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Multi-brand Firms and Brand Acquisition: The Impact of Trade Liberalization on Reallocation of Brand Equity

다브랜드 기업과 브랜드 인수: 무역자유화가 브랜드 자산의 재분배에 미치는 효과에 대해

2015 년 8 월

서울대학교 대학원
경제학부
라 미 링
Abstract

Multi-brand Firms and Brand Acquisition: The Impact of Trade Liberalization on Reallocation of Brand Equity

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Recent economic research on the effect of trade liberalization has focused on the firm-level reallocation of resources induced by free trade. The studies have developed theoretical models of multi-product firms and shown that trade liberalization induces resource reallocation across firms and within firms. However, these studies have only investigated the transfer of tangible resources, such as labor or capital. Only scarce attention has been paid to the transfer of intangible resources. This paper, however, analyzes the reallocation of brand equity between firms by introducing the possibility of brand acquisition. To analyze the effect of trade liberalization on brand acquisition in a general equilibrium setup, I extend the within-brand cannibalization model of Agur (2010) and Dhingra (2013). In the presence of the within-brand
cannibalization effect, consumers consider products to be more substitutable within brands than across brands. Therefore, when a firm introduces a new product variety, the demand for its original varieties falls more than the demand for the varieties of other brands. Under this effect, productive firms cannot increase their scope sufficiently. Through the acquisition of an existing brand, however, productive firms can expand their production without profit loss from within-brand cannibalization. In this model, acquisition can be used as another decision variable of expanding production.

Market expansion through international trade strengthens the incentive for more productive firms to acquire brand equity of less productive firms as it raises the surplus from brand acquisition for more productive ones. That is, trade liberalization will reallocate market shares from less productive to more productive firms; then the more productive firms will expand their production by purchasing other firms’ brands. Meanwhile the least productive firms will exit by selling their firms in the face of increased market competition.

In the empirical section of this paper, the main predictions of the theoretical model using USPTO Trademark Dataset are tested. Trademark data is suitable for the empirical test of brand acquisition, since trademark identifies and distinguishes the source of the goods of one party from those of others, and all associated trademarks are transferred when a brand is sold. This study provides a broad picture of trademark assignment activities, and shows how trade liberalization led to the reallocation of brand equity among firms during 1979-2000.
Keywords: Trade liberalization, Brand Acquisition, Within-brand cannibalization, Multi-brand firm, Trademark assignment, Market restructuring, Globalization.
Student Number: 2008-30803
Contents

Abstract .............................................................................................................. i
List of Tables .................................................................................................... vi
List of Figures ............................................................................................... vii

1. Introduction .................................................................................................. 1
   1.1. Background of research ................................................................. 1
   1.2. Related Literatures ............................................................................ 3

2. Motivating Observations ........................................................................... 7

3. Theoretical Model with CES Preference ........................................... 26
   3.1. Closed Economy .............................................................................. 26
       3.1.1. Preferences ............................................................................... 26
       3.1.2. Production Technology .............................................................. 29
       3.1.3. Acquisition of Brand .................................................................. 32
       3.1.4. Firm Entry and Exit ................................................................. 35
       3.1.5. Aggregation ............................................................................... 39
   3.2. Open Economy with Costly Trade ................................................. 42
       3.2.1. Equilibrium in Open Economy ................................................ 42
       3.2.2. Firm Entry and Exit in Open Economy ..................................... 47
       3.2.3. Aggregation ............................................................................... 52
   3.3. Impact of Freer Trade ......................................................................... 56
   3.4. Impact of “Love of brand” ................................................................. 59
   3.5. Numerical Solution ............................................................................ 62
4. Theoretical Model with Quasi Linear Preference .... 65
   4.1. Closed Economy ............................................................. 65
       4.1.1. Preferences and Production Technology .................. 65
       4.1.2. Industry Equilibrium ............................................ 71
   4.2. Free Trade Equilibrium ................................................. 73
   4.3. Numerical Solution ...................................................... 75

5. Empirics ........................................................................ 80
   5.1. Properties of Trademark .............................................. 80
   5.2. Indirect Measure of $\gamma$ .......................................... 82
   5.3. Test Results .............................................................. 87

6. Conclusion ...................................................................... 95

References ................................................................. 96

Appendix ........................................................................ 99
   A1. Definition and Example of Brand .................................... 99
   A2. Model with CES Preference .......................................... 99
   A3. Model with Quasi Linear Preference ......................... 111
   A4. Data ........................................................................ 118
List of Tables

Table 1. Different choice of the pre- and post-FTA periods ......................... 15
Table 2. Acquisition ratio (%) of the pre- and post-FTA periods .................. 15
Table 3. Acquisition ratio (%) ..................................................................... 16
Table 4. Average acquisition ratio (%) ...................................................... 17
Table 5. Average acquisition ratio (%) ...................................................... 17
Table 6. Share of firms who registered multiple trademarks and registered over multiple industries ............................................................... 18
Table 7. Share of assignees who registered multiple trademarks and registered over multiple industries .............................................. 18
Table 8. Top 28 firms’ trademarks during 1950 - 2013 ............................. 20
Table 9. List of Nice codes of industries ................................................... 22
Table 10. Correlation among different measures of $\gamma$ ......................... 87
Table 11. Detailed results using DID, 1980-1989 ....................................... 89
Table 12. Detailed results with time-variant $\gamma$ using DID, 1980-1989 .... 91
Table 13. Detailed results with prior ratio, 1980-1989 ................................ 92
Table 14. Detailed results with different measures of $\gamma$, 1980-1989 ...... 93
Table 15. Property-level results, 1980-1989 ............................................. 94
List of Figures

Figure 1. Number of assigned trademarks ................................................... 8
Figure 2. The rate of assigned trademarks relative to the existing trademarks ................................................................. 9
Figure 3. Merger versus assigned trademarks ........................................... 11
Figure 4. The rate of transferred trademarks by mergers and assignments 11
Figure 5. The average Canadian tariff rate against ..................................... 12
Figure 6. The average U.S. tariff rate against ............................................. 13
Figure 7. Openness Index against Canada .................................................. 13
Figure 8. Openness Index against NAFTA and EU .................................... 14
Figure 9. Acquisition cutoffs with $\alpha \in [2.1, 2.25]$ and $\tau \in [1.1, 1.7]$ .......... 63
Figure 10. Proportion of acquisition with $\alpha \in [2.1, 2.25]$ and $\tau \in [1.1, 1.7]$ . 63
Figure 11. Proportion of acquisition with $\theta \in [3.4, 3.8]$ and $\tau \in [1.1, 1.7]$ ...64
Figure 12. Proportion of acquisition with $\gamma \in [0.05, 0.25]$ and $L \in [1000, 10000]$ ....................................................................... 76
Figure 13. Proportion of acquisition with $\gamma = 0.05$ .................................. 77
Figure 14. Proportion of acquisition with $\gamma = 0.25$ .................................. 77
Figure 15. Cost cutoff with $\gamma \in [0.05, 0.25]$ and $L \in [1000, 10000]$ ............ 78
Figure 16. Acquisition cutoff with $\gamma \in [0.05, 0.25]$ and $L \in [1000, 10000]$...78
Figure 17. Average cost with $\gamma \in [0.05, 0.25]$ and $L \in [1000, 10000]$ .......... 79
Figure 18. Hirschmann-Herfindahl Index at brand-level............................. 83
Figure 19. Hirschmann-Herfindahl Index at firm-level................................. 84
Figure 20. Relation between prior ratio and average value of age ............... 86
1. Introduction

1.1. Background of research

Recent economic research on the effect of trade liberalization has focused on the firm-level adjustment processes by considering the various decision margins of firms. These literatures examine the change of firms’ decisions over entry and exit, whether to export, product range, technology adoption and innovation, etc. induced by freer trade. The central implication of these studies is that trade liberalization leads low productivity firms to exit and high productivity firms to expand to enter export markets. The export market entry increase the return to production expansion and productivity enhancing investment asymmetrically, as a result to within-industry reallocation of resources which raise average industry efficiency.

Many related literatures show how trade liberalization encourages the expansion decision of productive firms, but they do not consider another important channel of expansion - the acquisition of an existing establishment. In real economy, efficient firms often take over the valuable assets from inefficient firms which results in the restructuring of that industry. Especially, established intellectual property is more easily transferred from one firm to another, which affects overall efficiency of the industry. Therefore, analyses of the flow of intellectual property, for instance, trademark which is dealt with in this paper helps us understand an additional channel of resource reallocation in globalization. Understanding this additional margins can be important for
evaluating the new gains of trade.

This paper assumes monopolistic competition and introduces firm heterogeneity to explain the incentive of acquisition in the model. This paper also assumes each firm has its own brand and can produce multiple products within the brand which distinguish one firm’s product from those of other.\(^1\) In this study, brand acquisition is the transfer of such a brand among firms. Throughout the paper, two different theoretical models which use the preference structure, as seen in Agur (2010) and Dhingra (2013), are discussed. The common feature of the preference structure of these models is that consumers consider products to be more substitutable within brands than across brands. Under this feature, when a firm increases its product variety, the market shares of its existing products are cannibalized as consumers choose to select the firm’s new products over the older ones. Dhingra refers to this fall in demand as “within-brand cannibalization”. In the presence of this effect, productive firms cannot increase their scope sufficiently. Through the acquisition of an existing brand, however, productive firms can expand their production without profit loss resulting from cannibalization.

This paper yields three sets of core implications which will be examined with empirical analyses. The first is that export opportunity increases the return to acquiring a new brand. The second is that increased import competition lowers the incentive of acquisition. From these two implications, we can see that acquisition incentive for the most-efficient firms which are affected mainly

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\(^1\) The definition of brand and specific example of brand are described in Appendix A1.
by market expansion increases, but acquisition incentive for mid-efficient firms is ambiguous after trade liberalization. The final implication is that acquisition incentive is higher in an industry with a high degree of within-brand cannibalization. The results of the empirical test are consistent with these theoretical predictions.

The remainder of this paper is structured as follows. I first present the related literature, then show some motivating evidences which are descriptive and anecdotal evidences. Then I set up the multi-brand firm model under the CES preference and the quasi linear preference. Furthermore, I analyze the effect of trade liberalization using comparative statics, and then provide a visual representation of equilibrium with simulation results. In the empirical part, I test the core implications with U.S. trademark data. The proofs and mathematical equations are collected in the appendix.

1.2. Related Literatures

The main contribution of this paper is to systematically examine how trade liberalization affects the transfer of brand equity. In contrast to the classical literature of M&A², I focus on the brand acquisition and solve general equilibrium model under monopolistic competition.³ This article is related to the exiting theoretical works on multiple-product firms in the field of

---

² Spearot (2012) investigates the decision of acquisition and new investment using quasi linear preference with within-brand cannibalization. His work is closely related to my theoretical work, but he only analyzes the partial equilibrium by assuming the range of product is fixed.
³ To understand the effect of new entry in the acquisition model, I use the monopolistic competition model rather than oligopolistic competition model.
international trade literatures which study monopolistic competitive firms with free entry.

The models of this paper are based on the seminal implications of Melitz (2003) and Melitz and Ottaviano (2008). In the Melitz model, single-product firms produce a horizontally differentiated variety and differ in their productivity. Exporting involves a sunk cost, which leads only the most productive firms to export. The theoretical models of this paper have similar mechanism of Melitz (2003). Acquisition involves a sunk cost, which leads to results that only most productive firms acquire. Melitz and Ottaviano (2008) uses the quasi linear preference instead of CES to investigate the market size effect. In the section 4, this paper also uses the quasi linear preference to understand the effect of product market competition.

The concept of cannibalization came from Eckel and Neary (2010) which considers homogeneous multi-product firms under oligopolistic competition. They show that optimal variety of firms depends on both core competencies and cannibalization. Allanson and Montagna (2010), Agur (2010) work with nested-CES preference, in which the varieties of a firm are closer substitutes than the varieties of different firms. This causes the cannibalization effect, so I use their nested-CES preference in section 3. This property not only bounds a

---

4 Dhingra (2013) sets up the linear demand model with “within-cannibalization effect”, so I use her utility function in section 4.
5 Under the core competencies assumption, additional products raise marginal costs. Throughout the paper I assume there is no core competencies, but this property can be another driving force of brand acquisition.
6 Empirical evidence for these higher substitutability within brands is provided by Broda and Weinstein (2010) and Hui (2004).
firm’s optimal range of variety but also gives the incentive of acquiring an existing brand. Bernard, Redding and Schott (2010, 2011) also uses nested-CES preference, and studies multi-product firms’ scope choices and within-firm rationalization in response to trade liberalization under the assumption of heterogeneity among firms and each firm’s varieties.

There is other recent paper which developed monopolistically competitive models of multiple-product firms without cannibalization effect. Mayer, Metliz and Ottaviano (2014) shows that increased competition by freer trade leads a firm to skew its sales towards its best performing products, so all firms reduce their scope. Similar to Mayer et al (2014), I find that trade liberalization reduces the efficient firm’s range of varieties even though this paper does not have heterogeneity among varieties of a firm. This reduced sets of varieties comes from the tradeoff between scope of varieties and scope of brands.

On the empirical side, there is small sets of studies which investigate the brand acquisition. Some literatures point out that M&As can play a substantial role in restructuring industries. Jovanovic and Rousseau (2002) and Andrade and Stafford (2004) demonstrate that M&As are frequently used as a means of firm expansion and complement internal investment. In the international trade literature, Breinlich (2008) shows that M&A activity in Canada rose sharply after CUSFTA and the magnitude of this increase is related to the extent of tariff cuts across industries. Because of complexity of defining brand acquisition in data, transfer of intangible assets by brand acquisition has not yet been addressed in a rigorous way. Frey and Ansar (2013) uses Bureau van Dijk’s
Zephyr database and identifies brand-driven acquisitions by searching a number of brand-related keywords. They find about 1000-1700 brand-related deals per year, and provide the value of the average brand-driven M&A transaction is approximately 10 to 12 times higher than the value of the average global M&A deal. They also find out that markets for brand-driven M&A transactions are largely domestic. Fee, Hadlock and Pierce (2012) uses Competitive Media Reporting/Taylor Nelson Sofres (CMR/TNS) which issues the information on brand ownership. For instance, if reported owner of a microbrand in 2003 differs from its 1998 owner, they conduct news searches in Factiva to determine whether there was a substantive change in ownership. In contrast, I use the data of brand element, i.e., trademark, to focus on the transfer of intangible assets between firms. Trademark data do not capture relationships among trademarks, making it impossible to identify a brands. Thus this is somewhat far from the brand acquisition, but we can track the transfer of small sets of brand elements by examining trademark-level data.

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7 The main implication of their study is how advertisement investment is changed after acquisition.
2. Motivating Observations

Before move to the theoretical model, I present some motivating observations which give the outline of the data. The detailed data and regression results are illustrated in section 5. The most tangible evidence of this brand acquisition is a legal change in ownership of a trademark that is recorded by the United States Patent and Trademark Office (USPTO). This is because a trademark identifies and distinguishes the source of the goods of one party from those of others, and all associated trademarks are transferred when a brand is sold. Therefore, examinations of the number of transactions recorded by the USPTO provide a broad picture of brand acquisition activities. Figure 1 shows the number of trademark assignments since 1979. It demonstrates an increasing trend of trademark assignments, which implies the increased frequency of brand acquisitions.

---

8 A trademark assignment is the transfer of ownership of a trademark from one entity to another. The entity who sell a trademark is named as an assignor, and the entity who purchase a trademark is named as an assignee.
It is important to note that brand acquisitions can take many forms. Brands can be acquired as part of a company acquisition, or be acquired by themselves. Brand acquisitions sometimes accompany the acquisition of all the assets associated with a brand, such as the facilities, the management team, distribution network, etc. In some cases, brand acquisition includes only the brand elements, such as the name, logo, etc. and other intellectual assets associated with the brand. Because the first form of brand acquisition as part of a company acquisition might be taken for other purposes, here I exclude transactions which are described as a “mergers” to rule out cases where brand acquisitions are followed by M&As of the business establishments. This study only uses “assignment” deals yielding approximately 625,000 transacted

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9 In 2008 Kellogg Co. acquired Mother’s Cake and Cookie brand from Archway & Mother’s Cake and Cookie Co. receiving nothing but trademarks and the recipes.
Another thing to consider is that the increasing pattern of assignments might come from the increasing number of existing trademarks. Thus I calculate the ratio of assigned trademarks relative to the existing trademarks in each year. Then we obtain Figure 2, which indicates two peaks around 1986 and 1996. This shows the slightly increasing patterns of the acquisition ratio over time, and it also gives the prediction that the peaks around 1986 and 1996 are closely related to the CUSFTA, NAFTA and commencement of the WTO.

Figure 2. The rate of assigned trademarks relative to the existing trademarks

However, the steep decline of the acquisition ratio around 1990 is related to the institutional change of the renewal system of trademarks. While registrations were renewed for a 20-year term before November 1989, registered trademarks...
were renewed for a 10-year term after November 1989. This might reduce the expected value of trademarks and thus lower the incentive of acquiring existing trademarks. Therefore, I divide the periods into 1980-1989 and 1990-1999 to examine the effects of CUSFTA and NAFTA.

Figure 3 compares the assignments of trademarks with mergers. As you can see, assignments precede mergers. This is reasonable because the transfer of intangible assets is relatively easy, but a merger is often a lengthy process to reach. Thus if firms expect trade liberalization as a free trade agreement starts, then the acquiring of brands as response to freer trade appear more quickly than other transactions. The following Figure 4 illustrates the combined ratio of assignments and mergers.

---

10 Because of this property, the choices of the pre- and post-FTA periods are carefully handled. Throughout the difference in difference regression, I use two different definitions of the pre- and post-FTA periods.
Figure 3. Merger versus assigned trademarks

Figure 4. The rate of transferred trademarks by mergers and assignments
The following figures plot the variables which measure the degree of trade liberalization. The FTA reduce tariffs against its members without reducing tariffs against the rest of the world. So, we define trade liberalization measures like Trefler (2004) as follows. The FTA-mandated Canadian tariff concessions granted to the United States are given by

\[ \tau_{it}^{CA} = \tau_{it}^{CA,ROW} - \tau_{it}^{CA,US} \]  (2.1)

which is the gap between the solid line and the broken line in Figure 5. The FTA-mandated US tariff concessions granted to Canada are

\[ \tau_{it}^{US} = \tau_{it}^{US,ROW} - \tau_{it}^{US,CA} \]  (2.2)

which is the gap between the blue solid line and red broken line in Figure 6.

Figure 5. The average Canadian tariff rate against
Figure 6. The average U.S. tariff rate against Canada

Figure 7. Openness Index against Canada
The Figures 7 and 8 depict the trade-to-GDP ratio, i.e., openness index of the U.S. against Canada, NAFTA member countries and EU. These show the relative value of trade volume against Canada, NAFTA members, and EU with respect to the gross production of the U.S. We can see that the degree of openness against Canada increased since 1988 and those against NAFTA member countries increased since 1995.12

Now I present tables comparing the acquisition ratios of the pre-CUSFTA, post-CUSFTA, pre-NAFTA and post-NAFTA periods. I report the

---

11 Openness index is calculated as the sum of exports plus imports divided by the gross domestic product.
acquisition ratios at the different choices of the pre- and post-FTA periods. First I define the pre-FTA period as the years before the FTA negotiation started, then define the pre-FTA period as the years before the agreement went into effect. The pre- and post-FTA periods defined by the first definition are listed in the first row of Table 1 labeled as [1].

<table>
<thead>
<tr>
<th></th>
<th>Pre-CUSFTA</th>
<th>Post-CUSFTA</th>
<th>Pre-NAFTA</th>
<th>Post-NAFTA</th>
</tr>
</thead>
</table>

Table 2 shows the mean and median values of acquisition ratios by two different definitions of the pre- and post-FTA periods.

<table>
<thead>
<tr>
<th></th>
<th>Pre-FTA</th>
<th>Post-FTA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>CUSFTA [1]</td>
<td>2.563</td>
<td>2.564</td>
</tr>
<tr>
<td>NAFTA [1]</td>
<td>2.933</td>
<td>2.754</td>
</tr>
</tbody>
</table>

In both cases, we can see that the post-FTA periods have higher values of acquisition ratios. Furthermore, it is shown that the acquisition ratio is highest during the negotiation period. The mean value of acquisition ratio during the negotiation period, 1987-1988, is 3.493% and the median value is 3.381%. So
we can conjecture that the rational expectation of firms about trade liberalization also affects the acquisition ratio.

A notable thing is that WTO officially commenced on January 1995, thereby replacing GATT. Table 3 indicates the average acquisition ratios during the pre- and post-WTO periods. The results are qualitatively the same with the CUSFTA and NAFTA cases. Since post-NAFTA period and post-WTO period are overlap, there might exists a mixed effect in the period 1995-1999. Therefore, I only focus on the CUSFTA period in section 5.

Table 3. Acquisition ratio (%)

<table>
<thead>
<tr>
<th></th>
<th>Acquisition ratio (%)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td></td>
</tr>
<tr>
<td>Pre-WTO (1990-1994)</td>
<td>2.909</td>
<td>2.770</td>
<td></td>
</tr>
</tbody>
</table>

The next tables summarize the acquisition ratios according to the measure of within-brand cannibalization. As predicted, the acquisition rate is higher in an industry with high cannibalization effect, but the increment induced by free trade is not always higher in those industries. If we use the indirect measure of $\gamma$ calculated from the prior ratio$^{13}$, the increment is higher in the industries with high values of $\gamma$, but the growth rate is lower. This ambiguity regarding the cross-partial effect of $\gamma$ and trade liberalization is described in section 3.

$^{13}$ An explanation of how the indirect measure of $\gamma$ was constructed is described in section 5.
Table 4. Average acquisition ratio (%)

<table>
<thead>
<tr>
<th></th>
<th>Pre-NAFTA</th>
<th>Post-NAFTA</th>
<th>Difference</th>
<th>Growth rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>3.100 (2.885)</td>
<td>3.485 (3.399)</td>
<td>0.385 (0.514)</td>
<td>12.43 (17.80)</td>
</tr>
<tr>
<td>Low</td>
<td>2.875 (2.627)</td>
<td>3.318 (3.247)</td>
<td>0.443 (0.619)</td>
<td>15.41 (23.57)</td>
</tr>
</tbody>
</table>

Notes: Pre-NAFTA is the period from 1990 to 1993, and Post-NAFTA is the period from 1994 to 1999. The values in parenthesis are the median values of acquisition ratios. This table uses the indirect measure of $\gamma$ calculated by the residuals of regression (5.1), and “high” represents the top 20% industries having highest within-brand cannibalization.

Table 5. Average acquisition ratio (%)

<table>
<thead>
<tr>
<th></th>
<th>Pre-NAFTA</th>
<th>Post-NAFTA</th>
<th>Difference</th>
<th>Growth rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>3.176 (2.891)</td>
<td>3.622 (3.583)</td>
<td>0.446 (0.692)</td>
<td>14.04 (23.95)</td>
</tr>
<tr>
<td>Low</td>
<td>2.847 (2.622)</td>
<td>3.275 (3.258)</td>
<td>0.429 (0.635)</td>
<td>15.05 (24.23)</td>
</tr>
</tbody>
</table>

Notes: This table uses the prior ratio as the indirect measure of $\gamma$, and “high” represents the top 20% industries having highest within-brand cannibalization.

This paper focuses on the acquisition incentive arising from within-brand cannibalization. Thus this kind of acquisition must take place within the same industry. If the share of acquisitions which take place across the industry is relatively large, then the acquisitions coming from within-brand cannibalization is less important, and we can conclude that the major incentive of acquisition can be found in something else, for instance, a way of entering a new industry. To illustrate this, I use the ownership data during 1950 - 2013. 73.01% of the assignments have ownership data, and 83.32% of those transactions occurred within the same set of industries. That is, 83.32% of the assignees acquired trademarks of the same industries which its owned...
trademarks are classified into.\textsuperscript{14} Moreover, the proportion of within-industry transaction is stable over time. Specific values by different choices of periods are listed in Table A.4.1.

It should be noted that the owner of 75\% of the assigned trademarks are entrant firms, and thus it is implied that the majority of assignors are entrant firms. In contrast, the majority of assignees are identified multi-product firms. The properties of the assignees are as follows:

Table 6. Share of firms who registered multiple trademarks and registered over multiple industries

<table>
<thead>
<tr>
<th>Percent of firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple registration</td>
<td>36.38</td>
</tr>
<tr>
<td>Multiple industry</td>
<td>29.69</td>
</tr>
</tbody>
</table>

Table 7. Share of assignees who registered multiple trademarks and registered over multiple industries

<table>
<thead>
<tr>
<th>Percent of firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple registration</td>
<td>81.94</td>
</tr>
<tr>
<td>Multiple industry</td>
<td>70.29</td>
</tr>
</tbody>
</table>

Notes: 64.09\% of the assignee have information about registration and industry.

Table 6 shows the average ratio of multiple registrants in trademarks and industries. That is, 36.38\% of firms registered multiple trademarks and 29.69\% of firms registered their trademarks over multiple industries. On the contrary,

\textsuperscript{14} The actual rate will be higher than 83.32\%, because of typo, the relation between parent subsidiary firms and the mismatch between registered trademarks and assigned trademarks underestimate the value.
81.94% of the assignees are firms who registered multiple trademarks and 70.29% of the assignees already used their trademarks in multiple industries according to Table 7. These numbers show that the assignees have a high tendency of registering multiple trademarks which are used in multiple industries. This implies that the assignee is more productive firms than the average firms, which is consistent with the prediction of the theoretical model.

To see the assignments of the trademarks more specifically, I collected data on the top 28 U.S. companies, then summarized the assignment records in the following table. The firms’ names are listed in the order of average sales during 1980-2005.\textsuperscript{15} All numbers include the trademarks which is inactive now. The first column, \textit{Number owned} contains every owned trademark which have applied, registered and transferred from others. The second column, \textit{Number registered} represents the number of trademarks applied and registered by each firm. The third column, \textit{Number assigned} represents the number of trademarks acquired, but the ones transferred by mergers are excluded in this column. The last column, \textit{Ratio assigned} is the value of \textit{Number owned} divided by \textit{Number assigned}.

