Patent Ladder in an Endogenous Growth Model

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A dynamic general equilibrium model is developed to solve some empirical puzzles in the standard endogenous growth literature. The empirical data show that the growth rate of per capita output, and the patent rate or the rate of new invention, are constant, but research and development (R&D) expenditures, and the number of scientists and researchers are exponentially growing. This contradicts the main implications of endogenous growth models, which mean that scale effects exist. A model is developed to explain this empirical puzzle. The growth rate is made to depend only upon the productivity parameter in the invention production function, and the share parameter between the labor and the aggregate capital in the final good production. The scale effects are removed. The policy of subsidizing invention production can make the economy grow faster. However, the effect on welfare depends on whether negative externality, rent seeking, or market structure effects are stronger. Subsidies to the purchase of intermediate goods or to investment expenditures do not have any growth effects. (*JEL* Classification: O40)

I. Introduction

The main idea of endogenous growth models, both the quantity variety model and the quality ladder model, is that the growth rate can be expressed by economic parameters given in the model, rather than exogenously given technological progress rate or population growth rate. Almost all the literature on endogenous

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function of fundamental parameters, like the time preference rate and the risk aversion parameter. However, it is also a function of the total labor force. This fact causes problems in the interpretation of the result: The growth rate becomes higher as the size of fixed factors, for example, the labor force or the human capital stock becomes larger. The cases of India give the counterexample. Moreover, the empirical literature on patents, or invention production, shows the followings: First, research and development (R&D) expenditure or the number of the scientists and engineers has grown; second, Total Factor Productivity (TFP) in the manufacturing sector has grown more slowly, yet it does display a persistent upward trend; and third, the annual number of successful domestic patent applications has displayed little upward trends relative to its fluctuations.[1]

The quantity expansion model of Romer (1990) and Grossman and Helpman (1991a), and the quality ladder model of Aghion and Howitt (1992) and Grossman and Helpman (1991b) do not avoid this *scale effect problem*.[2] Even worse, these models could not explain the above three facts.

In the first version of the quantity variety model, the lab equipment model, if R&D expenditure (the investment share multiplied by the final output) is increasing by the increase in the output level due to the increase in the total labor, then the growth rate should also increase even though the share of investment remains the same. In the second version, the knowledge driven model, if the number of researchers and scientists (the share of the human capital devoted to R&D, multiplied by the human capital) increases by the increase of the total human capital stock, then the growth rate is also increasing. This is usually expressed as *the scale effect problem* because it has the implication that the country or the region which has a larger population level (or residents) grows faster. There are counterexamples to this argument, most notably, India.[3] Moreover, the knowledge driven model has the

[1]These were stated in Kortum (1994).

[2]A more concrete definition is given in Jovanovic (1996). A scale effect is defined as a change in some per capita variables—productivity level or productivity growth—that comes about if we increase the economy's scale, the number of agents in the economy, and their endowments, while assuming that the distribution of the agent's actions and endowments are unchanged.

implication that the number of ideas and patents increases as time passes, by the externality term, even though the human capital devoted to R&D remains the same.[4] These are counterfactual in the sense that the empirical data shows that the number of scientists and researchers (also the R&D investment, in terms of the output) is growing but the rate of new inventions or new patents is constant. The lab equipment model is also counterfactual because the introduction of new inventions is increasing as time passes, by the total number of the invention good in the reduced form of the final good production function.[5]

The quality ladder model has the same prediction that the increase in the total human capital stock in the economy will make the growth rate higher. The rate of the introduction of new goods or patents is, however, constant as time passes in this quality ladder model because there is no externality term involved in the patent production function.[6]

[3]Following the comments from the referee, I delete China from this category because the growth rate of China has been very high for the last ten years. I am still curious whether this is just an exception.

[4]The growth factor comes from different sources. The lab equipment model does not assume spillover effects, yet the economy grows by the automatic increase in investment even though the share stays the same. This occurs because final output is growing by the increase in the number of the variety. The knowledge driven model assumes the spillover effects directly. Therefore, the economy grows by the externality term (spillover effects).

[5]The data below were cited in Kortum (1994).

	Level in 1957	Level in 1989	Growth Rate 1957-89
TFP, Manufacturing			2.0
TFP, Private Business			1.3
Output per Hour, Private Business			2.0
Industry R&D Scientists & Engineers (000's)	229.4	720.2	3.6
Civilian Employment (000's, over age 16)	64,071	117,342	1.9
Total Industry R&D Expenditure ($ Billions)	7.7	101.9	
Compensation on Employees ($ Billion)	256.5	3,100	
Successful US Priority Patent Applications (000's)	39.2	58.5	1.3

[6]The quality ladder model also actually depends upon the strong externality. The new invention uses previously accumulated knowledge stock

To remedy these problems, Jones (1994, 1995b) and Kortum (1994) reintroduce the exogenous population growth rate in the two sector growth model. By removing the scale effect and the counterfactual implications of the endogenous growth model, the population growth rate plays the most important role. However, the growth rate of the economy depends totally upon the exogenous population growth rate, even though the share parameter of the labor between the R&D sector and the output sector is a various function of the economic parameters in the two sector growth model.[7] The main point is that even though they use the endogenous growth framework, the most important variable, the growth rate, depends upon the exogenously given population growth rate.[8] Therefore, government policy cannot play any role in the determination of the long run growth rate.[9]

