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경제학박사학위논문

Analyzing the Capital Market Instability Based on Agent-Based Model

Agent-Based Model에 기반한 자본시장 불안정성 연구

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이 선 영

Abstract

Analyzing the Capital Market Instability Based on an Agent-Based Model

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This study investigates capital market instabilities based on an agent-based model (ABM). In the first part, we show that herding behavior causes instability in the asset market by increasing the volatility of the asset price. While most previous studies investigating financial markets based on an ABM explain the main causes of the instability of the asset markets, such as bubbles and crashes with the existence of chartist, we additionally consider the expectations for a "tipping point," which is the threshold that the agent believes to be the maximum and minimum that the price can

attain. It could be heterogeneous for all agents; however, in some cases, it is also possible that some agents share the same tipping point and move together simultaneously like a group called “herding behavior.” As a result, we find that the existence of expectations for the tipping point can mitigate capital market instability by reducing the volatility of the asset price. However, when we allow the herding behavior among agents, we generate extreme instability where the asset price fluctuates very rapidly in a short period of time. With respect to the obtaining of wealth from trading, herding behavior does not always guarantee higher returns than individual traders, but on average, there are some advantages. This is especially true when the ratio of agents who share the same belief about expectations for the tipping point is 2.5%–3.3% of total number of agents. When this happens, the probability of winning is highest, whereas when the ratio of agents using herding behavior exceeds 20%, its advantages almost disappear.

In the second part, we build an artificial fund market where investors change their decisions for the investment of the fund depending on the short-term performance of the fund. We focus on the incentives to increase the proportion of risky assets to attract more investors, such as enhancing the performance of the fund they manage and investigate how this incentive for risky assets affects the bankruptcy risk of the fund using an ABM. As a result, we find that the incentives for risky assets increase the bankruptcy risk of the fund by increasing the probability of bankruptcy and shortening the time to reach bankruptcy. In addition, by simulating the relationships between the size of each fee, which consists of the payoff scheme for the fund manager and the

bankruptcy risk of the fund, we confirm that the size of the management fee has a bigger impact on the risk of the fund. We can infer that the reason behind the result is the absurdity of the payoff scheme for the fund manager, which gives relatively small degree of responsibility on the negative performance of the fund. The result of this study is consistent with the finding that the impact of the management fee is less imputed for the loss of the fund. To summarize, fund managers can have incentives for increasing the proportion of risky assets based on the desire of investors. This is done in an environment where investors make decisions for the investment based on the recent performance of the fund, and this incentive increases the risks of the fund. Also, the payoff scheme being less important for the negative performance of the fund can be a significant reason for the exacerbation of the risk of the fund and capital market stability.

Keywords : herding behavior, volatility, incentives to the risky asset, bankruptcy risk of the fund, payoff scheme, capital market instability

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Ⅰ. Introduction

In economics, the financial sector and real economy are so closely linked that we cannot think of the two parts separately. Instabilities in the financial market can seriously affect the growth of the real economy in a variety of ways by changing investment, consumption, and export patterns. In addition, because of the development of various financial instruments such as derivatives, financial markets become more complex and their impact on the real economy becomes increasingly significant. Thus, we cannot explain the economy without including the financial sector. Also, as seen in the case of the global financial crisis caused by US sub-prime mortgages in 2008, financial instability is not confined to its place of origin but affects the entire global economy because of the globalization of financial capital. In this respect, in order to promote stable and long-term economy growth, we should understand the instabilities in the financial market and prevent a similar crisis from happening in the future.

Although we observe lots of cases of crises in the financial sector, such as bank runs, herding behaviors, currency crises, bubbles, and crashes, we do not have sufficient economic theories or methods, which can explain the complex financial system. For example, the efficient market hypothesis, which has been accepted as a prominent financial theory since 1960s, excludes the internal forces, which can generate crises and disrupt equilibriums. According to this theory, large price changes or bubbles are just the outcome of new information or are responses of market participants to changes in

market fundamentals. Thus, though financial instability is nothing new and is certain to continue in the future, traditional economic thinking, which sticks to the steady state equilibrium concept and representative agents with perfect rationality, is unable to fully explain events in the financial markets.

Especially after the global financial crisis in 2008, there is a consensus that we need an alternative approach to explain the types of events that are hard to investigate with traditional economics. Agent-based modeling (ABM) has been proposed as an alternative. Unlike the traditional general equilibrium approach, it takes a bottom-up approach, which assigns particular behavioral rules to each bounded rational agent and investigates the outcome as a result of interaction among agents. It does not require strong assumptions such as representative agent, perfect rationality, and steady state equilibrium. In this light, ABM is appropriate for analyzing the complex system or events of the financial market.

In this study, we investigate two types of instabilities in the capital market and try to find the policy alternatives, which may reduce risk. In the first part, we show that herding behavior can exacerbate the instability of the capital market by increasing the volatility of the asset price. In the previous literature—which focuses on the financial market, including the capital market—it is found that the chartists who trade assets based on the history of the market price can exacerbate the stability of asset prices. However, in this study, we also consider the existence of a threshold, which can be interpreted as the expectations for the maximum and minimum level the price can attain. Thus, it can serve as a criterion for changing the trading rules from buying to selling or

from selling to buying. The threshold can be heterogeneous for all agents, and it is also possible that some agents share the same beliefs for the future asset price and thus move together like a group; this is called herding behavior.

