Optimal Trade Policy in an International Mixed Oligopoly

Nobuko Serizawa

This paper investigates the effects of trade policies on the home country where homogeneous goods are supplied by a public firm, domestic private firms, and foreign private firms. It is shown that the home government always has an incentive to introduce positive tariffs in opening the economy regardless of whether there are public firms. Furthermore, if the optimal positive tariff is strategically introduced in privatizing the public firm, then privatization can improve welfare even when there are only a few firms in the market. This is in contrast to the existing literature that states that privatization worsens welfare in a closed mixed oligopoly with very few firms.

Keywords: International mixed oligopoly, Trade policy, Privatization

JEL Classification: D43, F12, H20, L13, L32

I. Introduction

Recently we can find many countries, such as those in Western Europe and Asia, where partial or full privatization of public firms is undertaken in oligopolistic industries while also allowing foreign firms to operate. In such markets, state-owned welfare-maximizing public firms compete with profit-maximizing private firms of different nationalities. For example, Portugal privatized all the public banks except the Caixa Geral de Depo’sitos, which competes with foreign banks. In a very recent case, in the Japanese medical

9Associate Professor, Faculty of Economics, Niigata University, Niigata, 9502181, Japan. (Tel/Fax) +81-25-262-6568, (E-mail) serizawa@econ.niigata-u.ac.jp. The author expresses her profound gratitude to constructive comments by two anonymous referees.

industry, an American organization is announced to operate in the
domestic mixed oligopoly where public goods are supplied. At the
same time, many governments have changed their industrial and
trade policies in accordance with changes in the market structure.
Is there any rationale for their actions or are there any strategic
aspects in privatizing public firms while introducing trade policies?

While in recent years there has been a great deal of research on
mixed oligopolies consisting of a public firm and private firms (e.g.
DeFraja and Delbono 1989; Fershtman 1990; Barros 1995; and
White 1996), little attention has been paid in context of an
international setting. Fjell and Pal (1996) analyzed a mixed oligop-
oly in the presence of foreign private firms, which we call an
international mixed oligopoly. They showed the positive effect of an
increase in the number of foreign firms on the output of the public
firm. As mentioned, however, the general effects of this on
equilibrium total outputs and welfare are ambiguous, when the
public firm’s and the all-private firms’ reaction functions are down-
ward-sloping. The stability of the equilibrium has not been explicitly
analyzed. More precisely, the tangency of players’ reaction functions
have not been studied explicitly. Especially when economic policies
can affect players’ decisions, the slope of the reaction function
plays a crucial roll in determining the equilibrium. Nor have
systematic welfare comparisons been made in the previous
literature partly because the sign of their difference depends on the
values of several parameters.

The purpose of this paper is to analyze how the different types of
trade policies affect welfare and to compare welfare under combina-
tions of different policies in an international mixed oligopoly. Also
investigated is whether there are strategic aspects in privatizing a
public firm in relation with such trade policies. The stability condi-
tion is also mentioned in an international mixed oligopoly under a

\footnote{See Barros and Modesto (1999).}

\footnote{When the market is in imperfect competition and consists of both public and private firms, we refer to this a mixed oligopoly.}

\footnote{As in ordinal oligopoly models, the reaction function of identical firms do not always have the same tangency in mixed oligopoly models, which include those consisting of profit-maximizing firms and labor-managed firms. For an analysis of the set of different signs of the reaction function’s tangency in an international oligopoly, see, for example, Okuguchi and Serizawa (1996).}
more general framework. In a regime without tariff policy, either in an open economy or in a closed economy, a simple Cournot-Nash game is constructed and solved. In a regime of regulated trade, however, a two-stage game is constructed in which the government sets the tariff for the outputs of foreign firms’ in the first stage, and in the second stage all firms observe the tariff and simultaneously choose their production levels. For tractability, we assume there are no risks in the market.