To get out of the exogenous growth rate model of Jones (1994, 1995b) and Kortum (1994) (they call it a semi-endogenous growth model), and also to solve the counterfactual implications of Romer's (1990), Grossman and Helpman's (1991a, b), and Aghion and Howitt's (1992) models, I will develop a general one sector growth model. This model will have the features that the growth rate depends only upon the endogenous parameters and that the constant patent rate, constantly growing R&D expenditure, and

for free and improves upon the previous new invention. The new invention stands upon the shoulders of the Giant's.

[7]Jones (1995b) used the two sector quantity variety model and derived the result that the growth rate of all the variables is $\lambda n/(1-\phi)$, where λ is the optimal degree of duplication and overlap in R&D, n is the population growth rate, and ϕ is the external contribution of the previous knowledge to the R&D. Kortum (1994) used the two sector quality ladder model and derived the productivity growth rate as λn and the rate of the patenting as Jn, where λ is the distribution parameter in the search, n is the population growth rate, and J is the number of the firms in the economy. In both models, the most important parameter, the growth rate, depends upon the exogenously given population growth rate.

[8]In a conversation with Romer, he pointed out that there is no clear cut distinction between the exogenous growth model and the endogenous growth model. He states that the exponent of the externality parameter in the invention production function, in other words the degree of the externality, is assumed to be one for convenience because in the case where it is close to one but still less than one it takes quite a long time to return to the original steady state.

[9]Whether there is any role for government policy is the key to distinguish between the exogenous growth model and the endogenous growth model.

constant per capita output growth rate can be explained.

The continuous time overlapping generation model is assumed, where parents invest in their child's education in order to increase his/her human capital. Both quantity variety and quality ladder characteristics are assumed. The final good production function has the characteristics that it depends upon the number of the intermediate good and that newer goods are of a better quality, and are therefore more productive in the final good production. The productivity of the old good is slowing compared with the new intermediate good but is still demanded by the final good production firms. This is because all the intermediate goods are imperfectly substitutable goods in the production of the final good. The functional form of the patent or the invention production has the characteristics that the patent rate is constant at the steady state equilibrium. It is assumed to be an increasing function of the current investment but also to be a decreasing function of the previous R&D efforts based on the following rationale: The more ideas are developed in the past, the more difficult it is to find new ideas. This has the stronger implication that there is a negative externality to the production of a new invention. The exponents are assumed to exactly cancel out the two effects in this model in order to guarantee the balanced growth path.[10]

The model's implication at the steady state equilibrium is very simple. The new introduction of goods (the patent rate) is constant. The economy grows by the improvement in the quality of the new intermediate goods. The growth rate depends only upon the share parameters between the labor and the intermediate goods in the final good production function and the productivity parameter in the invention production function. It does not depend upon the total labor force, the time preference rate, the risk aversion parameter, nor the productivity parameter in the production of the final good, as in the standard endogenous growth model. The policy to subsidize the invention production to increase the productivity can increase the growth rate, but the effects on the welfare depend upon the combined effects of the market structure, the negative externality, and the rent seeking behavior. The subsidy on the investment in invention production cannot have any effects because

[10]The justification for this assumption is fully explained in Kortum (1994) and Jones (1995b).

it is washed out through the negative externality term in the invention production. Neither the subsidy to the purchase of the intermediate good or the subsidy to the production of the final good help make the economy grow.[11]

The paper proceeds as follows. Section II describes the basic setup of the model, comparing it to current literature. The simple overlapping generations model with the patent production function is assumed to have strong negative effects from the previous invention. In other words, the more difficult it is to create the new invention, the more advanced the current state of the art of the technology. Section III derives the balanced growth equilibrium path and the growth rates of all the variables. Section IV analyzes the empirical puzzles within this framework and government policy. Section V concludes and suggests further extensions for future research.

II. The Setup of the Model[12]

A. The Household's Behavior

The agents are in the interval [0, 1] and each is endowed with L units of labor. The agent lives at time t, and also takes care of the utility of his or her offsprings. The discount rate of the individual for the offspring is ρ. The agent has labor income and capital income from the purchase of the stock of the intermediate good firm. The agent spends their labor income and capital income to

[11]Barro and Sala-i-Martin (1994) gave a policy description of the subsidy or tax to achieve the socially optimal path. The right subsidy to the purchase of the intermediate good can make the economy get on to the optimal path in the lab equipment model. The subsidy to the production of the final good also has the same effect. This one policy can cure both static and dynamic distortions caused by monopolistic competition. The subsidy to the R&D alone, however, cannot make the economy grow at the optimal rate because it cures only the static distortion of monopolistic competition. In the two sector knowledge driven model, two policy measures are necessary to make the economy achieve the optimality: One for the distortion caused by the monopolistic competition market structure and the other for the distortion caused by the externality in the invention production function.