In the real asset market, we can easily find examples of the herding behavior, and institutional investors in the stock market might be one instance. Since their asset amount is much larger than that of individuals, their impact on the price movement in the asset market is significant. As a result of this study, we find that the heterogeneous expectations across agents about the top and bottom level a price can be expected to reach can reduce the volatility of the market price. However, the result is different when we allow for herding behavior. We find some extremely unstable intervals where price fluctuates very rapidly in the short-term and the volatility, measured by the maximum–minimum of the price and standard deviation, increases. This means that herding behavior can undermine capital market instability. We find a contribution in that we find one of the disturbing factors in capital markets without debt or the existence of chartists seen as primary causes of the instability in financial market in much of the previous literature.

Also, in the simulation comparing the average wealth of agents in a group participating in herding behavior, and individual agents who have a heterogeneous threshold and move independently, it is true that agents can be placed in an advantageous position to obtain more wealth through herding behavior. This is especially true when the ratio of agents in a group is between 2.5%–3.3% of total number of agents; this is where the probability of obtaining wealth is highest. On the

other hand, if the ratio of agents in a group exceeds 20%, then the advantages from herding behavior almost disappears.

Finally, in the simulation where we divided agents in groups with equals, we also found that high volatility emerges when more agents share the same belief and move together. Based on these simulation results, caution against sudden changes in decisions for the trading assets when herding behavior is used is needed. We especially need to consider policies to prevent agents from becoming short-term speculative forces that disturb the capital market.

In the second part, we investigate how incentive for the fund manager to increase risk affects the probability of bankruptcy. We built an artificial capital market consisting of two types of agents as follows: investors and a fund manager. We assume that the investors are greatly affected by the recent rate of the return of the fund when they make decisions for the investments of the fund. Thus, if some losses from the fund occur, some investors may choose to withdraw money from the fund. These decisions by the investors decrease the total amount of funds managed by the fund manager. As a result, the revenue of the fund manager, which is determined based on the size of the fund, managed by the fund manager and the performance of the fund is reduced. Then, the fund manager has some incentive to increase the proportion of risky assets to increase their revenue in order to attract more investors with high rates of return. This is particularly true after several withdrawals from the fund when the fund manager is desperate for a high rate of return. We focus on how such incentives for the fund manager to increase the proportion of risky assets on the basis of the inflow or outflow

to the fund can affect the bankruptcy risk of the fund based on ABM. Through the simulation analysis, we find that the incentives to increase risky assets can increase the probability of bankruptcy of the fund. Also, the impact on the bankruptcy risk of the fund increases as the incentives to increase risky assets rise.

In addition, we investigate the relationships between the payoff scheme of the fund manager, which consists of the management fee and the performance fee, and the bankruptcy risk of the fund. As a result, the management fee has more impact on the bankruptcy risk of fund. This result is consistent with the fact that the fund manager does not take heavy responsibility for losses. When losses occur in the fund, most of it is passed on to the investors. The only responsibility of the fund manager for the loss of the fund is that they receive no performance fee. This small degree of responsibility makes fund managers engage in more risky behaviors. In this model, fund managers have less responsibility in that they always receive fees unless bankruptcy of the fund occurs. We can thus conclude that payoff schemes for the fund manager need to be reformed and be based on long-term performance rather than short-term profits. More responsibility needs to be placed on the fund manager not only for the good performance, but also for the negative performance.

Ⅱ. Herding Behavior Leading to Asset Market Instability based on an Agent-Based Model

1. Introduction

The efficient market hypothesis (EMH) had emerged in the mid-1960s to gain general acceptance as a prominent financial theory. The main idea of the EMH is that asset prices are always at the correct value of the asset, markets move toward equilibrium, and after reaching equilibrium they remain in this state until influenced by an unexpected exogenous shock. Thus, according to this theory, financial markets cannot generate their own internal forces to disrupt the equilibrium, and large price changes are just the result of markets responding to new information from the outside or to changing fundamentals. Therefore, according to EMH, there is no room for asset price bubbles or crashes to occur[1].

However, history and past data show that the EMH is not always successful in explaining phenomena in financial markets. From the Dutch tulip bubble in 1626, the British South Sea Company bubble, the French Mississippi Company bubble of the early 18th century, and the Japanese bubble in 1980s to recent U.S. sub-prime mortgage and the financial crisis of 2008, bubbles and crashes in financial markets have occurred and continued to occur.

[1] For more details, see Cooper, George (2008).