\footnotesize \begin{center} \begin{tabular}{lrrr} \\
\hline
Firm & Number owned & Number registered & Number assigned & Ratio assigned \\
\hline
\end{tabular} \end{center} \normalsize

\textsuperscript{15} The average sales are calculated using Compustat North American Industrial database, and the top 28 firms are chosen depending on these values of average sales.
<table>
<thead>
<tr>
<th>Company Name</th>
<th>Number Owned</th>
<th>Number Registered</th>
<th>Number Assigned</th>
<th>Ratio Assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Motors</td>
<td>2339</td>
<td>2315</td>
<td>24</td>
<td>1.03</td>
</tr>
<tr>
<td>Exxon Mobil</td>
<td>985</td>
<td>779</td>
<td>206</td>
<td>20.91</td>
</tr>
<tr>
<td>Ford Motor</td>
<td>3619</td>
<td>3594</td>
<td>25</td>
<td>0.69</td>
</tr>
<tr>
<td>Chevrontexaco</td>
<td>1107</td>
<td>1003</td>
<td>104</td>
<td>9.39</td>
</tr>
<tr>
<td>General Electric</td>
<td>2470</td>
<td>2344</td>
<td>126</td>
<td>5.10</td>
</tr>
<tr>
<td>Altria Group</td>
<td>1185</td>
<td>1113</td>
<td>72</td>
<td>6.08</td>
</tr>
<tr>
<td>Chrysler</td>
<td>42</td>
<td>42</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>Boeing</td>
<td>575</td>
<td>561</td>
<td>14</td>
<td>2.43</td>
</tr>
<tr>
<td>Hewlett-Packard</td>
<td>2109</td>
<td>1983</td>
<td>126</td>
<td>5.97</td>
</tr>
<tr>
<td>Procter and Gamble</td>
<td>6581</td>
<td>6301</td>
<td>280</td>
<td>4.25</td>
</tr>
<tr>
<td>Koch Industries</td>
<td>91</td>
<td>76</td>
<td>15</td>
<td>16.48</td>
</tr>
<tr>
<td>Weyerhaeuser</td>
<td>640</td>
<td>611</td>
<td>29</td>
<td>4.53</td>
</tr>
<tr>
<td>United Technologies</td>
<td>950</td>
<td>914</td>
<td>36</td>
<td>3.79</td>
</tr>
<tr>
<td>Caterpillar</td>
<td>188</td>
<td>169</td>
<td>19</td>
<td>10.11</td>
</tr>
<tr>
<td>Dow Chemical</td>
<td>3013</td>
<td>2748</td>
<td>265</td>
<td>8.80</td>
</tr>
<tr>
<td>Pepsico</td>
<td>5105</td>
<td>4742</td>
<td>363</td>
<td>7.11</td>
</tr>
<tr>
<td>Johnson and Johnson</td>
<td>6832</td>
<td>6407</td>
<td>425</td>
<td>6.22</td>
</tr>
<tr>
<td>Visteon</td>
<td>108</td>
<td>90</td>
<td>18</td>
<td>16.67</td>
</tr>
<tr>
<td>Lockheed Martin</td>
<td>966</td>
<td>959</td>
<td>7</td>
<td>0.72</td>
</tr>
<tr>
<td>Dell</td>
<td>632</td>
<td>540</td>
<td>92</td>
<td>14.56</td>
</tr>
<tr>
<td>Merck and Co</td>
<td>3269</td>
<td>3026</td>
<td>243</td>
<td>7.43</td>
</tr>
<tr>
<td>Conagra Foods</td>
<td>2860</td>
<td>2506</td>
<td>354</td>
<td>12.38</td>
</tr>
<tr>
<td>Kraft General Foods</td>
<td>1283</td>
<td>1252</td>
<td>31</td>
<td>2.42</td>
</tr>
<tr>
<td>Company Name</td>
<td>Number Owned</td>
<td>Number Registered</td>
<td>Number Assigned</td>
<td>Ratio Assigned</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------</td>
<td>-------------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Pfizer</td>
<td>8150</td>
<td>7554</td>
<td>596</td>
<td>7.31</td>
</tr>
<tr>
<td>Coca-Cola</td>
<td>1772</td>
<td>1725</td>
<td>47</td>
<td>2.65</td>
</tr>
<tr>
<td>Intel</td>
<td>669</td>
<td>556</td>
<td>113</td>
<td>16.89</td>
</tr>
<tr>
<td>3M</td>
<td>2529</td>
<td>1940</td>
<td>589</td>
<td>23.29</td>
</tr>
</tbody>
</table>

Note: The name of original owner, that is, the name of registrant is listed in Table A.4.2.

96.43% of the top 28 firms have acquired trademarks during 1950-2013. For instance, the trademark “Quick Lube,” which has the serial numbers of “73197080” and “73197082,” transferred from “Quick Lube & Oil Inc.” to “General Motors” in 1996. This trademark is for automobile maintenance services, the installation of oil filters, air filters, and lubrication and oil changes. General Motors already has 79 other trademarks which are related to the same industry. The trademark “Signature” for tires of motor vehicles, which has the serial number of “73477688,” transferred from “Atlas Supply Company” to “Exxon Mobil” in 1985. The trademark “XPS” for a computer system, which has the serial number of “73602827,” transferred from “Xerox Corporation” to “Dell Inc.” in 2007. Table 9 shows the list of the industries of trademarks, and shows that most transactions occur within the same industries. Most firms acquire trademarks which are used in the same industry as the owned trademarks. Only “P&G,” “Merck and Co.” and “Conagra Foods” acquired the trademarks outside of the industries that they

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16 Only “Chrysler” did not acquire any trademark during that period. There is no trademark assigned to “Chrysler,” but still 26 trademarks are transferred to “Chrysler” by mergers.
had already offered products. The 2-digit Nice codes underlined in the second column represent the industries which “P&G,” “Merck and Co.” and “Conagra Foods” entered into by acquiring others’ trademarks. The descriptions of those industries classified 13, 14, 19 and 33 are below the Table 9. Third column represents the ratio of transactions occurred within-industry. The value in parenthesis is the number of trademarks transferred across industries.

<table>
<thead>
<tr>
<th>Company Name</th>
<th>Industries of trademarks registered</th>
<th>Industries of trademarks assigned</th>
<th>Ratio of within-industry transactions (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Motors</td>
<td>1-32, 34-43, 45</td>
<td>1, 4, 7-12, 25, 28, 36-37, 41</td>
<td>100</td>
</tr>
<tr>
<td>Exxon Mobil</td>
<td>1-12, 14, 16-19, 21-25, 27-28, 30, 32, 34-43, 45</td>
<td>1-5, 8-9, 12, 15-17, 19, 22-24, 27, 35, 37, 41-42</td>
<td>100</td>
</tr>
<tr>
<td>Ford Motor</td>
<td>1-21, 24-30, 32, 34-43, 45</td>
<td>6-9, 12, 16, 20, 24-25, 39, 42</td>
<td>100</td>
</tr>
<tr>
<td>Chevrontexaco</td>
<td>1-9, 11-12, 14, 16-22, 24-28, 30, 35-42</td>
<td>1-9, 11-12, 14, 16-17, 19-21, 36-37, 39, 42</td>
<td>100</td>
</tr>
<tr>
<td>General Electric</td>
<td>1-21, 25, 27-28, 31, 34-45</td>
<td>1-12, 15-17, 19-21, 27, 35, 36-38, 40-42, 44-45</td>
<td>100</td>
</tr>
<tr>
<td>Altria Group</td>
<td>3, 5, 8-12, 14, 16, 18, 20-21, 24-25, 28, 30, 32-36, 39, 41-44</td>
<td>3, 25, 30, 32-34</td>
<td>100</td>
</tr>
<tr>
<td>Company Name</td>
<td>Industries of trademarks registered</td>
<td>Industries of trademarks assigned</td>
<td>Ratio of within-industry transactions (%)</td>
</tr>
<tr>
<td>---------------------</td>
<td>-------------------------------------</td>
<td>----------------------------------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>Chrysler</td>
<td>1-4, 7, 9, 11-12, 21, 25-26, 36-37, 41-42</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>Boeing</td>
<td>1, 2, 4-12, 14, 16-18, 20-22, 24-28, 30, 34-42</td>
<td>2, 9, 12, 16, 25, 37, 38-39, 41</td>
<td>100</td>
</tr>
<tr>
<td>Hewlett-Packard</td>
<td>1-2, 5, 7-12, 14, 16, 17, 20-21, 24, 25, 28, 35-43, 45</td>
<td>1-2, 5, 7, 9-10, 16, 24, 35-42</td>
<td>100</td>
</tr>
<tr>
<td>P&amp;G</td>
<td>1-12, 14, 16-22, 24-32, 34-45</td>
<td>1, 3-5, 7-12, 16-18, 21, 25-26, 29-33, 35, 41-42</td>
<td>99.64 (1)</td>
</tr>
<tr>
<td>Koch Industries</td>
<td>1, 6-7, 9, 11, 16-17, 19-22, 35-37, 40-42</td>
<td>1, 9, 16, 19, 37</td>
<td>100</td>
</tr>
<tr>
<td>Weyerhaeuser</td>
<td>1-2, 4-7, 9-12, 16-17, 19-22, 24-25, 27-29, 31, 35-37, 39-42, 44</td>
<td>2, 6, 9, 16-17, 19-20, 31, 35, 40, 42</td>
<td>100</td>
</tr>
<tr>
<td>United Technologies</td>
<td>1, 3-4, 6-14, 16-18, 20-22, 24-26, 28-30, 32, 34-44</td>
<td>1, 6-9, 11-12, 16, 20-21, 28, 37, 42</td>
<td>100</td>
</tr>
<tr>
<td>Caterpillar</td>
<td>1-2, 4, 6-9, 11-12, 14, 16-18, 20-21, 25-26, 28, 34-37, 39-42, 44-45</td>
<td>4, 6-9, 11-12, 16-17, 20-21, 25, 28, 36, 41</td>
<td>100</td>
</tr>
<tr>
<td>Dow Chemical</td>
<td>1-13, 16-32, 35-45</td>
<td>1-5, 7, 9-11, 16-17, 19-20, 25, 30-31, 35, 42, 44</td>
<td>100</td>
</tr>
<tr>
<td>Company Name</td>
<td>Industries of trademarks registered</td>
<td>Industries of trademarks assigned</td>
<td>Ratio of within-industry transactions (%)</td>
</tr>
<tr>
<td>---------------------</td>
<td>------------------------------------</td>
<td>----------------------------------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>Pepsico</td>
<td>1-9, 11-12, 14-16, 18, 20-44</td>
<td>1, 5, 8, 14, 16, 18, 21-22, 25, 28-33, 35, 40-42, 44</td>
<td>100</td>
</tr>
<tr>
<td>Johnson and Johnson</td>
<td>1-12, 14, 16-22, 24-30, 32, 35-45</td>
<td>1, 3, 5, 8-12, 16-17, 21-22, 24-27, 29-30, 32, 35, 38, 40-42, 44</td>
<td>100</td>
</tr>
<tr>
<td>Visteon</td>
<td>1, 4, 6, 7, 9, 11-12, 16-17, 19-21, 25, 28, 35-36, 40, 42</td>
<td>1, 4, 6, 7, 9, 11-12, 17, 19, 21, 25, 28, 35-36, 40, 42</td>
<td>100</td>
</tr>
<tr>
<td>Lockheed Martin</td>
<td>1-2, 6-9, 11-14, 16-18, 20-22, 24-26, 28, 35-42, 45</td>
<td>9, 16, 25, 28, 35, 41</td>
<td>100</td>
</tr>
<tr>
<td>Dell</td>
<td>2, 9, 11, 16, 20, 25, 35-43, 45</td>
<td>7, 9, 35, 38, 39-42, 45</td>
<td>100</td>
</tr>
<tr>
<td>Conagra Foods</td>
<td>1-12, 15-22, 24-32, 34-44</td>
<td>1-9, 11-13, 14, 16-18, 20-21, 24-26, 28-33, 34-36, 39-42</td>
<td>98.87 (4)</td>
</tr>
<tr>
<td>Kraft General Foods</td>
<td>1, 5, 7-11, 14, 16, 20-21, 25, 28-32, 35-36, 41-43</td>
<td>1, 5, 16, 29-32, 42</td>
<td>100</td>
</tr>
<tr>
<td>Pfizer</td>
<td>1-22, 25-26, 28-32, 34-45</td>
<td>1-12, 14, 16, 18-21, 28-32, 35-36, 38, 41-42, 44-45</td>
<td>100</td>
</tr>
<tr>
<td>Company Name</td>
<td>Industries of trademarks registered</td>
<td>Industries of trademarks assigned</td>
<td>Ratio of within-industry transactions (%)</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------------------------------</td>
<td>----------------------------------</td>
<td>-------------------------------------------</td>
</tr>
<tr>
<td>Coca-Cola</td>
<td>1, 3-9, 11-12, 14-16, 18, 20-21, 24-45</td>
<td>25, 29-30, 32-33, 36, 39-41</td>
<td>100</td>
</tr>
<tr>
<td>Intel</td>
<td>6, 7, 9, 11-12, 14-16, 18, 20-21, 24-25, 28, 35-39, 41-42, 44-45</td>
<td>9, 16, 28, 35-38, 41-42, 45</td>
<td>100</td>
</tr>
</tbody>
</table>

Notes: International trademark classification are established by the “Committee of Experts of the Nice Union,” and set forth in the “International Classification of Goods and Services.” It is published by the World Intellectual Property Organization, which is available at [http://www.wipo.int/classifications/en/index.html](http://www.wipo.int/classifications/en/index.html). Class 13 is “Firearms.” Class 14 is “Jewelry.” Class 19 is “Non-metallic building material.” Class 33 is “Alcoholic beverage (except beer).”

For comparison, I conduct similar works on bottom 354 firms which make average sales below 1 million dollar. From that, we can observe that 13.28% of the bottom firms acquired other firms’ trademarks. The bottom firms have owned 3036 trademarks, and 106 trademarks of them had been assigned from others. That is, 3.49% of the trademarks were transferred from others, and these whole transactions occurred within the same industries.
3. Theoretical Model with CES Preference

In this section, I outline a theoretical model of multi-product firms and brand acquisition that is an extension of Agur (2010). The goal of this theoretical model is to introduce the simplest model necessary for investigating brand acquisition. Toward that end, I employ a number of simplifying assumptions, such as ruling out the vertical differentiation of brands, ruling out the bargaining situation in acquisition market and strategic incentive of acquisition. I return to a discussion of how the model might be generalized, and discuss alternative approaches.

3.1. Closed Economy

3.1.1. Preferences

Labor is the only fact of production and is inelastically supplied in a competitive market. Consumer’s preferences are given by a nested version of the standard CES Dixit–Stiglitz formulation. For simplicity, I assume that the first stage of utility takes Cobb-Douglas form. Consumer decides consumption optimally between the quantity index of a differentiated good Q and numeraire good Q₀ in the first stage.

\[ U = Q_0^\eta Q^{1-\eta}, \quad 0 < \eta < 1 \]  

(3.1)

By optimization, consumer spends \((1 - \eta)I\) on differentiated goods which is constant share of consumer income I. Additionally assume that the numeraire good is produced with identical constant returns to scale technology.
everywhere and is freely traded. This leads to international wage equalization.

The second stage utility function is given by the industry-level quantity index of the differentiated good.

\[ Q = \left( \int_{j=0}^{m} q_j \frac{\alpha-1}{\alpha} dj \right)^{\frac{\alpha}{\alpha-1}} \]  

(3.2)

where \( \alpha > 1 \) is the elasticity of substitution between brands and \( m \) is the mass of available brands. The quantity index of brand \( j \in [0, m], q_j \), corresponds to the third stage sub-utility function and is given by

\[ q_j = \left( \int_{i=0}^{h_j} q_{ji} \frac{\sigma-1}{\sigma} di \right)^{\frac{\sigma}{\sigma-1}} \]  

(3.3)

where \( \sigma > 1 \) is the elasticity of substitution between varieties within a brand. \( h_j \) is the mass of available varieties of brand \( j \) and \( q_{ji} \) is consumption of a typical variety \( i \in [0, h_j] \). In this paper, assume that \( \sigma > \alpha \) like Agur (2010), Allason and Montagna (2005), and let \( \gamma \equiv \sigma - \alpha \). This implies that a variety is more substitutable with the variety from same brand than the one from different brand. Under this assumption, when a firm add new variety into its brand the demand of existing varieties decreases, that is within-brand cannibalization occurs. The value of \( \gamma \) captures the degree of within-brand cannibalization with \( \gamma = 0 \) implying no cannibalization. \(^{17}\) The rate of cannibalization from total brand quantity \( q_j \) is calculated by

\[ \frac{\partial p_{ji}}{\partial q_j} \frac{q_j}{p_{ji}} = -\frac{\gamma}{\sigma \alpha} \]  

(3.4)

\(^{17}\) Further interpretation about \( \gamma \) is illustrated in section 3.4.
If there is no within-brand cannibalization effect, that is \( \sigma = \alpha \), then all varieties are equally substitutable and nested-CES utility function reduces to a single stage CES function. Furthermore, we can easily verify that there is no acquisition incentive.\(^{18}\)

The price indices are respectively given by

\[
P = \left( \int_{j=0}^{m} p_j^{1-\alpha} \, dj \right)^{\frac{1}{1-\alpha}} \tag{3.5}
\]

\[
p_j = \left( \int_{i=0}^{h_j} p_{ji}^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}} \tag{3.6}
\]

where \( P \) is the price index of differentiated goods, \( p_j \) is the price index of the varieties within the brand \( j \) and \( p_{ji} \) is the price of variety \( i \) of brand \( j \). From utility maximization,

\[
\max_{q_j} Q = \left( \int_{j=0}^{m} q_j^{\frac{\alpha-1}{\alpha}} \, dj \right)^{\frac{\alpha}{\alpha-1}} \text{ subject to } (1 - \eta)l = R = P \cdot Q
\]

\[
q_j = Q \left( \frac{p_j}{P} \right)^{-\alpha} \tag{3.7}
\]

and

\[
\max_{q_{ji}} q_j = \left( \int_{i=0}^{h_j} q_{ji}^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}} \text{ subject to } p_j q_j = \int_{i=0}^{h_j} p_{ji} q_{ji} \, di
\]

\[
q_{ji} = q_j \left( \frac{p_{ji}}{p_j} \right)^{-\alpha} = Q \left( \frac{p_{ji}}{p_j} \right)^{-\alpha} \left( \frac{p_{ji}}{p_j} \right)^{-\sigma} \tag{3.8}
\]

The demand for the individual variety depends negatively on its price and positively on both the brand-level and industry-level price indices.

---

\(^{18}\) Under \( k > 1 \), however, there still exists firms that produce multiple varieties.
3.1.2. Production Technology

There is a continuum of firms, which choose to produce different sets of varieties. Production requires only one factor, labor. Firms draw their productivity level \( \varphi \) only after making the irreversible investment \( f_e \) required for entry. Incurring the sunk entry cost creates unique brand which is horizontally differentiated among brands. After entry, a firm decides range of varieties by incurring R&D cost, then decides the quantities of each variety given its brand.\(^{19}\) The total cost function of a firm \( j \) that produces \( h_j \) varieties of the goods is given by

\[
c_j = f + \lambda(h_j)^k + \frac{w}{\varphi_j} \left( \int_{i=0}^{h_j} q_{ji} \, di \right) \quad \text{where } k > 1
\]

where \( \varphi_j \) is the productivity of firm \( j \) and \( w \) is common wage rate hereafter normalized to one. \( \lambda(h_j)^k \) is variety-specific R&D cost related to creating a new variety.\(^{20}\) \( k>1 \) implies that marginal variety-specific cost increases as the number of products increases. This means that it gets harder to create additional variety differentiated from its old varieties. Or this can be interpreted as managing additional variety gets costly as the total number of varieties increases. Suppose that productivity \( \varphi \) is Pareto distributed with the following density function:\(^{21}\)

\[
g(\varphi) = \theta \varphi_{min}^\theta \varphi^{-(\theta+1)}, \quad \varphi_{min} > 0, \quad \theta > 0
\]

---

\(^{19}\) The simultaneous decision of \( h_j \) and \( q_{ji} \) gives the same results.

\(^{20}\) This cost is also related to advertisement, management, and maintenance of distribution network.

\(^{21}\) Del Gatto, Mion & Ottaviano (2006) shows that Pareto distribution provides a very good fit for firm productivity across sectors and countries.
Given the demand functions, firm $j$’s profit is given by

$$
\Pi_j^N = \int_{i=0}^{h_j} \left( p_{ji} - \frac{1}{\varphi_j} \right) q_{ji} \, di - \lambda(h_j)^k - f \equiv \pi_j^N - f \tag{3.11}
$$

This yields the first order condition for variety price, and we get

$$
p_{ji} = \frac{\sigma}{\varphi_j(\sigma-1)} \text{ for all } i \in [0, h_j] \tag{3.12}
$$

and

$$
p_j = \left( \int_{i=0}^{h_j} p_{ji}^{1-\sigma} \, di \right)^{\frac{1}{\sigma-1}} = \frac{\frac{1}{\varphi_j(\sigma-1)}}{\frac{1}{h_j^{\frac{1}{\sigma-1}}}} \tag{3.13}
$$

where $\sigma/(\sigma - 1)$ is the constant mark-up over marginal cost. From above pricing rule, we can derive the relation between quantity of each variety and scope.

$$
q_{ji} = R\varphi_j^{\sigma-1} \left( p_j \right)^{\frac{\sigma}{\sigma-1}} = R\varphi_j^{\sigma-1} \left( \frac{\varphi_j}{\sigma-1} \right)^{-\frac{\sigma}{\sigma-1}}
$$
This implies that optimal quantity of variety $i$ decreases as the range of product $h_j$ increases. The profit function can be rewritten as:

$$\Pi_j^N = R P^{\alpha - 1} (h_j)^{\frac{\alpha - 1}{\sigma - 1}} \sigma^{-\alpha} (\sigma - 1)^{1 - \alpha} (\varphi_j)^{\alpha - 1} - \lambda(h_j)^k - f$$

And we have

$$h_j = \left( R P^{\alpha - 1} \right)^{\frac{\alpha - 1}{\sigma - 1}} \frac{\sigma - 1}{\sigma - 1} \frac{1}{k \lambda} \frac{\lambda - \alpha(\sigma - 1)}{(\sigma - 1)(\alpha - 1)} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}}$$

$$p_j = \left( R P^{\alpha - 1} \right)^{\frac{\alpha - 1}{\sigma - 1}} \frac{\sigma - 1}{\sigma - 1} \frac{1}{k \lambda} \frac{\lambda - \alpha(\sigma - 1)}{(\sigma - 1)(\alpha - 1)} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}}$$

CES preference and monopolistic competition imply that prices are a constant mark-up over marginal costs. These property gives that the relative quantities, scopes and revenues depend solely on the relative firm abilities:

$$\frac{q_j(\varphi)}{q_j(\varphi')} = \left( \frac{\varphi}{\varphi'} \right)^{\frac{\alpha k(\sigma - 1)}{(\sigma - 1)(\alpha - 1)}} \frac{h_j(\varphi)}{h_j(\varphi')} = \left( \frac{\varphi}{\varphi'} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{(\sigma - 1)(\alpha - 1)}} \frac{\sigma + (k - 1)(\sigma - 1)}{\sigma + (k - 1)(\sigma - 1)} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}} \left( \frac{\alpha - 1}{\sigma - 1} \right)^{\frac{(\sigma - 1)(\alpha - 1)}{\sigma - 1}}$$

That is, firms with lower abilities supply smaller products to market and have lower profits. For sufficiently low value of $\varphi$, the variable profits cannot cover the fixed production cost. Therefore there is a zero-profit cutoff for firm productivity, $\varphi^*$, such that a firm exit if its productivity is lower than $\varphi^*$. Such
cutoff is defined by the following zero-profit condition.\(^{22}\)

\[
\Pi_j^N(\varphi^*) = \frac{r_j(\varphi^*)}{\alpha} - \lambda \left(h_j(\varphi^*)\right)^k - f = 0
\]  

(3.15)

3.1.3. Acquisition of Brand

Now introduce the possibilities of brand acquisition into the basic model. Then the timing of the model is changed as follows. In stage one, a firm decides whether to enter, and after paying entry cost \(f_i\), firm receives its brand name and observes its productivity level \(\varphi\). Then each firm chooses the product range \(h_j\) by incurring the investment cost. In the second stage, acquisition decision is made and the adjusted \(h_j\) for acquired brand is decided subsequently. In this model, I assume that acquirer can produce varieties of acquired brand with its own productivity. Finally, in stage three, each active firm supplies its varieties to the product market.\(^{23}\) For simplicity assume no discount factor. In the acquisition stage, firms must choose between three options: selling their brand by exit the market, doing nothing, or buying other firm’s brand.\(^{24}\) This decision depends on the acquisition price which is determined in the acquisition market. Here this paper only considers the case where acquisition price is zero, because of the complexity of determination of acquisition price. When the number of

\(^{22}\) Using \(\varphi^*\) we can rewrite \(h_j\) as follows:

\[
h_j = \left(\frac{f k}{(1-\alpha)+k(\sigma-1)}\right)^{\frac{1}{k}} \left(\frac{\alpha-1}{k\lambda}\right)^{\frac{1}{k}} \left(\frac{\varphi_j}{\varphi^*}\right)^{\frac{(\alpha-1)(\sigma-1)}{(1-\alpha)+k(\sigma-1)}}
\]

\(^{23}\) Simultaneous decision of \(q\) and \(h\) will not make any difference in the case of zero acquisition price.

\(^{24}\) Creating new brand by incumbent firm is not considered in this model, but the creating new brand by new entry is considered at the industry equilibrium.
firms who want to sell their brand is bigger than the number of ones who want to buy, the acquisition price is determined by zero. In this case, some firms cannot sell their brand and are forced to exit. Unlike this, the case with positive acquisition prices generates some selling firms who would produce profitably in the production market.