[12]The details are shown in the Appendix in the original paper. This will be distributed upon request.

consume or to invest in their offspring's education. The decision-making of the consumer is described as follows:

$$\underset{\{c_t,\ i_t\}}{\text{Max}}\ W(\bullet) = U(c_t) + \exp^{-\rho} \times W(\bullet), \tag{1}$$

$$s.t.\quad 0 = w_t L + \pi_t - P_t c_t - P_t i_t, \tag{2}$$

where ρ is the time preference rate of the agents, c is consumption, i is investment, w is the wage rate, L is labor, π is the profit income from the firm, and P is the price level.[13]

The agent is assumed to have only one child and investment is made for this child's education. However, even though the agent educates their child and makes him acquire more human capital, the child cannot always invent new inventions due to the crowdedness in the invention sector between young generations. Thus, the agent can become a loser in the game of making new inventions.[14] When the agent invests the final output into their child's education, this can affect the quality of human capital and the number of the inventions that their offspring can make. Moreover, the agent knows and takes into account this fact when investing. In this sense, the previous generation in the economy behaves as a social planner who takes care of the child's utility.

The arbitrage condition is that one unit cost of the final good should be equal to the market value of the new invention that the human capital, produced by one unit of the final good, can invent. In this model, as in Ciccone (1993), the increase in human capital means an increase in the quality of the human capital rather than an increase in the quantity of the human capital, in other words, the increase in the number of people.[15] The old bequests the

[13]This form of the welfare function is adopted from Ciccone (1993), who used a continuous time overlapping generations model and the functional form of welfare $W_t(h) = v \times U(c_t) + \exp^{-\rho v} \times W_{t+v}(h)$, where h is household and v is the life time the agent lives, and is assumed to be infinitesimal to make differentiation possible.

[14]Jones (1995b) takes into account the crowdedness and the externality of the people in the production function of the invention.

[15]Usually in the models of Romer (1990) and Grossman and Helpman (1991a, b), the agent cannot affect either the wage rate nor the interest rate. Therefore he maximizes his utility, taking as given the price variables. The most contrasting feature of the overlapping generations model of Ciccone (1993) is that the decision-maker who invests in the child's education takes into account the fact that he can affect the wage rate that the young receives. This makes identical the planning problem and the

property right for the previous inventions to his child. He also educates his offspring, and the quality of his child's ability increases as the parents are spending resources on his education. Therefore the child has three endowments. One is the raw labor, another is the inherited property right for the previous invention from his parents, and the third is the increased quality in the human capital. He behaves as a laborer by supplying the raw labor and as a entrepreneur by making the inherited, previously invented, good. Moreover, there is a chance that he can invent a new and more advanced good. Then he can also become the supplier of the new and more advanced good. These new features, new inventions, and newly developed intermediate goods especially will be the engines of growth.[16] The accumulation of human capital and the new invention production function are as follows:

$$\dot{HK}_t = \left(\frac{i_t}{L}\right)^{\alpha} \times L^{\beta}, \tag{3}$$

$$\dot{N}_t = A \times \dot{HK}_t^{\gamma} \times \left(\int_{-\infty}^{t} \dot{HK}_s \times \dot{N}_s ds\right)^{\delta} = A \times i_t \times \left(\int_{-\infty}^{t} i_s \dot{N}_s ds\right)^{-1}, \tag{4}$$

where i is investment done by the old, L is the labor hour, HK is the human capital stock, A is the productivity parameter in the invention production, and N is the number of the invention. α is a exponent of the per capita investment in the production of human capital, β represents the exponent of overcrowdedness in human capital production, γ is an exponent of human capital in the invention production, and δ represents the exponent of accumulated human capital.

Equation (3) is a human capital accumulation function. The increase in the quality of human capital is restricted by the aggregate labor hour, which creates the negative externality.[17] This shows that there is crowdedness between the young, and some of the young may lose the game in the competition with his colleagues. β catches this negative externality to the improvement

competitive problem.

[16]There are heterogeneity problems in this setup. The children who are equipped with human capital cannot always develop new inventions. Some of them can develop the new good, but others cannot.

[17]The total labor hour in this economy is $\int_0^1 L dn = L$, and the investment per labor hour is $i_t / \int_0^1 L dn = i_t / L$

of human capital. Invention is increasing with respect to the investment per labor hour and decreasing with respect to the aggregate labor hour. We assume, however, that the exponent to the labor hour becomes 0, and this means that the effects of labor are canceled out.

Equation (4) is an invention production function, and the reduced form on the right hand side comes from the two assumptions below. The invention production function has the same functional form as human capital production. The new invention or the number of new products is also bounded by the accumulated numbers of the invention multiplied by the quality of the human capital stock that makes that invention. As the quality of the previous human capital stock increases, the higher is the quality of the previous intermediate good and the more difficult it is to find newer and better intermediate goods. δ reflects the negative externality to the finding of the new intermediate good. The functional form assumes that it is an increasing function of the newly improved human capital stock, but is a decreasing function of the previously inherited invention goods multiplied by the human capital stock. I will assume that $\alpha + \beta = 0$, $\alpha = 1$, and $\gamma = -\delta = 1$. These are quite restrictive assumptions, but are simplified like this to derive the closed form solution at the steady state equilibrium.

