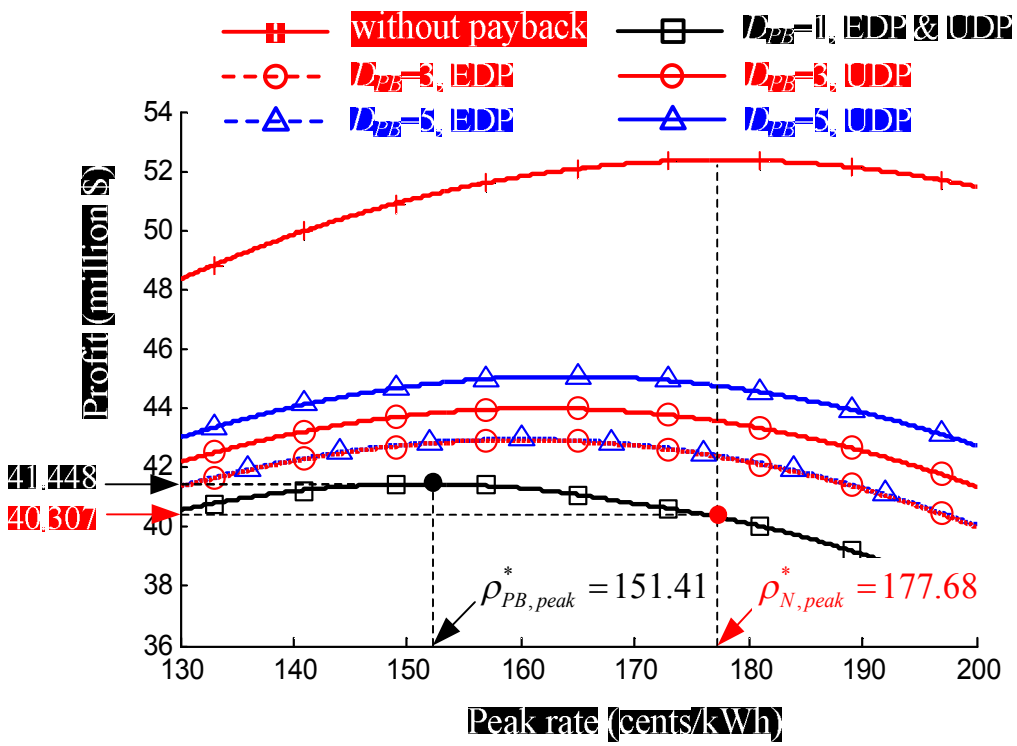

Figure 5. 12 Profit of the LSE with respect to the peak rate for the case without payback and several cases with payback

The profit without payback is also shown in Figure 6. The values for ρ^*_{PEAK} and $\rho^*_{PB,PEAK}$ determined via the simulation are clearly the extreme ones, yielding maximum profits. The existence of payback decreases profits in all cases. To emphasize the significance of this analysis, the profits resulting when $\rho^*_{PEAK} = 177.68$ cents/kWh and $\rho^*_{PB,PEAK} = 151.41$ cents/kWh for $D_{PB} = 1$ and $\alpha_{PB} = 1$ are indicated in Figure 6. (Note that the functions for the two payback patterns are the same for $D_{PB} = 1$). Comparing the

two profits reveals that using $\rho^*_{PB,PEAK}$ increases profits by 2.83% (from \$40.307 million for ρ^*_{PEAK} to \$41.448 million for $\rho^*_{PB,PEAK}$). In practical terms, properly designing a CPP scheme by considering payback effects could lead to a significant, if not dramatic, increase in profits for the LSE.

CHAPTER 6. CONCLUSIONS AND FUTURE WORKS

6.1 CONCLUSIONS

When a CPP scheme is used by LSEs that operate in a deregulated retail sector, it should be appropriately designed to meet the profit maximization objective of such entities. An effective CPP design means selecting parameters such as peak rate, number of events, and event duration. Based on the price responsiveness of customers, how these parameters affect profit was analytically investigated in this thesis, allowing us to offer the following three guidelines. First, the optimal peak rate is approximately inversely proportional to price responsiveness of customers. Second, the optimal peak rate changes little as the number of events varies. Third, the optimal peak rate when $N_{CPP} = 1$ can be safely used even when $N_{CPP} > 1$ in terms of the profit. Fourth, there exists a minimum number of events to avoid losing profit compared the uniform pricing. Finally, it is a reasonable choice to set $D_{CPP} = 1$ and $N_{CPP} = H_{CPP}$ instead of evaluating each combination of N_{CPP} and D_{CPP}. These findings were then verified by using the numerical examples.