The problem of non-participant in acquisition stage is the same with the problem of basic model. Potential acquirer $j$’s profit will be given by:

$$\Pi^A_j = \int_{i=0}^{h_j} \left( p_{ji} - \frac{1}{\phi_{ji}} \right) q_{ji} \, di + v_j \int_{i=0}^{h_j} \left( p_{ji}^a - \frac{1}{\phi_{ji}} \right) q_{ji}^a \, di - \lambda (h_j + v_j h_j^a)^k - f(1 + v_j)$$

(3.16)

where $\Pi^A_j \equiv \pi^A_j - f(1 + v_j)$. In this setting, I assume that variety-specific cost not only depends on the number of varieties but also the number of brands.\textsuperscript{25}

Using FOCs, we can derive the price of varieties which are the same for its original brand and acquired brand.

$$p_{ji} = \frac{\sigma}{\phi_{ji}(\sigma-1)} \text{ for all } i \in [0, h_j]$$

$$p_{ji}^a = \frac{\sigma}{\phi_{ji}(\sigma-1)} \text{ for all } i \in [0, h_j^a]$$

We can derive the relation between scope and the number of brands.

$$(h_j)^k (1 + v_j)^{k-1} = \frac{f}{k \lambda} \cdot \frac{\sigma-1}{\sigma-\alpha}$$

(3.17)

\textsuperscript{25} Combining this property with within-brand cannibalization effect, the incentive of acquiring another brand arises. Without this assumption, brand acquisition arise only because of productivity differences, and this cannot explain the tradeoff between adding variety and adding brand.
\[h_j = h^a_j = \left\{ f \left( \frac{1}{R P^{a-1}} \right) \frac{(\sigma-1)}{\sigma-\alpha} \right\}^{\frac{\sigma-1}{\alpha-1}} \frac{\alpha \left(1-\sigma\right)}{\alpha-1} (\sigma - 1)^{1-\sigma} (\varphi_j)^{1-\sigma}\]

\[1 + v_j = (RP^{a-1})^{k(\sigma-1)} \left( \frac{1}{(\sigma-1)(k-1)} \right) \frac{1}{k} \left( \frac{\alpha-1}{k \lambda} \right)^{k-1} \left( \frac{1}{(\alpha-1)(k-1)^\alpha-1} \right) \frac{1}{f} \left( \frac{1-\alpha}{\alpha-1} \right) \frac{1}{f} \left( \frac{1-\alpha}{\alpha-1} \right) \frac{1}{(\sigma-1)(k-1)} \frac{1}{(\sigma-1)(k-1)} (\sigma - 1)^{1-\sigma} \]

We can see that expanding scope and acquiring existing brand is substitute as a way of expanding production. The optimal scope decreases as the number of brands increases. Total number of varieties which includes the varieties of acquired brand is given by

\[h^A_j \equiv h_j + v_j h^a_j = (RP^{a-1})^{k(\sigma-1)} \left( \frac{1}{(\sigma-1)(k-1)} \right) \frac{1}{k} \left( \frac{\alpha-1}{k \lambda} \right)^{k-1} \left( \frac{1}{(\alpha-1)(k-1)^\alpha-1} \right) \frac{1}{f} \left( \frac{1-\alpha}{\alpha-1} \right) \frac{1}{f} \left( \frac{1-\alpha}{\alpha-1} \right) \frac{1}{(\sigma-1)(k-1)} \frac{1}{(\sigma-1)(k-1)} (\sigma - 1)^{1-\sigma} \]

The number of managing brands increases as the firm ability increases, but varieties per brand decreases. Together, total number of varieties increases as the firm ability increases. Sales of varieties and sales of acquiring firms are given by

\[r_{ji} = r^a_{ji} = (RP^{a-1})^{k(\sigma-1)} \left( \frac{1}{(\sigma-1)(k-1)} \right) \frac{1}{k} \left( \frac{1-\alpha}{\alpha-1} \right) \frac{1}{f} \left( \frac{1-\alpha}{\alpha-1} \right) \frac{1}{f} \left( \frac{1-\alpha}{\alpha-1} \right) \frac{1}{(\sigma-1)(k-1)} \frac{1}{(\sigma-1)(k-1)} (\sigma - 1)^{1-\sigma} \]

\[r^A_{ji} \equiv r_j + v_j r^a_j = (RP^{a-1})^{k(\sigma-1)} \left( \frac{1}{(\sigma-1)(k-1)} \right) \frac{1}{k} \left( \frac{1-\alpha}{\alpha-1} \right) \frac{1}{f} \left( \frac{1-\alpha}{\alpha-1} \right) \frac{1}{f} \left( \frac{1-\alpha}{\alpha-1} \right) \frac{1}{(\sigma-1)(k-1)} \frac{1}{(\sigma-1)(k-1)} (\sigma - 1)^{1-\sigma} \]

Because of CES preference and monopolistic competition, the relative
quantities, revenues, product ranges and number of brands within the same
groups depend solely on relative firm abilities:

\[
\frac{q_j^A(\varphi)}{q_j^R(\varphi')} = \frac{r_j^A(\varphi)}{r_j^R(\varphi')} = 1
\]

\[
\frac{r_j^A(\varphi)}{r_j^R(\varphi')} = \left(\frac{\varphi}{\varphi'}\right)^{\frac{k(\sigma-1)}{k-1}}, \quad \frac{\pi_j^A(\varphi)}{\pi_j^R(\varphi')} = \left(\frac{\varphi}{\varphi'}\right)^{\frac{k(\sigma-1)}{k-1}},
\]

\[
\frac{h_j^A(\varphi)}{h_j^R(\varphi')} = \left(\frac{\varphi}{\varphi'}\right)^{1-\sigma}, \quad \frac{1+v_j(\varphi)}{1+v_j(\varphi')} = \left(\frac{\varphi}{\varphi'}\right)^{\frac{k(\sigma-1)}{k-1}}
\]

Supplying market with multiple brands generates additional fixed cost. Thus
there is zero acquisition profit cutoff for firm productivity, \( \varphi_a^* \), such that a firm
only participates in acquisition if it draws a value of \( \varphi \) equal to or greater than
\( \varphi_a^* \). This acquisition cutoff will be determined by

\[
\Pi_j^A(\varphi_a^*) = \Pi_j^N(\varphi_a^*)
\]

\[
\varphi_a^* = \Gamma \varphi^* \text{ where } \Gamma = \left\{\frac{(1-\alpha)+k(\sigma-1)}{(\sigma-\alpha)k}\right\}^{\frac{1}{k(\sigma-1)}}
\]

Since \( \frac{(1-\alpha)+k(\sigma-1)}{(\sigma-\alpha)k} > 1 \) and \( \frac{(1-\alpha)+k(\sigma-1)}{k(\alpha-1)(\sigma-1)} > 0 \), \( \varphi_a^* > \varphi^* \) is satisfied, which
implies firm selection into acquisition markets.\(^{26}\)

### 3.1.4. Firm Entry and Exit

To enter, firms must make an initial investment, a fixed entry cost \( f_e >

\[\]

\(^{26}\) Using \( \varphi_a^* \) we can rewrite \( h_j^A \) as follows:

\[
h_j^A \equiv h_j + v_j h_j = \left(\frac{a-1}{\delta}\right)^{\frac{1}{\delta}} \left(\frac{f}{\varphi_a^*}\right)^{\frac{1}{\delta}} \left(\frac{\varphi_j}{\varphi_a^*}\right)^{\frac{\sigma-1}{k-1}}
\]
which is measured in units of labor. Firms then draw their initial productivity parameter \( \varphi \) from a common distribution \( g(\varphi) \) which has a continuous cumulative distribution, \( G(\varphi) \). Any entrant firms drawing a productivity level \( \varphi < \varphi^* \) will immediately exit or sell their brands. Hence, firm level conditional distribution of \( g(\varphi) \) on \((\varphi^*, \infty)\):

\[
\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi)}, & \varphi \geq \varphi^* \\ 0, & Otherwise \end{cases}
\] (3.21)

\( 1 - G(\varphi^*) \) is the ex-ante probability of successful entry. The weighted average of firm productivity levels \( \tilde{\varphi}^N \) and \( \tilde{\varphi}^A \) are defined as:

\[
\tilde{\varphi}^N \equiv \left\{ \frac{1}{G(\varphi^*_A) - G(\varphi^*)} \int_{\varphi^*_A}^{\infty} g(\varphi)^{k(\sigma-1)(\alpha-1)} \left( \frac{k(\sigma-1)(\alpha-1)}{1-\alpha+k(\sigma-1)} \right) d\varphi \right\}^{(1-\alpha)+k(\sigma-1)}
\] (3.22)

\[
\tilde{\varphi}^A \equiv \left\{ \frac{1}{1-G(\varphi^*_A)} \int_{\varphi^*_A}^{\infty} g(\varphi)^{k(\sigma-1)} \left( \frac{k(\sigma-1)}{1-\alpha+k(\sigma-1)} \right) d\varphi \right\}^{k-1}
\] (3.23)

where \( \tilde{\varphi}^N \) is the weighted average productivity within non-participants, and \( \tilde{\varphi}^A \) is the weighted average productivity within acquirers.\(^{27}\) Note that exponents of weighted average of productivity satisfy \( \frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)} < \frac{k(\sigma-1)}{k-1} \), and these numbers reflect the relative sales of firms with different productivity levels.

\[
\frac{r^A_j(\varphi)}{r^A_j(\varphi^*)} = \left( \frac{\varphi}{\varphi^*} \right)^{k(\sigma-1)} \left( \frac{k(\sigma-1)}{1-\alpha+k(\sigma-1)} \right), \quad \frac{r^N_j(\varphi)}{r^N_j(\varphi^*)} = \left( \frac{\varphi}{\varphi^*} \right)^{k(\sigma-1)(\alpha-1)} \left( \frac{k(\sigma-1)(\alpha-1)}{1-\alpha+k(\sigma-1)} \right)
\]

The average revenue will be rewritten as

\(^{27}\) Throughout this paper, superscript ‘A’ stands for acquirer related value.
where \( \bar{r}^N \) is average revenue of non-participants and \( \bar{r}^A \) is average revenue of acquirers.

\[
\bar{r}^N \equiv \frac{1}{G(\hat{\phi}_a) - G(\hat{\phi}_*)} \int_{\phi^*}^{\phi_a} r_j(\phi) g(\phi) \, d\phi = r^N(\bar{\phi}^N)
\]

\[
\bar{r}^A \equiv \frac{1}{1 - G(\hat{\phi}_a)} \int_{\phi^*}^{\phi_a} r_j^A(\phi) g(\phi) \, d\phi = r^A(\bar{\phi}^A)
\]

Using the properties of (3.14) and (3.18), we obtain

\[
\bar{r}^N = r^N(\bar{\phi}^N) = \left( \frac{\bar{\phi}^N}{\phi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{1-\sigma+k(\sigma-1)}} r^N(\phi^*)
\]

\[
\bar{r}^A = r^A(\bar{\phi}^A) = \left( \frac{\bar{\phi}^A}{\phi_a^*} \right)^{\frac{k(\sigma-1)}{k-1}} r^A(\phi_a^*)
\]

Similarly, the average profit will be rewritten as:

\[
\bar{\Pi} = \frac{1}{1-G(\phi^*)} \int_{\phi^*}^{\phi_a} \Pi_j g(\phi) \, d\phi
\]

\[
\leftrightarrow \bar{\Pi} = \frac{G(\phi_a^*)-G(\phi^*)}{1-G(\phi^*)} \bar{\Pi}^N + \frac{1-G(\phi_a^*)}{1-G(\phi^*)} \bar{\Pi}^A
\]

where \( \bar{\Pi}^N \) is average profit of non-participants and \( \bar{\Pi}^A \) is average profit of acquirers. Then the average profit of non-participants is

\[
\bar{\Pi}^N = \bar{\pi}^N - f
\]

\[
\bar{\pi}^N = \frac{1}{G(\phi_a^*)-G(\phi^*)} \int_{\phi^*}^{\phi_a^*} \pi_j^N(\phi) g(\phi) \, d\phi = \pi^N(\bar{\phi}^N).
\]
\[ \bar{\pi}^N = \pi^N(\phi^N) = \left( \frac{\phi^N}{\varphi^*} \right)^{k(\sigma-1)(\alpha-1)} (1-\alpha+k(\sigma-1)) \pi^N(\varphi^*) = \left( \frac{\phi^N}{\varphi^*} \right)^{k(\sigma-1)(\alpha-1)} (1-\alpha+k(\sigma-1)) f. \]

The average profit of acquirers is

\[ \bar{\Pi}^A = \bar{\pi}^A - f(1 + \bar{v}) \]

\[ \bar{\pi}^A = \frac{1}{1-G(\varphi^*_a)} \int_{\varphi^*_a}^{\infty} \pi^A(\varphi) g(\varphi) d\varphi = \pi^A(\bar{\phi}^A) \]

\[ \pi^A(\bar{\phi}^A) = \left( \frac{\varphi^A}{\varphi^*_a} \right)^{k(\sigma-1)} \pi^A(\varphi^*_a) = \left( \frac{\varphi^A}{\varphi^*_a} \right)^{k(\sigma-1)} f \frac{(1-\alpha)+k(\sigma-1)}{(\sigma-\alpha)k}. \]

The average number of acquisitions will be

\[ 1 + \bar{v} = 1 + v(\bar{\phi}^A) \]

\[ = \left( \frac{\varphi^A}{\varphi^*_a} \right)^{k(\sigma-1)} \left( 1 + v(\varphi^*_a) \right) = \left( \frac{\varphi^A}{\varphi^*_a} \right)^{k(\sigma-1)} \left( 1 + v(\varphi^*_a) \right). \] (3.26)

Since the average productivity level is determined by the cutoff productivity level \( \varphi^* \) and \( \varphi^*_a \), the average revenues and profit levels are also tied to the cutoff levels.

\[ \bar{\pi}^N = \left( \frac{\phi^N}{\varphi^*} \right)^{k(\sigma-1)(\alpha-1)} (1-\alpha+k(\sigma-1)) f \]

\[ \bar{\pi}^A = \left( \frac{\varphi^A}{\varphi^*_a} \right)^{k(\sigma-1)} f \frac{(1-\alpha)+k(\sigma-1)}{(\sigma-\alpha)k}. \]

Firms decide whether to enter based on a comparison of the expected value of entry and the sunk entry cost. The free entry condition takes the following form:

\[ V_e = \{1 - G(\varphi^*)\} \bar{\Pi} = f_e \] (3.27)

where
\[ V_e = \{1 - G(\varphi^*)\} \left[ \frac{G(\varphi_a) - G(\varphi^*)}{1 - G(\varphi^*)} (\bar{\pi}^N - f) + \frac{1 - G(\varphi_a)}{1 - G(\varphi^*)} (\bar{\pi}^A - f (1 + \bar{v})) \right] \]

Substitute \( \varphi_a = \Gamma \varphi^* \) then, FE condition can be expressed in terms of a single unknown \( \varphi^* \)

\[
f \frac{(1-\alpha) + k(\sigma - 1)}{(\sigma - \alpha)k} \int_{\varphi^*}^{\Gamma \varphi^*} \left\{ \left( \frac{\varphi}{\Gamma \varphi^*} \right)^{\frac{k(\sigma - 1)(\alpha - 1)}{1-\alpha + k(\sigma - 1)}} - \frac{(\sigma - \alpha)k}{(1-\alpha + k(\sigma - 1))} \right\} g(\varphi) \, d\varphi + \frac{f(\alpha - 1)(k-1)}{(\sigma - \alpha)k} \int_{\varphi^*}^{\infty} \left( \frac{\varphi}{\Gamma \varphi^*} \right)^{\frac{k(\sigma - 1)}{k-1}} g(\varphi) \, d\varphi = f_e \quad (3.28) \]

This free entry condition has the following properties. (i) The right-hand side is the constant sunk entry cost \( f_e > 0 \); (ii) As \( \varphi^* \to 0 \), the left-hand side converges towards infinity; (iii) As \( \varphi^* \to \infty \), the left-hand side converges towards zero; (iv) The left-hand side is monotonically decreasing in \( \varphi^* \). It follows that there exists a unique fixed point, where the expected value of entry \( V_e \) is equal to the sunk entry cost \( f_e \).

**Proposition 1.** There exists a unique equilibrium with zero acquisition price, referenced by a vector \( \{ \varphi^*, \Gamma \} \).

**Proof.** See Appendix.

### 3.1.5. Aggregation

An equilibrium will be characterized by a mass of brands and a distribution \( g(\varphi) \) of productivity levels. Let \( M^a \) be a mass of acquired brands, and \( M^A \) be a mass of acquisition related brands, that is, the mass of brands of acquirers. \( M^N \) is a mass of brands which are not involved in acquisition, and \( M_e \) is a mass of brands entering. Let \( N^A \) be a mass of acquirers, \( N^N \) be a mass of non-participants, and \( N_e \) be a mass of entrants.
Every firm receives a brand when it pays entry cost, thus the number of entrants and the mass of brand entering are equal, \( M_e = N_e \). Additionally \( M^H \) represents total mass of brands which is sum of \( M^A \) and \( M^N \). Similarly \( N^H \) represents total number of firms which is sum of \( N^A \) and \( N^N \). Aggregate price index \( P \) is defined by (3.5), thus \( P \) is given by

\[
P = \left[ \int_{\phi^*}^{\phi} p_j^N(\phi) N^N \frac{g(\phi)}{g(\phi^*)} d\phi + \int_{\phi^*}^{\infty} p_j^A(\phi)^{1-\alpha} \left\{ \frac{1}{1-\alpha} \left[ \frac{v_j(\phi) N^A}{1-G(\phi^*)} d\phi \right] \right\} \right]^{1-\alpha} \tag{3.29}
\]

and which is rewritten as:

\[
P = \left\{ M^N p_j^N(\bar{\phi}^N)^{1-\alpha} + M^A p_j^A(\bar{\phi}^A)^{1-\alpha} \right\}^{1-\alpha}
\]

Let’s define \( P^N ( P^A ) \) is an aggregate price index of non-participants’ (acquirers’) brands as follows:

\[
P^N = \left( M^N p_j^N(\bar{\phi}^N)^{1-\alpha} \right)^{\frac{1}{1-\alpha}} = (M^N)^{\frac{1}{1-\alpha}} p_j^N(\bar{\phi}^N)
\]

\[
P^A = \left( M^A p_j^A(\bar{\phi}^A)^{1-\alpha} \right)^{\frac{1}{1-\alpha}} = (M^A)^{\frac{1}{1-\alpha}} p_j^A(\bar{\phi}^A)
\]

Then we have,

\[
P = \left\{ (P^N)^{1-\alpha} + (P^A)^{1-\alpha} \right\}^{\frac{1}{1-\alpha}}.
\]

In equilibrium, the mass of new entrants is determined by

\[
N^H = (1 - G(\phi^*)) N_e.
\]

Aggregate number of brands \( M^H \) will be the sum of successful brands and acquired brands.
\[ M^H = N^N + (1 + \bar{\nu})N^A \]

Aggregate labor used for investment by entrant is

\[ L_e = M_e f_e = N_e \{1 - G(\varphi^*)\} \bar{N} = N \bar{N} \tag{3.30} \]

and aggregate labor used for production and R&D investment is

\[ L_p = R - N^N \bar{N}^N - N^A \bar{N}^A \tag{3.31} \]

where

\[ N^N \bar{N}^N + N^A \bar{N}^A = \frac{G(\varphi^*_d) - G(\varphi^*)}{1 - G(\varphi^*)} N \bar{N}^N + \frac{1 - G(\varphi^*_d)}{1 - G(\varphi^*)} N \bar{N}^A \]

\[ = N \left\{ \frac{G(\varphi^*_d) - G(\varphi^*)}{1 - G(\varphi^*)} \bar{N}^N + \frac{1 - G(\varphi^*_d)}{1 - G(\varphi^*)} \bar{N}^A \right\} = N \bar{N} \]

Therefore, we can derive following condition.

\[ L_e + L_p = R = (1 - \eta)L \tag{3.32} \]

Equation (3.32) implies that aggregate revenue in differentiated good sector must equal to the total payments to labor \( L \) which is employed that sector. The mass of firms can be determined from

\[ N = \frac{R}{\bar{r}} = \frac{(1 - \eta)L}{\bar{r}} \tag{3.33} \]

where average revenue is given by\(^{28}\)

\[^{28}\text{The average revenue is given by } \bar{r} = \frac{G(\varphi^*_d) - G(\varphi^*)}{1 - G(\varphi^*)} \bar{r}^N + \frac{1 - G(\varphi^*_d)}{1 - G(\varphi^*)} \bar{r}^A \text{ where } \bar{r}^A = \]

\[ \left( \frac{\varphi^*_A}{\varphi^*_d} \right)^{\frac{k(\sigma - 1)}{k - 1}} \frac{\sigma(\sigma - 1)}{\sigma - \alpha} \text{ and } \bar{r}^N = \left( \frac{\varphi^*_N}{\varphi^*_d} \right)^{\frac{k(\sigma - 1)(\alpha - 1)}{(1 - \alpha) + k(\sigma - 1)}} \frac{f k \sigma(\sigma - 1)}{(1 - \alpha) + k(\sigma - 1)} \]
\[ \tilde{r} = \frac{G(\varphi^*_a) - G(\varphi^*)}{1 - G(\varphi^*)} \left( \frac{\varphi^N}{\varphi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} \frac{f\kappa\sigma(\sigma-1)}{(1-\alpha)+k(\sigma-1)} + \frac{1-G(\varphi^a)}{1-G(\varphi^*)} \right) \]

Additionally, for zero acquisition price to arise as an equilibrium, the following condition must be satisfied.

\[ \tilde{v}\{1 - G(\varphi^a)\} < G(\varphi^*) \quad (3.34) \]

### 3.2. Open Economy with Costly Trade

In this section, consider the open economy with costly trade. For simplicity, I assume symmetry among \( n \) number of countries. This symmetry assumption ensures that all countries share the same wage and the same aggregate variables. Each firm’s pricing rule in its domestic market is given, as before. Firms who export will set higher prices in the foreign markets that reflects the increased marginal cost \( \tau > 1 \) of serving foreign markets. Additionally, assume that a firm must incur a fixed exporting cost \( f_x \) per export country and per brand.

#### 3.2.1. Equilibrium in Open Economy

In open economy model, firms face additional decision margin, whether to export or not. Take account of its exporting and acquisition decisions together, we can divide firms into four groups referenced by superscripts \( \{Naut, Nopen, Aaut, Aopen\} \). Superscript \( Naut \) represents non-exporting non-participant, and \( Nopen \) represents exporting non-participant. \( Aaut \) indicates
non-exporting acquirer, and \( A_{\text{open}} \) indicates exporting acquirer. Since the profit function of non-exporter is the same with the one in the Section 3.1, optimal decisions of these firms will be the same. Therefore it is needed to consider the decisions of exporting firms.

The profit function of exporting non-participant is given by

\[
\Pi_j^{\text{Open}}(\phi) = \frac{r^d(\phi)}{\sigma} + n \frac{r^x(\phi)}{\sigma} - \lambda \{h_j(\phi)\}^k - f - nf_x
\]  

(3.35)

where

\[
\frac{r^d(\phi)}{\sigma} = h_j \left(p_{ji} - \frac{1}{\phi_j}\right) q_{ji}, \quad \frac{r^x(\phi)}{\sigma} = h_j \left(p_{ji}^x - \frac{\tau}{\phi_j}\right) q_j^x.
\]

Superscript “d” represents domestic related value and “x” stands for exporting related one. Every exporting firm will produce for its domestic market, since domestic variable profit is always positive under CES preference and the production cost \( f \) is already incurred. Similarly, it is always profitable to export all the varieties after incurring a R&D cost and developing varieties, since there is no additional variety specific exporting cost and marginal revenue of each variety is always positive. Therefore, firm provides the same range of products to foreign markets and domestic market. As firms face the same elasticity of demand in foreign markets, export prices are a constant multiple of domestic prices due to the variable trade costs:

\[
p_{ji}^x = \frac{\tau \sigma}{\phi_j(\sigma-1)} \text{ for all } i \in [0, h_j]
\]  

(3.36)

thus

\[
p_j^x = \left( \int_{l=0}^{h_j} p_{ji}^x \frac{1}{\sigma} \, dl \right)^{\frac{1}{1-\sigma}} = \frac{\tau \sigma (h_j)^{\frac{1}{\sigma}}}{\phi_j(\sigma-1)}
\]
Using the above pricing rule, the optimal scope of non-participant \( j \) is:

\[
h_j = \left\{ RP^{\alpha-1} \left( \frac{\alpha-1}{k\lambda} \right) \sigma^{-\alpha} (\sigma - 1)^{\alpha-2} (1 + nt^{1-\alpha})(\varphi_j)^{\alpha-1} \right\}^{\frac{\sigma-1}{(1-\alpha)+k(\sigma-1)}}
\]

In a similar way, exporting acquirer \( j \)'s profit is given by

\[
\Pi_j^{\text{Open}} = \frac{r_j^{Ad}(\varphi)}{\sigma} + n \frac{r_j^{Ax}(\varphi)}{\sigma} - \lambda \{ h_j^{A}(\varphi) \}^k - f \{ 1 + v_j(\varphi) \} - n f_x \{ 1 + v_j(\varphi) \}
\]

where

\[
r_j^{Ad}(\varphi) = r_j^{d}(\varphi) \{ 1 + v_j(\varphi) \}, \quad h_j^{A}(\varphi) = h_j(\varphi) \{ 1 + v_j(\varphi) \}
\]

Using pricing rule, acquirer’s profit function is rewritten as:

\[
\Pi_j^{\text{Open}} = (1 + nt^{1-\alpha})(1 + v_j) RP^{\alpha-1} h_j^{\alpha-1} \sigma^{-\alpha} (\sigma - 1)^{\alpha-1} (\varphi_j)^{\alpha-1} - \lambda \{ (1 + v_j) h_j \}^k - f \{ 1 + v_j \} - n f_x \{ 1 + v_j \}
\]

Then the optimal decisions of scope and the number of acquisitions will be determined by

\[
h_j = (1 + nt^{1-\alpha})^{-\frac{(\sigma-1)}{\alpha-1}} f + n f_x \frac{\sigma-1}{\alpha-1} (RP^{\alpha-1})^{-\frac{(\sigma-1)}{\alpha-1} (\sigma - \alpha)^{\frac{(\sigma-1)}{\alpha-1} (\sigma - \alpha - 1)}}{\frac{\sigma(\sigma-1)(\sigma-2)}{k\lambda}} (\varphi_j)^{1-\sigma}
\]

\[
1 + v_j = (1 + nt^{1-\alpha})^{\frac{k(\sigma-1)}{(\alpha-1)(k-1)}} (f + n f_x \frac{\sigma-1}{\alpha-1} (RP^{\alpha-1})^{-\frac{(\sigma-1)}{\alpha-1} (\sigma - \alpha)^{\frac{(\sigma-1)}{\alpha-1} (\sigma - \alpha - 1)}}{\frac{\sigma(\sigma-1)(\sigma-2)}{k\lambda}} (\varphi_j)^{1-\sigma})^{\frac{(\alpha-1)-k(\sigma-1)}{(\alpha-1)(k-1)}}
\]

Then, total number of varieties is given by

\[
h_j^A \equiv h_j(1 + v_j) = (1 + nt^{1-\alpha})^{\frac{(\sigma-1)}{(\alpha-1)(k-1)}} (f + n f_x \frac{\sigma-1}{\alpha-1} (RP^{\alpha-1})^{-\frac{(\sigma-1)}{\alpha-1} (\sigma - \alpha)^{\frac{(\sigma-1)}{\alpha-1} (\sigma - \alpha - 1)}}{\frac{\sigma(\sigma-1)(\sigma-2)}{k\lambda}} (\varphi_j)^{1-\sigma})^{\frac{(\alpha-1)-k(\sigma-1)}{(\alpha-1)(k-1)}} \frac{k(\sigma-1)}{k\lambda} (\varphi_j)^{\frac{k(\sigma-1)}{k\lambda}}.
\]
\[ n_{f_x}^{-\frac{(\sigma-\alpha)}{(\alpha-1)(k-1)}} (RP^{\alpha-1})^{\frac{(\alpha-1)}{\alpha-1}(k-1)} \left( \frac{1}{k} \right)^{\frac{1}{k-1}} \sigma - \alpha \frac{(\sigma-1)}{(\alpha-1)(k-1)} \frac{1}{1} \left( \frac{\sigma-1}{\alpha-1} \right) k^{-1} \right) \]

Similar to the autarky equilibrium, \( \varphi^* \) identifies the cutoff level of productivity for successful entry, and \( \varphi^*_a \) identifies the cutoff level of productivity for profitable acquisition. Additionally, \( \varphi^*_x \) represents the cutoff level of productivity for exporting firms. In the open economy, these cutoff levels \( \varphi^*, \varphi^*_x \) and \( \varphi^*_a \) describe the differentiated good industry equilibrium, and there are partitioning of firms by acquisition decision and exporting decision. Throughout the paper, I assume selection into export markets like Melitz (2003). Then there are two possible cases where \( \varphi^* < \varphi^*_x < \varphi^*_a \) and \( \varphi^* < \varphi^*_a < \varphi^*_x \). From now consider the case of \( \varphi^* < \varphi^*_x < \varphi^*_a \) where only exporting firms participate in brand acquisition.\(^{30}\) In this case, cutoff for successful entry \( \varphi^* \) is determined by the equation (3.15), and cutoffs \( \varphi^*_x \) and \( \varphi^*_a \) are determined by

\[
\Pi_j^{\text{Open}}(\varphi^*_x) = \Pi_j^{\text{Naut}}(\varphi^*_x) \tag{3.38}
\]

and

\[
\Pi_j^{\text{Open}}(\varphi^*_a) = \Pi_j^{\text{Open}}(\varphi^*_a) \tag{3.39}
\]

\(^{29}\) Note that

\[
r^d_j(\varphi) = \frac{(f+n_{f_x})\sigma(\sigma-1)}{(\sigma-\alpha)(1+n(\sigma-\alpha))}, \quad r^x_j(\varphi) = \frac{r^{1-a}(f+n_{f_x})\sigma(\sigma-1)}{(\sigma-\alpha)(1+n(\sigma-\alpha))}
\]

\[
r^d_j(\varphi) = r^d_j(\varphi)(1+v_j), \quad r^x_j(\varphi) = r^x_j(\varphi)(1+v_j)
\]

\[
r_j^{\text{Open}}(\varphi) = r_j^{\text{Ad}}(\varphi) + n r_j^{\text{Ax}}(\varphi),
\]

\[
\pi_j^{\text{Open}}(\varphi) = \frac{r_j^{\text{Ad}}(\varphi)}{\sigma} + n \frac{r_j^{\text{Ax}}(\varphi)}{\sigma} - \lambda \left( h^A(\varphi) \right)^k
\]

\[
\Pi_j^{\text{Open}} = \pi_j^{\text{Open}}(\varphi) - f \left( 1 + v_j(\varphi) \right) - n f_x \left( 1 + v_j(\varphi) \right)
\]

\(^{30}\) There is no unique equilibrium of second case. The proof of this is in the Appendix.
\[ \leftrightarrow v_j^{op en}(\varphi_a^*) = 0. \]

Then we have,

\[ \varphi_x^* = \Gamma_x \varphi^* \quad (3.40) \]

\[ \varphi_a^* = \Gamma_a \varphi^* \quad (3.41) \]

where

\[ \Gamma_x \equiv \left\{ (1 + n \tau^{1-\alpha})\frac{k(\sigma-1)}{(1-\alpha) + k(\sigma-1)} - 1 \right\}^{(1-\alpha)+k(\sigma-1)} \left( \frac{n f_x}{f} \right)^{(1-\alpha)+k(\sigma-1)} \frac{k(\sigma-1)}{k(\sigma-1)(\sigma-1)} \]

\[ \Gamma_a \equiv (1 + n \tau^{1-\alpha})^{-\frac{1}{\alpha-1}} \left\{ \frac{(1-\alpha)+k(\sigma-1)}{k(\sigma-\alpha)} \frac{f + n f_x}{f} \right\}^{(1-\alpha)+k(\sigma-1)} \frac{k(\sigma-1)}{k(\sigma-1)(\sigma-1)} \]