The investment purpose is the same as that of Ciccone (1993) and Easterly *et al.* (1994). Ciccone (1993) combines the concept of human capital and the new knowledge of how to produce a new intermediate good. In Ciccone (1993), the investment to human capital is done by the parents who take care of their children. It increases the human capital stock, or in other words, the quality of the given number of the population rather than the quantity of the human capital stock. This increases the boundary of the products to be used by that human capital. In the end it affects the wage rate that the raw labor can earn in the next period. The parent's decision-making affects the price parameters, especially the wage rate that the child receives. Therefore, when he is maximizing his behavior, he takes into account that his behavior can affect the future outcomes. In this sense, the parent behaves as a social planner. In Ciccone (1993), the human capital stock and the new technology affect the wage rate linearly via the interaction of the final good production sector and the capital good production sector.

The wage rate is the return to raw labor.[18] Easterly *et al.* (1994) also consider the same kind of problem and deal with the problem of interpretation. The individual spends the output for the import of the new intermediate goods from abroad. The intermediate good can be purchased from abroad by paying a price. It costs more, however, to adopt the intermediate good. Training human capital for the use of the new good involves a cost. This, however, can increase the ability to use more of the newly imported advanced intermediate goods. It increases the wage rate that the individual can receive.[19] Unlike the Ciccone (1993) model, the agent should take the wage rate as given even though he affects the wage rate in the equilibrium. This is because there is no mechanism to guarantee that the agent takes into account the fact that he can affect the wage or other price parameters. In other words, the solution in this model looks much more similar to the social planner's problem than the competitive one. This is because he solves the problem taking into account the change of the wage rate.[20] Both models adopt the setup that the increase of the human capital stock is done through the investment of the output, even though Ciccone (1993) is a two sector model and Easterly *et al.* (1994) is a one sector model, and the rate of return is the induced increase in the wage rate differentiated by the number of the intermediate goods or technology.

[18]There is confusion over whether it is a competitive solution or the social planner's problem. Because the individual choice affects the price parameters, there is no clear distinction over whether it is a social planner's problem or an individual maximization problem.

[19]Also it has the same problem of Ciccone (1993). Because it affects the wage rate, it cannot become the choice variable of the individual problem. Of course, the social planner considers the increase in the wage rate implicitly by using the reduced form production. Then the technology parameter, the number of inventions, can be got out of the integral. However, when the consumer is also the producer and he behaves as a monopolistic competitor, he can consider the effects on the wage rate or the interest rate that are not considered in the competitive market. This means he cannot control price variables as a consumer, but can affect the variables as a producer.

[20]Rebelo states later that human capital involves the skill needed to use the newer technology. Therefore, the social planning problem is the same as the competitive equilibrium solution.

B. The Firm's Behavior

There are two types of firms: One is the final good making firm, which acts in the competitive market, and the other is the intermediate good making firm, which also acts in the competitive market. The intermediate good making firm takes into consideration the demand of the final good producers when they make decisions about prices.

a) The maximization of the final good making firm

The final good making firm's profit maximization is set-up as follows:

$$\underset{\{x_t(q),L\}}{\text{Max}}\ P_tY_t-\int_{-\infty}^{N_t}\{R_t(q)\times x_t(q)\}dq-w_tL, \tag{5}$$

$$s.t.\quad Y_t=L^{\xi}\times\left[\int_{-\infty}^{N_t}\{\exp^{q}\times x_t(q)\}^{\varepsilon}dq\right]^{\frac{1-\xi}{\varepsilon}}, \tag{6}$$

where Y is the final good, ε is the elasticity of substitution between intermediate goods, ξ is the factor share of labor out of the final good, $x_t(q)$ is an intermediate good which has quality q, and L is the labor hour.[21]

The final good producers produce the good by hiring the raw labor and by getting an intermediate good which has quality q from the laborers and producers, and by paying the competitive price w_t and $R_t(q)$. The market for the final good is competitive. Therefore, the factors are compensated at their marginal products. The first order conditions show that the wage rate and the price of the intermediate good become the marginal product of labor and the marginal product of the intermediate good, respectively, as follows:

$$w_t=\xi\times L^{\xi-1}\times\left[\int_{-\infty}^{N_t}\{\exp^{q}\times x_t(q)\}^{\varepsilon}dq\right]^{\frac{1-\xi}{\varepsilon}}\times P_t, \tag{7}$$

[21]Caballero and Jaffe (1993) used the same functional form for the composite consumption good. In their model, the labor supply decision was not included. I did not include the productivity parameter in the production function because the major concern was to analyze the growth effects, rather than to look at the level of the variables. The results show that the growth rate does not depend upon the other parameters, except for the productivity coefficient in the invention production and the share parameter in the production. However, the levels will be affected by the productivity parameter in the production function.