In addition to examining the profit maximization of the LSE, we also explored the benefits that a CPP scheme would afford customers. The results showed that CPP could be advantageous to both customers and the LSE if the customers were sufficiently responsive to price fluctuations. It should be emphasized that such a win-win situation may be only achieved if the CPP scheme is properly designed; this, in turn, requires an analysis of the effects of the CPP parameters on profit and a methodology for selecting appropriate parameter values, which were presented in this thesis.

Further in this thesis, we pointed out that the result of such design scheme may not be an optimal anymore if the payback phenomenon takes place. Actually, the profit of the

LSE is later reduced by the payback phenomenon as we can see from several examples in Chapter 4. This study thus expanded research range to finding optimal strategies for designing a profit-maximizing CPP scheme taking payback into consideration. After characterizing payback through the appropriate parameters, the resulting change in optimal event scheduling was demonstrated, and the optimal peak rate under payback was analytically derived. The validity of this analysis was then verified through numerical simulations.

The results yield certain practical suggestions for designing a CPP scheme in a payback scenario. When payback occurs, it is better to set the peak rate to a lower value than would be optimal without payback. Moreover, if the paid-back demand is expected to be concentrated in the time periods soon after a critical event, the peak rate should be set at an even lower value. As long as the optimal event schedule does not change, payback results in a slight linear decrease in the optimal peak rate. However, if the schedule changes, there is a step change in the optimal peak rate. Consequently, the LSE should jointly optimize the event schedule and the peak rate.

6.2 FUTURE WORKS

Although the results of the proposed method in this thesis are helpful for designing a CPP scheme for the profit-maximizing LSEs, there remain open questions regarding following issues. We leaves it as future works.

Implementation Issues in the Real-World Situation

The first question is implementation possibility and feasibility of the design scheme in real-world situation. In particular, the availability of payback parameters, which are not constant but depend on the levels of demand and price, can be challenging in the implementation of a CPP scheme. Simultaneously, working with forecasted information on demand and RTMCP may require including some probability functions regarding risk generated from forecasting error in the objective function. It cannot be perfectly addressed in deterministic optimization scheme which requires further stochastic approaches

As one more issue related to the real-world implementation, the effects of nonlinear and unpredictable behavior resulting from different compositions of customer loads also need to be examined. Further, penetration of the smart devices in customer-side which induces huge change of the response characteristics of the customers could make the analyses more difficult in designing process of the profit-maximizing CPP scheme. We tried to solve this problem in our recent research [57], however, could not reach a satisfactory conclusion that whether the penetration will be a chance for the LSE or not because we could access few empirical evidences for actual response of the customers who use such kind of devices. Therefore, further empirical research will be necessary to demonstrate the practical implications and real-world effectiveness of the CPP design

strategy presented here, especially finding evidence for customers' response characteristics and payback tendencies.

Impacts on the Market Dynamics

In this study, it is assumed that the LSE was a price-taker and that the change in demand in response to the peak price would not, itself, affect market prices. However, when the change in demand is synchronized, it may be significant enough to influence market prices, which would, in turn, affect the CPP. Such an interaction between the CPP and the market price would be a continuously repeating process, which should be modeled as market dynamics in the form of, e.g., differential equations [58] or gaming model between several entities [6]. Therefore, further research is needed to establish a method for designing a CPP scheme that takes into account the unique dynamics of the market.

Participating in Other Energy Markets

For the profit-maximizing LSE, there are several other electricity markets that it can participate in. For example, many market operators provided day-ahead market or bilateral market service to give a chance to participants for hedging their risk from volatility of the RTMCP. Further, ancillary service markets (e.g., reserve or frequency regulation market) are also opened to LSE in some electricity market such as PJM and NordPool. In [59], we developed optimal participating strategies in imbalance market, a kind of the ancillary service market, for the profit-maximizing LSE perspective. Akin to the study, many analyses and developments can be conducted for the LSEs who intend to participate in various kind of the electricity market based on models provided in this thesis.