First, it is shown that the optimal tariff is strictly positive in an international mixed oligopoly, i.e. before privatization. This is also valid for the case after privatization. Second, when the optimal tariff can be strategically introduced in privatizing the public firm, privatization improves welfare even though there are only a few firms in the market. This is in contrast to DeFraja and Delbono (1989), who assert that privatization worsens welfare when the number of the entrants is very small in a mixed oligopoly.

In Section II we present the model and solve it. Comparisons of and comments on different regimes are presented in Section III, and Section IV concludes the paper.

II. The Model

We assume that there are one public firm, \( n \) domestic private firms and \( m \) foreign private firms in the domestic market. All \((1+n+m)\) firms supply the home market with homogeneous goods and have identical technologies. The inverse demand function is given by \( p = a - X \), where \( X = x_0 + \sum_{i=1}^{n} x_i + \sum_{i=1}^{m} x_i^* \) and \( x_0 \) represents the output of the public firm. In the following analysis, variables with asterisks refer to the foreign firm. The cost function of a firm is assumed to be \( C(x) = b(x^2) / 2 \), where \( b > 0 \) is a constant.\( ^4 \) We assume that all solutions are interior.

Each domestic private firm and foreign private firm choose their output to maximize their own profit defined as

\( ^4 \)As the public firm is often pointed out for its production inefficiency, we implicitly assume that there are some incentive problems in a firm. To illustrate this situation, we introduce increasing marginal cost in which the incentive cost is expressed by the parameter \( b \). Under constant and increasing marginal cost, the public firm monopolizes the market in an international mixed oligopoly.
\[ \pi_i = x_i p - \frac{b x_i^2}{2}, \quad i = 1, \ldots, n. \quad (1) \]

and

\[ \pi_j^* = x_j^* p - \frac{b x_j^{*2}}{2} - t x_j^*, \quad j = 1, \ldots, m. \quad (2) \]

respectively, where \( t \) is the tariff rate when the government allows import with trade policy. \(^5\)

The public firm’s objective, however, is to maximize domestic welfare. We define welfare as the sum of the consumer surplus, total profits of the domestic firms, and tariff revenue:

\[ W = CS + \pi_0 + \sum_{i=1}^n \pi_i + t \sum_{j=1}^m \pi_j^*. \quad (3) \]

where \( CS = \frac{x_0^2}{2} \) is the consumer surplus and \( \pi_0 = x_0 p - b(x_0)^2/2 \) is the profit of the public firm.

In the international mixed oligopoly models, the public firm chooses \( x_0 \) to maximize welfare (3), while in the international private oligopoly model, the firm is privatized and therefore maximizes its own profit (1).

\textbf{A. Before Privatization: Autarky and Free Trade}

As a benchmark, we first present the results in an autarky where the industry is a mixed oligopoly of \((1+n)\) domestic firms. One public firm and \( n \) domestic private firms maximize (3) and (1) with \( m=t=0 \), respectively. Solving these simultaneously, we obtained the following Cournot-Nash equilibrium values for outputs and welfare:

\[ x_0^{\text{Cour}} = \frac{a(1+b)}{g}, \quad x_j^{\text{Cour}} = \frac{ab}{g}. \]

\(^5\)Our results are also valid for other trade policies, such as with local-content policy and when allowing foreign owners to acquire domestic firms.
\[ X_{\text{O:MO}} = \frac{a[1 + b(1 + n)]}{g}, \quad p_{\text{O:MO}} = \frac{ab(1 + b)}{g}, \quad (4) \]

\[ W_{\text{A:MO}} = \frac{a^2[1 + b(1 + n) + 2b(3 + 4n + r^2) + b(3 + 2n)]}{2g^2}, \]

where \( g = 1 + b^2 + b(2 + n) \). The public firm produces more than the private firm, \( x_{0,\text{O:MO}} > x_{1,\text{O:MO}} \). Note that the public firm chooses an output such that its marginal cost equals price, even though it does not monopolize the market.\(^6\) The superscripts indicate the nature of the oligopoly under some trade and industrial policies ((A: MO) for mixed oligopoly under autarky; (IMO:FT) for free trade international mixed oligopoly; (IMO:It) for international mixed oligopoly with tariffs; and (IPOz) for international private oligopoly with tariffs).\(^7\)