$$R_t(q) = P_t(1-\xi) \times Y_t^{\frac{1-\xi-\varepsilon}{1-\xi}} \times L^{\frac{\varepsilon}{1-\xi}} \times \exp^{q\varepsilon} \times x_t(q)^{\varepsilon-1}. \tag{8}$$

As was explained in the introduction, the final good production function has the characteristics that it depends upon the number of the intermediate goods and that the newer intermediate goods are more productive in the final good production. The old and less productive intermediate goods are still demanded by the final good making firms because they are imperfectly substitutable goods in the production of the final goods.

b) The maximization of the intermediate good making firm

The intermediate good producer takes into account the demand condition when making the production decision, and behaves as monopolistic competitor. He prices following the markup rule. The entrepreneur who develops or renews intermediate goods will make profits behaving as a monopolistic competitor. To produce one unit of the intermediate good, η units of the final good and the knowledge of how to make it are needed. Therefore, the value of one invention is a discounted stream of profits that it can earn afterwards. The profit stream goes down as time passes because the newly introduced intermediate good has a better quality and replaces the older and lower quality good. However, the older and lower quality good is still demanded because the intermediate good is an imperfectly substitutable good. The profit of the firm which has quality q, and the value of the firm which has the highest quality is:

$$\pi_t(q) = R_t(q) \times x_t(q) - \frac{P_t \times x_t(q)}{\eta}, \tag{9}$$

$$V_t(N_t) = \int_t^\infty \exp^{-r(\tau - t)} \times \pi_\tau(N_t) d\tau . \tag{10}$$

c) Free entry in the research and development sector

The free entry condition into the R&D sector in this economy says that the cost of one unit of the final good should be the same as the return from it. The return is the value of one invention multiplied by the number of the invention that one unit of the final good can produce,

$$V_t \times \frac{\partial \dot{N}_{t+1}}{\partial i_t} = P_t . \tag{11}$$

The equilibrium condition is that the final output is allocated to consumption, investment, and to the resources that make the intermediate good. The labor market clears automatically because all the human capital in this economy is allocated to the research and development sector and all the raw labor hours are allocated to the final good production function. The market clearing condition is:

$$P_t Y_t = P_t c_t + P_t i_t + P_t \int_{-\infty}^{N_t} x_t(q) dq. \tag{12}$$

Having set up the model, the next step is to find the equilibrium of this economy and characterize that equilibrium and the steady state of that economy.

III. Characterization of the Balanced Growth Equilibrium Path[22]

As is described in detail in the Appendix, the free entry condition into the R&D sector derives that the value of the firm becomes:

$$V_t = P_t \times A^{-1} \times \int_{-\infty}^{t} i_s \dot{N}_s ds\,. \tag{13}$$

The consumer maximization problem results when the growth rate of the consumption is

$$g_c = \frac{1}{\sigma}\left(r - \frac{\xi - 1}{\xi} \times \dot{N}_t - \rho \right). \tag{14}$$

The maximization condition of the final good producer and the intermediate good producer says that output can be expressed as:

$$Y_t = B \times L^{\frac{\xi(1-\varepsilon)+\varepsilon}{\xi}} \times \exp^{\frac{1-\xi}{\xi} N_t}, \tag{15}$$

where B is a function of the various economic parameters.

The price level can be expressed as:

$$P_t = C \times L^{\frac{\varepsilon\xi - \varepsilon}{\xi}} \times \exp^{\frac{\xi - 1}{\xi} N_t}, \tag{16}$$

where C is a function of the economic parameters.

The profit level of the intermediate good making firm becomes:

[22]The details are proved in the Appendix.

$$\pi_t(q) = E \times L^{\frac{\varepsilon\xi - \varepsilon}{\xi}} \times L^{(\varepsilon-1)\{-(\varepsilon-1)\xi - \xi^2 + \varepsilon\}} \times \frac{1-\varepsilon}{\varepsilon\eta} \times \exp^{\frac{\varepsilon}{\varepsilon-1}(N_t - q)}, \tag{17}$$

where E is also a combination of the technology and preference parameters.

Moreover a market value of the newer intermediate good can be simplified as follows:

$$\begin{aligned} V_t(N_t) &= \int_t^\infty \exp^{-r(\tau-t)} \times \pi_\tau(N_t) \mathrm{d}\tau \\ &= \int_t^\infty \exp^{-r(\tau-t)} \times E \times L^{\frac{\varepsilon\xi-\varepsilon}{\xi}} \times L^{(\varepsilon-1)\{-(\varepsilon-1)\xi-\xi^2+\varepsilon\}} \\ &\quad \times \left(\frac{1-\varepsilon}{\varepsilon\eta}\right) \times \exp^{\frac{\varepsilon}{\varepsilon-1}(N_\tau - N_t)} \, d\tau . \end{aligned} \tag{18}$$

The above relationship makes it possible to derive the equilibrium growth rate. At the balanced growth equilibrium path, the price level grows at the rate of $\{(\xi-1)/\xi \times \dot{N}_t\}$, because the total number of labor hours is constant, the final good grows at the rate of $\{(1-\xi)/\xi \times \dot{N}_t\}$, and the growth rate of the patent is $\dot{N}_t = A \times i_t \times \left(\int_{-\infty}^t i_s \dot{N}_s ds\right)^{-1}$.