APPENDIX

A. NOVEL ALGORITHM FOR EVENTS SCHEDULING PROBLEM

In the Appendix A, we suggest novel algorithm developed based on the definition of the profit index, Eq. (3.1), to solve events scheduling problem more easily compared to existing time consuming methodologies such as dynamic programming or integer programming. The algorithm is helpful especially in case of facing enormous number of the calculation when LSE performs design of the profit-maximizing CPP scheme. Existing methods gives optimal solution in anytime, however, it is well known that such approaches may incurs curse of dimensionality problem if the number of variables is very large [60]. Furthermore, the constraints in Eq. (2.6) and (2.9) are nonlinear, which enhance the complexity of the problem. Thus, following algorithm can be utilized if the size of the problem is very large, in other words, N_{CPP} has very large number.

Recall the definition of the profit index in Eq. (3.1) and objective function which means profit of the LSE in Eq. (3.2). The profit index represents the additional profit generated from the triggering of a critical event in time period k. Suppose that a critical event is triggered during the scheduling horizon. Then, value of the Eq. (3.2) is maximized if the critical event is triggered when the profit index is at its largest. Thus, solving the event scheduling problem reduces to finding the N_{CPP} numbers of time periods consecutively among the sorted profit indexes without violating the constraints. The proposed method is efficient because calculating and sorting the profit indexes is straightforward while it is not for existing methodologies because of complex calculating process entailed in it. The specific procedures of this method are described in following Algorithm 1.

Algorithm 1: Method for solving event scheduling problem

1: Calculate PI_k for all $k \in \{1, 2, \cdots, N - D_{CPP} + 1\}$.
2: Compose a sorted time set $TS = \{k_1, k_2, \cdots, k_{N-D_{CPP}+1}\}$, where $PI_{k_1} \geq PI_{k_2} \geq \cdots \geq PI_{k_{N-D_{CPP}+1}}$.
3: Initialize a feasible set $FS = \varnothing$ and a candidate set $CS = \varnothing$.
4: **while** the number of elements in FS is less than N_{CPP}
5: Pick $k_P \in TS$, which is the first element in TS.
6: Delete k_P from TS.
7: **if** $PI_{k_P} \leq 0$
8: **break**
9: **end**
10: Save k_P into FS and CS.
11: **if** (15) is violated among the elements in FS
12: Delete k_P from FS **.**
13: **end**
14:**end**
15: Set N_{CS} as the number of elements in CS.
16: **if** $N_{CS} \leq N_{CPP}$
17: FS is determined as the optimal solution OS^*.
18:**else**
19: Compose a set $\{OS_m\}$ of sorted optimal candidate sets with N_{CPP} elements determined from the combinations of CS, where $m \in \left\{1, 2, 3, \cdots, \frac{N_{CS}!}{N_{CPP}!(N_{CS} - N_{CPP})!}\right\}$ and $\sum_{k \in OS_1} PI_k \geq \sum_{k \in OS_2} PI_k \geq \sum_{k \in OS_3} PI_k \geq \cdots$.
20: **for** each element set in $\{OS_m\}$
21: **if** (15) is NOT violated among the elements in OS_m
22: OS_m is determined as the optimal solution OS^*.
23: **break**
24: **end**
25: **end**
26:**end**

After step 14 of Algorithm 1, there is at least one solution in *CS*, which is *FS*. However, because *FS* is not guaranteed to be globally optimal, steps 15 to 26 are followed to find the globally optimal solution by composing a set of combinatorial sets with N_{CPP} elements from *CS*. If N_{CPP} is large, the proposed method also suffers from the dimensionality problem mentioned above because step 19 contains a combinatorial procedure. However, N_{CPP} is too small compared with the entire scheduling time horizon *N*. For example, SDG&E, a utility providing gas and electricity service in San Diego area, sets $N_{CPP} = 18$ during one year (*N*=8,760 time periods) in its CPP program [61]. Meanwhile, an optimization method is likely to be effective when problem-specific knowledge is incorporated [62]. For the case of the proposed method, the knowledge corresponds to the fact that N_{CPP} is small. Consequently, Algorithm 1 is able to solve the event scheduling problem more efficiently than when using the dynamic and integer programming methods.