Selection into export markets, \( \varphi_x^* > \varphi^* \), requires

\[ 1 + \frac{n f_x}{f} > (1 + n \tau^{1-\alpha})\frac{k(\sigma-1)}{(1-\alpha) + k(\sigma-1)} \quad (3.42) \]

That means export related costs \( f_x \) and \( \tau \) must be high enough to prevent some unproductive firms from exporting. Selection into acquisition markets \( \varphi_a^* > \varphi_x^* \) requires

\[ \frac{(1-\alpha)+k(\sigma-1)}{k(\sigma-\alpha)} \left( \frac{f + n f_x}{f} \right) (1 + n \tau^{1-\alpha})^{-\frac{k(\sigma-1)}{(1-\alpha) + k(\sigma-1)}} > \left\{ (1 + n \tau^{1-\alpha})\frac{k(\sigma-1)}{(1-\alpha) + k(\sigma-1)} - 1 \right\} \frac{n f_x}{f} \quad (3.43) \]

Then \( \varphi^* < \varphi_x^* < \varphi_a^* \) will be satisfied when the following condition holds.

\[ \frac{(f + n f_x)((1-\alpha)+k(\sigma-1))}{f((1-\alpha) + k(\sigma-1) + n f_x(k-1)(\alpha-1))} < (1 + n \tau^{1-\alpha})\frac{k(\sigma-1)}{(1-\alpha) + k(\sigma-1)} < \frac{f + n f_x}{f} \quad (3.44) \]

This can be satisfied since
\[
\frac{(f + nf_x)((1-\alpha) + k(\sigma-1))}{f((1-\alpha) + k(\sigma-1)) + nf_x(k-1)(\alpha-1)} < 1 + \frac{nf_x}{f}
\]
holds. \footnote{Note that \(0 < \frac{(1-\alpha) + k(\sigma-1)}{k(\sigma-1)} < 1.\)}

### 3.2.2. Firm Entry and Exit in Open Economy

Similar to the close economy model, defines the weighted average productivity. For non-participants, let \( \tilde{\varphi}^{Naut}_{\alpha} \) be the weighted average productivity within non-exporters, and \( \tilde{\varphi}^{Open}_{\alpha} \) be the weighted average productivity within exporters. \( \tilde{\varphi}^{Aopen}_{\alpha} \) is defined as the weighted average productivity within exporting acquirers.

\[
\tilde{\varphi}^{Naut} = \left\{ \frac{1}{G(\varphi^*_\alpha) - G(\varphi^*)} \int \varphi^*_\alpha \left( \frac{k}{1-\alpha + k(\sigma-1)} \right)^{(1-\alpha)(\alpha-1)\frac{k-1}{k}} \varphi^* \right\} \int \varphi^*_\alpha \left( \frac{k}{1-\alpha + k(\sigma-1)} \right)^{(1-\alpha)(\alpha-1)\frac{k-1}{k}} g(\varphi) d\varphi
\]

\[
\tilde{\varphi}^{Open} = \left\{ \frac{1}{G(\varphi^*_\alpha) - G(\varphi^*)} \int \varphi^*_\alpha \left( \frac{k}{1-\alpha + k(\sigma-1)} \right)^{(1-\alpha)(\alpha-1)\frac{k-1}{k}} \varphi^* \right\} \int \varphi^*_\alpha \left( \frac{k}{1-\alpha + k(\sigma-1)} \right)^{(1-\alpha)(\alpha-1)\frac{k-1}{k}} g(\varphi) d\varphi
\]

\( h_{\alpha}^{Naut} = \left( f + nf_x \right) \frac{1}{\sigma-1} \left( \frac{1}{1-\alpha + k(\sigma-1)} \right) ^{-1} \left( \frac{1}{\alpha} \right) \left( \frac{1}{\varphi^*} \right) \left( \frac{1}{\varphi^*_\alpha} \right) \left( \frac{1}{\alpha} \right) \left( \frac{1}{\varphi^*_\alpha} \right) \left( \frac{1}{\alpha} \right) \left( \frac{1}{\varphi^*_\alpha} \right) \left( \frac{1}{\alpha} \right)
\]

\( h_{\alpha}^{Open} = \left( f + nf_x \right) ^{-1} \left( \sigma - \alpha \right) \left( \frac{1}{k} \right) ^{-1} \left( \frac{1}{\varphi^*} \right) \left( \frac{1}{\varphi^*_\alpha} \right) \left( \frac{1}{\alpha} \right) \left( \frac{1}{\varphi^*_\alpha} \right) \left( \frac{1}{\alpha} \right)
\]

\( h_{\alpha}^{Aopen} = h_{\alpha}^{a}(1 + v_j) = \left( f + nf_x \right) ^{-1} \left( \sigma - \alpha \right) \left( \frac{1}{k} \right) ^{-1} \left( \frac{1}{\varphi^*} \right) \left( \frac{1}{\alpha} \right) \left( \frac{1}{\varphi^*_\alpha} \right) \left( \frac{1}{\alpha} \right)
\]
\[
\bar{\phi}^{Aopen} = \left\{ \frac{1}{1-G(\bar{\phi}^*_a)} \int_{\phi_a^*}^{\infty} (\phi)^{k(\sigma-1)} (e) \cdot k^{-1} \cdot g(\phi) \, d\phi \right\}^{k-1}
\]

The exponents of weighted average productivities reflect the relative sales of firms with different productivity levels.

\[
\frac{r_j^{Naut}(\phi)}{r_j^{Naut}(\phi')} = \left( \frac{\phi}{\phi'} \right)^{k(\sigma-1)(\alpha-1)} \frac{(1-\alpha) + k(\sigma-1)}{1(1-\alpha) + k(\sigma-1)} , \quad \frac{\pi_j^{Naut}(\phi)}{\pi_j^{Naut}(\phi')} = \left( \frac{\phi}{\phi'} \right)^{k(\sigma-1)(\alpha-1)} \frac{(1-\alpha) + k(\sigma-1)}{1(1-\alpha) + k(\sigma-1)}
\]

\[
\frac{r_j^{Open}(\phi)}{r_j^{Open}(\phi')} = \left( \frac{\phi}{\phi'} \right)^{k(\sigma-1)(\alpha-1)} \frac{(1-\alpha) + k(\sigma-1)}{1(1-\alpha) + k(\sigma-1)} , \quad \frac{\pi_j^{Open}(\phi)}{\pi_j^{Open}(\phi')} = \left( \frac{\phi}{\phi'} \right)^{k(\sigma-1)(\alpha-1)} \frac{(1-\alpha) + k(\sigma-1)}{1(1-\alpha) + k(\sigma-1)}
\]

The average revenue will be rewritten as

\[
\bar{\tau} = \frac{G(\phi^*_x) - G(\phi^*)}{1-G(\phi^*)} \bar{\tau}^{Naut} + \frac{G(\phi^*_x) - G(\phi^*)}{1-G(\phi^*)} \bar{\tau}^{Nopen} + \frac{1-G(\phi^*_a)}{1-G(\phi^*)} \bar{\tau}^{Aopen}
\]

where \( \bar{\tau}^{Naut} \) is average revenue of non-exporting non-participants and \( \bar{\tau}^{Nopen} \) is average revenue of exporting non-participants and \( \bar{\tau}^{Aopen} \) is average revenue of exporting acquirers.

Using the properties of relative sales, we obtain

\[
\bar{r}^{Naut} = \frac{1}{G(\phi^*_x) - G(\phi^*)} \int_{\phi^*_x}^{\phi^*} r_j^{Naut}(\phi) g(\phi) \, d\phi = r^{Naut}(\bar{\phi}^{Naut})
\]

\[
\bar{r}^{Nopen} = \frac{1}{G(\phi^*_a) - G(\phi^*_x)} \int_{\phi^*_x}^{\phi^*_a} r_j^{Nopen}(\phi) g(\phi) \, d\phi = r^{Nopen}(\bar{\phi}^{Nopen})
\]

\[
\bar{r}^{Aopen} = \frac{1}{1-G(\phi^*_a)} \int_{\phi^*_a}^{\infty} r_j^{Aopen}(\phi) g(\phi) \, d\phi = r^{Aopen}(\bar{\phi}^{Aopen})
\]

Using the properties of relative sales, we obtain

\[
r^{Naut}(\bar{\phi}^{Naut}) = \left( \frac{\phi^{Naut}}{\phi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} r^{Naut}(\phi^*)
\]
\[ r^{N_{open}}(\bar{\phi}^{N_{open}}) = \left(\frac{\bar{\phi}^{N_{open}}}{\varphi^*_x}\right)^{k(\sigma-1)(\alpha-1)/(1-\alpha+k(\sigma-1)/1)} r^{N_{open}}(\varphi^*_x) \]

\[ r^{A_{open}}(\bar{\phi}^{A_{open}}) = \left(\frac{\bar{\phi}^{A_{open}}}{\varphi^*_a}\right)^{k(\sigma-1)/k-1} r^{A_{open}}(\varphi^*_a) \]

Like the equation (3.25), average profit will be rewritten as:

\[ \bar{\Pi} = \frac{G(\varphi^*_x)-G(\varphi^*)}{1-G(\varphi^*)} \bar{\Pi}^{N_{naut}} + \frac{G(\varphi^*_a)-G(\varphi^*_x)}{1-G(\varphi^*)} \bar{\Pi}^{N_{open}} + \frac{1-G(\varphi^*_a)}{1-G(\varphi^*)} \bar{\Pi}^{A_{open}} \]  

(3.50)

where

\[ \bar{\Pi}^{N_{naut}} = \bar{\Pi}^{N_{naut}} - f, \quad \bar{\Pi}^{N_{open}} = \bar{\Pi}^{N_{open}} - f - nf_x \]

\[ \bar{\Pi}^{A_{open}} = \bar{\Pi}^{A_{open}} - (f + nf_x)(1 + \bar{v}). \]

Note that

\[ \bar{\Pi}^{N_{naut}} = \frac{1}{G(\varphi^*_x)-G(\varphi^*)} \int_{\varphi^*_a}^{\varphi^*_x} \bar{\Pi}^{N_{naut}}(\varphi) g(\varphi) d\varphi = \pi^{N_{naut}}(\bar{\phi}^{N_{naut}}) \]

\[ \bar{\Pi}^{N_{open}} = \frac{1}{G(\varphi^*_a)-G(\varphi^*_x)} \int_{\varphi^*_a}^{\varphi^*_x} \pi^{N_{open}}(\varphi) g(\varphi) d\varphi = \pi^{N_{open}}(\bar{\phi}^{N_{open}}) \]

\[ \bar{\Pi}^{A_{open}} = \frac{1}{1-G(\varphi^*_a)} \int_{\varphi^*_a}^{\varphi^*_a} \pi^{A_{open}}(\varphi) g(\varphi) d\varphi = \pi^{A_{open}}(\bar{\phi}^{A_{open}}) \]

Using the properties of (3.48), we have

\[ \bar{\Pi}^{N_{naut}} = \left(\frac{\bar{\phi}^{N_{naut}}}{\varphi^*}\right)^{k(\sigma-1)(\alpha-1)/(1-\alpha+k(\sigma-1))} \pi^{N_{naut}}(\varphi^*) \]

\[ \bar{\Pi}^{N_{open}} = \left(\frac{\bar{\phi}^{N_{open}}}{\varphi^*_x}\right)^{k(\sigma-1)(\alpha-1)/(1-\alpha+k(\sigma-1))} \pi^{N_{open}}(\varphi^*_x) \]

\[ \bar{\Pi}^{A_{open}} = \left(\frac{\bar{\phi}^{A_{open}}}{\varphi^*_a}\right)^{k(\sigma-1)/k-1} \pi^{A_{open}}(\varphi^*_a) \]
The average number of acquisitions will be same with the equation (3.26).

\[ 1 + \bar{\phi} = \left( \frac{\phi_{A_{open}}}{\phi_a} \right)^{\frac{k(\sigma-1)}{k-1}} \]

Since the average productivity level is completely determined by the cutoff productivity level \( \phi^* \) and \( \phi_a^* \), the average revenue and profit levels are also tied to the cutoff levels. Then rewrite the equations (3.38) and (3.39) using weighted average productivities, we have:

\[ \Pi_{Naut} = \left( \frac{\phi_{Naut}}{\phi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} f \]

\[ \Pi_{Open} = \left( \frac{\phi_{Open}}{\phi_x^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} n f_x \left\{ \frac{(1+n\tau^{1-\alpha})(1-\alpha)+k(\sigma-1)}{k(\sigma-1)} \right\} \]

\[ \Pi_{A_{open}} = \left( \frac{\phi_{A_{open}}}{\phi_a^*} \right)^{\frac{k(\sigma-1)}{k-1}} \left( f + n f_x \right) \left\{ \frac{1}{k(\sigma-1)} \right\} \]

Therefore, the average profit is given by

\[ \bar{\Pi} = \frac{G(\phi^*_x) - G(\phi^*)}{1-G(\phi^*)} f \left\{ \left( \frac{\phi_{Naut}}{\phi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} - 1 \right\} + \]

\[ \frac{G(\phi_a^*) - G(\phi_x^*)}{1-G(\phi^*)} n f_x \left( \frac{\phi_{Open}}{\phi_x^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} \left( \frac{1+n\tau^{1-\alpha}(1-\alpha)+k(\sigma-1)}{k(\sigma-1)} \right) - \]

\[ \frac{G(\phi_a^*) - G(\phi_a^*)}{1-G(\phi^*)} \left( f + n f_x \right) + \frac{1-G(\phi_a^*)}{1-G(\phi^*)} \left( \frac{\phi_{A_{open}}}{\phi_a^*} \right)^{\frac{k(\sigma-1)}{k-1}} \left( f + n f_x \right) \left\{ \frac{(k-1)(\alpha-1)}{k(\sigma-1)} \right\} \]

Firms decide whether to enter based on a comparison of the expected value of
entry and the sunk entry cost. The free entry condition (3.27) in open economy takes the following form:

\[
f [G(\varphi_x^*) - G(\varphi^*)] \left\{ \phi^{N_{\text{aut}}} \frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)} - 1 \right\} + n f x [G(\varphi_a^*) - \\
G(\varphi_x^*)] \left( \frac{\phi^{N_{\text{open}}}}{\varphi_x^*} \right)^{k(\sigma-1)(\alpha-1)} \frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)} \left\{ \frac{(1+n \tau)^{1-\alpha}}{\frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)}} \right\} - (f + n f_x \{G(\varphi_a^*) - G(\varphi_x^*)\}) + (f + n f_x) \left[ 1 - \\
G(\varphi_a^*) \right) \left( \frac{\phi^{A_{\text{open}}}}{\varphi_a^*} \right)^{k-1} \left\{ \frac{(k-1)(\alpha-1)}{k(\sigma-1)} \right\} = f_e .
\]

Substitute (3.40) and (3.41) into equation (3.51) then FE condition can be expressed in terms of a single unknown \( \varphi^* \) as follows:

\[
f \int_{\varphi_x^*}^{\Gamma_x \varphi^*} \left( \frac{\varphi}{\varphi^*} \right)^{k(\sigma-1)(\alpha-1)} \frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)} \left\{ \frac{(1+n \tau)^{1-\alpha}}{\frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)}} \right\} - \\
g(\varphi) d \varphi - f \{G(\Gamma_x \varphi^*) - G(\varphi^*)\} + f (1 + \\
n t \tau^{1-\alpha}) \frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)} \int_{\varphi_x^*}^{\Gamma_a \varphi^*} \left( \frac{\varphi}{\varphi^*} \right)^{k(\sigma-1)(\alpha-1)} \frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)} \left\{ \frac{(k-1)(\alpha-1)}{k(\sigma-1)} \right\} - \\
G(\varphi_x^*) \left[ f + n f_x \right] + (f + n f_x) \frac{k(\sigma-1)}{(1-\alpha)(k-1)}(1 + \\
n t \tau^{1-\alpha}) \frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)} \left\{ \frac{(1-\alpha)+k(\sigma-1)}{fk(\sigma-\alpha)} \right\} \frac{(k-1)(\alpha-1)}{k(\sigma-\alpha)} \left\{ \frac{(k-1)(\alpha-1)}{k(\sigma-\alpha)} \right\} - \\
f \int_{\varphi^*}^{\infty} \frac{\varphi}{\varphi^*} \frac{k(\sigma-1)}{k(\sigma-\alpha)} g(\varphi) d \varphi = f_e
\]

This free entry condition has the following properties. (i) The right-hand side is the constant sunk entry cost \( f_e > 0 \); (ii) As \( \varphi^* \rightarrow 0 \), the left-hand side converges towards infinity; (iii) As \( \varphi^* \rightarrow \infty \), the left-hand side converges towards zero; (iv) The left-hand side is monotonically decreasing in \( \varphi^* \). It follows that there exists a unique fixed point, where the expected value of entry \( V_e \) is equal to the sunk entry cost \( f_e \).
**Proposition 2.** There exists a unique open economy equilibrium with zero acquisition price, referenced by the vector \( \{ \varphi^*, \Gamma_x, \Gamma_a \} \).

**proof.** See Appendix

### 3.2.3. Aggregation

The open economy equilibrium will be characterized by a mass of brands and a distribution \( g(\varphi) \) of productivity levels over a subset of \((0, \infty)\). \( M^{\text{Naut}} \) is a mass of brands which are not involved in acquisition and exporting, \( M^{\text{Open}} \) is a mass of brands which are not involved in acquisition but are involved in exporting, \( M^a \) is the mass of acquired brands, and \( M^{\text{Aopen}} \) is total mass of brands which acquirers own. Like the previous section, \( M_e \) is a mass of brands entering and \( N_e \) is a mass of entrants. \( N^{\text{Naut}} \) is a mass of non-participants who do not export, \( N^{\text{Open}} \) is a mass of non-participants who serve foreign market, and \( N^{\text{Aopen}} \) is a mass of acquirers. Under open economy, aggregate price index \( P^{\text{open}} \) is defined by

\[
P^{\text{open}} = \left\{ \int_{j=0}^{m^H} \left( p_j^H \right)^{1-\alpha} dj + n \int_{j=0}^{m^F} \left( p_j^F \right)^{1-\alpha} dj \right\}^{\frac{1}{1-\alpha}}
\]

\[
= \left[ M^H \left( p_j^H (\tilde{\varphi}_j^H) \right)^{1-\alpha} + n M^F \left( p_j^F (\tilde{\varphi}_j^F) \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}
\]

(3.52)

where \( p_j^H \) is the price index of brand \( j \) of domestic firm, and \( p_j^F \) is the price index of brand \( j \) of foreign firm. \( M^H \) is domestic mass of brands and \( M^F \) is foreign mass of brands serving domestic market such that:

\[
M^H = M^{\text{Naut}} + M^{\text{Open}} + M^{\text{Aopen}}
\]

(3.53)
\[ M^F = nM^{open} + nM^{Aopen} \]

Since every country is symmetric, the mass of foreign firms serving domestic market equals to the mass of domestic firms exporting multiplied by the number of countries. Then the mass of brands available in every country will be \( M = M^H + M^F \). Therefore, in such equilibrium, \( p^{open} \) is rewritten by:

\[
p^{open} = \left[ \int_{\varphi^*}^{\varphi^a} \{ p_j^{Naut}(\varphi) \}^{1-\alpha} N^{Naut} \frac{g(\varphi)}{G(\varphi^*) - G(\varphi)} d\varphi + \int_{\varphi^a}^{\varphi^d} \{ p_j^{Nd}(\varphi) \}^{1-\alpha} N^{Nopen} \frac{g(\varphi)}{G(\varphi^a) - G(\varphi^d)} d\varphi + \int_{\varphi^d}^{\varphi^x} \{ p_j^{Nx}(\varphi) \}^{1-\alpha} N^{Nopen} \frac{g(\varphi)}{G(\varphi^d) - G(\varphi^x)} d\varphi + \int_{\varphi^x}^{\varphi} \{ p_j^{Ax}(\varphi) \}^{1-\alpha} N^{Aopen} \frac{g(\varphi)}{1-G(\varphi)} d\varphi \right]^{1-\alpha} \{ 1 + v_j(\varphi) \} nN^{Aopen} \frac{g(\varphi)}{1-G(\varphi)} d\varphi + \int_{\varphi}^{\varphi^*} \{ p_j^{Naut}(\varphi) \}^{1-\alpha} N^{Naut} \frac{g(\varphi)}{G(\varphi^*) - G(\varphi^a)} d\varphi \]

where superscript \( Nd \) represents domestic price of non-participants and \( Nx \) represents exporting price of non-participants. Similarly, \( Ad \) and \( Ax \) indicates domestic and exporting price of acquirers. Using (3.53) and pricing rule, aggregate price index is rewritten as:

\[
p^{open} = \left[ M^{Naut} \{ p_j^{Naut}(\varphi^{Naut}) \}^{1-\alpha} + M^{Nopen} (1 + n\tau^{1-\alpha}) \{ p_j^{Nd}(\varphi^{Nopen}) \}^{1-\alpha} + M^{Aopen} (1 + n\tau^{1-\alpha}) \{ p_j^{Ax}(\varphi^{Aopen}) \}^{1-\alpha} \right]^{1-\alpha}
\]

Let’s define aggregate price indices according to its exporting decision and acquisition decision. Then we have:
Using above price indices by groups of firms, rewrite the price index as

\[ p^\text{open} = \left( (p^\text{Naut})^{1-\alpha} + (p^\text{Nopen})^{1-\alpha} + (p^\text{Aopen})^{1-\alpha} \right)^{\frac{1}{1-\alpha}}. \]

Aggregate number of successful domestic firms will be

\[ N^H = N^\text{Naut} + N^\text{Nopen} + N^\text{Aopen} \]

where

\[ N^\text{Naut} = \frac{G(\phi^*_x) - G(\phi^*)}{1 - G(\phi^*)} N^H, \quad N^\text{Nopen} = \frac{G(\phi^*_a) - G(\phi^*_x)}{1 - G(\phi^*_x)} N^H, \]

\[ N^\text{Aopen} = \frac{1 - G(\phi^*_a)}{1 - G(\phi^*)} N^H. \] (3.54)

From \( N_e = M_e \) and \( N^H = [1 - G(\phi^*)]N_e \), domestic mass of brands is represented by

\[ M^H = \{1 - G(\phi^*)\}M_e + \bar{\nu}N^\text{Aopen} \]

\[ = \left( [1 - G(\phi^*)] + \bar{\nu} [1 - G(\phi^*_a)] \right)M_e. \]

Note that
\[ M^{Naut} = \frac{G(\phi_a^*) - G(\phi^*)}{1 - G(\phi^*) + \bar{\theta}(1 - G(\phi_a^*))} M^H, \]
\[ M^{Nopen} = \frac{G(\phi_a^*) - G(\phi^*)}{1 - G(\phi^*) + \bar{\theta}(1 - G(\phi_a^*))} M^H, \]
\[ M^{Aopen} = \frac{(1 + \bar{\theta})(1 - G(\phi_a^*))}{1 - G(\phi^*) + \bar{\theta}(1 - G(\phi_a^*))} M^H, \]
\[ M^H = \left[ 1 + \frac{\bar{\theta}(1 - G(\phi_a^*))}{1 - G(\phi^*)} \right] N^H. \]

Like equation (3.30) aggregate labor used for investment by entrant is \( N^H \bar{\Pi}, \) and aggregate labor used for production, R&D investment and exporting investment is given by

\[ L_p = R - N^{Naut} \bar{\Pi}^{Naut} - N^{Nopen} \bar{\Pi}^{Nopen} - N^{Aopen} \bar{\Pi}^{Aopen} \]
\[ = R - N^H \bar{\Pi} \quad (3.55) \]

Like closed economy case, we can derive the equation (3.32), which is \( L_e + L_p = R = (1 - \eta)L. \) Aggregate revenue in differentiated good sector must equal the total payments to labor \( L \) which is employed that sector. Because of symmetry among countries, the mass of firms is simply determined from the equation (3.33), \( N^H = R / \bar{r}. \) Then, total number of acquisitions will be determined by

\[ Num_{acq} = M^{Aopen} - N^{Aopen} = \bar{\theta} N^{Aopen} = \bar{\theta} (1 - G(\phi_a^*)) N_e \quad (3.56) \]

Then the proportion of acquisition is calculated by

\[ Prop_{acq} = \frac{\bar{\theta} N^{Aopen}}{M_e} = \bar{\theta} (1 - G(\phi_a^*)) \quad (3.57) \]

Additionally, for acquisition price to be equalized to zero, condition (3.34) must
be satisfied too.

\[
\int_{\varphi_a}^{\infty} \left\{\left(\frac{\varphi}{\varphi_a}\right)^{k(\sigma-1)} - 1\right\} g(\varphi) \, d\varphi < \int_{0}^{\varphi_a} g(\varphi) \, d\varphi
\]

3.3. Impact of Freer Trade

In this section, I consider the impact of trade liberalization on the level of firm productivity cutoffs and industry-level transfers of brand in equilibrium. The effects of two mechanisms are investigated: a decrease in variable trade cost, \(\tau\), and decrease in fixed trade cost, \(f_x\). These two investigations involve comparative statics of the open economy variables with respect to \(\tau\) and \(f_x\), which generate similar results each other. In all case, trade liberalization will force the least productive firms to exit and will reallocate resources from less productive to more productive firms not only labor but also intangible asset, brand equity.

Now consider the effect of decrease in variable trade cost \(\tau\) on the level of productivity cutoffs.

**Proposition 3.** When the condition for zero acquisition price holds, a reduction in variable trade cost: (i) increases the productivity cutoff level \(\varphi^*\); (ii) decreases the productivity cutoff level for exporting \(\varphi_x^*\); (iii) decreases the productivity cutoff level for acquisition \(\varphi_a^*\); (iv) increases the proportion of acquired brands relative to all entered brands

**Proof.** See Appendix.

As Melitz (2003), this model shows that trade liberalization induces the least productive firms to exit and generate entry of new firms into the export market,
\( d\varphi^*/d\tau < 0 \) and \( d\varphi^*_x/d\tau > 0 \). Add to that, this model shows that freer trade induces entry of new firms into the acquisition market, \( d\varphi^*_a/d\tau > 0 \), and raises the probability of being acquired, \( d\text{Prop}_{acq}/d\tau < 0 \). That is, the share of brands transferred in acquisitions market increases in the face of trade liberalization.

Now consider the effect of decrease in fixed exporting cost \( f_x \) on the productivity cutoffs.

**Proposition 4.** When the condition for zero acquisition price holds, a reduction in fixed trade cost: (i) increases the productivity cutoff level \( \varphi^* \); (ii) decreases the productivity cutoff level for exporting \( \varphi^*_x \); (iii) decrease the productivity cutoff level for acquisition \( \varphi^*_a \); (iv) increases the proportion of acquired brands relative to all entered brands.

**Proof.** See Appendix.

Like variable trade cost, \( \tau \), decreased \( f_x \) induces the similar effects on productivity cutoffs such as, \( d\varphi^*/df_x < 0 \), \( d\varphi^*_x/df_x > 0 \) and \( d\varphi^*_a/df_x > 0 \). Also the probability of being acquired increases as fixed trade cost decreases, \( d\text{Prop}_{acq}/df_x < 0 \).

The basic intuitions of these results are quite similar with the previous literatures. Trade liberalization increases the expected value of entry, as a result to increased entry, which enhances factor market competition and raises the zero-profit cutoff. Lowered trade barriers widen the range of firm productivity to export, and hence increase the return to production expansion through brand acquisition. In addition, the share of transferred brands increases, since the range of firm productivity to acquire and the optimal number of acquisition of
high-efficient firms increase.