Next, allowing imports without tariffs, i.e., \( m > 0 \) and \( t = 0 \), was called a free-trade international mixed oligopoly. Then, solving the maximization problems of (1), (2), and (3) with \( t = 0 \), gave the following results:

\[ x_0^{\text{IMO:FT}} = \frac{a[1 + b + m]}{h}, \quad x_1^{\text{IMO:FT}} = \frac{ab}{h}, \]

\[ X^{\text{IMO:FT}} = \frac{a[1 + b(1 + n + m)]}{h}, \quad p^{\text{IMO:FT}} = \frac{ab(1 + b)}{h}. \quad (5) \]

\[ W^{\text{IMO:FT}} = \frac{a^2[b(1 + n) + b^2(3 + 4n + n^2 + 2m(1 + n + m) + b(1 + m)(3 + 2n + m) + (1 + m)^2)]}{2h^2}, \]

where \( h = 1 + b^2 + b(2 + n + m) + m \). From (5), we observe that the public firm has the highest level of production, and thus its

\(^6\) With a constant and increasing marginal cost, the public firm monopolizes the market by setting the price to equalize its marginal cost in an international mixed oligopoly.

\(^7\) Mixed oligopoly under autarky and free-trade international mixed oligopoly are interpreted as in De Fraja and Delbono (1989) and Fjell and Pal (1996), respectively.
marginal cost is also the highest, \( x_{0}^{(1+0.5T)} > x_{0}^{(1+0.5T)} = x_{0}^{(1+0.5T)} \). As the foreign firms’ profits drop out from domestic welfare \((3)\), the public firm trying to recover this loss produces at a level in which its marginal cost exceeds price.

**B. Before Privatization: Tariffs in an International Mixed Oligopoly**

We will now consider the effects of trade policies in an international mixed oligopoly of \((1+n+m)\) firms when the government introduces tariffs. We have to solve the two-stage model by backward induction. First we maximize \((1)-(3)\) and solve them simultaneously for the second-stage Cournot-Nash equilibrium outputs in terms of the tariff \( t \):

\[
\begin{align*}
    x_{0} &= \frac{\alpha(1+b)(1+b+m)-nmt}{(1+b)h}, \\
    x_{i} &= \frac{ab+mt}{h}, \\
    x_{j}^{*} &= \frac{ab(1+b)-(1+b^{2}+b(2+n)t)}{(1+b)h}, \\
    X &= \frac{\alpha[1+m+b(1+n+m)]-(1+b)mt}{h}.
\end{align*}
\]

Though the output of the domestic private firm is positively related to the tariff, total output as well as the outputs of the public firm and the foreign firm have a negative relation. Therefore, the pure consumer surplus decreases with the tariff. Each foreign firm reduces output by the direct effect of the tariff on the marginal cost. By changing from free trade to tariff policy, the tariff revenue as well as industry-marginal-cost-consciousness motivates the public firm to reduce its output by increasing the output of each domestic private firm whose marginal cost is lower than that of the public firm under free trade, \( x_{0}^{(0.05T)} > x_{0}^{(0.05T)} = x_{0}^{(0.05T)} \).

We then solve for the optimal tariff in the first stage. As the government maximizes \((3)\) taking into account the second-stage outcomes of \((6)\), the optimal tariff is given as

---

\(^8\)These relations are derived from the fact that reaction functions slope downward. For the discussion on the slope of each firm’s reaction function, see Appendix A.
\[ \tilde{\lambda} (\text{IMO}) = \frac{ab(1+b)[1+b^2+b^3(3+n)+b(3+2n)+n(1+m)]}{k} > 0, \] (7)

where \( k = 2+b^2+b^3(10+4n+n^2)+b^3(10+6n+n^2)+m(4+n)+2b^3(10+6n+n^2)+m(3+n)+b(10+4n+m(4+n+n^2))+m \) and tilde “\( \sim \)” over variable means the equilibrium value.