B. NOMENCLATURE

B.1 VARIABLES

S_k	net benefit of customers in time period *k*
$B(q_k)$	benefit of customers from consuming q_k amount of electricity in time period *k*
ρ_{BASE}	off-peak rate of a critical peak pricing scheme
ρ_{PEAK}	peak rate of a critical peak pricing scheme
ρ_k	electricity price in time period *k*

$\rho_{LSE,k}$	retail rate of the load serving entity
$\rho_{CPP,k}$	price rate of a critical peak pricing scheme in time period *k*
$\rho_{TOU,k}$	price rate of a time-of-use pricing scheme in time period *k*
$\rho_{TOU-HIGH}$	high price level of a time-of-use pricing scheme in time period *k*
$\rho_{TOU-LOW}$	low price level of a time-of-use pricing scheme in time period *k*
$\rho_{RTP,k}$	price rate of a real-time pricing scheme in time period *k*
ρ_{U}	price rate of a uniform pricing scheme
$\rho_{RTMCP,k}$	real-time market clearing price in time period *k*
ρ^{*}_{PEAK}	optimal peak rate for a normal situation without payback
$\rho^{*}_{PB,PEAK}$	optimal peak rate considering payback effects
$\Delta\rho_{PB,PEAK}$	difference of $\rho^{*}_{PB,PEAK}$ from ρ^{*}_{PEAK}
q_{k}	consumption of customers in time period *k*
$q_{0,k}$	nominal consumption of customers in time period *k*
$q_{CR,k}$	consumption of customers in time period *k* if a critical event is triggered
$q_{TOU,k}$	consumption of customers in period *k* under a time-of-use pricing scheme
$q_{RTP,k}$	consumption of customers in period *k* under a real-time pricing scheme
$q_{PB,k}$	recovered demand due to payback in time period *k*
$Q_{0,k}$	cumulative consumption during the critical event periods starting from time period *k*
$Q_{CUR,k}$	cumulative curtailed demand for a critical event starting in time period *k*

$Q_{PB,k}$	paid-back demand for the critical event in time period k
R_k	revenue of a load serving entity in time period k
C_k	cost of a load serving entity in time period k
PI_k	profit index in time period k
$PI_{N,k}$	profit index in a normal situation without payback in time period k
$PI_{PB,k}$	profit index considering payback effects in time period k
u_k	binary event decision variable in time period k
N_{CPP}	number of critical events
N_{CPP}^{min}	minimum number of critical events
D_{CPP}	duration of the critical event
H_{CPP}	maximum total event time
D_{PB}	payback duration
$f_{PB}(n)$	payback function
$f_{PB}^{U}(n)$	normalized payback function
$\mathbf{K}^*$	solution of the events scheduling problem for the optimal peak rate
OS^*	solution of the events scheduling problem
k^*	an element of $\mathbf{K}^*$
$t_{TOU-HIGH}$	time periods that high price level is applied in time-of-use pricing scheme
$t_{TOU-LOW}$	time periods that low price level is applied in time-of-use pricing scheme
α_{PB}	payback ratio

β	price elasticity of customers
m_k	design parameter of real-time pricing in time period k
N	scheduling time horizon of the event scheduling problem
Δk	minimum interval between successive events
λ	constant for the exponentially decreasing payback function
c	constant for the uniformly distributed payback function
x	variable to be forecasted in autoregressive moving average model
ext, ϕ, θ	constants in the autoregressive moving average model
ε	zero-mean white noise

B.2 List of Abbreviations

ANN	artificial neural network
AR	autoregressive
ARIMA	autoregressive integrated moving average
ARMA	autoregressive moving average
CPP	critical peak pricing
DR	demand response
EDP	exponentially decreasing payback
GenCo	generation corporation
LSE	load serving entity

PG&E	Pacific Gas and Electric Company
PJM	Pennsylvania-New Jersey-Maryland Interconnection
RTMCP	real-time market clearing price
RTP	real-time pricing
SCE	Southern California Edison
TOU	time-of-use
UDP	uniformly distributed payback

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논문 초록

최근 전력산업에서 화두가 되고 있는 탈규제화는 소매사업자에게 자신의 이윤을 추구할 수 있는 기회를 제공하고 있다. 이에 따라 소매사업자인 부하공급자 (LSE, Load Serving Entity)의 입장에서는 자신의 이익을 최대화하기 위한 동적요금제 (dynamic pricing scheme)의 설계가 필요하게 되었다.

본 논문에서는 동적요금제의 한 종류인 피크요금제 (CPP, Critical Peak Pricing)를 도입하고자 하는 부하공급자가 탈규제화 된 시장환경에서 자신의 이익최대화를 위해 어떻게 피크요금제를 설계해야 하는지에 대한 전략을 제시하였다. 이 설계에는 특히 소비자 가격탄력성으로 대표되는 소비자들의 가격변화에 대한 반응모형이 고려되었다.