To determine the effect of trade liberalization on aggregate variables, we need to use Pareto distribution for productivity distribution. Then first we obtain that

\[
\frac{d}{d\tau} \left\{ \frac{G(\Gamma x \varphi^*) - G(\varphi^*)}{1 - G(\varphi^*)} \right\} > 0, \quad \frac{d}{d\tau} \left\{ \frac{G(\Gamma a \varphi^*) - G(\Gamma x \varphi^*)}{1 - G(\varphi^*)} \right\} < 0, \quad \frac{d}{d\tau} \left\{ \frac{1 - G(\Gamma a \varphi^*)}{1 - G(\varphi^*)} \right\} < 0.
\]

That is, the share of firms labeled as Naut decreases and the share of other groups of firms labeled as Nopen and Aopen increases. Using Pareto distribution, we can also rewrite the condition for zero acquisition price as,

\[
\frac{k(\sigma-1)}{\theta(k-1) - k(\sigma-1)} < \left( \Gamma a \right)^\theta \left\{ (\varphi^*)^\theta - 1 \right\}
\]

\[
\leftrightarrow 1 + \frac{k(\sigma-1)(\Gamma a)^{-\theta}}{\theta(k-1) - k(\sigma-1)} < (\varphi^*)^\theta
\]

(3.58)

According to (3.58), condition for zero acquisition price holds when \( \Gamma_a \) is high enough, that is, the range of firm productivity to acquire brand profitably is relatively small.

For the mass of firms, we have

\[
\frac{dN^H}{d\tau} = -\frac{(1-\eta)L}{(r)^2} \frac{dr}{d\tau} > 0,
\]

since the sign of \( d\bar{r}/d\tau \) is negative.

\[
\frac{dr}{d\tau} = \frac{\theta k_0 \sigma (\sigma-1) f(x \varphi^*)^ {\theta-1}}{(1-\alpha) + k(\sigma-1)} \frac{d\varphi^*}{d\tau} + \theta (f + nf_x) \left\{ 1 - \right\}
\]
\[
\left(\frac{k(\sigma-1)}{\theta(k-1)}\right)^{-1}\} \Gamma_a - \theta \left\{ \frac{k \sigma (\sigma-1)}{(1-\alpha)+k(\sigma-1)} \right\} - \theta n f_{x1} \Gamma_x - \theta \left\{ \frac{k \sigma (\sigma-1)}{(1-\alpha)+k(\sigma-1)} \right\} < 0.33
\]

The effect of decreased variable trade cost on aggregate number of firms is negative, \(dN^H/d\tau < 0\). The comparative statics related to the aggregate number of domestic brands is as in the following.

\[
\frac{dM^H}{d\tau} = - \frac{(1-\eta) L d\rho}{(\rho)^2} \frac{d\rho}{d\tau} - \frac{(1-\eta) L \frac{d\rho}{d\tau}}{(\rho)^2} \left( \Gamma_a \right)^{-\theta} \left\{ \frac{k(\sigma-1)}{\theta(k-1)-k(\sigma-1)} \right\} - \theta \left( \Gamma_a \right)^{-\theta-1} \frac{d\Gamma_a}{d\tau} \left\{ \frac{k(\sigma-1)}{\theta(k-1)-k(\sigma-1)} \right\} > 0
\]

Even though the share of acquirer increases as variable trade cost decreases, the total number of domestic brands decreases. This is because of the assumption of zero acquisition price. The number of exit brands is always greater than the number of acquired brands under zero acquisition price case.

The effect of fixed trade cost on aggregate variables are similar with the one of variable trade cost.

\[
\frac{dN^H}{df_x} > 0, \quad \frac{dM^H}{df_x} > 0.
\]

### 3.4. Impact of “Love of brand”

The two stage CES utility function is given by (3.2) and (3.3), and \(\gamma\) captures the degree of cannibalization with \(\gamma = 0\) implying no cannibalization. When \(\gamma > 0\), there exists the effect of within-brand cannibalization. The rate

\[\frac{d\phi^*}{d\tau} < 0, \quad \Gamma_a > 1, \quad \frac{d\Gamma_a}{d\tau} > 0, \quad \frac{d\Gamma_x}{d\tau} > 0 \text{ and } 1 - \left\{ \frac{k(\sigma-1)}{\theta(k-1)} \right\}^{-1} < 0.\]
of cannibalization from total brand quantity \( q_j \) is in (3.4). We can see that the rate of cannibalization increases as \( \sigma \) increases, or \( \alpha \) decreases. From now we will consider the case where \( \alpha \) decreases and \( \sigma \) is constant by deriving the comparative statics with respect to \( \alpha \). Decreased \( \alpha \) implies that the increased degree of “love of brand,” and increased degree of within-brand cannibalization.

In autarky, the acquirer’s optimal scope and the number of acquisitions respond to the degree of “love of brand” as follows.

\[
\frac{\partial h_j}{\partial \alpha} = h_j \left( \frac{\sigma-1}{\alpha-1} \right) \left\{ \ln \left( \frac{p_j}{p} \right) + \frac{1}{\sigma-\alpha} \right\} > 0
\]

\[
\frac{\partial (1+v_j)}{\partial \alpha} = \frac{1+v_j}{k-1} \left\{ -k \frac{\partial h_j}{h_j} + \frac{(\sigma-1)}{(\alpha-1)(\sigma-\alpha)} \right\}
\]

\[
= \frac{1+v_j}{k-1} \left\{ -k \left( \frac{\sigma-1}{\alpha-1} \right) \ln \left( \frac{p_j}{p} \right) - \frac{(k-1)(\sigma-1)}{(\alpha-1)(\sigma-\alpha)} \right\} < 0
\]

The number of acquisitions for all acquirers increases and scope decreases as \( \alpha \) decreases. In open economy, the acquirer’s optimal scope and the number of acquisitions respond to the degree of “love of brand” as follows.

\[
\frac{\partial h_j}{\partial \alpha} = h_j \left( \frac{\sigma-1}{\alpha-1} \right) \left\{ \ln \left( \frac{p_j}{p} \right) + \frac{1}{\sigma-\alpha} + \frac{nr^{1-\alpha} \ln r}{(1+nr^{1-\alpha})} \right\} > 0
\]

\[
\frac{\partial (1+v_j)}{\partial \alpha} = \frac{1+v_j}{k-1} \left\{ -k \frac{\partial h_j}{h_j} + \frac{(\sigma-1)}{(\alpha-1)(\sigma-\alpha)} \right\} < 0
\]

Like the autarky case, the number of acquisitions for all acquirers increases and scope decreases as \( \alpha \) decreases. The incentive of acquiring additional brand increases and the incentive of adding new product within its own brand
decreases as the degree of “love of brand” increases.

Now consider the effect of higher “love of brand” on the productivity cutoffs. The following results are derived from simulation in section 3.5.

\[
\frac{d \phi'}{d \alpha} > 0
\]

\[
\frac{d \phi'_{a}}{d \alpha} > 0
\]

Like the effect of “love of varieties,” “love of brand” eases the market competition. When consumers love the diversity of brands, the inefficient firms can survive. Furthermore, increased “love of brand” raises the incentive of acquisition by increase the degree of within-brand cannibalization. We can see that acquisition cutoff decreases and the proportion of transferred brand increases as the parameter \( \alpha \) decreases.

**Proposition 5.** When the condition for zero acquisition price holds, the higher degree of “love of brand,” the relatively more brands are transferred in the market, that is, \( dProp_{acq}/d\alpha < 0 \), if \( d\phi'_{a}/d\alpha > 0 \).

**proof.** See Appendix

Furthermore, the cross partial derivatives of \( \alpha \) and \( \tau \) on productivity cutoffs and the share of brands participated in acquisition are given by

\[
\frac{\partial^2 \phi'}{\partial \tau \partial \alpha}, \quad \frac{\partial^2 \phi'_{a}}{\partial \tau \partial \alpha}, \quad \frac{\partial^2 Prop_{acq}}{\partial \tau \partial \alpha}.
\]

These are ambiguous in sign. Thus we cannot sure the effect of within-brand cannibalization on the marginal impacts of globalization.
3.5. Numerical Solution

In this section, I parameterize the open economy model and solve it numerically. These solutions provide a visual representation of the equilibrium and enable us to see the interaction of “love of brand” and trade liberalization. The choice of parameter values is follows:

- Variety specific cost $\lambda=5$
- Sunk entry cost $f_e=2$
- Fixed production cost $f=1.5$
- Fixed exporting cost $f_x=2$
- Exponent of variety-specific cost function $k=3$
- The number of trading partners $n=2$
- Aggregate revenue $R=10000$
- Pareto shape parameter $\theta=3.4$
- Pareto scale parameter $\varphi_{min}=1$
- Measure of “love of variety” $\delta=3$
- Measure of “love of brand” $\alpha \in [2.1, 2.25]$
- Variable trade cost $\tau \in [1.1, 1.7]$

These parameters are chosen to ensure that there is an interior equilibrium of the previous theoretical model where only some of exporting firms acquire brands and all acquisition prices are determined to zero. These are somewhat arbitrary numbers, thus more accurate simulation must be conducted by using estimated parameters.

The following figures describe the numerical solution with different value of $\alpha$ and $\tau$. These results are the same with the Propositions 3 and 5.
Figure 9. Acquisition cutoffs with $\alpha \in [2.1, 2.25]$ and $\tau \in [1.1, 1.7]$

Figure 10. Proportion of acquisition with $\alpha \in [2.1, 2.25]$ and $\tau \in [1.1, 1.7]$
Note that the cross partial effect of $\tau$ and $\alpha$ does not have any consistent direction. Even though share of acquisition is high in the industry with low value of $\alpha$, the marginal effect of trade liberalization on acquisition isn’t high in that industry. The sign of cross partial effect depends on the value of parameters and functional form of the cumulative distribution function for firm’s productivity.

In addition, I draw a graph describes the relation between proportion of acquisition and firm distribution. Pareto shape parameter $\theta$ determines the degree of heterogeneity, and smaller $\theta$ implies wider distribution of productivity. As we can see in Figure 11, the share of acquired brand increases as $\theta$ decreases, since the share of acquirers increases.

Figure 11. Proportion of acquisition with $\theta \in [3.4, 3.8]$ and $\tau \in [1.1, 1.7]$
4. Theoretical Model with Quasi Linear Preference

In the previous section, I have focused on how trade liberalization affects the industry equilibrium by redistributing resources across heterogeneous firms. The intensified competition in the product market, however, does not considered in the previous model as a CES preference is used. The mark up of each variety is always constant in closed and open economy. In this section, therefore, I consider how the effect of increased product market competition affect the decision of firms by assuming quasi linear preference. The theoretical model of this section is an extension of Dhingra (2013), and only deals with the case of zero acquisition price. Like the other literatures with linear demand, I focus on the closed economy equilibrium, then investigate the free trade equilibrium.

4.1. Closed Economy

4.1.1. Preferences and Production Technology

Like the CES model, preferences are defined over a continuum of differentiated varieties and a homogeneous good chosen as numeraire. All consumers share the same quasi linear utility function given by:

\[ U^k \equiv q_0^k + \beta Q^k - \frac{\delta}{2} \int_j \int_i (q_{ij}^k)^2 \, di \, dj - \frac{\gamma}{2} \int_j (q_i^k)^2 \, dj - \frac{\eta}{2} (Q^k)^2 \]  

(4.1)

where \( q_0^k \) and \( q_{ij}^k \) represent the individual consumption levels of the
numeraire good and product $i \in \Omega_j$ of brand $j \in J$ of the differentiated good. Consumer $k$’s total consumption of brand $j$ goods is

$$q_j^k \equiv \int q_{ji}^k \, di.$$  

Her consumption of differentiated goods of all brands is

$$Q_k^k \equiv \int q_{j}^k \, dj.$$  

The demand parameters $\alpha, \delta, \gamma$ and $\eta$ are all strictly positive. The parameters $\alpha$ and $\eta$ index the substitution pattern between the homogeneous and differentiated good. Parameter $\delta$ determines the degree of love of varieties. In the limit when $\delta=0$, consumers have no taste for diversity in products. $\gamma$ captures the degree of differentiation across brands with $\gamma=0$ implying no brand differentiation. I assume that consumers have positive demand for the numeraire good. The consumer $k$’s inverse demand for product $i$ of brand $j$ is then given by

$$p_{ji} = \beta - \delta q_{ji}^k - \gamma q_j^k - \eta Q_k^k$$

Then the total demand for product $i$ of brand $j$ is

$$p_{ji} = \beta - \frac{\eta Q_j^k}{L} - \frac{\delta q_{ji}^k}{L} - \frac{\gamma q_j^k}{L}.$$  

where $\alpha \equiv \beta - \frac{\eta Q_j^k}{L}$

As you can see, an increase in consumption of the product $i' \neq i$ of brand $j$ reduces demand for product $i$ more than demand for other brands’ products. The
degree of this cannibalization effect depends on the parameter $\gamma$.\textsuperscript{34}

Labor is the only factor of production and production exhibits constant return to scale at marginal cost $c$ which equal to unit labor requirement. Firms learn about his cost level only after making the irreversible investment $f_e$ required for entry. Like the previous section, let this as a draw from a Pareto distribution $G(c)$ with support on $(0, c_M]$\textsuperscript{35}

$$G(c) = \left(\frac{c}{c_M}\right)^\theta, \ g(c) = \theta \left(\frac{1}{c_M}\right)^\theta c^{\theta-1} \quad (4.3)$$

A firm $j$ takes demand intercept ‘$\alpha$’ as given when choosing its quantity of product $i$, $q_{ji}$. Each firm chooses the range of products $h_j$ by paying a cost $\lambda$ per product like the previous model. A firm $j$ with unit cost $c_j$, faces the following profit maximization problem. The profits of the firms which do nothing in the acquisition market are\textsuperscript{36}

$$\Pi_j^N = h_j(p_{ji} - c_j)q_{ji} - \lambda(h_j)^2 \quad (4.4)$$

Then optimal decisions satisfy the following properties.

$$\frac{\partial q_{ji}}{\partial c_j} = -\frac{r_hL^2}{3\delta y q_{ji}^2 + 2\delta r_hL} < 0$$

$$\frac{\partial h_j}{\partial c_j} = -\frac{\delta L(a-c_j)}{4r_h(3y^2h_j^2 + 4\delta y h_j + \delta^2)} < 0$$

\textsuperscript{34} Dhingra(2013) shows that when $\gamma>0$, within-brand cross elasticity exceeds across-brand cross elasticity.

\textsuperscript{35} In the linear demand model, I use cost as a measure of ability of a firm like Swati(2013) and Melitz & Ottaviano (2009). This unit cost is inversely related to a firm’s productivity, $c = w/\varphi$.

\textsuperscript{36} Particularly I assume $k=2$ for the simplification of the problem.
It is clear that a firm chooses the same quantities for its varieties under the symmetry among varieties given a brand. Industry cost cutoff $c_D$ for successful entry will be determined by

$$\Pi^N_j(c_D) = 0$$

and we have $c_D = a$. Potential acquirer $j$ maximizes the following profit.

$$\Pi^A_j = h_j(p_{ji} - c_j)q_{ji} + \int_0^{v_j} h_j^k(p_{ji}^k - c_j)q_{ji}^k - R_{jk}dk - r_h(h_j + \int_0^{v_j} h_j^k dk)^2 - v_j f_m$$

where $k \in \Omega^A_j$ which is the acquired brand sets of firm $j$, and $R_{jk}$ is acquisition price for acquired brand $k$. Similar to the CES model, I consider the case where acquisition price is zero. Then we have equation (4.6)’

$$\Pi^A_j = h_j(p_{ji} - c_j)q_{ji} + v_j[h_j^k(p_{ji}^k - c_j)q_{ji}^k] - r_h(h_j + v_j h_j^k)^2 - v_j f_m$$

From the first order condition, we can see that optimal decisions of original brand is the same with decisions of acquired brand.

$$q_{ji} = q_{ji}^k, \quad h_j = h_j^k, \quad p_{ji} = p_{ji}^k$$

The optimal decisions are\(^{37}\)

$$p_{ji} - c_j = \mu_{ji} = \frac{a - c_j}{2}$$

\(^{37}\) Note that $\frac{\partial h_j}{\partial c_j} < 0$. 
\[ q_{ji} = \frac{L}{\delta} \left\{ \frac{a-c_j}{2} - \left( \frac{yf_m}{L} \right)^{\frac{1}{2}} \right\}, \quad h_j = \delta \left( \frac{f_m}{yL} \right)^{\frac{1}{2}} \left\{ \frac{a-c_j}{2} - \left( \frac{yf_m}{L} \right)^{\frac{1}{2}} \right\}^{-1} \] (4.8)

\[ v_j = \frac{L}{2r_h \delta^2} \left( \frac{yL}{f_m} \right)^{\frac{1}{2}} \left\{ \frac{a-c_j}{2} - \left( \frac{yf_m}{L} \right)^{\frac{1}{2}} \right\}^{\frac{3}{2}} - 1 \] (4.9)

Total number of varieties of firm \( j \) is given by:

\[ h_j + v_j h_j^k = h_j(1 + v_j) = \frac{L}{2r_h \delta} \left\{ \frac{a-c_j}{2} - \left( \frac{yf_m}{L} \right)^{\frac{1}{2}} \right\}^2 \]

Like the previous model, there is tradeoff between product range and the number of acquisitions. Also it is clear that incentive of acquisition increases as a firm ability increases. Total net profit of acquirer \( j \) is

\[ \Pi_j^A - f_e = \frac{L^2}{4r_h \delta^2} \left\{ \frac{a-c_j}{2} - \left( \frac{yf_m}{L} \right)^{\frac{1}{2}} \right\}^{\frac{4}{3}} + f_m - f_e \]

Cost cutoff for profitable acquisition will be determined by

\[ v(c_a) = \frac{L}{2r_h \delta^2} \left( \frac{yL}{f_m} \right)^{\frac{1}{2}} \left\{ \frac{a-c_a}{2} - \left( \frac{yf_m}{L} \right)^{\frac{1}{2}} \right\}^{\frac{3}{2}} - 1 = 0 \] (4.10)

so we have the acquisition cutoff

\[ c_a = c_D - 2 \left\{ \left( \frac{yf_m}{L} \right)^{\frac{1}{2}} + \left( \frac{2r_h \delta^2}{L} \right)^{\frac{1}{3}} \left( \frac{f_m}{yL} \right)^{\frac{1}{3}} \right\} \] (4.11)

For the existence of acquisition, acquisition cutoff must be positive. This can be rewritten by:
If the degree of product market competition is excessively high, that is, $c_D$ is low enough, then acquisition is not profitable for any firms. Thus for the existence of acquisitions, equation (4.12) must be satisfied. The proportion of acquired brands relative to all brands which entered into market is given by

$$Prop_{acq} = \bar{v} \cdot G(c_a) = \int_0^{c_a} v(c) g(c) \, dc,$$

and for the zero acquisition price, the following condition must hold.

$$Prop_{acq} < 1 - G(c_D)$$

We can easily see that the following properties of acquisition for acquirer $j$

$$\frac{\partial v_j}{\partial y} > 0 \quad \text{for} \quad c_j < c_D - \left(\frac{y f_m}{L}\right)^{\frac{1}{2}}$$

$$\frac{\partial v_j}{\partial L} > 0 \quad \text{for} \quad c_j < c_D - 2 \left(\frac{y f_m}{L}\right)^{\frac{1}{2}}$$

$$\frac{\partial h_j(1+v_j)}{\partial L} > 0 \quad \text{for all} \quad c_j < c_a$$

Given product market competition, the incentive of acquisition for most productive firm increases as the degree of within-cannibalization increases. Also, increased market size, that is, increased opportunity to produce raises the incentive of acquiring other firms’ brands for all acquirers. In the extreme case

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38 Note that $\frac{\partial v_j}{\partial L} > 0 \quad \text{for all} \quad c_j < c_a$. 

70
where $\gamma$ equals to zero, brand acquisition cannot arise. Without within-brand cannibalization, the decisions of firms are given by

$$c_a = 0, \ v_j = 0 \ for \ all \ j, \ c_D = a,$$

$$h_j = \frac{L}{2r_h\delta} \left(\frac{c_D - c_j}{2}\right)^2, \ q_{jl} = \frac{L}{\delta} \left(\frac{c_D - c_j}{2}\right), \ \mu_{jl} = \frac{c_D - c_j}{2}.$$

### 4.1.2. Industry Equilibrium

In an equilibrium with positive entry, the expected value of entry must equal to the sunk entry cost, which requires the following free entry condition to hold:

$$\int_0^{c_a} \Pi^A_j \ g(c_j) dc + \int_{c_a}^{c_D} \Pi^N_j \ g(c_j) dc = f_e \quad (4.15)$$

The cost cutoffs and free entry condition jointly determine industry equilibrium. Because of the complexity, I assume that the cost distribution follows uniform distribution, that is, $\theta = 1$. Then the probability of being acquired is rewritten by

$$Prop_{acq} = \frac{L}{4r_h\delta^2} \left(\frac{\gamma L}{f_m}\right)^{\frac{1}{2}} \left(\frac{c_D}{2} - \left(\frac{\gamma f_m}{L}\right)^{\frac{1}{2}}\right)^4 - \left(\frac{2r_h\delta^2}{L}\right)^{\frac{1}{2}} \left(\frac{f_m}{\gamma L}\right)^{\frac{1}{2}} 4 - \frac{c_a}{c_m}$$

Under the uniform distribution of firms’ cost, the average values of firm level are defined as follows:

$$\bar{c} = \frac{1}{\mathcal{G}(c_D)} \left\{ \int_0^{c_a} c_j \left(1 + v(c_j)\right) g(c_j) \ dc + \int_{c_a}^{c_D} c_j g(c_j) \ dc \right\}$$

$$\bar{h} = \frac{1}{\mathcal{G}(c_D)} \left\{ \int_0^{c_a} h_j \left(1 + v(c_j)\right) g(c_j) \ dc + \int_{c_a}^{c_D} h_j g(c_j) \ dc \right\}$$
The average values of brand level are

$$\bar{\xi} = \frac{1}{G(c_D) + \varphi G(c_a)} \left\{ \int_{c_a}^{c_p} c_j \left( 1 + \nu(c_j) \right) g(c_j) \, dc + \int_{c_a}^{c_p} c_j g(c_j) \, dc \right\}$$

$$\bar{h} = \frac{1}{G(c_D) + \varphi G(c_a)} \left\{ \int_{c_a}^{c_p} h_j \left( 1 + \nu(c_j) \right) g(c_j) \, dc + \int_{c_a}^{c_p} h_j g(c_j) \, dc \right\}$$

Aggregate output is given by

$$Q = \frac{L}{\eta} (\beta - c_D)$$  \hspace{1cm} (4.16)

and, the total number of firms in the market is given as follows:

$$N^H = \frac{Q}{\bar{q}}$$  \hspace{1cm} (4.17)

where

$$\bar{q} = \frac{1}{G(c_D)} \left\{ \int_{c_a}^{c_p} q_{ji} \left( 1 + \nu(c_j) \right) g(c_j) \, dc + \int_{c_a}^{c_p} q_{ji} g(c_j) \, dc \right\}$$

is the average quantity at the variety level. Then we can derive the number of brands, $M^H$, the mass of acquired brands, $M^a$ and the number of entrants, $N_e$ from the following equations.

$$N^H = G(c_D)N_e,$$

$$M^a = \bar{\nu} G(c_a)N_e,$$

$$M^H = N^H + M^a$$

In this linear model, I assume that consumers have positive demands for the numeraire good. Therefore it need to ensure that each consumer spends on the differentiated varieties less than its entire labor income. That is, spending per consumer on the varieties is less than 1.
4.2. Free Trade Equilibrium

Now suppose that trade liberalization reduces all fixed and variable trade costs to zero. Then the problem is identical to an increase in market size, since markets are perfectly integrated through trade. Using this property, we analyze the effect of trade liberalization by investigating the effect of increased market size. The main results of free trade are summarized in the proposition 6.

Proposition 6. When the condition for zero acquisition price holds, a move from autarky to free trade brings the following results.

(i) Cutoff for successful entry, $c_D$ drops. Thus the least efficient firms exit.
(ii) Most efficient firms always increases their number of brands by acquiring other firm’s brand equity.
(iii) The move of acquisition cutoff, $c_a$ is ambiguous. If the effect of intensified competition is high enough, then the acquisition cutoff may decrease.
(iv) The move of overall proportion of acquisition is ambiguous which is depending on the value of parameters and functional form of the cost distribution function. In summary, if the effect of intensified competition is high enough, the acquisition proportion may decrease.
(v) Aggregate productivity increases.
(vi) Low efficiency firms reduce scope and high efficiency firms expand total range of product (including acquired brand).

Proof. See Appendix

More varieties are sold in each country, so the demand for each product, represented by ‘a’ drops after trade liberalization. This increased competition induces the lower value of cost cutoff $c_D$.

Trade liberalization works differently among firms. Trade brings
about exporting opportunities as well as the intensified competition. Low productive firms are mostly affected by intensified competition, but high productive firms are mostly affected by increased market. That is, the profitability of low efficient firm decreases, on the contrary, the profitability of high efficient firm increases. The profitability of acquisition for high efficiency firms also increases, thus they expand their production by acquiring other firms. However, the net effect of trade on the mid productive firms is ambiguous. If the competition is high enough, then the mid productive firms will decide the smaller number of brands and the cost cutoff for acquisition, \( c_a \) would decrease. In spite of ambiguity of acquisition cutoff, however, it is clear that brand equity moves from low to high efficiency firms. Moreover we can see that average productivity increases after trade liberalization from numerical solution in section 4.3.

\[
\frac{dc}{dt} < 0
\]

Now consider the effect of within-brand cannibalization effect. Under positive value of \( \gamma \), consumers want to consume various brand, so that unprofitable firms can survive. Even if productive firms tend to acquire more in equilibrium, the overall effect of \( \gamma \) on market competition is still negative. In the industry with high \( \gamma \), unproductive firms can produce more by taking advantage of lowered competition. High productive firms mostly get hurt by high degree of within-brand cannibalization, so they acquire more brands and decrease the scope of each brand. Consequently, the distribution of sales per brand has flatter shape in industry with high \( \gamma \).
Proposition 7. When the condition for zero acquisition price holds, the industry with high value of $\gamma$ satisfies the following properties.

(i) Cutoff cost for successful entry, $c_D$ is higher.

(ii) The acquisition cutoff cost, $c_a$ is ambiguous.

(iii) The number of acquired brands for the most efficient firms is higher.

(iv) The overall proportion of acquired brand is ambiguous which is depending on the value of parameters and functional form of the cost distribution function.

Proof. See the Appendix

4.3. Numerical Solution

In this section, I parameterize the theoretical model and solve it numerically. These solutions provide a visual representation of the equilibrium and enable us to compare the equilibrium of linear model with CES model. By doing that we can see how the overall effect of product market competition changes the interaction of acquisition and trade liberalization. The choice of parameter values is as follows:

Variety specific cost, $\lambda=5$

Sunk entry cost, $f_e=10$

Fixed acquisition cost, $f_m=20$

Pareto parameter, $\theta=2$. ³⁹

³⁹ Del Gatto, Mion and Ottaviano (2006) estimate the distribution of total factor productivity using firm-level data for a panel of 11 EU countries and 18 manufacturing sectors. The average $\theta$ is estimated to be close to 2. Under $\theta = 1$ and the above specific parameter values, acquisition proportion and acquisition cutoff increase after trade liberalization.
Upper bound of cost, $c_M=1$

Measure of “love of variety,” $\delta=0.5$

Measure of “love of brand,” $\gamma \in [0.05, 0.25]$

Market size, $L \in [1000, 10000]$

The following figures describe the numerical solution with different values of $\gamma$ and $\tau$. These results are the same with the Proposition 6 and 7.

Figure 12. Proportion of acquisition with $\gamma \in [0.05, 0.25]$ and $L \in [1000, 10000]$

As you can see in Figures 12, 13 and 14, the proportion of acquisition increases on the most of $\gamma$s as market size increases, but it decreases as market size increases on some range of $\gamma$. 
Figure 13. Proportion of acquisition with $\gamma=0.05$

Figure 14. Proportion of acquisition with $\gamma=0.25$
Like CES model, exit rate decreases as $\gamma$ increases, and this rate increases in the exposure to trade. The range of firm cost to acquire brand is reduced in globalization under given parameters.