The government has an incentive to introduce positive tariffs regardless of the values of parameters when the market is an international mixed oligopoly. The effects of changes in the number of domestic and foreign private firms are ambiguous.

All firms choose output levels observing this policy. Substituting (7) into (6), we obtain the following subgame perfect Nash equilibrium (SPNE) outcomes:

\[ x_i^{\text{(IMO)}} = \frac{a(1+b)[1+2b+m][1+b^2+b(2+n)]}{k}, \]

\[ x_f^{\text{(IMO)}} = \frac{ab(1+2b^3+b^2(3+n)+2b(3+n)+nm)}{k}, \]

\[ x_j^{\text{(IMO)}} = \frac{a(1+b)[1+b^2+b(2+n)-n]}{k}. \] (8)

\[ X^{\text{IMO}} = \frac{b(1+b)[1+2b+m][1+b^2+b(2+n)+n(4+n)]+b(1+b)[1+b^2+b(2+n)+n(4+n)]+b(1+b)[1+b^2+b(2+n)+n(4+n)]+b(1+b)[1+b^2+b(2+n)+n(4+n)]+m}{k}, \]

\[ W^{\text{IMO}} = \frac{a(1+b)[1+2b+m][1+b^2+b(2+n)+n(4+n)]+b(1+b)[1+b^2+b(2+n)+n(4+n)]+b(1+b)[1+b^2+b(2+n)+n(4+n)]+b(1+b)[1+b^2+b(2+n)+n(4+n)]+m}{2k}, \]

where \( x_i^{\text{(IMO)}} > x_j^{\text{(IMO)}} > x_f^{\text{(IMO)}} \). We note that foreign firms may exit from the market depending on the number of domestic private firms and the cost parameter \( b \). The smaller the cost parameter under a relatively large number of domestic private firms, or equally when the market is domestically competitive, the smaller is the opportunity for foreign firms to make a profit. But, if the cost parameter is large enough (\( b \leq 1 \)), then foreign firms enter the market regardless of the number of incumbent domestic private
firms, because their cost is more advantageous than those of public firm’s and domestic private firms’.

C. After Privatization: Tariffs in an International Private Oligopoly

When the government privatizes the public firm while allowing imports with tariffs, all \((1+n)\) domestic private firms and \(m\) foreign private firms maximize (1) and (2), respectively. In this case, welfare becomes

\[
W = CS + \sum_{i=1}^{1+n} \pi_i + t \sum_{j=1}^{m} x_j^*.
\]

(9)

Following the backward induction as in Section II, B, we solve for the second-stage equilibrium outputs in terms of \(t\):

\[
x^*_i = \frac{a(1+b) + mt}{(1+b)(2+b+n+m)}, \quad x_j^* = \frac{a(1+b) - (2+b+n)t}{(1+b)(2+b+n+m)},
\]

\[
X = \frac{a(1+n+m) - mt}{(2+b+n+m)}.
\]

(10)

Then, the optimal tariff in the first-stage is given by maximizing (9) based on the equilibrium outputs of (10):

\[
\tilde{t}^{(IPO)} = \frac{a(1+b)[3+b^2+b(3+n)+2n]}{v} > 0.
\]

(11)

where \(v = 2b^2 + b^2(10 + 2n + m) + b[2(8 + 6n + n^2) + m(3 + n)] + 2(2 + n)^3 + m\).

After privatization, the optimal tariff is also positive. From (11), we have \(\partial \tilde{t} / \partial n \geq 0\) and \(\partial \tilde{t} / \partial m < 0\). When the relative number of the incumbent foreign firms to the domestic firms is very large, the optimal tariff rate increases with additional domestic firms, \(\partial \tilde{t} / \partial n > 0\). That is, the government tries to increase the share of the domestic firms and support its industry by raising the optimal tariff rate in the regime after privatization.