Moreover, it was derived that the consumption grows at the rate of $g_c = (1/\sigma)[r - \{(\xi-1)/\xi\}\dot{N}_t - \rho]$, based on the first order conditions of the consumer maximization problem. By differentiating the value of the firm with respect to time, we know that the value of the firm evolves, following the law of motions as follows:

$$\dot{V}_t = \left(r + \frac{\varepsilon}{1-\varepsilon} \times \dot{N}_t\right) \times V_t - E \times L^{\frac{\varepsilon\xi-1}{\varepsilon}} \times L^{(\varepsilon-1)\{-(\varepsilon-1)\xi-\xi^2+\varepsilon\}} \times \frac{1-\varepsilon}{\varepsilon\eta}, \tag{19}$$

and the value of the newer intermediate good is also known to be:

$$P_t \times \mathrm{A}^{-1} \times \int_{-\infty}^t i_s \dot{N}_s d_s, \tag{13}$$

from the free entry condition into the research and development market.

At the balanced growth path equilibrium, the growth rate of the consumption and the final good are the same by the aggregate resource constraint described by equation (12). By equating the two growth rates, we get the first relationship between the interest rate and the invention rate:

$$\dot{N}_t = \frac{r-\rho}{\sigma-1} \cdot \frac{\xi}{1-\xi}. \tag{20}$$

Also from the invention equation, using the fact that at the balanced growth path equilibrium, the investment becomes a fixed share of the final output, the second relationship is derived as follows:

$$\begin{aligned}
\dot{N}_t &= A \times i_t \times \left(\int_{-\infty}^{t} i_s \dot{N}_s ds \right)^{-1} \\
&= A \times (\phi \times Y_t) \times \left(\int_{-\infty}^{t} \phi\, Y_s \dot{N}_s ds \right)^{-1} \\
&= A \times \exp^{\frac{1-\xi}{\xi} N_t} \times \left(\int_{-\infty}^{t} \exp^{\frac{1-\xi}{\xi} N_s} \times \dot{N}_s ds \right)^{-1} \\
&= A \times \exp^{\frac{1-\xi}{\xi} N_t} \times \left(\frac{\xi}{1-\xi} \times \exp^{\frac{1-\xi}{\xi} N_t} \right)^{-1} \\
&= A \times \frac{1-\xi}{\xi},
\end{aligned} \tag{21}$$

where ϕ is a fixed share of the investment of the final good production.

This result is quite surprising in the sense that the patent rate is constant and that it does not depend upon the preference side at all. Moreover, it depends only upon the productivity parameter in the invention production and the share parameter between the labor and the aggregate capital stock.

Equating the above two relationships, equations (20) and (21), we get the following simple result for the interest rate. It is determined by both the preference side parameters and the production side parameters. The result is:

$$r = A \times \left(\frac{1-\xi}{\xi} \right)^2 \times (\sigma - 1) + \rho\,. \tag{22}$$

Inserting this steady state relationship into the value of the firm equation, and using the fact that at the balanced growth path equilibrium, the value of the newer intermediate good is not changing, $\dot{V}_t = 0$ in equation (19), then the value of the newer intermediate good becomes:

$$V_t = C \times L^{\frac{\varepsilon \xi - \varepsilon}{\xi}} \times \exp^{\frac{\xi - 1}{\xi} N_t} \times A^{-1} \times \int_{-\infty}^{t} i_s \dot{N}_s ds\,, \tag{23}$$

and we get the closed form of the value of the newer intermediate good:

$$V_t=\left\{A\times\left(\frac{1-\xi}{\xi}\right)^2\times(\sigma-1)+\rho+\frac{\varepsilon}{1-\varepsilon}\times A\times\frac{1-\xi}{\xi}\right\}^{-1} \times E\times L^{\frac{\varepsilon\xi-\varepsilon}{\xi}}\times L^{(\varepsilon-1)\{-(\varepsilon-1)\xi-\xi^2+\varepsilon\}}\times\frac{1-\varepsilon}{\varepsilon\eta}. \tag{24}$$

From the above relationships, we also get the accumulated knowledge stock, weighted by the human capital stock at that time. This gives the negative externalities to the production of the new invention good. These are shown as follows:

$$\int_{-\infty}^{t} i_s\dot{N}_s ds=\left\{A\times\left(\frac{1-\xi}{\xi}\right)^2\times(\sigma-1)+\rho+\frac{\varepsilon}{1-\varepsilon}\times A\times\frac{1-\xi}{\xi}\right\}^{-1} \times\frac{E}{C}\times L^{(\varepsilon-1)\{-(\varepsilon-1)\xi-\xi^2+\varepsilon\}}\times\frac{1-\varepsilon}{\varepsilon\eta}\times\exp^{\frac{1-\xi}{\xi}N_t}\times A. \tag{25}$$

This grows at the rate of $\{(1-\xi)/\xi\times\dot{N}_t\}$, as can be seen from the exponential term, which is the same growth rate as the final good.