Figure 15. Cost cutoff with $\gamma \in [0.05, 0.25]$ and $L \in [1000,10000]$

Figure 16. Acquisition cutoff with $\gamma \in [0.05, 0.25]$ and $L \in [1000,10000]$
Figure 17. Average cost with $\gamma \in [0.05, 0.25]$ and $L \in [1000,10000]$

From Figure 17, we can conjecture that overall industry efficiency is improved after trade liberalization. The average cost might decrease as market size increases, since existing brand equity moves from unproductive to more productive firms.
5. Empirics

Throughout this section, I analyze the effect of trade liberalization on brand acquisitions by investigating the effect of CUSFTA. Since post-NAFTA period and post-WTO period overlap, there might exist a mixed effect in the period 1995-1999. Therefore I only conduct an empirical analysis of the effect of CUSFTA during 1980-1989. To trace the transfer of brands between firms, I use USPTO trademark application data and assignment data. The data on tariffs are from Trefler (2004), which defined tariffs as duties divided by imports. The industrial U.S. trade data are from Schott (2010)’s U.S. Manufacturing Imports and Exports database, and the other U.S. industrial data are from NBER-CES Manufacturing Industry Database (2013).

5.1. Properties of Trademark

“A trademark is a word, phrase, symbol, design, color, smell, sound, or combination thereof that identifies and distinguishes the goods and services of one party from those of others.”40 That is, trademarks play a role as a source of identifier of brand to consumer. Furthermore, when a brand is sold, all associated trademarks are transferred. This is why trademark assignment data are suitable for investigation of brand acquisition.

To understand the properties of trademark, first see the life cycle of a registered trademark as follows: An owner files an application to register some mark with the USPTO, then the application goes through examination and

40 See 15 U.S.C. §1127
publication for opposition. If application satisfies legal requirements and there is no acceptable opposition, the trademark is registered, and the registered trademark must be renewed for ten-year term. Prior November 1989, registration were renewed for 20 year term, thereafter, registered trademark were renewed for 10 year term. In the 6 year after registration, the owner must file an acceptable affidavit or declaration of continued use or excusable nonuse with USPTO, and if not, the registration is cancelled. In addition, the owner must file an application for renewal and pay some fees between the 9th and 10th years after the registration date. If the renewal application is not filed within the time periods, the registration expires. The change on the lifecycle of trademarks in 1989 must be considered when analyzing the effect of CUSFTA. More frequent declaration of use and application of renewal could raise the death rate of trademarks, therefore affect the expected value of each trademark.

Generally, ownership transfer of trademark involves the transfer of the right, title, and interest on a mark. Assignment apart from goodwill is invalid in United States, and interpretation of goodwill has evolved over time. Before 1946 this rule required full transfer of business ownership including tangible assets. After that this relaxed and allowed for transfer without tangible assets as long as the assignee’s products were similar in kind. Therefore assignment related transactions can capture the incentive of acquiring intangible assets apart from the incentive of acquiring establishments e.g., plant and machines.

The USPTO uses International Classification of Goods and Services

41 There is no legal requirement for parties to report trademark assignments to the USPTO. However, an assignee must record the assignment to make legal claims or take action.
under Nice Agreement so called Nice Classification to assign the appropriate classification to the goods and services since 1973. There are 45 classes, including 34 goods classes and 11 services classes. The applicant initially designates the class numbers he deems appropriate. To expand protection of the mark for use on other products, the owner must apply for a new registrations for the same mark. For instance, the “FORD” is registered in different 19 classes between 1917 and 1997. Because of this industry classification differ with Standard Industrial Classification (SIC) of trade datasets, I construct mappings linking SIC 4 digit codes to Nice codes. As Nice Classification has broad coverage and has the different classification system from SIC, the concordance between Nice and SIC can mislead the results of empirical test. More exquisite mapping is needed for further investigation.

5.2. Indirect Measure of $\gamma$

Throughout the theoretical model, the parameter $\gamma$ is important variable governs the incentive of acquisition. The parameter $\gamma$ can be derived from the elasticity of substitution between varieties and the elasticity of substitution between brands, but it is difficult to estimate the value of elasticity. Hence, I take the various way of constructing indirect measure of $\gamma$ in this section to examine the testable implication of theoretical model.

First, I construct the value of $\gamma$ as the same way which Spearot (2012) uses. He use the fact that when varieties within the differentiated industry are

42 See Graham, Hancock, Marco and Myers (2013)
43 This converter does not have weight. Every 4 digit SIC code has the same weight in the mappings.
more substitutable, observed sales heterogeneity should be larger. Like Spearot (2012), I first derive the following relationship between Hirschmann-Herfindahl Index (HHI) and $\gamma$ using simulation in section 3.5 and 4.3.

**Hypothesis.** Holding the set of brands in industry $k$ in year $t$,

$$\frac{\partial HHI_{k,t}}{\partial \gamma} < 0 \quad \text{where} \quad HHI_{k,t} = \frac{\sum_{j \in S_{k,t}} SALE_{j,t}^2}{(\sum_{j \in S_{k,t}} SALE_{j,t})^2}$$

Y-axis of Figure 18 is Hirschmann-Herfindahl Index (HHI) calculated at brand level, and one of Figure 19 is calculated at firm level. Like **Hypothesis**, both figures illustrate that industrial concentration is lower in the industry with high value of $\gamma$.

![Figure 18. Hirschmann-Herfindahl Index at brand-level](image)

---

44 These figures represent the numerical solutions under CES preference.
To recover $\gamma$, I collect the residuals after regressing the following equation:

$$ln(HHI_{k,t}) = \alpha + \beta_1 \ln(N_{k,t}) + \beta_2 \ln(Skew_{k,t}) + \beta_3 \ln(Average\;sale/Labor_{k,t})$$

(5.1)

where $N_{k,t}$ is the number of firms in SIC industry $k$ in year $t$, $Average\;sale/LABOR_{k,t}$ is average value of sales per labor, $Skew_{k,t}$ is ratio of maximum productivity to median productivity for industry $k$ in year $t$. The $Sale/Labor$ is used as a primary productivity measure. After regressing log HHI on these explanatory variables, I collect the residuals which capture the unexplained factor of sale’s distribution. Because high $\gamma$ tends to make the flat-shaped distribution of sales, the low value of residuals imply the high value of $\gamma$. Thus, I use inverse value of these residuals as an indirect measure of $\gamma$. According to this measure, class 14 and class 24 is the industry with highest
The second indirect measure of $\gamma$ comes from results of Hui (2004). Using personal computers data, he shows the existence of cannibalization effect for branded multiproduct firms. According to his work, firms with high brand reputation faces bigger cannibalization effect. Thus, I use the age of brands as an indirect measure of $\gamma$. I construct the values by taking average on the ages of trademarks for each industry. According to this measure, class 30 and class 32 are the industry with highest value of $\gamma$. In a similar way, the ratio of trademarks which have prior trademarks is used as an indirect measure of $\gamma$. “Prior mark” variable in the Casefile database provides the information of whether each trademark’s owner has the pre-existent trademarks. From this variables, we can identify whether each trademark is new entrant or not. Using the data, I construct prior ratio, which is average share of incumbent trademarks for each industry. According to this measure, class 30, class 32 and class 29 are the industries with highest $\gamma$. Note that these two indirect measures of $\gamma$ are closely related to each other. See Figure 20, then we find the strong correlation between prior ratio and ages of trademarks.

---

45 The class 14 includes Jewelry products, such as precious stones, watches and precious metals. The class 24 includes mainly textile goods.

46 The class 30 includes mainly processed foods, such as coffee, tea, bread, cereals, sugar, salt, mustard, sauces and spices. The class 32 includes light beverages, such as beers, fruit juices, mineral waters.

47 However, this should be cautious. Since citation of prior registrations or pending applications is not mandatory, there exists selection bias.

48 The class 29 includes mainly meat and processed foods, such as jellies, jams, eggs, milk, milk products, edible oils and fats.
The third indirect measure of $\gamma$ is derived from the idea that $\gamma$ is higher in the final (finished) product class than the intermediate (unworked/semi-worked) product class. For example, Class 1 includes mainly chemical products used in industry, science and agriculture, thus this is classified into industries with low $\gamma$. On the contrary, Class 30 including processed foods like coffee, tea, bread, sugar, salt and sauce is labeled as industries with high $\gamma$.

The relations among these indirect measures of $\gamma$ are described in the following table.
Table 10. Correlation among different measures of $\gamma$

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Residual</th>
<th>Age</th>
<th>Prior ratio</th>
<th>Final/Intermediate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>-0.192</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prior ratio</td>
<td>-0.312</td>
<td>0.652</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Final/Intermediate</td>
<td>-0.022</td>
<td>0.427</td>
<td>0.144</td>
<td>1.000</td>
</tr>
</tbody>
</table>

*Notes:* To construct the measures, I use the data during 1980-1989. The column “Residual” indicates the value of residuals from the regression of (5.1), and the column “Final/Intermediate” indicates the dummy value which is constructed as above.

Since the “Residual” is inversely related to the true $\gamma$, the negative correlation among other measures is consistent with the prediction.

### 5.3. Test Results

This section examines the role of within-brand cannibalization and the effect of trade liberalization on decisions of acquisition which are summarized in Propositions 3, 5, 6 and 7. The main testable implication from those propositions are described as follows:

(i) Increased export opportunity raises the return to acquiring a brand equity;

(ii) Increased import competition lowers the return to acquiring a brand equity;

(iii) Acquisition incentive is higher in an industry with a high degree of within-brand cannibalization effect.

Bilateral tariff reductions induced by free trade agreement have joint effects, such as increased exporting opportunity and challenge of intensified
competition, and those effects work in the opposite way on acquisition incentive. Thus, not only FTA-mandated Canadian tariff rate, $\tau_{kt}^{CA}$, but also U.S. tariff rate, $\tau_{kt}^{US}$, is added into empirical analyses as a measure of trade liberalization. In addition, indirect measure of $\gamma$ is included into analyses, since the degree of cannibalization effects affect the return of acquisitions like (iii). Throughout this section, I conduct various econometric analyses for testing the testable implications at the industry- and trademark-level. Empirical strategies at the industry-level involve “differences-in-differences,” “fixed effect” and “random effect” specifications. At the trademark-level, I use “linear probability,” “probit” and “logit” specifications.

At first, the linear regression equation is given by

$$acq_{kt} = \alpha + \beta_1 Ex_{kt} + \beta_2 Im_{kt} + \beta_3 Cann_k + \delta d_t + u_k + e_{kt}$$ \hspace{1cm} (5.2)

where $acq_{kt}$ is acquisition ratio of industry $k$ in time period $t$, which are calculated by

$$acq_{kt} = \frac{\text{Number of assigned trademarks}_{kt}}{\text{Number of existing trademarks}_{kt}}, \hspace{0.2cm} t = 0, 1.$$ 

$Ex_{kt}$ is a dummy variable that equals 1 if an industry $k$ experienced above-median Canadian tariff cuts in time $t=1$. $Im_{kt}$ is a dummy variable that equals 1 if an industry $k$ experienced above-median U.S. tariff reductions in time $t=1$. $Cann_k$ is also dummy variable that equals 1 if an industry faced high within-brand cannibalization effect. Finally, $d_t$ equals 1 for the years of post-FTA period. $u_k$ is industry fixed effects that control for unobserved heterogeneity that affects the decision of acquisition. Taking first differences across periods
t=0 and t=1 yields the following regression specification.

\[ \Delta acq_k = \beta_1 \Delta Ex_k + \beta_2 \Delta Im_k + \delta + \Delta e_{kt} \]  

(5.3)

Now we have \( \Delta Ex_k = 1 \) if industry k is the 50% of the industries with highest Canadian tariff cut, and \( \Delta Im_k = 1 \) if industry k is the 50% of the industries with highest U.S. tariff cut. This specification controls for specific industry trends by differencing out \( u_k \), but also differencing out the effect of cannibalization, \( Cann_k \). Then the results of OLS regression of (5.3) are summarized as follows:

<table>
<thead>
<tr>
<th></th>
<th>[1]</th>
<th>[2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta Ex )</td>
<td>0.001 (0.001)*</td>
<td>0.002 (0.001)**</td>
</tr>
<tr>
<td>( \Delta Im )</td>
<td>-0.002 (0.001)**</td>
<td>-0.002 (0.001)**</td>
</tr>
<tr>
<td>Constant</td>
<td>0.011 (0.001)**</td>
<td>0.008 (0.001)**</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is the change of acquisition ratio between t=0 and t=1, \( \Delta acq_k \). The estimating equation is (5.3) and use CUSFTA-mandated tariffs, \( \tau_{kt}^{CA} \) and \( \tau_{kt}^{US} \) to construct dummy variables Ex and Im. Standard errors for each coefficient belong in parentheses. *, **, *** indicate statistical significance at the 10%, 5% and 1%.

The column [1] shows result of the case where FTA-negotiation period, 1986-1987 is included in the Post-FTA periods, the column [2] shows result of the case where negotiation period is included in the pre-FTA periods. According to the theoretical prediction, the export market entry increases acquisition rate and the intensified competition induced by lowered trade barriers decreases
acquisition rate. Like the predictions, Table 11 shows that acquisition increases in the industries experiencing above-median Canadian tariff reductions, and acquisition decreases in the industries experiencing above-median U.S. tariff reductions after CUSFTA. The results of these simple DID estimators are consistent with the testable implications.

To investigate the effect of $\gamma$ in the “differences in differences” estimation, we can think of the case where $\gamma$ is time-variant. In such case, the regression equation (5.2) can be rewritten as

$$\text{acq}_{kt} = \alpha + \beta_1 E_{kt} + \beta_2 I_{kt} + \beta_3 \text{Cann}_{kt} + u_k + \delta \text{Post}_t + \epsilon_{kt}$$ (5.4)

where $\text{Cann}_{kt}$ is a dummy variable that equals 1 if an industry $k$ is one of the 50% industries with highest $\gamma$ in period $t$. Taking first differences between periods $t=0$ and $t=1$, then we have

$$\Delta \text{acq}_k = \beta_1 \Delta E_k + \beta_2 \Delta I_k + \beta_3 \Delta \text{Cann}_k + \delta + \Delta \epsilon_{kt}$$ (5.5)

where $\Delta \text{Cann}_k = \text{Cann}_{k1} - \text{Cann}_{k0}$ equals 1 if an industry experienced increase of $\gamma$, and equals 0 if an industry does not experience any change in $\text{Cann}$. $\Delta \text{Cann}_k$ equals -1 if an industry experienced decrease of $\gamma$ after Free Trade Agreement. The OLS results of (5.5) using residuals, average age and prior ratio are summarized in Table 12. In this table, the post-FTA period is defined as the years of 1988-1989.
Table 12. Detailed results with time-variant $\gamma$ using DID, 1980-1989

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Ex$</td>
<td>0.002 (0.001)**</td>
<td>0.002 (0.001)**</td>
<td>0.002 (0.001)***</td>
</tr>
<tr>
<td>$\Delta Im$</td>
<td>-0.002 (0.001)**</td>
<td>-0.002 (0.001)**</td>
<td>-0.002 (0.001)**</td>
</tr>
<tr>
<td>$\Delta Cann$</td>
<td>-0.004 (0.001)***</td>
<td>-0.000 (0.002)</td>
<td>0.004 (0.001)***</td>
</tr>
<tr>
<td>Constant</td>
<td>0.008 (0.001)***</td>
<td>0.008 (0.001)***</td>
<td>0.007 (0.001)***</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is the change of acquisition ratio between $t=0$ and $t=1$, $\Delta acq_k$. The estimating equation is (5.5) and use CUSFTA-mandated tariffs, $\tau^{CA}_{kt}$ and $\tau^{US}_{kt}$ to construct dummy variables $Ex$ and $Im$. Each column reports the results of “differences in differences” using residuals, average age and prior ratio as an time-variant measure of $\gamma$. Standard errors for each coefficient belong in parentheses. *, **, *** indicate statistical significance at the 10%, 5% and 1%.

The results are consistent with predictions, but the coefficient of time-variant $\gamma$ is insignificant in [2]. This might come from the fact that the indirect measure of $\gamma$ is time-invariant variables. Actually, the share of industries which experienced the change of $Cann_{kt}$ is 83.78% in “Residual,” 89.19% in “Prior ratio,” but 94.59% in “Age.”

Now go over more precise empirical specification. Let $k$ represents industries, and $t$ indicates years, then baseline specification can be represented as the following regression:

$$acq_{kt} = \alpha + \beta_1 \tau^{CA}_{kt} + \beta_2 \tau^{US}_{kt} + \beta_3 Gamma_{kt} + \beta_4 B_{kt} + \epsilon_{kt}$$

where $\epsilon_{kt} = u_k + e_{kt}$

(5.6)

$\tau^{CA}_{kt}$ is the FTA-mandated Canadian tariff concessions granted to the United States, and $\tau^{US}_{kt}$ is the FTA-mandated US tariff concessions granted to Canada.
\( \Gamma_{kt} \) is the indirect measure of \( \gamma \). \( BC_{kt} \) is the variable governs the business cycle which is the industrial value added data from NBER-CES Manufacturing Industry Database (2013). The error term is composed of \( u_k \) and \( e_{kt} \). \( u_k \) is industry fixed effect in fixed effect model, while \( u_k \) is random variables satisfying \( u_k \sim (0, \sigma_u^2) \) in random effect model. The idiosyncratic error term, \( e_{kt} \) is satisfying \( e_{kt} \sim (0, \sigma^2_e) \). For the baseline test, I use GLS instead of OLS because of the heteroscedasticity. The results using prior ratio as \( \Gamma_{kt} \) are given following tables.

**Table 13. Detailed results with prior ratio, 1980-1989**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{kt}^{CA} )</td>
<td>0.295 (0.041)***</td>
<td>0.157 (0.060)***</td>
<td>0.3012 (0.052)***</td>
</tr>
<tr>
<td>( \tau_{kt}^{US} )</td>
<td>-0.044 (0.038)</td>
<td>-0.068 (0.063)</td>
<td>-0.022 (0.052)</td>
</tr>
<tr>
<td>( \Gamma_{kt} )</td>
<td>0.314 (0.030)***</td>
<td>1.515 (0.278)***</td>
<td>0.310 (0.060)***</td>
</tr>
<tr>
<td>( BC_{kt} )</td>
<td>0.097 (0.030)***</td>
<td>0.762 (0.162)***</td>
<td>0.102 (0.060)*</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.073 (0.033)***</td>
<td>-0.000 (0.044)***</td>
<td>0.000 (0.060)***</td>
</tr>
</tbody>
</table>

*Notes:* The dependent variable is the acquisition ratio. Standard errors for each coefficient belong in parentheses. *, **, *** indicate statistical significance at the 10%, 5% and 1%. All coefficients are the standardized values.

These results also coincide with the prediction of theoretical model and are statistically significant. Only the effect of increased competition, coefficient of \( \tau_{kt}^{US} \) is insignificant. According to the result of Hausman test, there exists endogeneity between explanatory variables and unobserved variable \( u_k \).\(^{49}\)

\(^{49}\) In such case, where the condition \( \text{cov}(x_{kt}, u_k) = 0 \) does not hold, the estimator of random effect cannot be consistent estimator.
Thus, I only report the results of fixed effect estimation using various measures of \( \gamma \).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{kt}^{CA} )</td>
<td>0.289 (0.051)***</td>
<td>0.220 (0.058)***</td>
</tr>
<tr>
<td>( \tau_{kt}^{US} )</td>
<td>-0.051 (0.050)</td>
<td>-0.109 (0.065)*</td>
</tr>
<tr>
<td>( Gamma_{kt} )</td>
<td>-0.112 (0.051)**</td>
<td>1.148 (0.267)***</td>
</tr>
<tr>
<td>( BC_{kt} )</td>
<td>0.051 (0.053)</td>
<td>0.721 (0.165)***</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>Omitted</td>
<td>-0.000 (0.044)***</td>
</tr>
</tbody>
</table>

*Notes: The dependent variable is the acquisition ratio. I use the average value of residuals during 1980-1989 as an indirect measure of \( \gamma \) in column [1]. Because \( Gamma_{kt} \) is dummy variable in [1] and [3], I use LSDV method. Standard errors for each coefficient belong in parentheses. *, **, *** indicate statistical significance at the 10%, 5% and 1%. All coefficients are the standardized values.*

Now go over the property-level data. Letting \( j \) index trademarks, the baseline property-level specification is given by

\[
ass_{jkt} = \begin{cases} 
1 & \text{if } ass_{jkt}^* > 0 \\
0 & \text{if } ass_{jkt}^* \leq 0 
\end{cases}
\]

\[
ass_{jkt}^* = \alpha + \beta_1 \tau_{kt}^{CA} + \beta_2 \tau_{kt}^{US} + \beta_3 Gamma_{kt} + \beta_4 BC_{kt} + \epsilon_{jkt}
\]

where dependent variable \( ass_{jt} \) equals to 1 if the trademark \( j \) is assigned in time \( t \). Then the regression results are summarized in Table 15.
Table 15. Property-level results, 1980-1989

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{kt}^{CA}$</td>
<td>0.001 (0.000)***</td>
<td>0.032 (0.001)***</td>
<td>0.076 (0.003)***</td>
</tr>
<tr>
<td>$\tau_{kt}^{US}$</td>
<td>-0.000 (0.000)***</td>
<td>-0.005 (0.001)***</td>
<td>-0.011 (0.002)***</td>
</tr>
<tr>
<td>Gamma$_{kt}$</td>
<td>0.004 (0.000)***</td>
<td>0.060 (0.001)***</td>
<td>0.136 (0.002)***</td>
</tr>
<tr>
<td>BC$_{kt}$</td>
<td>0.000 (0.000)***</td>
<td>0.006 (0.001)***</td>
<td>0.014 (0.002)***</td>
</tr>
<tr>
<td>Constant</td>
<td>0.026 (0.000)***</td>
<td>-1.943 (0.001)***</td>
<td>-3.623 (0.002)***</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is the acquisition ratio. I use “age” as an indirect measure of $\gamma$. The results using other measures are similar with this result. Column [1] is the coefficients of linear probability model. Standard errors for each coefficient belong in parentheses. *, **, *** indicate statistical significance at the 10%, 5% and 1%.

The results are significant and coincide with the predictions. At the property-level estimation, the effect of intensified competition, $\tau_{kt}^{US}$ is consistent with the theoretical implication but also statistically significant.
6. Conclusion

This paper has shown that globalization lead more productive firms to expand their production by acquiring other firms’ brand, and reallocate brand equity from less productive to more productive firms. By introducing additional decision margin of firms, this paper provides additional channel of resource reallocation induced by trade liberalization. In addition, this shows that the reallocation of intellectual property between firms is more active in the industry with high within-brand cannibalization.

On the empirical side, the interaction of brand acquisition and trade liberalization has investigated using U.S. trade data and USPTO trademark data. It provides broad pictures of trademark assignment activities, and show how trade liberalization leads to rise in acquisition during the period of 1980-1989.

However, there is limitation because the theoretical model of this paper only solve the case with zero acquisition price. The consideration of positive acquisition price will be needed for rich analysis. Also this study does not consider the quality difference among brands, and only solve the static model. The process of building reputation of brand is important to investigate the transactions of brand equity more precisely. Thus extension to dynamic model with qualitatively different brand can be widen the implication of this model. Furthermore, financial data of firms are necessary to conduct the structural estimation. The mapping of USPTO data with firm-level financial data will be required for further empirical work.
References


Melitz, Marc J, and Gianmarco I.P. Ottaviano, “Market Size, Trade, and
Appendix

**A1. Definition and Example of Brand**

Brand is unique design, sign, symbol, words, or a combination of these, employed in creating an image that identifies a product and differentiates it from its competitors. Throughout this paper, I use “brand” to refer to the perception customers have about that product or service, and to refer to the name signifying the source of a product or service.

For example, “Procter & Gamble Co.” owns brands such as, “Anna Sui,” “Braun,” “Cover Girl,” “Dolce & Gabbana,” “Downy,” “Gillette,” “Olay,” “Oral-B,” “Wella,” etc. Each brand has various product line, for instance, the brand “Gillette” produces different types of shavers and razors. I refer to “Procter & Gamble Co.” as the owner of the brand, “Gillette,” and refer to its shavers and razors as the varieties in this paper.

**A2. Model with CES Preference**

**A2.1. Existence and Uniqueness of Equilibrium in Closed Economy**

Proof of Proposition 1.

To establish the proposition, need to show 
\[
\frac{dV_e}{d\varphi^*} < 0
\]

Note that

\[
\frac{\partial \varphi^N}{\partial \varphi^*} = \frac{\{(1-\alpha)+k(\sigma-1)\varphi^N\}}{k(\sigma-1)(\alpha-1)(G(\Gamma \varphi^*)-G(\varphi^*))} \times \left[ \Gamma g(\Gamma \varphi^*) \left( \frac{\Gamma \varphi^*}{\varphi^N} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} - 1 \right] - g(\varphi^*) \left( \frac{\varphi^*}{\varphi^N} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} - 1 \]
and also,

\[ \frac{\partial \bar{\phi}^A}{\partial \varphi^*} = - \frac{(k-1)G\varphi}{k(\sigma-1)(1-G\varphi^*)} \left( \frac{G\varphi^*}{\bar{\phi}^A} \right)^{k(\sigma-1)} \]

The derivative of expected value of entry, \( V_e \) with respect to \( \varphi^* \) will be

\[ \frac{\partial V_e}{\partial \varphi^*} = -f \Gamma (\sigma-1)(\alpha-1) \frac{\varphi^*}{\sigma-\alpha} \left[ \{ G(\Gamma \varphi^*) - G(\varphi^*) \} \left( \frac{\varphi^N}{\Gamma \varphi^*} \right)^{k(\sigma-1)} \right] < 0 \]

The unique equilibrium value of \( \varphi_a^* \) follows immediately from

\[ \varphi_a^* = \varphi^* - k(\sigma-1) \frac{1-\alpha}{\sigma-1} \frac{1}{\alpha-1} - k(\sigma-1) \frac{1-\alpha}{\sigma-1} \frac{(1-\sigma)}{\alpha-1} + k(\sigma-1) \frac{1}{\alpha-1} \frac{(1-\sigma)}{\alpha-1} \]

**A2.2. Another Way of Selection in Acquisition and Export**

Throughout the paper, \( \varphi^* < \varphi_x^* < \varphi_a^* \) is considered. Here, I consider another case where \( \varphi^* < \varphi_a^* < \varphi_x^* \). In order to support this equilibrium, cost cutoff \( \varphi_a^* \) must satisfy

\[ \Pi_j^{A,aut}(\varphi_a^*) = \Pi_j^{N,aut}(\varphi_a^*) \quad (A.1) \]

Therefore, the following relation holds

\[ \varphi_a^* = \varphi^* \left( \frac{(1-\alpha)+k(\sigma-1)}{(\sigma-\alpha)k} \right)^{k(\sigma-1)} \]

and from \( \frac{(1-\alpha)+k(\sigma-1)}{(\sigma-\alpha)k} > 1 \) and \( \frac{(1-\alpha)+k(\sigma-1)}{k(\alpha-1)(\sigma-1)} > 0 \), \( \varphi_a^* > \varphi^* \) is satisfied. The export productivity cutoff \( \varphi_x^* \) is determined by following equation:
\[ \Pi_j^{A,\text{open}}(\varphi_x^*) = \Pi_j^{A,\text{aut}}(\varphi_x^*) \]  
(A.2)

where

\[ \Pi_j^{A,\text{open}}(\varphi) = B \left(1 + nt^{1-\alpha}\frac{k(\sigma-1)}{(\alpha-1)(k-1)}\left(f+nfx \frac{f}{f} \frac{1}{(a-1)(k-1)}\right) \right) \times \frac{\Pi_j^{A,\text{aut}}(\varphi)}{B} \]  
(A.3)

Note that \( \Pi_j^{A,\text{open}}(\varphi) \) is multiplication of \( \Pi_j^{A,\text{aut}}(\varphi) \). If the term B equals to 1, then \( \varphi_x^* \) is not uniquely defined. If the term B is greater than 1, then exporting is profitable for all acquirers, thus this is contradictory to \( \varphi_a^* < \varphi_x^* \). If the term B is smaller than 1, then every firm will not export. That is, there is no unique equilibrium in the second case.