이러한 목적 하에 피크요금제의 다양한 설계가능 요소들인 피크요율, 이벤트 횟수, 이벤트의 길이 등의 변화가 부하공급자의 이익에 미치는 영향이 분석되었다. 그 결과 본 논문에서는 다음과 같은 사항들을 기존 연구들과 차별화된 기여사항으로 제시하였다. 첫째, 최적 피크요율은 소비자들의 반응성향과 반비례하는 경향성을 가진다. 둘째, 최적 피크요율은 이벤트 횟수의 변화에 대해 크게 변하지 않는 특성을 가진다. 셋째, 이벤트 횟수가 한 번일 때 계산한 최적

피크요율을 이벤트 횟수가 여러 번일 때도 사용 가능하다. 넷째, 고정요율 요금제 (UP, Uniform Pricing)를 사용할 때보다 더 많은 이익을 내기 위한 최소 이벤트 횟수가 반드시 존재한다. 다섯째, 총 이벤트 시간이 제한되어 있다면 이벤트의 길이는 한 시간으로, 그리고 이벤트의 횟수는 총 이벤트 시간과 같은 숫자로 정하는 것이 이익 측면에서 최적의 결과를 제공한다. 이러한 기여사항들은 수치적 예제들을 통해 다시 한 번 검증되었다.

본 연구에서는 부하공급자의 이익 측면에 중점을 맞추었지만 다른 한 편으로 소비자가 피크요금제를 받아들였을 때 얻을 수 있는 이익에 대해서도 살펴보았다. 이는 탈규제화로 인한 경쟁환경에서 소비자 또한 기존에 사용하던 고정요율 요금제보다 더 낮은 비용을 낼 경우에만 피크요금제를 선택하기 때문이다. 계산 결과는 만약 본 연구에서 제안한 구조 하에서 설계된 피크요금제가 부하공급자의 이익을 증가시킬 뿐만 아니라 소비자의 에너지 사용비용 또한 줄여줄 수 있는, 양쪽 모두가 이익을 얻을 수 있는 구조가 가능함을 보여주었다. 그리고 그 상호 이익의 정도는 소비자의 가격에 대한 반응성이 높을수록 더 높게 나타났다.

지금까지 언급한 바에 더하여 본 연구에서는 추가적으로 소비자의 부하이전현상 (payback phenomenon)이 피크요금제의 설계에 미치는 영향에 대해서도 살펴보았다. 다른 많은 수요반응 (DR, Demand Response) 프로그램과

마찬가지로 피크요금제 또한 부하이전현상으로부터 자유로울 수 없으며 이 현상이 부하공급자의 이익에 부정적인 영향을 미침을 예제를 통해 보여주었다. 또한 적절한 가이드라인 하에서 설계된 피크요금제라 하더라도 만약 부하이전현상으로 고려하지 않은 상태에서 설계되었다면 부하이전현상이 일어날 경우 적용하였을 때 더 이상 최적값을 보장할 수 없음도 증명하였다. 이러한 상황에도 불구하고 부하이전현상을 고려한 피크요금제 설계에 대한 연구가 충분히 이루어지지 않았기 때문에 본 연구에서는 다음과 같은 작업들을 통해 부하이전현상을 고려한 최적 피크요금제 설계전략을 수립하고자 하였다. 첫째로 부하이전현상을 해석적으로 표현하기 위하여 페이백 기간, 양, 그리고 패턴과 같은 몇 가지 파라미터들을 이용하여 특징화 (characterization)하였다. 그리고 이를 도구로 사용하여 피크요금제 설계의 가장 중요한 두 가지 문제인 최적 이벤트 결정문제와 최적 피크요율 결정문제에 부하이전현상이 미치는 현상을 검증하였다.

부하이전현상에는 다양한 형태가 존재하지만 많은 기존의 실증적 연구들을 통해 제시된 지수적 감소 (exponentially decreasing)와 균일 분포 (uniformly distributed) 두 가지 형태의 부하이전패턴이 고려되었다. 이 결과를 통해 부하이전현상을 고려한 설계가 그렇지 않을 때보다 최대 2.83%까지 부하공급자의 이익을 증가시킬 수 있음을 수치적으로 보여주었다.

주요어:

피크요금제, 수요반응, 탈규제화, 수요탄력성, 소비자 가격반응, 수요이전현상

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