In a broader context, the failure of the mainstream economic theory to predict financial crises has been pointed out (Colander et al. 2009; Buchanan 2009), such that while it has been partially successful in predicting events in the near future using past data, it has failed in the face of the changes of a much bigger scale. The theory of dynamic stochastic general equilibrium (DSGE) also has limitations due to its dependence on strong assumptions, such as perfect rationality of the representative agent and the predetermined equilibrium concept for analytic solutions. In addition, because they exclude crises of the type that have occurred before and that we are experiencing now, it is not possible to predict a crisis using these models. Now, there is a growing recognition that an alternative theory or model is needed to explain the types of events that conventional economic models have never sufficiently explained. Agent-based modeling (ABM) has been proposed as an alternative (Farmer and Foley 2009; LeBaron 2006; The Economist 2010).

ABM is a computational methodology that allows an analyst to create, analyze, and experiment with artificial worlds composed of agents that interact within a specific environment.[2] It is characterized by key assumptions such as bounded rationality of agents, routine-based or adaptive behavioral rules, heterogeneity of agents, and learning process (Yoon and Lee 2009). Namely, in contrast to the DSGE, ABM allows a high degree of heterogeneity of agents and takes a bottom-up approach which assigns particular behavioral rules to each agent and generate dynamic path of an economy out of interaction among agents. In this light, ABM is appropriate for analyzing the financial

[2] See Gilbert (2007).

market as an outcome of heterogeneous agents and their interactions and for investigating economic phenomena such as financial crisis, which is hard to explain using mainstream economics tools.

Some previous studies have analyzed the financial markets based on ABM. The Santa Fe Artificial Stock Market, SF-ASM, is one of the earliest representative models. The SF-ASM model has evolved from the initial version of Palmer et al. (1994) and LeBaron et al. (1999) to the modified version by Ehrentreich (2004). Palmer et al. (1994) set up a simple stock market model where independent adaptive agents trade stock on a central market. They showed that price can display bubbles, crashes, and continued high trading volume. LeBaron et al. (1999) present an experimental computer simulated stock market to show that for some parameters the market generates interesting features, which appear to replicate some of those found in real financial time series data.

A growing number of models have been developed to consider the heterogeneity of the agents and interactions among them, such as Chiarella and Iori (2002), Chiarella et al. (2006), Chiarella et al. (2009), and Malek and Ezzeddine (2011). Chiarella and Iori (2002) show how the trading strategies of noise traders, fundamentalists and chartists have an effect on the price, bid-ask spreads, trading volume and volatility; Fundamentalists are agents who make trading decisions with a belief that an asset has its own intrinsic value (equilibrium price) at any point of time, whereas chartists use past price trends as basis for decisions with a belief that the history of the price changes is useful information for making meaningful predictions for the future price of the asset. They show that the presence of chartist plays a critical role in the volatility of price and

trading volume, and investigate the correlation between them. Chiarella et al. (2009) set up the model with agents assumed to have three components of the expectation of future asset returns, namely fundamentalist, chartist and noise trader, and analyzed the impact of these three components of trading strategies on the statistical properties of prices and order flows. Through this model, it turns out that fat tails are caused by the chartist strategy. Malek and Ezzeddine (2011) consider three different types of investors; fundamental agents, irrational agents, and loss adverse agents, and show that irrational investors can explain the excess volatility of stock prices. They focus on the volatility puzzle which is hard to explain with traditional financial theories including efficient market hypothesis. Chiarella et al. (2006) consider a financial market with both a risky asset and a safe asset with a view to explain asset price and wealth dynamics as a result of the interaction between two groups of agents, fundamentalists and chartists. Lee (2013) also point out that the cooperation of rational agents with bounded rational agents can make a bubble in the asset market.

Besides the papers which consider heterogeneity of the agents, the studies on asset market using ABM have been extended in a variety of ways. Harras and Sornette (2011) focus on the source of information. They endogenize three sources of information to the decisions of agents to show that a random sequence of the same signed news pushes price in one direction, and that through the coordination process among agents, the price reaches to an unsustainable level. Also, they show that once the agents have invested all their cash into the stock, just a little negative news can cause the price to collapse.

Taken together, the previous studies on asset market based on ABM mainly focused on the heterogeneity of agents and their interactions, and some of these studies were extended to consider the source of information that agents can access. Now, while the aim of the current study is also to explain the mechanisms of how bubbles and crashes emerge in asset markets with heterogeneous agents based on the ABM, our distinctions are as follows. First, while most of the previous literatures considered the heterogeneity of agents in forming expectations for future asset price or returns, and explained the dynamics of asset price as a consequence of the presence of chartists (Chiarella and Iori 2002; Chiarella et al. 2009), we expand upon this by considering the role of different expectations for the 'tipping point' by each agent as a cause of the collapse of the asset prices in the financial markets. As previous studies have pointed out, chartists can be expected to buy additional assets even at the high price level if they believe that the asset price will rise further. These behaviors of agents push the market price to a higher level, and bubbles can occur. However, during the process, if the price rises above certain level that the agent believes to be the maximum, the agent will change his behavior from buying to selling due to a concern for loss of capital. Also, with the same logic, if the price falls below certain level that the agent believes to be the minimum, the agent is expecting a price rises and will change his behavior from selling to buying. Namely, each agent can have different or heterogeneous expectations about the 'tipping point' for the asset price. In some cases, it also possible that some agents share the same belief about the tipping point like a group, called herding behavior. The existence of both heterogeneous expectations about the tipping point and herding behavior can affect

the dynamics of asset price but in different directions. However, so far, most of the previous studies have not considered this. In this paper, we reflect this aspect into the model to investigate how different expectations about tipping point and herding behavior could have an effect on the dynamics of an asset price.