**A2.3. Existence and Uniqueness of Equilibrium in Open Economy**

**Proof of Proposition 2.** To establish the proposition, need to show \( dV_e/d\varphi^* < 0 \).

Note that

\[ \frac{\partial \phi^{N,\text{aut}}}{\partial \varphi^*} = \frac{1}{k(\sigma-1)(\alpha-1)} \left[ \frac{G_x^* (G_x^*)}{G_x^* - G_x^*} \right] \left[ \frac{k(\sigma-1)(\alpha-1)}{(1-\alpha) + k(\sigma-1)} \left(1 - \frac{G_x^*}{G_x^*} \right) \right] \]

\[ \frac{\partial \phi^{N,\text{open}}}{\partial \varphi^*} = \frac{1}{k(\sigma-1)(\alpha-1)} \left[ \frac{G_x^* (G_x^*)}{G_x^* - G_x^*} \right] \left[ \frac{G_x^*}{G_x^*} \right] \left[ \frac{k(\sigma-1)(\alpha-1)}{(1-\alpha) + k(\sigma-1)} \left(1 - \frac{G_x^*}{G_x^*} \right) \right] \]

\[ \frac{\partial \phi^{A,\text{open}}}{\partial \varphi^*} = \frac{1}{k(\sigma-1)(1-G_x^*)} \left[ \frac{G_x^*}{G_x^*} \right] \left[ \frac{k(\sigma-1)(\alpha-1)}{(1-\alpha) + k(\sigma-1)} \left(1 - \frac{G_x^*}{G_x^*} \right) \right] \]
The derivative of expected value of entry, $V_e$ with respect to $\varphi^*$ will be

$$\frac{\partial V_e}{\partial \varphi^*} = -f\{G(\Gamma_x \varphi^*) - G(\varphi^*)\} \left(\frac{\varphi^{\text{N,aut}}}{\varphi^*}\right)^{k(\sigma-1)(\alpha-1)} \frac{k(\sigma-1)(\alpha-1)}{\varphi^*\{(1-\alpha)+k(\sigma-1)\}} - f\{G(\Gamma_a \varphi^*) - G(\Gamma_x \varphi^*)\}(1 +$$

$$nt^{1-\alpha})\frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)} \left(\frac{q_{\text{N,open}}^{\Gamma}}{\varphi^*}\right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} \frac{k(\sigma-1)(\alpha-1)}{\varphi^*\{(1-\alpha)+k(\sigma-1)\}} - (f + nf_x){1 -$$

$$G(\Gamma_a \varphi^*)}\left(\frac{\varphi^{\text{A,open}}}{\Gamma_a \varphi^*}\right)^{\frac{k(\sigma-1)}{1-\alpha}} \left\{\frac{(\sigma-1)(\alpha-1)}{\varphi^*\{(\sigma-1)\}}\right\} < 0$$

The unique equilibrium value of $\varphi_a^*$, $\varphi_x^*$ follows immediately from $\varphi_a^* = \Gamma_a \varphi^*$ and $\varphi_x^* = \Gamma_x \varphi^*$.

A2.4. Impact of Trade Liberalization

**Proposition 3.** The comparative statics of cutoffs with respect to variable trade cost $\tau$ are:

(i) $\frac{d\varphi^*}{d\tau} < 0$ (ii) $\frac{d\varphi_x^*}{d\tau} > 0$ (iii) $\frac{d\varphi_a^*}{d\tau} > 0$ (iv) $\frac{d\text{Propacq}}{d\tau} < 0$.

**Proof of Proposition 3.** The comparative statics of zero-profit productivity cutoff is given by

$$\frac{d\varphi^*}{d\tau} = -\frac{\partial V_e/\partial \tau}{\partial V_e/\partial \varphi^*} = -\frac{\partial V_e/\partial \varphi^*}{\partial V_e/\partial \varphi^*} \quad (A.4)$$

where

$$\frac{\partial V_e}{\partial \tau} = -(f + nf_x){1 - G(\Gamma_a \varphi^*)}(1 +$$

$$nt^{1-\alpha})^{-1}nt^{-\alpha} \left\{\frac{(\sigma-1)(\alpha-1)}{(1-\alpha)}\right\} \left(\frac{q_{\text{N,open}}^{\Gamma}}{\varphi^*}\right)^{\frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)}} - f\{G(\Gamma_a \varphi^*) - G(\Gamma_x \varphi^*)\}(1 +$$

$$nt^{1-\alpha})^{\frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)}} \left(\frac{q_{\text{N,open}}^{\Gamma}}{\varphi^*}\right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} \frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)} < 0.$$
Therefore, using $\frac{\partial V_e}{\partial \phi^*} < 0$ and $\frac{\partial V_e}{\partial \tau}$, we have $\frac{d \phi^*}{d \tau} < 0$.

Now consider the impact of trade cost on the other cutoff levels $\phi_x^*$. 

\[
\frac{d \phi^*_x}{d \tau} = \frac{d (f_{x, \phi^*})}{d \tau} = \frac{d f_{x, \phi^*}}{d \phi^*} + \frac{d \phi^*_x}{d \phi^*} f_{x, \phi^*}
\]

\[= f_{x, \phi^*} \left\{\left(1 + n \tau^{1-\alpha}\right) \frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)} - 1\right\}^{-1} n \tau^{-\alpha} \left(1 + \frac{(a-1)}{(1-\alpha)+k(\sigma-1)} \phi^* \frac{\partial V_e}{\partial \phi^*} - \frac{\partial V_e}{\partial \tau}\right) + \frac{\partial V_e}{\partial \tau}
\]

We already know that $\frac{\partial \phi^*}{\partial V_e} < 0$, we only need to derive the sign of the following function.

\[\left\{\left(1 + n \tau^{1-\alpha}\right) \frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)} - 1\right\}^{-1} n \tau^{-\alpha} \left(1 + n \tau^{1-\alpha}\right) \frac{(a-1)}{(1-\alpha)+k(\sigma-1)} \phi^* \frac{\partial V_e}{\partial \phi^*} - \frac{\partial V_e}{\partial \tau}\]

\[\left\{\left[1 + n \tau^{1-\alpha}\right] \frac{f_{x, \phi^*}}{\phi^*} \frac{k(\sigma-1)(\sigma-1)}{(1-\alpha)+k(\sigma-1)} \left\{1 + \left(n \tau^{1-\alpha}\right) \frac{G(\Gamma_x \phi^*)}{\phi^*} \frac{k(\sigma-1)(\sigma-1)}{(1-\alpha)+k(\sigma-1)} \left\{1 + \left(n \tau^{1-\alpha}\right) \frac{G(\Gamma_x \phi^*)}{\phi^*} \frac{k(\sigma-1)(\sigma-1)}{(1-\alpha)+k(\sigma-1)} \left\{(1 + \left(n \tau^{1-\alpha}\right) \frac{G(\Gamma_x \phi^*)}{\phi^*} \frac{k(\sigma-1)(\sigma-1)}{(1-\alpha)+k(\sigma-1)} \right\}\right\}\right\}\right\}^{-1} < 0
\]
Therefore, we have \( \frac{d\phi^*_x}{dt} > 0 \).

The comparative statics of acquisition productivity cutoff is given by

\[
\frac{d\phi^*_a}{dt} = \frac{d(\Gamma_a \phi^*)}{dt} = \frac{d\Gamma_a}{dt} \phi^*_a + \frac{d\phi^*_a}{dt} \Gamma_a ^{positive} + \frac{d\phi^*_a}{dt} \Gamma_a ^{negative}
\]

\[
= \Gamma_a ^{positive} \frac{\partial \phi^*_a}{\partial \phi^*} \left\{ (1 + n\tau^{1-\alpha})^{-1} n\tau^{-\alpha} \phi^*_a \frac{\partial V_e}{\partial \phi^*} - \frac{\partial V_e}{\partial \tau} \right\}
\]

We already know that \( \frac{\partial \phi^*_a}{\partial \phi^*} < 0 \), we only need to derive the sign of the following function.

\[
(1 + n\tau^{1-\alpha})^{-1} n\tau^{-\alpha} \phi^*_a \frac{\partial V_e}{\partial \phi^*} - \frac{\partial V_e}{\partial \tau} = -f \{ G(\Gamma_a \phi^*) - G(\phi^*) \} (1 + \\
\frac{k}{1-\alpha + k(\sigma-1)} < 0
\]

Therefore, we have \( \frac{d\phi^*_a}{dt} > 0 \).

Finally, we have,

\[
\frac{dProp_{acq}}{dt} = -\{1 - G(\phi^*_a)\} \left( \frac{\phi^*_a}{\phi^*_a} \right) \frac{k}{k-1} \frac{1}{\phi^*_a} \frac{d\phi^*_a}{dt} < 0
\]

**Proposition 4.** The comparative statics of cutoffs with respect to fixed trade cost \( f_x \) are:

(i) \( \frac{d\phi^*_a}{df_x} < 0 \)  
(ii) \( \frac{d\phi^*_a}{df_x} > 0 \)  
(iii) \( \frac{d\phi^*_a}{df_x} > 0 \)  
(iv) \( \frac{dProp_{acq}}{df_x} < 0 \)

**Proof of Proposition 4.** The comparative statics of zero-profit productivity cutoff is given by

\[
\frac{d\phi^*_a}{df_x} = -\frac{\partial V_e/\partial f_x}{\partial \phi^*} = -\frac{\partial V_e}{\partial f_x} \frac{\partial \phi^*_a}{\partial \phi^*} 
\]

\[
(A.7)
\]
where

\[
\frac{\partial v_e}{\partial f_x} = -n\{1 - G(\Gamma_a \varphi^*)\} \left( \frac{\phi_{A,open}}{\Gamma_a \varphi^*} \right)^{\frac{k(\sigma-1)}{k-1}} - n\{G(\Gamma_a \varphi^*) - G(\Gamma_x \varphi^*)\} < 0.
\]

Therefore, we have \( d\varphi^*/df_x < 0 \).

Like variable trade cost \( \tau \), decreased \( f_x \) induce the least productive firms to exit. Now consider the impact of trade cost on the cutoff levels \( \varphi^*_x \).

\[
\frac{d\varphi^*_x}{df_x} = \frac{d(\Gamma_x \varphi^*)}{df_x} = \frac{d\Gamma_x}{df_x} \varphi^* + \frac{d\varphi^*_x}{df_x} \Gamma_x
\]

\[
= \Gamma_x \left( \varphi^*_x \frac{\partial v_e}{\partial \varphi^*} f_x (1-\alpha + k(\sigma-1)) - \frac{\partial v_e}{\partial f_x} \varphi^*_x k(\sigma-1)(\alpha-1) \right) - \frac{1}{f_x} \left\{ G(\Gamma_a \varphi^*) - G(\Gamma_x \varphi^*) \right\} \left( \frac{\phi_{N,aut}}{\Gamma_a \varphi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{k-1}} - n\{1 -
\]

\[
G(\Gamma_a \varphi^*) \left( \frac{\phi_{A,open}}{\Gamma_a \varphi^*} \right)^{\frac{k(\sigma-1)}{k-1}} \left( \frac{(1-\alpha)k(\sigma-1)}{k(\sigma-\alpha)} \right) - \frac{f}{f_x} \left\{ G(\Gamma_a \varphi^*) - G(\Gamma_x \varphi^*) \right\} (1 + n\tau^{1-\alpha}) \left( \frac{\phi_{N,open}}{\varphi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{k-1}} + n\{G(\Gamma_a \varphi^*) - G(\Gamma_x \varphi^*)\} < 0.
\]

This is because,

\[
- \frac{f}{f_x} \left\{ G(\Gamma_a \varphi^*) - G(\Gamma_x \varphi^*) \right\} (1 + n\tau^{1-\alpha}) \left( \frac{\phi_{N,open}}{\varphi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{k-1}} + n\{G(\Gamma_a \varphi^*) - G(\Gamma_x \varphi^*)\}.
\]
\[ n\tau^{1-\alpha} \left( \frac{\phi^{N,\text{open}}}{\phi^*} \right)^{\frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)}} < 0 \]

from

\[ \Pi^{N,\text{open}} = \left( \frac{\phi^{N,\text{open}}}{\phi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} f(1 + n\tau^{1-\alpha})^{\frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)}} - (f + nf_x) > 0. \]

For acquisition productivity cutoff,

\[
\begin{align*}
\frac{d\phi^*_a}{df_x} &= \frac{d(\Gamma_a\phi^*_a)}{df_x} = \frac{d\Gamma_a}{df_x}\phi^* + \frac{d\phi^*_a}{df_x} \Gamma_a, \\
\Gamma_a &= \frac{\partial \phi^*}{\partial \phi_a} \left( \frac{\phi^{N,\text{aut}}}{\phi^*} \right)^{\frac{1}{(1-\alpha)+k(\sigma-1)}} - \frac{nf}{(f+nf_x)} \left( G(\Gamma_x\phi^*) - G(\Gamma_a\phi^*) \right) (1 + \frac{n}{f+nf_x}) \phi^{N,\text{open}} \left( \frac{\phi}{\phi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} - n\{1 - G(\Gamma_a\phi^*) - G(\Gamma_x\phi^*)\} < 0,
\end{align*}
\]

where

\[
\frac{\partial V_e}{\partial \phi^*} \left( \frac{\phi^{N,\text{aut}}}{\phi^*} \right)^{\frac{1}{(1-\alpha)+k(\sigma-1)}} - \frac{nf}{(f+nf_x)} \left( G(\Gamma_a\phi^*) - G(\Gamma_x\phi^*) \right) (1 + \frac{n}{f+nf_x}) \phi^{N,\text{open}} \left( \frac{\phi}{\phi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} - n\{1 - G(\Gamma_a\phi^*) - G(\Gamma_x\phi^*)\} < 0,
\]

since

\[
\begin{align*}
- \frac{nf}{(f+nf_x)} \left( G(\Gamma_a\phi^*) - G(\Gamma_x\phi^*) \right) (1 + n\tau^{1-\alpha})^{\frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)}} &+ \phi^{N,\text{open}} \left( \frac{\phi}{\phi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} < 0, \\
n\{G(\Gamma_a\phi^*) - G(\Gamma_x\phi^*)\} &= \frac{n}{(f+nf_x)} \left( G(\Gamma_a\phi^*) - G(\Gamma_x\phi^*) \right) \left( f + nf_x \right) - f(1 + n\tau^{1-\alpha})^{\frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)}} \phi^{N,\text{open}} \left( \frac{\phi}{\phi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} < 0,
\end{align*}
\]

from

\[
f(1 + n\tau^{1-\alpha})^{\frac{k(\sigma-1)}{(1-\alpha)+k(\sigma-1)}} \phi^{N,\text{open}} \left( \frac{\phi}{\phi^*} \right)^{\frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k(\sigma-1)}} > (f + nf_x).\]
Therefore, we can see that \( \frac{d\varphi_x}{df_x} > 0 \) and \( \frac{d\varphi_a}{df_x} > 0 \).

Finally, for the proportion of acquisition, we have

\[
\frac{d\text{Prop}_{acq}}{df_x} = -\left\{ \left( 1 - G(\varphi_a^*) \right) \frac{k(\sigma - 1)}{k - 1} \left( \frac{\varphi_a^{\text{open}}}{\varphi_a} \right)^{k - 1} \frac{1}{\varphi_a} \frac{d\varphi_a}{df_x} \right\} < 0.
\]

### A2.5. Computation of Aggregate variables

From Free entry condition and average revenue, we can rewrite average revenue as given

\[
\bar{e} = \frac{f_e k \sigma (\sigma - 1)}{(1 - \alpha) + k(\sigma - 1)} \frac{1}{1 - G(\varphi^*)} + \frac{1 - G(\Gamma_a \varphi^*)}{1 - G(\varphi^*)} \left( \frac{\varphi_a^{\text{open}}}{\Gamma_a \varphi^*} \right)^{k(\sigma - 1)} (f + nf_x) \frac{k \sigma (\sigma - 1)}{(1 - \alpha) + k(\sigma - 1)} + \frac{G(\Gamma_a \varphi^* - 1)}{1 - G(\varphi^*)} \left( f + nf_x \right) \frac{k \sigma (\sigma - 1)}{(1 - \alpha) + k(\sigma - 1)}
\]

Then, implementing the Pareto distribution and solving algebraically, rewrite average revenue as given:

\[
\bar{e} = \frac{f_e k \sigma (\sigma - 1)}{(1 - \alpha) + k(\sigma - 1)} \left( \varphi^* \right)^{\theta} + \left( \Gamma_a \right)^{-\theta} \left\{ \theta - \frac{k(\sigma - 1)}{k - 1} \right\}^{-1} (f + nf_x) \frac{k \sigma (\sigma - 1)}{(1 - \alpha) + k(\sigma - 1)} + \frac{G(\Gamma_a \varphi^* - 1)}{1 - G(\varphi^*)} \left( f + nf_x \right) \frac{k \sigma (\sigma - 1)}{(1 - \alpha) + k(\sigma - 1)}
\]

\[
\{1 - (\Gamma_x)^{-\theta}\} \frac{f_e k \sigma (\sigma - 1)}{(1 - \alpha) + k(\sigma - 1)} + \left\{ (\Gamma_x)^{-\theta} - (\Gamma_a)^{-\theta}\right\} \frac{(f + nf_x) k \sigma (\sigma - 1)}{(1 - \alpha) + k(\sigma - 1)}
\]

Now, using the property of \( p_j = \left( \int_{i=0}^{h_j} p_j^{1-\sigma} di \right)^{1-\sigma} = \sigma \cdot h_j^{1-\sigma} / \{\varphi_j(\sigma - 1)\} \)

we can rewrite aggregate price index as

\[
P = \frac{\sigma}{(\sigma - 1)} \left[ M^{N,\text{aut}} \left( \varphi_a^{\text{open}} \right)^{\sigma - 1} \left\{ h_j^{N,\text{aut}} \left( \varphi_a^{\text{open}} \right)^{\sigma - 1} + M^{N,\text{open}} (1 +
\right) \right]
\]
\[ nτ^{1-α}(\tilde{φ}^N,\text{open})^{α-1}\{h_j^N,\text{open} (\tilde{φ}^N,\text{open})\}^{\frac{α−1}{σ−1}} + M^A,\text{open} (1 + \]

\[ nτ^{1-α}(\tilde{φ}^A,\text{open})^{α-1}\{h_j^a,\text{open} (\tilde{φ}^A,\text{open})\}^{\frac{α−1}{σ−1}} \]

where

\[ M^{N,\text{aut}} = \frac{G(φ′_\alpha)-G(φ′)}{1-G(φ′)} N^H, \quad M^{N,\text{open}} = \frac{G(φ′_a)-G(φ′)}{1-G(φ′)} N^H, \]

\[ M^A,\text{open} = \frac{(1+\bar{v})(1-G(φ′_a))}{1-G(φ′)} N^H, \]

\[ h_j^{N,\text{aut}} = \left(\frac{f}{(1-α)+k(σ-1)}\right)^{\frac{1}{k}} \left(\frac{α-1}{λ}\right)^{\frac{1}{k}} \left(\frac{φ_j}{φ′}\right)^{\frac{(α-1)(σ-1)}{(1-α)+k(σ-1)}}, \]

\[ h_j^{N,\text{open}} = (1 + nτ^{1-α})^{\frac{σ-1}{(1-α)+k(σ-1)}} \left(\frac{f}{(1-α)+k(σ-1)}\right)^{\frac{1}{k}} \left(\frac{α-1}{λ}\right)^{\frac{1}{k}} \left(\frac{φ_j}{φ′}\right)^{\frac{(α-1)(σ-1)}{(1-α)+k(σ-1)}}, \]

\[ h_j^{a,\text{open}} = \left(\frac{f+nτ}{σ-α}\right)^{\frac{1}{k}} \left(\frac{α-1}{λ}\right)^{\frac{1}{k}} \left(\frac{φ_j}{φ′}\right)^{1-σ}, \quad h_j^{A,\text{open}} = (1 + \bar{v})h_j^{a,\text{open}}. \]

Then price index is given by

\[ P = \frac{σ}{(σ-1)} (N^H)^{\frac{1}{1-α}} \left[ \frac{G(φ′_\alpha)-G(φ′)}{1-G(φ′)} (\tilde{φ}^N,\text{aut})^{α-1}\{h_j^{N,\text{aut}} (\tilde{φ}^N,\text{aut})\}^{\frac{α−1}{σ−1}} + \right. \]

\[ \left. \frac{G(φ′_a)-G(φ′)}{1-G(φ′)} (1 + nτ^{1-α})(\tilde{φ}^N,\text{open})^{α-1}\{h_j^{N,\text{open}} (\tilde{φ}^N,\text{open})\}^{\frac{α−1}{σ−1}} + \right. \]

\[ \left. \frac{(1+\bar{v})(1-G(φ′_a))}{1-G(φ′)} (1 + nτ^{1-α})(\tilde{φ}^A,\text{open})^{α-1}\{h_j^{a,\text{open}} (\tilde{φ}^A,\text{open})\}^{\frac{α−1}{σ−1}} \right]^{\frac{1}{1-α}} \]

\[ \equiv \frac{σ}{(σ-1)} (N^H)^{\frac{1}{1-α}} E^{\frac{1}{1-α}} \]

where
\[ E \equiv \frac{G(\varphi^*_x) - G(\varphi^*_a)}{1 - G(\varphi^*)} \left( \tilde{\varphi}^{N, aut} \right)^{\alpha - 1} \left\{ h_j^{N, aut} (\tilde{\varphi}^{N, aut}) \right\}^{-1} + \frac{G(\varphi_*^{a, open}) - G(\varphi^*_x)}{1 - G(\varphi^*)} \left( 1 + \right) \]

\[ n\tau^{1 - \alpha}) (\tilde{\varphi}^{N, open})^{\alpha - 1} \left\{ h_j^{N, open} (\tilde{\varphi}^{N, open}) \right\}^{-1} + \frac{(1 + \beta)(1 - G(\varphi_*^{a, open}))}{1 - G(\varphi^*)} \left( 1 + \right) \]

\[ n\tau^{1 - \alpha}) (\tilde{\varphi}^{A, open})^{\alpha - 1} \left\{ h_j^{a, open} (\tilde{\varphi}^{A, open}) \right\}^{-1}. \]

Then implementing the Pareto distribution and solving algebraically, rewrite \( E \) as given:

\[ -\theta \left\{ \frac{k(\sigma - 1)(\alpha - 1)}{(1 - \alpha) + k(\sigma - 1)} - \right\}

\[ \theta \left\{ \frac{f k}{(1 - \alpha) + k(\sigma - 1)} \right\}^{1 - \alpha} \left( \frac{\alpha - 1}{\lambda k} \right)^{\alpha - 1} \left( \frac{\alpha - 1}{k(\sigma - 1)} \right) (\Gamma_x)^{-\theta}(\varphi^*)^{\alpha - 1} + \theta \left\{ \frac{k(\sigma - 1)(\alpha - 1)}{(1 - \alpha) + k(\sigma - 1)} - \right\}

\[ \theta \left\{ \theta - \right\}

\[ \frac{k(\sigma - 1)}{k - 1} \left\{ \frac{f + nf_x}{f} \right\}^{1 - \alpha} \left( \frac{f k}{(1 - \alpha) + k(\sigma - 1)} \right) \left( \frac{\alpha - 1}{\lambda k} \right)^{\alpha - 1} \left( \frac{\alpha - 1}{k(\sigma - 1)} \right) (\Gamma_a)^{-\theta}(\varphi^*)^{\alpha - 1}. \]

A2.6. Impact of “Love of brand”

**Proof of Proposition 5.** Need to show \( dProp_{acq}/d\alpha < 0 \). The proportion of acquisition is given by

\[ Prop_{acq} = \frac{k(\sigma - 1)}{\theta(k - 1) - k(\sigma - 1)} (\varphi_{min})^\theta (\varphi_*^a)^{-\theta} \]

Therefore, the sign of \( dProp_{acq}/d\alpha \) is opposite sign of \( d\varphi_*^a/d\alpha \). Then the
sign of $d\varphi_a^*/d\alpha$ is

$$
\frac{d\varphi_a^*}{d\alpha} = \frac{\partial\varphi_a^*}{\partial V_e} \left\{ \frac{\partial\varphi_e}{\partial \alpha} \cdot \frac{\partial V_e}{\partial \varphi^*} \varphi^* - \frac{\partial V_e}{\partial \alpha} \Gamma_a \right\}
$$

where

$$
\frac{\partial \Gamma_a}{\partial \alpha} \cdot \frac{\partial V_e}{\partial \varphi^*} \varphi^* + \frac{1}{\theta} \left( \frac{\varphi_{\text{min}}}{\varphi^*} \right)^{-\theta} - \frac{\partial V_e}{\partial \alpha} \Gamma_a \left( \frac{\varphi_{\text{min}}}{\varphi^*} \right)^{-\theta} =
$$

$$
-\frac{f}{k(\sigma-1)(\alpha-\theta(1-\alpha)+k(\sigma-1))} \Gamma_a \left\{ \frac{1}{(1-\alpha)+k(\sigma-1)} \ln(1 + n\tau^{1-\alpha}) - \frac{n\tau^{1-\alpha}}{(1+n\tau^{1-\alpha})} \ln \tau \right\} +
$$

$$
n_f \frac{k(\sigma-1)}{k(\sigma-1)(\alpha-\theta(1-\alpha)+k(\sigma-1))} \Gamma_a (\Gamma_x)^{-\theta} \left\{ \frac{1}{(1-\alpha)+k(\sigma-1)} \ln(1 + n\tau^{1-\alpha}) - \frac{n\tau^{1-\alpha}}{1+n\tau^{1-\alpha}} \ln \tau \right\} = f \frac{k(\sigma-1)}{k(\sigma-1)(\alpha-\theta(1-\alpha)+k(\sigma-1))} \Gamma_a \ln \Gamma_a -
$$

$$
n_f \frac{k(\sigma-1)}{k(\sigma-1)(\alpha-\theta(1-\alpha)+k(\sigma-1))} \Gamma_a (\Gamma_x)^{-\theta} \ln \Gamma_a +
$$

$$
n_f \frac{k(\sigma-1)}{k(\sigma-1)(\alpha-\theta(1-\alpha)+k(\sigma-1))} \Gamma_a (\Gamma_x)^{-\theta} \ln \Gamma_a - (f + n_f) \frac{k(\sigma-1)}{k(\sigma-1)(\alpha-\theta(1-\alpha)+k(\sigma-1))} \Gamma_a \ln \Gamma_a -
$$

$$
\frac{k(\sigma-1)}{k(\sigma-1)(\alpha-\theta(1-\alpha)+k(\sigma-1))} \Gamma_a \ln \Gamma_a - f \left\{ \frac{k(\sigma-1)}{k(\sigma-1)(\alpha-\theta(1-\alpha)+k(\sigma-1))} \Gamma_a \ln \Gamma_a -
$$

$$
n_f \left\{ \frac{1}{\theta(k-1)-k(\sigma-1)} \left( \frac{k(\sigma-1)}{k(\sigma-1)^2} \Gamma_a \right)^{-\theta} - f \Gamma_a \right\} \left\{ \frac{1}{(1-\alpha)+k(\sigma-1)} \ln(1 + n\tau^{1-\alpha}) -
$$

$$
= \frac{k(\sigma-1)}{k(\sigma-1)(\alpha-\theta(1-\alpha)+k(\sigma-1))} \left\{ n_f \Gamma_a (\Gamma_x)^{-\theta} - f \Gamma_a \right\} \left\{ \frac{1}{(1-\alpha)+k(\sigma-1)} \ln(1 + n\tau^{1-\alpha}) -
$$

$$
\right\}
$$
\[
\frac{n\tau^{1-\alpha}}{1 + n\tau^{1-\alpha}} \ln \tau = f \frac{k(\sigma-1)}{k(\sigma-1)(\alpha-1) - \theta((1-\alpha)+k\sigma(1))} \left(\frac{k(\sigma-1)}{(1-\alpha)+k\sigma(1)}\Gamma_a \ln \Gamma_a + \frac{n_f x}{k(\sigma-1)(\alpha-1) - \theta((1-\alpha)+k\sigma(1))} \Gamma_a (\Gamma_x)^{-\theta} \{\ln \Gamma_x - \ln \Gamma_a\} + \frac{k(\sigma-1)}{k(\sigma-1)(\alpha-1) - \theta((1-\alpha)+k\sigma(1))} \left(\frac{k(\sigma-1)}{(1-\alpha)+k\sigma(1)}\right) \frac{n_f x (\Gamma_x)^{-\theta}}{k(\sigma-1)} \Gamma_a \Gamma_a^{-\theta} \right)
\]

From
\[
\theta > \frac{k(\sigma-1)}{k-1} > \frac{k(\sigma-1)(\alpha-1)}{(1-\alpha)+k\sigma(1)}
\]
we can see that
\[
\frac{k(\sigma-1)}{k(\sigma-1)(\alpha-1) - \theta((1-\alpha)+k\sigma(1))} < 0
\]

A3. Model with Quasi Linear Preference

We can see that the slope of free entry condition is positive.

\[
\frac{\partial}{\partial c_D} \int_0^{c_a} \Pi_j^A \ g(c) \ dc + \frac{\partial}{\partial c_D} \int_{c_a}^{c_D} \Pi_j^N \ g(c) \ dc > 0
\]

In order to ensure that \( c_D < c_m \), the following free entry condition value must be positive for \( c_D = c_m \).

\[
FE\big|_{c_D \to c_m} \int_0^{c_a} \Pi_j^A \ g(c) \ dc + \int_{c_a}^{c_D} \Pi_j^N \ g(c) \ dc - f_e > 0
\]

Also, in order to ensure that (5.12),

\[
FE\big|_{c_D \to 2\left(\frac{\gamma m}{L}\right)^{rac{1}{2}} + 2\left(\frac{2r h \delta^2}{L}\right)^{rac{1}{2}} \left(\frac{f m}{L}\right)^{rac{1}{2}}} \int_0^{c_a} \Pi_j^A \ g(c) \ dc + \int_{c_a}^{c_D} \Pi_j^N \ g(c) \ dc < 0
\]
A3.1. Impact of Trade Liberalization

**Proposition 6.** A move from autarky to free trade brings the following results:

(i) \[ \frac{dC_D}{dL} < 0 \]

(ii) \[ \frac{dv_j}{dL} \bigg|_{c_j=0} > 0 \]

(iii) The sign of \[ \frac{dc_a}{dL} \] is ambiguous

(iv) The sign of \[ \frac{d\text{Prop acq}}{dL} \] is ambiguous

(v) \[ \frac{dc}{dL} > 0 \] (vi) \[ \frac{dh_j(1+v_j)}{dL} \bigg|_{c_j=0} > 0 \] and \[ \frac{dh_j}{dL} \bigg|_{c_j=c_D} < 0 \].