To summarize, the price level is decreasing at the rate of $A\times\{(1-\xi)/\xi\}^2$, while the output and consumption is growing at the rate of $A\times\{(1-\xi)/\xi\}^2$. The patent rate is constant at the rate of $A\times\{(1-\xi)/\xi\}$. The value of the firm is constant and the weighted previous knowledge stock grows at the rate of $A\times\{(1-\xi)/\xi\}^2$.

IV. Explanation of the Empirical Puzzles and Policy Implications

In the exogenous growth literature (a typical example is the Solow (1956) model), the growth rate is determined outside of the economic model, for example, the population growth rate and the technological progress, which the agents in the economy cannot control. This growth rate determines the interest rate in the economy and then the growth rate of the other variables in the economy. Government policy cannot affect the growth rate of the economy. It can only affect the level of the capital stock or the output level. The Solow (1956) model gets out of the scale effects problems because the growth rate depends only on the exogenous growth rate.

In the endogenous growth framework, the one sector lab equipment model (Rivera-Batiz and Romer (1991) is a prototype model) implies that the interest rate is determined from the production side, which dictates the growth rate in the preference side. The two sector knowledge driven model (Romer (1990) and Grossman and Helpman (1991a) are examples) implies that there is one relationship between the interest rate and the growth rate from the production side and another relationship between them from the preference side. The interest rate and the growth rate are determined together by both sides. The one sector quality ladder model (Barro and Sala-i-Martin 1994) also has two relationships between the two variables. The production side has one relationship between the growth rate and the interest rate, by combining two equations which relate the probability of success, the interest rate, and the growth rate. There is another relationship between the growth rate and the interest rate in the preference side. Two unknowns are determined by two equations. The two sector quality ladder model (Grossman and Helpman 1991b) also has two relationships from both sides, and the two unknowns are determined simultaneously. The government can exercise a policy in order to boost the economy and increase the growth rate, such as subsidizing research and development activities and/or subsidizing the production of intermediate goods to remove distortions created by the market structure. The market equilibrium is not Pareto Optimal because the market for the intermediate goods is not competitive and there is one more distortion, the externality in the invention production in the two sector model. Therefore, there is room for government policy to remove distortions in order to achieve the optimal equilibrium. Moreover, all of these endogenous growth models cannot get rid of the scale effect problem because the growth rate depends on the economic parameters and the fixed factors.

In this model, the growth rate is determined from the production side only, and the interest rate in the preference side is dictated by the growth rate in the production side. The growth rate is only a function of the productivity parameter in the invention production and the relative share between capital (aggregates of the intermediate goods) and labor in the final good production.[23] The

[23]Kim (1997) analyzes the small open economy, and the derived growth rate depends only upon production side parameters, the step of the quality, and the price of the imported intermediate goods.

main reason for this, compared with the Barro and Sala-i-Martin (1994) model, is that the externality term in the invention production function washes out all the parameter terms in the investment through the functional form of the final good, except for the technology parameter in invention creation and the relative share between labor and capital. This is because the externality term is composed of all previously accumulated knowledge which is a sum of the investment multiplied by the number of inventions, over time, rather than just the negative of the growing factor.[24]

There are several interesting features in this model compared with other model in the empirical literature on the R&D share, the investment expenditure share, the patent rate, per capita output growth, and other models in the endogenous growth literature.

The first is that the total labor force does not affect the interest rate or the growth rate in our model. This means there is no scale effect, compared with other endogenous growth models except Easterly *et al.* (1994).[25] Jones (1995b) and Kortum (1994) removed the scale effect and other counterfactual implications by reintroducing an exogenous population growth rate. These are, however, exogenous growth models (they named it a “semi-endogenous growth model” because the main engine of the growth comes from population growth or other exogenous technological changes). Our model eliminates both the scale effect and the counterfactual implications of the endogenous growth models. It does this in the framework of the endogenous growth model, and without introducing the population growth rate or exogenous technological progress. The growth rate depends only upon the share parameters of the production function between labor and aggregate capital (the sum of the intermediate goods) and the productivity parameters in the production of new invention good. This derivation comes from the fact that we assume the negative influence of the previous accumulated knowledge on the current innovation and the specific coefficients in that function, which can guarantee the balanced growth path. Moreover, it is not a function of the time preference parameter or the risk aversion parameter. It does not depend upon the exogenous population growth rate or exogenous technological progress.

[24]In Barro and Sala-i-Martin (1994), the externality term consists of the terms which only cancel out the growth factor, $\phi(\kappa_j) = (1/\zeta) \times q^{-(\kappa_j+1)\alpha/(1-\alpha)}$.

[25]The removal of the scale effects has been surveyed in Jovanovic (1996).

The second is that the new invention rate per time period or the patent rate per time period is constant. As has been explained in the introduction, the patent rate (fluctuating around the constant mean), the constantly growing investment expenditure, and the constant growth rate are empirical facts. The results of this model are consistent with the real world phenomena. The constant patent rate fits the real data since the patent rate just fluctuates around the constant mean. The growth is generated by the quality improvement of the intermediate goods, even though the introduction of the new patent rate is constant. The growth rate of the output is constant, which also fits the empirical data. The investment expenditure into the new innovation is also constantly increasing, in that it is a fixed proportion of the final output. Therefore, it also fits the real data which says that the investment expenditure is constantly growing.