Second, we do not divide agents into either fundamentalists or chartists. In the real world, if agents have access to information about both the fundamental values of the asset and the historical price change data, they will use both of these to predict the future price rather than using just one type of information. Thus, we assume that each agent has the weight for each component used for anticipating the future prices.

Third, we use simple but intuitive trading rules for each agent, rather than assuming certain types of expected utility functions such as CARA (Arthur et al. 1997; Chen and Yeh 2001; Lettau 1997) and CRRA (Bullard and Duffy 2001; LeBaron 2001; LeBaron 2006; Chiarella et al. 2006), to avoid adding unnecessary assumptions to derive simple demand function from it.

The ABM simulation in this paper is to show how the heterogeneous expectations across groups of agents about the tipping point affect price dynamics and maximum price level. It will be first shown in a benchmark model without grouping of agents where every single agent have different expectation that the collapse of the price does not emerge automatically, and price fluctuations are often small and even some (seemingly) flat intervals appear.

However, this pattern of dynamics for asset price change when we reduce the heterogeneity in forming the expectations about tipping points and allow herding

behavior among agents. In other words, we investigate how herding behaviors of agents can affect asset price dynamics by divide agents into several groups with the same expectation about the tipping point so that agents in the same group move together or collectively but agents in different groups behave differently. By changing the size of groups (number of agents in each group) or changing the number of groups, we will generate a case of extreme instability where the asset price fluctuates rapidly in large amount within a short period of time. Basically, price dynamics show higher volatility when more agents share same thresholds, which is the case of a smaller number of groups or larger size of members in each group. In sum, we show that volatility of prices is more like to emerge when heterogeneous expectation about prices are combined with herding behavior among agents, so that agents in the same group share the similar expectations about the price changes.

In addition, in the perspective of advantage to obtain wealth from trading, we find that agents who share the same belief with others and thus move together have some advantages in probability to win in the wealth competition. One interesting thing is that when the ratio of agents for herding behavior is 2.5-3.3% of total number of agents, agents in a group are most advantageous to win the wealth competition with individuals but if the level of agents whose tipping point is the same with some of other agents exceeds a certain level, 20% in this paper, the benefits from herding behavior seems to disappear. This is came from that even if agents can enhance the power to predict the future asset price through herding behavior, at the same time, they can lose some benefits resulting from distinction with others.

In another simulation, by changing upper bound and lower bound, we find that trading assets earlier than others can be a good strategy to obtain gains from trading. Especially, buying assets earlier than others and selling it at a rising price in the future is favorable to obtain gains.

This paper is organized as follows. In section 2, we present a basic model, and introduce the formations of the expectation for the asset price, trading rules, and equations for price determination. Section 3 provides the simulation results of price dynamics without tipping point and with 'heterogeneous expectation' for the tipping point. In section 4, we introduce a 'herding behavior' into the model. Finally, section 5 provides a conclusion.

2. The Model

At period 0, each agent is endowed with one unit of risky asset and some amount of cash which are randomly drawn from uniform distribution on the interval $[0, \bar{C}]$. We define the wealth of each agent at the initial period as

$$w_{i0} = c_{i0} + p_0 a_{i0} \tag{1}$$

where $w_{i0}, c_{i0}, a_{i0,}, p_0$ is the wealth, amount of cash, and amount of assets for agent i and asset price at initial period, respectively.

2.1. Heterogeneity of Agents

In the literature which studies the asset markets, two types of agents are considered: fundamentalists and chartists. Fundamentalists are agents who make trading decisions based on estimates of the fundamental value of an asset. Unlike fundamentalists, chartists use the past price trends as a basis for decisions. Now, if an agent knows both types of information, namely the fundamental value of the asset and the history of price change rate, it is reasonable to assume that one will use both information for anticipating the future asset price. Therefore, in this model, we do not distinguish between fundamentalists and chartists. Instead, we assume that all agents know both types of information for the asset price and thus use both to predict the future price of an asset. Yet the weight for each type of information is different for all agents according to their beliefs. In sum, the prediction of the asset price for the next period is determined by current fundamental value of the asset and the history of the asset price change.