**Proof of Proposition 6.**

(i) Totally differentiate free entry condition (5.15), then we have

\[
dC_D \left\{ \frac{\partial}{\partial c_D} \int_0^{c_a} \Pi_j^a dG(c) + \frac{\partial}{\partial c_D} \int_{c_a}^{c_D} \Pi_j^n dG(c) \right\} + dL \left\{ \frac{\partial}{\partial L} \int_0^{c_a} \Pi_j^a dG(c) + \frac{\partial}{\partial L} \int_{c_a}^{c_D} \Pi_j^n dG(c) \right\} = 0
\]

\[
\leftrightarrow dC_D \left( \int_0^{c_a} \frac{\partial \Pi_j^a}{\partial c_D} dc + \int_{c_a}^{c_D} \frac{\partial \Pi_j^n}{\partial c_D} dc \right) + dL \left( \int_0^{c_a} \frac{\partial \Pi_j^a}{\partial L} dc + \int_{c_a}^{c_D} \frac{\partial \Pi_j^n}{\partial L} dc \right) = 0.
\]

where

\[
\frac{\partial \Pi_j^n}{\partial L} = \frac{\partial \Pi_j^a}{\partial p_j} \frac{\partial p_j}{\partial L} = \left( \delta q_j + \gamma h_j q_j \right) \frac{h_j q_j}{L^2} > 0,
\]

\[
\frac{\partial \Pi_j^n}{\partial a} = \frac{\partial \Pi_j^a}{\partial p_j} \frac{\partial p_j}{\partial a} = h_j q_j > 0.
\]
and
\[
\frac{\partial n^a_j}{\partial L} = \frac{\partial n^o_j}{\partial p_j} = (v_j + 1)(\delta q_j + \gamma h_j q_j) \frac{h_j q_j}{L^2} > 0,
\]
\[
\frac{\partial n^a_j}{\partial a} = \frac{\partial n^o_j}{\partial p_j} = (v_j + 1) h_j q_j > 0.
\]

Then,
\[
\frac{dc_D}{dL} = \left\{ \int_0^1 c_a \frac{\partial n^o_j}{\partial L} d c_j + \int_c c_D \frac{\partial n^o_j}{\partial c} d c_j \right\} \left\{ \int_c c_a \frac{\partial n^o_j}{\partial c} d c_j + \int_c c_D \frac{\partial n^o_j}{\partial c} d c_j \right\} < 0.
\]

(ii) Since the comparative statics of optimal number of acquisitions for each firm j is
\[
\frac{dv_j}{dL} = \frac{3}{4r h \delta^2} \left( \frac{\gamma L}{f_m} \right)^{\frac{1}{2}} \left\{ \frac{c_D - c_j}{2} - \left( \frac{\gamma f_m}{L} \right)^{\frac{1}{2}} \right\} \left\{ \frac{dc_D}{dL} + \frac{1}{L} \left( \frac{c_D - c_j}{2} \right) \right\}.
\]
We have
\[
\left. \frac{dv_j}{dL} \right|_{c_j \to 0} = \frac{3}{4r h \delta^2} \left( \frac{\gamma L}{f_m} \right)^{\frac{1}{2}} \left\{ \frac{c_D}{2} - \left( \frac{\gamma f_m}{L} \right)^{\frac{1}{2}} \right\} \left\{ \frac{dc_D}{dL} + \frac{1}{L} \left( \frac{c_D}{2} \right) \right\} > 0.
\]

(iii) For acquisition cost cutoff, we have
\[
\frac{dc_a}{dL} = \frac{dc_D}{dL} + \frac{1}{L} \left( \frac{\gamma f_m}{L} \right)^{\frac{1}{2}} \left\{ \frac{dc_D}{dL} + \frac{1}{L} \left( \frac{c_D}{2} \right) \right\}
\]
\[
= \left[ \frac{L^2}{4r h \delta^2} \left( \frac{c_D}{2} - \left( \frac{\gamma f_m}{L} \right)^{\frac{1}{2}} \right)^4 - \left( \frac{2r h \delta^2}{L} \right)^{\frac{1}{3}} \left( \frac{f_m}{\gamma L} \right)^{\frac{1}{6}} \left( \frac{c_D - c_j}{2} \right) \right] + \int_c c_D \frac{c_D - c_j}{2} - \frac{\delta q_j}{L} \right\} \left\{ \left( \frac{c_D}{2} - \left( \frac{\gamma f_m}{L} \right)^{\frac{1}{2}} \right)^5 - \left( \frac{2r h \delta^2}{L} \right)^{\frac{1}{3}} \left( \frac{f_m}{\gamma L} \right)^{\frac{1}{6}} \right\} \left. \frac{dc_a}{dL} \right|_{c_j \to 0}^{-1} \times \left[ -\frac{L}{5r h \delta^2} \left( \frac{c_D}{2} - \left( \frac{\gamma f_m}{L} \right)^{\frac{1}{2}} \right)^5 - \left( \frac{2r h \delta^2}{L} \right)^{\frac{1}{3}} \left( \frac{f_m}{\gamma L} \right)^{\frac{1}{6}} \right]^{-1} - 113
\]
\[
\frac{L}{4r_h\delta^2} \left( \frac{yf_m}{L} \right)^\frac{1}{2} \left\{ \left( \frac{c_D}{2} - \left( \frac{yf_m}{L} \right)^\frac{1}{2} \right)^4 - \left( \left( \frac{2r_h\delta^2}{L} \right)^\frac{1}{3} \frac{f_m}{yL} \right)^\frac{1}{6} \right\} - \frac{1}{\gamma} \int_{c_a}^{c_D} \frac{(c_D-c_j)}{2} \left( \frac{c_D-c_j}{2} - \frac{\delta q_j}{L} \right) dc
\]

\[
\frac{\delta q_j}{L} dc + \frac{L}{4r_h\delta^2} \left( \frac{yf_m}{L} \right)^\frac{1}{2} + \left( \frac{2r_h\delta^2}{L} \right)^\frac{1}{3} \frac{f_m}{yL} \left\{ \left( \frac{c_D}{2} - \left( \frac{yf_m}{L} \right)^\frac{1}{2} \right)^4 - \left( \left( \frac{2r_h\delta^2}{L} \right)^\frac{1}{3} \frac{f_m}{yL} \right)^\frac{1}{6} \right\} \int_{c_a}^{c_D} \frac{L}{\gamma} \left( \frac{c_D-c_j}{2} - \frac{\delta q_j}{L} \right) dc
\]

which is ambiguous in sign.

(iv) The proportion of acquisition is

\[
Prop_{acq} = \int_{0}^{c_a} \nu(c) g(c) dc
\]

and

\[
\frac{dProp_{acq}}{dL} = \frac{L}{2r_h\delta^2} \left( \frac{yL}{f_m} \right)^\frac{1}{2} \frac{1}{c_m} \left\{ \left( \frac{c_D}{2} - \left( \frac{yf_m}{L} \right)^\frac{1}{2} \right)^3 - \left( \left( \frac{2r_h\delta^2}{L} \right)^\frac{1}{3} \frac{f_m}{yL} \right)^\frac{1}{6} \right\} \left\{ \frac{dc_D}{dL} + \right. \\
\left. \frac{1}{L} \left( \frac{yf_m}{L} \right)^\frac{1}{2} + \frac{3}{8r_h\delta^2} \frac{yL}{f_m} \frac{1}{c_m} \left\{ \left( \frac{c_D}{2} - \left( \frac{yf_m}{L} \right)^\frac{1}{2} \right)^4 - \left( \left( \frac{2r_h\delta^2}{L} \right)^\frac{1}{3} \frac{f_m}{yL} \right)^\frac{1}{6} \right\} \right\}
\]

where

\[
\frac{dc_D}{dL} = \frac{1}{L^2} \left( \frac{yf_m}{L} \right)^\frac{1}{2} \left\{ \left( \frac{c_D}{2} - \left( \frac{yf_m}{L} \right)^\frac{1}{2} \right)^4 - \left( \left( \frac{2r_h\delta^2}{L} \right)^\frac{1}{3} \frac{f_m}{yL} \right)^\frac{1}{6} \right\} +
\]

\[
\left[ \frac{1}{\gamma} \int_{c_a}^{c_D} \left( \frac{yf_m}{L} \right)^\frac{1}{2} - \left( \frac{c_D-c_j}{2} \right) \left( \frac{c_D-c_j}{2} - \frac{\delta q_j}{L} \right) dc \right]^{-1}
\]
\[
\frac{L}{5r_h\delta^2} \left\{ \left( \frac{c_D}{2} - \left( \frac{y_m}{L} \right)^{\frac{1}{2}} \right)^5 - \left( \frac{2r_h\delta^2}{L} \right)^{\frac{1}{3}} \left( \frac{y_m}{yL} \right)^{\frac{1}{2}} \right\} \right]
\]

which is ambiguous in sign.

(vi) The comparative statics of total product ranges with respect to market size \( L \) is given by

\[
\frac{dA}{dL} = \frac{\delta (c_D - c_j)}{4r_hy^{2}} \left\{ 3h_j^{2} + \frac{\delta}{y} h_j + \left( \frac{\delta}{y} \right)^{2} \right\}^{-1} \left\{ \frac{dc_D}{dL} + \frac{1}{L} \left( \frac{c_D - c_j}{2} \right) \right\}
\]

Thus we have

\[
\left. \frac{dh_j}{dL} \right|_{c_j \to 0} = \frac{L}{2r_h\delta} \left\{ \frac{a - c_j}{2} - \left( \frac{y_m}{L} \right)^{\frac{1}{2}} \right\} \left\{ \frac{dc_D}{dL} + \frac{1}{L} \left( \frac{c_D - c_j}{2} \right) \right\} > 0
\]

and

\[
\left. \frac{dh_j}{dL} \right|_{c_j \to c_D} < 0.
\]

A3.2. Impact of “Love of brand”

**Proposition 7.** In the industry with high value of \( \gamma \) the following properties are satisfied: (i) \( \frac{dc_D}{dy} > 0 \) (ii) The sign of \( \frac{dc_a}{dy} \) is ambiguous (iii) \( \frac{dv_j}{dy} \bigg|_{c_j \to 0} > 0 \) (iv) The sign of \( \frac{dProp_{Pa,ca}}{dy} \) is ambiguous.

**Proof of Proposition 7.**

(i) Totally differentiate free entry condition (5.15), then we have
\[ dc_D \left\{ \frac{\partial}{\partial c_D} \int_0^c \Pi_j^a \, dG(c) + \frac{\partial}{\partial c_D} \int_{c_D}^c \Pi_j^n \, dG(c) \right\} + dy \left\{ \frac{\partial}{\partial y} \int_0^c \Pi_j^a \, dG(c) + \frac{\partial}{\partial y} \int_{c_D}^c \Pi_j^n \, dG(c) \right\} = 0 \]

\[ \leftrightarrow dc_D \left\{ \int_0^c \frac{\partial \Pi_j^a}{\partial c_D} \, dc + c_D \frac{\partial \Pi_j^n}{\partial c_D} \right\} + dy \left\{ \int_0^c \frac{\partial \Pi_j^a}{\partial y} \, dc + c_D \frac{\partial \Pi_j^n}{\partial y} \right\} = 0 \]

where

\[ \frac{\partial \Pi_j^a}{\partial y} = \frac{\partial \Pi_j^n}{\partial p_j} \frac{\partial p_j}{\partial y} = - \frac{(h_j q_j)^2}{L} < 0 \quad \text{and} \quad \frac{\partial \Pi_j^a}{\partial y} = \frac{\partial \Pi_j^n}{\partial p_j} \frac{\partial p_j}{\partial y} = - \frac{(v_j + 1)(h_j q_j)^2}{L} < 0. \]

Therefore we have

\[ \frac{dc_D}{dy} = - \left\{ \int_0^c \frac{\partial \Pi_j^a}{\partial y} \, dc + c_D \frac{\partial \Pi_j^n}{\partial y} \right\} \bigg/ \left\{ \int_0^c \frac{\partial \Pi_j^a}{\partial c_D} \, dc + c_D \frac{\partial \Pi_j^n}{\partial c_D} \right\}. \]

(ii) The comparative statics of acquisition cost cut-off with respect to \( \gamma \) is given:

\[ \frac{dc_a}{dy} = \frac{dc_D}{dy} + \frac{1}{\gamma} \left\{ - \left( \frac{\gamma f_m}{L} \right)^{\frac{1}{2}} + \frac{1}{3} \left( \frac{\gamma f_m}{y L} \right)^{\frac{1}{2}} \right\} \]

which is rewritten by

\[ \frac{dc_a}{dy} = \left[ \frac{L^2}{4r_h \delta^2} \left( \frac{c_D}{2} - \left( \frac{\gamma f_m}{L} \right)^{\frac{1}{2}} \right)^4 - \left( \frac{2r_h \delta^2}{L} \right)^{\frac{1}{3}} \left( \frac{f_m}{y L} \right)^{\frac{1}{6}} \right] + \int_{c_a}^{c_D} \frac{dc_a}{dc_D} \left( \frac{c_D - c_j}{2} \right)^{-1} \left( \frac{L^2}{4r_h \delta^2} \right) \left( \frac{\gamma f_m}{L} \right)^{\frac{1}{2}} \left( \frac{1}{y} \right) \left\{ - \left( \frac{\gamma f_m}{L} \right)^{\frac{1}{2}} + \frac{1}{3} \left( \frac{2r_h \delta^2}{L} \right)^{\frac{1}{3}} \left( \frac{f_m}{y L} \right)^{\frac{1}{6}} \right\} \right\} \]

\[ \int_{c_a}^{c_D} \frac{dc_a}{y^2} \left( \frac{c_D - c_j}{2} - \frac{\delta q_j}{L} \right)^2 \, dc + \frac{1}{y} \frac{L^2}{4r_h \delta^2} \left\{ - \left( \frac{\gamma f_m}{L} \right)^{\frac{1}{2}} + \frac{1}{3} \left( \frac{2r_h \delta^2}{L} \right)^{\frac{1}{3}} \left( \frac{f_m}{y L} \right)^{\frac{1}{6}} \right\} \left( \frac{c_D}{2} - \right. \]
\[
\left( \frac{yf_m}{L} \right)^{\frac{2}{3}} - \left( \frac{2rh\delta^2}{L} \right)^{\frac{1}{3}} \left( \frac{f_m}{yL} \right)^{\frac{1}{6}} \right) \right] + \frac{1}{y} \left[ - \left( \frac{yf_m}{L} \right)^{\frac{1}{3}} \right] + \\
\frac{1}{3} \left( \frac{2rh\delta^2}{L} \right)^{\frac{1}{3}} \left( \frac{f_m}{yL} \right)^{\frac{1}{6}} \int c_D \cdot \frac{L}{y} \left( \frac{c_D - c_j}{2} - \delta a_j \right) \mu \right].
\]

Because of its complexity, the sign of \( \frac{dc}{dy} \) is ambiguous. However, we can see that \( \frac{dc}{dy} > 0 \) if the following condition is satisfied.

\[
\left( \frac{yf_m}{L} \right)^{\frac{2}{3}} < \frac{1}{3} \left( \frac{2rh\delta^2}{L} \right)^{\frac{1}{3}} \left( \frac{f_m}{yL} \right)^{\frac{1}{6}}
\]

(A.11)

The strong sufficient condition of \( \frac{dc}{dy} > 0 \) can be rewritten by

\[
27y^2f_m < 2rh\delta^2
\]

This condition implies that, under some large value of \( y \), the range of firms who acquire other firms’ brands decreases as \( y \) increases.

(iii) The comparative statics of optimal number of acquisitions for each firm \( j \) is

\[
\frac{dv_j}{dy} = \frac{L}{4rh\delta^2} \left( \frac{yL}{f_m} \right)^{\frac{1}{2}} \left[ \left( \frac{c_D - c_j}{2} - \left( \frac{yf_m}{L} \right)^{\frac{1}{2}} \right) \right] \cdot \left[ 3 \cdot \frac{dc}{dy} + \frac{1}{y} \left( \frac{c_D - c_j}{2} - 4 \left( \frac{yf_m}{L} \right)^{\frac{1}{2}} \right) \right]
\]

Therefore we have

\[
\frac{dv_j}{dy} \bigg|_{c_j \to 0} = \frac{L}{4rh\delta^2} \left( \frac{yL}{f_m} \right)^{\frac{1}{2}} \left[ \left( \frac{c_D}{2} - \left( \frac{yf_m}{L} \right)^{\frac{1}{2}} \right) \right] \cdot \left[ 3 \cdot \frac{dc}{dy} + \frac{1}{y} \left( \frac{c_D}{2} - 4 \left( \frac{yf_m}{L} \right)^{\frac{1}{2}} \right) \right] > 0.
\]

(iv) The overall proportion of acquisition is
\[
\frac{d\text{Prop}_{\text{acq}}}{dy} = \frac{L}{2r_h\delta^2} \left( \frac{\gamma L}{f_m} \right)^\frac{1}{2} \frac{1}{c_m} \left[ \left( \frac{c_D}{2} - \left( \frac{y f_m}{L} \right)^\frac{1}{2} \right)^3 - \left( \frac{2r_h\delta^2}{L} \right) \left( \frac{f_m}{y L} \right)^\frac{1}{6} \right] \right] \left( \frac{dc_D}{dy} - \right)
\]

\[
\frac{1}{y} \left( \frac{y f_m}{L} \right)^\frac{1}{2} + \frac{L}{8r_h\delta^2} \frac{1}{c_m} \left[ \left( \frac{c_D}{2} - \left( \frac{y f_m}{L} \right)^\frac{1}{2} \right)^4 - \left( \frac{2r_h\delta^2}{L} \right)^\frac{1}{3} \left( \frac{f_m}{y L} \right)^\frac{1}{4} \right]
\]

and which is ambiguous in sign

## A4. Data

Table A.4.1 describes the proportion of within-industry transactions during pre and post-FTA period, which are stable over periods.

<table>
<thead>
<tr>
<th></th>
<th>Pre-CUSFTA</th>
<th>Post-CUSFTA</th>
<th>Pre-NAFTA</th>
<th>Post-NAFTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>82.15</td>
<td>83.15</td>
<td>83.69</td>
<td>83.02</td>
</tr>
<tr>
<td>[2]</td>
<td>82.06</td>
<td>82.83</td>
<td>83.58</td>
<td>82.99</td>
</tr>
</tbody>
</table>

* [1] represents the pre- and post-FTA periods defined by the first row of Table 1.

Table A.4.2 represents the name of assignors which had assigned their trademarks to the top 28 firms listed in section 2.
Table A.4.2. List of original owner of assigned trademarks

<table>
<thead>
<tr>
<th>Company Name</th>
<th>Owner of assigned trademark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford Motor</td>
<td>Automobile Protection Corporation, Automotive Components Holdings LLC, Bear Stearns and Co Inc, Daimlerchrysler Ag, Electric Auto-Lite Company, Hayes Lemmerz International Inc, Holiday Rambler Corporation, Marquette Corporation, Meneil Ohio Corporation, Merchants Automotive Group Inc, Model E Corporation, Murray Ohio Manufacturing Co, People Of Labyrinths BV, Pivco As, Pivco Industries</td>
</tr>
<tr>
<td>Company 1</td>
<td>Company 2</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------</td>
</tr>
<tr>
<td>Chevrontexaco</td>
<td>General Electric</td>
</tr>
<tr>
<td>Corporation, Unisyn Medical Technologies Inc, Vip Enterprises Inc, Witco Corporation</td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td></td>
</tr>
<tr>
<td>None</td>
<td></td>
</tr>
<tr>
<td>Aircraft Performance Unlimited Inc, Bell Helicopter Textron Inc, Lockheed Aircraft Corporation, Rockwell International Corporation, Wigwam Mills Inc</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Koch Industries</td>
</tr>
<tr>
<td>Weyerhaeuser</td>
</tr>
<tr>
<td>United Technologies</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
</tbody>
</table>

124

Pepsico


<table>
<thead>
<tr>
<th>Visteon</th>
<th>Diesel Kiki Co Ltd, Emed Systems Inc, Florida Automotive Distributing Inc, Ford Motor Company, Zexel Corporation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Company</td>
<td>Coca-Cola</td>
</tr>
<tr>
<td>-----------------------</td>
<td>----------------------------------------------------------------</td>
</tr>
<tr>
<td>---</td>
<td></td>
</tr>
</tbody>
</table>
초 록

일반적으로 무역 자유화는 시장의 경쟁수준을 높이는데 주요한 역할을 한다. 개방화가 진전되면 개별 국가 내의 경쟁기업 및 제화의 수가 증가하므로 기업은 보다 경쟁적인 가격을 설정하게 되며 그 결과 시장의 왜곡이 감소하여 시장성과가 개선되는 것이다. 또한 무역자유화의 움직임은 자원의 이동에 대한 압력으로 작용한다. 경쟁력이 뛰어난 기업들은 확대된 시장을 바탕으로 생산을 늘리고 경쟁력이 떨어지는 기업들은 도태됨으로써, 자원이 생산성 높은 기업에게로 재분배되는 것이다. 최근 많은 경제학자들이 이와 같은 무역 자유화에 따른 자원의 재배분(resources reallocation) 과정을 분석하고 있으나, 기업의 진입과 퇴출, 제품 범위(scope)의 조정에 의한 재배분 과정을 설명하는 데에 그치고 있다. 현재까지 기업의 인수합병 등에 의한 산업구조 개편이나 지적재산권 등의 무형자산의 재배분 과정에 대해서는 이렇다 할 연구를 내놓지 못하고 있는 실정인 것이다. 따라서 본인은 기존의 일반모형에 브랜드 자산(brand equity) 개념을 도입하여 무역자유화에 따른 무형자산의 기업 간 재배분 과정을 살펴보았다. 특히 무역자유화가 상표권의 기업 간 이전(transfer)에 미치는 영향을 이론적, 실증적으로 연구함으로써, 무역자유화의 무형자산 재배분에 미치는 효과를 분석하였다.

본 논문의 이론모형은 브랜드 내 자기잡식효과(within-brand cannibalization)가 존재할 때, 독점적 경쟁시장(monopolistic
competition) 하에서 브랜드를 인수할 (brand acquisition) 유인이 발생함을 보이고 있다. 브랜드 내 자기잠식효과는 서로 다른 브랜드의 두 상품보다 동일한 브랜드의 두 상품 사이에 대체가능성 (substitutability)이 더 큰 경우 발생하게 된다. 이 경우 기업이 제품의 범위 (scope)를 증대시킬 때, 기존 제품에 대한 수요가 타 브랜드 제품에 대한 수요보다 더 큰 폭으로 감소하게 되어 브랜드 내 자기잠식효과가 발생하게 되는 것이다. 생산성이 높은 기업의 경우 생산을 증대시킬 유인이 높기에, 이러한 자기잠식 효과의 제약을 크게 받게 된다. 브랜드 인수는 이와 같은 자기잠식효과 없이 생산을 증대시킬 수 있는 방안으로, 생산성이 높은 기업은 브랜드 자산 인수를 통해 생산을 확장시킬 유인을 갖는다.

자유무역협정 (Free Trade Agreement, FTA)으로 인한 회원국 간의 무역장벽이 낮아지게 되면 수출시장 진출의 기회와 수입 증대로 인한 국내시장의 경쟁심화가 동시에 발생하게 되는데, 이는 기업의 브랜드 인수 결정에 반대로 영향을 미친다. 즉, 수출 기회는 생산을 증대시킬 유인을 제공하여 산업 내 브랜드 인수를 증가시키는 반면, 국내시장의 경쟁심화는 브랜드 인수 시 발생하는 수익을 낮추는 역할을 하고 그 결과 산업 전반의 브랜드 이전을 감소시키는 역할을 한다. 특히 생산성이 높은 기업들은 수출시장 진출 기회에 많은 영향을 받게 되므로 생산성이 낮아 퇴출되는 기업으로부터 브랜드를 구입하게 된다. 즉, 무역자유화에 따라 브랜드 자산이 생산성이 낮은 기업으로부터 생산성이 뛰어난 기업으로 이동하게 되는 것이다. 본 논문은 이러한 브랜드 인수가
본 논문은 미국 특허청(United State Patent and Trademark Office)의 상표권 자료(trademark data)를 이용하여 이론모형의 주요 결과를 실증 분석을 통해 검증하고 있다. 산업별, 상표권 단위별 패널데이터 실증분석을 이용하여 캐나다-미국 자유무역협정(Canada–United States Free Trade Agreement)이 미국 내 브랜드 자산 이전에 미치는 영향을 살펴보고, 그 결과 이론모형의 주된 예측이 현실과 일치함을 보였다. 즉, 현실에서 무역자유화 이후 브랜드 자산의 기업간 이동이 활발해짐을 확인하였다.

주요어: 무역자유화, 브랜드 인수, 다브랜드 기업, 브랜드 내 자기잠식효과, 상표권 이전, 산업구조 재편성, 지적재산권 이전.
학번: 2008-30803