Thus, the SPNE outcomes in this game are obtained by substituting (11) into (10), and we have:

\[
\frac{\partial \tilde{t}}{\partial n} = -\frac{2a(1+b)[4+b^3+2b^2(2+n)+b(6+n^2+6n-m+2n^2+6n-m)]/v^2}{0} \quad \text{and} \quad \frac{\partial \tilde{t}}{\partial m} = -\frac{k[1+b][1+b^2+b(3+n)][3+b^2+2n+b(3+n)]/v^2}{0}.
\]
\[ x_i^{\text{OPT}} = \frac{2a(1 + b)(2 + b + n)}{v}, \quad x_j^{\text{OPT}} = \frac{a(1 + b^2 + b(3 + n))}{v}, \]

\[ x_i^{\text{OPT}} = \frac{a[4 + b^2(2 + 2n + m) + b(3 + n)(2 + 2n + m) + 2n^2 + 6n + m]}{v}, \quad (12) \]

\[ W[\text{IPO;}d] = \frac{\alpha^2[b^2(2 + 2n + m) + b(2(2 + 5n + n^2) + m(3 + n)] + 2(3 + 4n + n^2) + m]}{2v}. \]

As seen from \( x_i^{\text{OPT}} > x_j^{\text{OPT}} \), clearly the domestic industry is less efficient than the foreign one, and foreign firms always enter the market even under the tariff policy, \( x_j^{\text{OPT}} > 0 \).

**Proposition 1**

In the open economy, the optimal tariff is strictly positive whether or not the public firm is privatized. After privatization, foreign firms always enter the market, however, they do so especially when the cost parameter is very large in an international mixed oligopoly under the tariff policy.

**III. Welfare Comparisons**

First, we look into the case before privatization and compare trade policies in an international mixed oligopoly: free trade (\( W[\text{IMO;}\text{FT}] \)), and the tariffs (\( W[\text{IMO;}d] \)). The difference between welfare under these different policies is:

\[ W[\text{IMO;}\text{FT}] - W[\text{IMO;}d] = \frac{-\alpha^2[b^2(1 + b^2 + b^2(3 + n) + b(3 + 2n) + n^2 + nm^2]}{2n^2 + k} < 0, \quad (13) \]

If optimal tariffs are imposed, then production by the public firm becomes less active because of the tariff revenue, \( x_0^{\text{IMO;}\text{FT}} > x_0^{\text{IMO;}d} \). On the other hand, the domestic private firms activate their production, \( x_0^{\text{IMO;}\text{FT}} < x_0^{\text{IMO;}d} \), which in turn increases their market share. Still, the equilibrium output of a domestic private firm is smaller than
that of the public firm, $x_0^{(MOFT)} > x_0^{(MOF)}$. Thus, industry inefficiency in the home country is alleviated by reducing production of the higher-marginal-cost public firm, which results in $W^{(IMO;FT)} < W^{(IMO)}$. We note that as seen in $x^{(MOFT)} > x^{(MOF)}$, this welfare improvement is brought in part by the sacrifice of consumer surplus when we neglect the positive effect of tariff revenue.

When we compare welfare under free trade ($W^{(IMO;FT)}$) and autarky ($W^{(A;MO)}$) with the public firm, we have:

$$W^{(IMO;FT)} - W^{(A;MO)} = \frac{c'b'n[(1+b)n(1+b)n(2-bm)+2(1+m)+b(2+2b+m)br]}{2g^2T^2} \leq 0. \tag{14}$$