To consider the fundamental value of the asset into our model, it is necessary to define the fundamental value of the asset. In many previous papers which deal with fundamentalists in the asset market, fundamental values are assumed to be a constant

(Chiarella and Iori 2002) or random walk process (Chiarella et al. 2009). In this paper, we assume that the fundamental value of the asset follows the random walk process with zero drift and volatility σ^2 to reflect the economic states, which are not constant. At time t, agents have the information for the fundamental value of the asset, p_t^f which determined by following equation and they use this information to predict the future price of the asset. Finally, we can define the fundamental value of the asset price at time t as following.

$$P_{t+1}^f = p_t^f + \varepsilon_t \text{ where } \varepsilon_t \sim N[0, \sigma^2] \tag{2}$$

Regarding the information on the history of the price change, the level of the current asset price and the price change rate may be considered. This means the level of the asset price for the next period is determined by the level of the current price asset and the trend. Here, γ is the weight for the price change rate. The information on the history of the price change which is available at time t can be expressed as following.

$$P_{t+1}^c = p_t \left[1 + \gamma \left(\frac{p_t - p_{t-1}}{p_{t-1}}\right)\right] \tag{3}$$

Finally, we have the prediction equation of the asset price for the next period, $\bar{p}_{it+1}$.

This equation represents that agents anticipate the future asset price by using both types of information; fundamental value, p_t^f and the price change rate, p_t^c. Here, α_i means the weight of the fundamental value between two types of information, which is different for all agents.

$$E_i[p_{t+1}] = \alpha_i p_{t+1}^f + (1-\alpha_i)p_{t+1}^c$$

$$= \alpha_i\left(p_t^f + \varepsilon_t\right) + (1-\alpha_i)p_t\left[1+\gamma\left(\frac{p_t - p_{t-1}}{p_{t-1}}\right)\right] \quad (4)$$

where $0 < \alpha_i < 1$, γ is constant for all i and t

The trading rule is simple. Because agents examined in this paper pursue capital gains, they want to buy an amount of asset only if they expect that the price will rise above the current level. i.e., they can expect capital gains through asset trading. Otherwise, they choose to sell some of the assets they hold to prevent capital loss.

The demand function for agents reflects this trading strategy. If agents expect the capital gains, $\bar{p}_{it+1} - p_t > 0$, they choose to buy an amounts of the asset with a fixed fraction g of cash. With the same logic, if agents anticipate the capital loss, $\bar{p}_{it+1} - p_t < 0$, they decide to sell g portions of assets they currently hold. In this model, because we impose constraint on borrowing and short-selling, agents can only trade assets when they have the cash for buying or assets for selling (Harras and Sornette

2011). One additional feature we need to notice is that when agents trade assets according to their prediction rules, they use only g fractions of their cash or asset. During the simulation we set $g = 0.1$. This reflects that agents hold both a risk free asset (cash) and risky assets in their portfolio, not confining these to one type of financial asset. The demand function could be summarized as following.

$$D_{it} = \begin{cases} (E_i[p_{t+1}] - p_t)gc_{it} = \left[\alpha_i(p_t^f - p_t) + (1-\alpha_i)\gamma p_t\left(\frac{p_t - p_{t-1}}{p_{t-1}}\right)\right]gc_{it}, & \text{if } E_i[p_{t+1}] \geq p_t \\ (E_i[p_{t+1}] - p_t)ga_{it} = \left[\alpha_i(p_t^f - p_t) + (1-\alpha_i)\gamma p_t\left(\frac{p_t - p_{t-1}}{p_{t-1}}\right)\right]ga_{it}, & \text{if } E_i[p_{t+1}] < p_t \end{cases} \quad (5)$$

2.2. The Existence of Threshold: Tipping Point

In this section, we consider the tipping point. Many previous studies, which deal with the financial markets, have pointed out that the existence of chartists and their strategy (technical trading) play a significant role in making volatility in asset markets (Joshi et al. 1998; Chiarella et al. 2009). That is to say, since they are speculators who seek excessive capital gains, they choose to buy additional assets if they believe that the price will rise even if the price is already high enough. During this process, the asset price reaches a very high level. At this point, there is a possibility that some agents might change their trading strategies from buying to selling due to a concern about a collapse

of the asset price bubble if the price level increases above the level that agents believe to be the maximum. That means agents have expectations about the 'tipping point' for the asset price, and this level can vary among agents. To reflect this idea, we impose certain thresholds $\overline{\omega}_i$ and $\underline{\omega}_i$ for the price level that agents believe to be the maximum and minimum, respectively. We model that this level, $\overline{\omega}_i$ and $\underline{\omega}_i$ are randomly drawn from uniform distribution on the interval $[\underline{\Omega}, \overline{\Omega}]$ and is different for all agents.

The trading rule for considering the tipping point is similar to that of when we did not consider it. If the current level of the asset price is lower than the expectations for the tipping point and the price is expected to increase, then agents decide to buy some amount of assets. On the other hand, if agents expect the future price to fall or the price level is high enough compared with the level they believe to be the maximum, they will choose to sell an amount of assets to prevent capital loss. The trading rule and demand function when we consider the expectations about the 'tipping point' of agents are the following.