The third feature is that the consumption and the output growth rate are the same and depend only upon the share parameter between labor and aggregate capital and the productivity parameter in the invention. Moreover, the price level is decreasing at the growth rate of the final good and the wage rate is fixed at one. The real interest rate and the value of the firm is also held constant at the balanced growth equilibrium path. The interest rate does not play any role in the determination of the growth rate in this economy. The growth rate and the other preference parameters determine the interest rate.

The aggregate level of labor, however, affects all the variables except the growth rate: The aggregate output level, the aggregate price level, the profit level, and the value of the firm. The results look very similar to those of the exogenous growth models, where the growth rate is determined from the outside and the fixed factors, such as the aggregate labor or the land size, affect only the level of those economic variables.[26] The growth rate is, however, determined only by the endogenous parameters in this model.

The fourth feature is that the total human capital stock, or the accumulated knowledge stock, is growing at the same rate as the final good or the consumption. This is shown in equation (25). The research and development expenditure on the invention is growing

[26]The effects of labor on the output level, price level, profit, and the value of the firm is positive, negative, negative, and negative, respectively.

at the rate of the final good. The human capital stock, or the accumulated knowledge stock, is also growing at the same rate. The last terms affect patent production by giving disexternality to the invention production and, as a result, making the patent rate constant. This is why our model can solve both empirical puzzles, the constant patent rate introduction, and the scale effect problem at the same time. The quality ladder model can solve the constant patent problem, but cannot solve the scale effects problem. The quantity expansion cannot get rid of both problems because the growth rate in this model depends upon the growing number of the patents, or the growing number of newer products.

The policy implication of our model is that the subsidy to the investment in the research sector, or the policy which will increase A (the productivity parameter in the invention production) has effects on the growth rate. The growth rate will be increased as occurs in the standard growth literature, but the effects on welfare can be positive or negative, depending upon whether negative externality, rent seeking behavior, and distorted market structure effects are stronger. In the case where the economy is in a state where the innovations are too low, then the subsidy will increase the welfare. In the reverse case where the innovations are too high, the subsidy will decrease the welfare but increase the growth rate.[27] Moreover, the subsidy to the purchase of the intermediate good and/or the subsidy to the investment expenditure does not have any growth effects. This occurs because effects are canceled out in the production of the invention through the negative externality term of the previous investment expenditure. The subsidy to the production of the output also has no effect because it is also washed out in the invention production through investment.

V. Conclusion

The empirical data, which are described as a constant patent rate per unit of time, as a constantly growing research and development expenditure, and as a constant per capita growth rate,

[27]In Barro and Sala-i-Martin (1994), all three effects are fully considered when evaluating welfare. In contrast, Grossman and Helpman (1991a), the market structure is not considered because they consider the consumption variety model. Here market distortion becomes internalized.

cannot be explained simultaneously within the exogenous growth model, in the quantity expansion endogenous growth model *a la* Romer (1990), or in the quality ladder model *a la* Grossman and Helpman (1991a, b). A more general model was set up and analyzed, focusing on these empirical facts. The growth rate only depends upon the productivity parameter in the invention production, and the relative ratio of the share parameter between labor and capital in the final good production function. The scale factor, the total labor, or other fixed factors cannot affect the growth rate. The patent introduction is derived to be constant. This explains the first empirical puzzle which says that the patent rate is fluctuating around the constant mean. The investment expenditure on research and development is growing constantly at the same rate as output growth. This also fits the empirical data, which says that the R&D expenditure is growing at a constant rate. Finally, the growth rate is also constant, fitting the actual data.

The main economic reason why our model could solve all these three puzzles at the same time lies in the invention production function, which has a stronger restriction in its functional form. It assumes a strong negative externality from the previous knowledge stock and/or human capital stock on the invention production. It reflects that the more advanced the current knowledge level, the more difficult it is to make a new invention. That assumption washes out all the parameters because negative externality takes into consideration all the terms in the final good production function, not just the growth factor of the production. The fixed factor can affect the level of output, the price level, the profit of the firm, and the value of the newer innovation, even though it does not affect the growth rate at all.

Our model can be seen as a mixture of the exogenous growth and endogenous growth models: Exogenous in the sense that the growth rate is determined by the production side only, and the fixed factor has only level effects; endogenous in the sense that the growth rate is determined within the model.[28]

The subsidy to the research and development to increase the productivity parameter increases the growth rate. However, the

[28]The growth rate depends upon only exogenous factors: the population growth rate and the technological progress, as may be seen in the Solow (1956) model.

welfare can increase or decrease depending upon the three effects: The negative externality, rent seeking behavior, and the market structure. The subsidy to the investment expenditure will be offset exactly by the negative externality of all previous investment expenditures. The subsidy to the purchase of the intermediate goods and the final good production is also washed out through the negative externality terms, and does not affect the growth rate.

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