When there is an open door policy without tariffs, the public firm must produce more to compensate for the welfare loss that results from the transfer of the foreign firms’ profits from the home country. Furthermore, the government cannot receive tariff revenue in a free-trade international mixed oligopoly. If these two negative pressures make the public firm overproduce, then welfare decreases under free trade $W^{(IMO;FT)} < W^{(A;MO)}$. Note that, however, free trade is superior to autarky when the home industry is monopolized by the public firm, that is, $W^{(IMO;FT)} > W^{(A;MO)}$ with $n=0$. The equilibrium output of the public firm in an autarky is bigger than that in an international mixed oligopoly without a tariff, which in turn is bigger than the foreign firm’s output in the same regime, i.e., $x_0^{(A;MO)} > x_0^{(MOFT)} > x_0^{(MOF)}$ with $n=0$. The marginal cost of the public firm is reduced in a free trade international mixed oligopoly. This is interpreted to mean that foreign entry alleviates production inefficiency of the monopolized industry. On the other hand, if $n > 0$, then the order of the outputs of the equilibrium outcomes becomes $x_0^{(A;MO)} > x_0^{(MOFT)} > x_0^{(MOF)} > x_0^{(MO)}$, and the sign of (14) depends on the relative number of domestic private firms to foreign private firms as well as the value $b$.\(^{10}\)

Next, we compare welfare in the case of opening with the public firm ($W^{(IMOx)}$) to that in case of opening without the public firm ($W^{(IPx)}$) under tariffs. We have:

\(^{10}\)The order of welfare in different regimes, i.e., before and after privatization, are presented in (16) and (17).
\[ W(\text{IMO}) - W(\text{IPO}) = \frac{\sigma(z^2)}{2k\nu} \geq 0, \tag{15} \]

where \( z = br(2 + b(2 + m))[(1 + m) + b(2 + (5 + 2b)m)] + bn(2 + b(2 + m))^2 + (1 + b)^2[n(2 + b(3 + b))] - 2(1 + b)m - 4(1 + b)^2 \geq 0, \quad k > 0, \text{ and } \nu > 0. \) The sign of (15) is negatively related to the sign of \( z, \) which depends on the relative number of domestic private firms to foreign private firms and the cost parameter. The negative effect of opening policy due to the transfer of the foreign firms' profits out of the home country, and the two positive effects of trade policy due to tariff revenue and production distribution from the public firm to the domestic private firms, by which the industry cost is reduced, are interrelated.

Note that privatization may increase welfare when the optimal tariff can be strategically introduced, i.e. \( W(\text{IMO}) < W(\text{IPO}) \). Privatization can increase welfare with the optimal tariff when there are as few as four firms in the market.\(^{11}\) Also, when we turn our attention to welfare comparisons in other combinations, we find very interesting results. These are the case of opening without the public firm under tariff policy (\( W(\text{IPO}) \)) and the cases of free trade (\( W(\text{IMO}:F) \)) and autarky (\( W(\text{A}:\text{MO}) \)) with the public firm. For example, as seen in Table 1, if the closed mixed duopoly \( (n=1) \) is opened, privatizing the public firm with the optimal tariff \( (n=m=1, \nu > 0) \), then welfare increases, i.e. \( W(\text{IPO}) > W(\text{A}:\text{MO}) > W(\text{IMO}:F) \) with \( b=1.12 \) This type of improvement is also realized when the closed public monopoly \( (n=0) \) is opened privatizing it with the optimal tariff \( (n=0, m=2, \nu > 0) \). In these cases, there are three firms in the market. These results are in sharp contrast to the existing literature, such as DeFraja and Delbono (1989), which asserts that privatization decreases welfare when there are only a few firms in a closed mixed oligopoly.

**Proposition 2**

Even when there are only a few firms in the market, privatizing the

---

\(^{11}\)This is the case when \( b=1, \ n=1 \) and \( m=2 \) with the public firm. As the effect of privatization on welfare is ambiguous, we utilize numerical examples in Table 1 to give policy implications more precisely.