1) If $\underline{\omega}_i \leq p_t \leq \overline{\omega}_i$

$$D_{it} = \begin{cases} (E_i[p_{t+1}] - p_t)gc_{it} = \left[\alpha_i(p_t^f - p_t) + (1-\alpha_i)\gamma p_t\left(\frac{p_t - p_{t-1}}{p_{t-1}}\right)\right]gc_{it}, & \text{if } E_i[p_{t+1}] \geq p_t \\ (E_i[p_{t+1}] - p_t)ga_{it} = \left[\alpha_i(p_t^f - p_t) + (1-\alpha_i)\gamma p_t\left(\frac{p_t - p_{t-1}}{p_{t-1}}\right)\right]ga_{it}, & \text{if } E_i[p_{t+1}] < p_t \end{cases}$$

2) If $p_t > \overline{\omega}_i$,

$$D_{it} = -ga_{it}$$

3) $\underline{\omega}_i > p_t$

$$D_{it} = gc_{it} \tag{6}$$

2.3. Price Determination

Once all agents make decisions, the returns and new asset price are determined. Returns are determined by the excess demand and asset price is determined by the asset price of the last period and the return.

$$R_t = \frac{1}{\lambda N}\sum_{i=1}^{N} D_{it} \tag{7}$$

$$p_t = p_{t-1}\exp(r_t) \tag{8}$$

where r(t) is the return at time t, and λ is the relative impact of the excess demand upon the price (Harras and Sornette 2011).

2.4. Cash and Asset Update

Finally, cash and asset are updated as a result of trading. New cash amounts are determined by the cost for purchasing new assets subtracted from the amount of cash held by agents. The new asset amounts are determined by the amount of asset agents held in the last period and the demand for asset in this period.

$$c_{it+1} = c_{it} - p_t D_{it}$$

$$a_{it+1} = a_{it} + D_{it} \tag{9}$$

3. Simulation Analysis

In the simulation, we set the number (N) of agents as 2,500, the initial amount of assets for each agent to $a_{i0} = 1$, the maximum amount of cash to $\bar{C} = 2$, the weight for fundamental value between two types of information to $\alpha_i \in [0,1]$ and the excess demand on the price of the asset to $\lambda = 0.25$. We also fix the fraction of trading to g=0.1, that means agents use 0.1 percent of their cash or asset when they buy or sell the assets, respectively. We assume that the fundamental value follows random walk with

initial value $p_0^f = 0.53$, zero drift and the volatility $\sigma^2 = 10^{-3}$. The initial price of the asset, p_0 is set to 0.5, and the expectations for the asset price at initial period are assumed to be randomly drawn from uniform distribution on the interval [0,1].

For the simulation considering the expectations about the tipping point, we assume that the expectation for the tipping point, $\overline{\omega}_i$ is randomly drawn from uniform distribution on the interval $[\underline{\Omega}, \overline{\Omega}]$. For the basic test, lower bound and upper bound for the tipping point is fixed to $\underline{\Omega} = 1$ and $\overline{\Omega} = 2$, respectively. We additionally run simulations with numbers of $\overline{\Omega}$ from 1.5 to 2.5 to test the effect of the level of the expectations about the tipping point on the asset price dynamics. All parameter values are summarized in Table 1.

Table 1 Summary of the parameters of the simulation

Description	Symbol	value
Number of agent	N	2500
Initial amount of asset	a_{i0}	1
The maximum amount of cash agent can hold	$\bar{C}$	2
weight for fundamental value between two types of information	α_i	$\alpha_i \in [0,1]$
Variance for random walk	σ^2	10^{-3}
Impact of the excess demand on the price	λ	0.25
The fraction of trading	g	0.1
Weight on price change rate	γ	1
Initial fundamental value	p_0^f	0.53
Initial asset price	p_0	0.5
Upper bound of the tipping point	$\overline{\omega}_i$	U[1, 2]
Lower bound of the tipping point	$\underline{\omega}_i$	U[0, 0.3]

3.1. Price Dynamics

In the basic simulation, we have considered two types of information, the fundamental value of the asset and the history of the price change rate. All agents in this simulation have both information, thus use both types of information but the weight for each component is different for all agents. The tipping point is not considered yet. Figure 1 shows asset price dynamics as a consequence of the behaviors of heterogeneous agents without considering the 'tipping point'. We reproduce well the major stylized facts of the price dynamics in the financial market in that the price continue to fluctuations in the long term with small variations in the short term.