\(^{12}\)In the closed economy, \( n=0 \) means the public monopoly. See Table 1 for numerical examples.
Table 1

<table>
<thead>
<tr>
<th>b</th>
<th>number of private firms</th>
<th>opened</th>
<th>closed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>domestic</td>
<td>foreign</td>
<td>(W[PO_2])</td>
</tr>
<tr>
<td>(b=1)</td>
<td>(n=1)</td>
<td>(m=1)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(n=1)</td>
<td>(m=2)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(n=0)</td>
<td>(m=2)</td>
<td>2</td>
</tr>
<tr>
<td>(b=0.1)</td>
<td>(n=1)</td>
<td>(m=1)</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>(n=1)</td>
<td>(m=2)</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>(n=0)</td>
<td>(m=2)</td>
<td>4</td>
</tr>
</tbody>
</table>

Note: The numbers shown in the welfare column express the order from highest to the lowest from 1 to 4. When \(n=0\), the domestic market is monopolized by the public firm.\(^{13}\)

Public firm in an open economy can strategically increase welfare.

Finally, we present overall comparisons among welfare in different regimes, i.e. before and after privatization:

\[
W[MO_{FT}] > \max\{W[A:MO], W[MO_{FT}]\}. \tag{16}
\]

and

\[
W[PO_2] > \max\{W[A:PO], W[PO_{FT}]\}. \tag{17}
\]

where \(W[A:PO]\) and \(W[PO_{FT}]\) are private oligopoly in autarky and free trade without the public firm, respectively.\(^{14}\) When the government can choose the optimal tariff rate, opening the economy achieves the highest welfare regardless of whether the public firm

\(^{13}\)Besides the importance of the relative number of the domestic private firms to foreign private firms, the cost parameter \(b\) plays a decisive role in determining the equilibrium in an international mixed oligopoly (See Barros (1995) for a mixed oligopoly model). So, we separate the cases for the higher cost parameter, i.e. organizational inefficiency may be serious, and the lower one, less serious. The details of the calculations will be provided by the author upon request.

\(^{14}\)(16) is derived from (13) and (14). For the derivation of (17), see Appendix B.
is or is not present, i.e. $W^{IMO2}$ and $W^{IPO2}$ are the highest in (16) and (17), respectively. But again from (15), the government must be very careful to privatize the public firm in an international mixed oligopoly under the optimal tariff policy.

**Proposition 3**
To achieve the highest welfare, the optimal positive tariff rate must be chosen whether or not the public firm is privatized. But, if the government considers privatization in the international mixed oligopoly, it must be careful about the relative number of domestic private firms to foreign private firms and the cost parameter so as not to decrease welfare.

**IV. Conclusions**

In an international mixed oligopoly, it is shown that the government always has an incentive to introduce positive tariffs. If the government allows free trade, then the higher-marginal-cost public firm must increase production to compensate for the welfare loss due to rent shifts. However, this higher industry cost of domestic firms' results in production inefficiency, and therefore worsens welfare. Autarky achieves higher welfare than free trade when negative effects of opening without tariff revenue make the public firm overproduce. If the closed public monopoly is opened under free trade, however, inefficient public firm's marginal cost is alleviated by the entry of foreign firms. In this case, welfare under autarky becomes smaller than that under free trade. Besides, the different objective functions of the firms, industry specific technology, i.e. cost functions, and the market's competitiveness, i.e. relative number of domestic private firms to foreign private firms, affect welfare when trade policies are introduced in an international mixed oligopoly. With this information, the government can strategically improve welfare by privatizing the public firm in an open economy when the positive tariff is optimally set.

We may extend our model in such a way, for example, that the cost parameter is controlled endogenously in an international mixed oligopoly. This model will have an advantage as it can incorporate asymmetric information.
Appendix

A. Slope of the Reaction Functions in a Generalized International Mixed Oligopoly

Even if we assume symmetric technology among firms, the tangency of firms’ reaction functions does not necessarily have the same sign as in oligopoly models without product differentiation. As seen in Section II, if trade policies can affect the equilibrium, we have to know the process of effects via the firms’ reaction functions on each firm’s output decision. So, we focus on the conditions that determine the slope of the reaction functions in a more general framework, namely whether or not the goods are strategic substitutes, assuming \( n = m = 1 \) and increasing marginal cost for all firms. But the inverse demand function is generalized under the assumptions of \( p’ + xp^* < 0 \) and \( p’ + xp^* < 0 \), which are widely used in analyzing the existence and stability of the Cournot oligopoly equilibrium \( \varphi(\sum y_i) + xp^* (\sum y_i) < 0 \) means that the marginal revenue of the \( i \)-th firm with respect to the increase in its own output is decreasing with respect to the increase in the \( j \)-th firm’s output \((i \neq j)\). We present the following condition when the downward-sloping reaction function of the public firm is expressed in terms of the outputs of joint private firms, and vice versa.

Remark

If and only if \( p^* > 0 \), the goods are strategic substitutes among every players’ pair in an international mixed oligopoly. When the inverse demand function is linear, \( p’ = 0 \), the public firm’s reaction function is vertical in the public and foreign private firms’ strategic space where the latter reaction function is downward-sloping (Figure 1).

B. Combinations of Trade and Industrial Policies

<table>
<thead>
<tr>
<th>Trade Policies</th>
<th>Before Privatization</th>
<th>After Privatization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed/Open</td>
<td>Tariff</td>
<td>MO/IMO</td>
</tr>
<tr>
<td>Autarky (( m = 0 ))</td>
<td>FT: ( t = 0 )</td>
<td>W(A:MO)</td>
</tr>
<tr>
<td>Open economy (( m &gt; 0 ))</td>
<td>FT: ( t = 0 )</td>
<td>W(IMO:FT)</td>
</tr>
<tr>
<td>Open economy (( m &gt; 0 ))</td>
<td>( t &gt; 0 )</td>
<td>W(IMO:2)</td>
</tr>
</tbody>
</table>

OPTIMAL TRADE POLICY

\[ X_c \]

Note: If the optimal tariff is positive, then the reduction in joint private firms' outputs \( X \) shifts its reaction function \( R(x) \) downward, where \( X_c = x + x^* \). Thus, the total industry outputs decrease despite the increase of the domestic private firms' output.

**FIGURE 1**

- \( W(A:MO) = \alpha^2 \left[ b^2(1 + n) + b^2(3 + 4n + n^2) + b(3 + 2n) \right] / 2g^2 \),
- \( W(A:PO) = \alpha^2 \left[ 1 + n + b(3 + 2n) \right] / 2(2 + b + n)^2 \),
- \( W(IMO:FT) = \alpha^2 \left[ b^2(1 + n) + b^2(3 + 4n + n^2 + 2m(3 + 2n + m(3 + 2n + m) + b(1 + n)(3 + 2n + m) + b(1 + n)) / 2(2 + b + n + m)^2 \right] / 2(2 + b + n + m)^2 \),
- \( W(IMO:FO) = \alpha^2 \left[ 2 + b^2(2 + 2n + m) + b^2(4 + 5n + n^2 + m[3 + n]) + b^2[2(6 + 2n + n^2 + m(3 + n)]) + 2(3 + 4n + n^2 + m) \right] / 2v \),
- \( W(IMO:FO) = \alpha^2 \left[ b^2(2 + 2n + m) + b[2(4 + 5n + n^2 + m[3 + n])] + 2(3 + 4n + n^2 + m) \right] / 2v \),
- \( W(IMO:FO) = \alpha^2 \left[ b^2(2 + 2n + m) + b[2(4 + 5n + n^2 + m[3 + n])] + 2(3 + 4n + n^2 + m) \right] / 2v \),

where \( g, h, k, \) and \( v > 0 \).

Following the same processes in Section II, A and C, \( W(A:PO) \) is derived by solving \( 1 + n \) maximization problems of \( 1 \)s, and \( W(IMO:FO) \) by solving maximization problems of \( 1 + n \) of \( 1 \)s and \( m \) of \( 2 \)s simultaneously.

(Received September, 1999; Revised January, 2000)

**References**


Oligopoly?" International Journal of Industrial Organization 17 (No. 6 1999): 869-86.