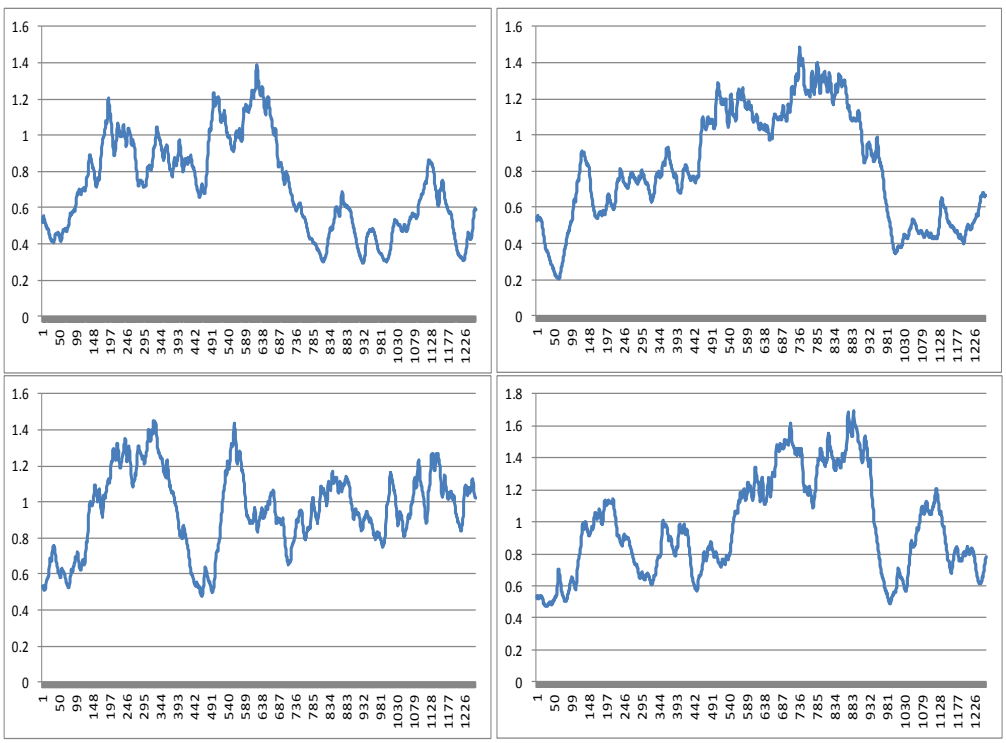


Figure 1 Asset price dynamics when agents use both types of information

3.2. The 'Tipping Point' Effect

Next, to investigate the role of expectations about the 'tipping point' on the dynamics of asset price, we impose certain thresholds representing the expectations for the tipping point to each agent, and compare the two results where it is considered and not considered. In this basic simulation, the expectations for the tipping point of agents are randomly drawn from uniform distribution on the interval [1,2], and once it has been imposed for each agent, we assume that it has not changed over the time. Namely, it could be interpreted as the innate characteristics of agents.

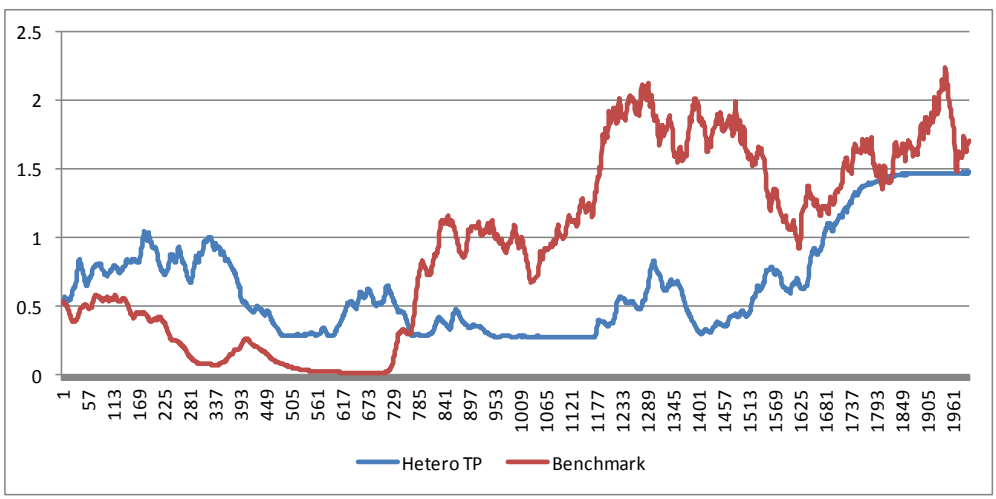


Figure 2 Asset price dynamics with and without tipping point

As seen in Figure 2, the existence of heterogeneous expectations for the tipping point can affect the dynamics of the price. Before the price attains the tipping point, the pattern of the price is similar to the case without it as shown in Figure 1. However, after

the asset price reaches above the certain level, there are some agents who start to sell some amount of asset due to a concern about the possible collapse of the asset price. As a result, the price does not rise sharply, and seemingly flat intervals are detected. The reason why the seemingly flat intervals occur is simple. Since all agents are heterogeneous in their expectations about the tipping point level, when the price attains certain levels some agents who have smaller expectations for the tipping point than the current asset price level change their strategy and begin to sell a portion of assets they hold. However, simultaneously, the level of the price is still not high enough to change the strategy of some agents who have a higher level of expectation for the tipping point. Namely, even in the same price level, the decisions for trading assets will diverge depending on the tipping point level that each agent expects. As a result, price fluctuations are small and may even they appear flat when tipping point exists.

To clarify the effect of existence of threshold on the volatility of the asset price, we run 50 times of the simulation to calculate the average value of the maximum, minimum and std. dev and compare it with the benchmark case. Maximum-minimum and std. dev value is used to measure the volatility of the price.

Table 2 Compare the volatility between benchmark case and heterogeneous TP case

	Benchmark case	Heterogeneous TP case
Max	1.570189	1.231103
Min	0.110962	0.287559
Std dev	0.320816	0.240052

Note: runs on 50 times

As seen above result, when we consider the heterogeneous tipping point during the simulation we have 1.2311 and 0.2875 as a maximum and minimum values which is lower than 1.5701 of the maximum value and higher than 0.1109 of the minimum value of the benchmark case. Also, we have smaller std. dev 0.1109 than 0.3208 of the benchmark case. To sum up the two results, we can conclude that the existence of tipping point can lower the volatility.

In addition, we simulate such as changing the upper bound for the uniform distribution, $\bar{\Omega}$, from 1.5 to 2.5 while keeping the lower bound is fixed at U[0, 0.3] and investigate how the level of the expected tipping point can affect price dynamics. The results are shown in Figure 3.

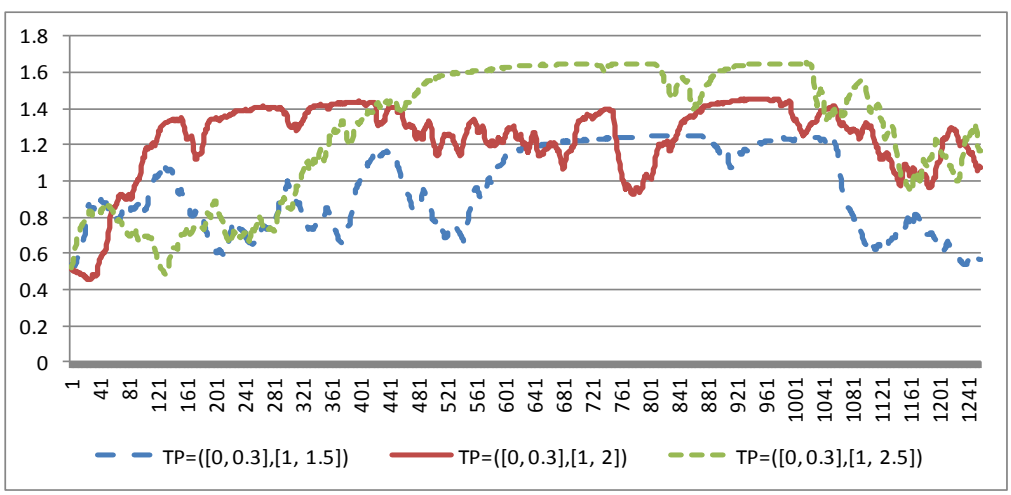


Figure 3 The effect of level of the upper bound of the tipping point on price dynamics with keeping the lower bound is fixed at U[0, 0.3]

Through Figure 3, we confirm that as the upper bound for the threshold get larger,

the peak the price can attain become higher. Some flat intervals still appear because of the existence of the expectation about the tipping point and heterogeneity of it.

With the same logic, as the lower bound get larger, the bottom level the price can fall become higher. This result can be seen in Figure 4.

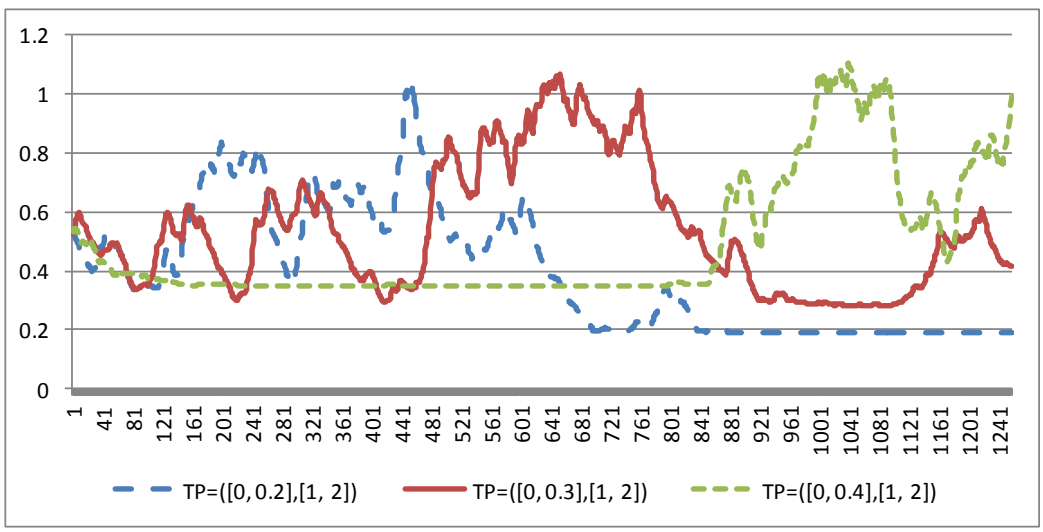


Figure 4 The effect of level of the lower bound of the tipping point on price dynamics with keeping the upper bound is fixed at U[1, 2]

Taken all together, we can draw an initial conclusion about the role of heterogeneous expectation for the tipping point of agents in asset markets; mainly it could play a role in mitigating the bubbles in financial markets and heterogeneity of this level prevents the price from falling too sharply.

4. Herding Behavior

As shown in the first result, the existence of tipping point plays a role as a psychological upper limit and lower limit and simultaneously the criterion for the decision of trading asset. Thus, it decreases the volatility of the asset price. One important thing is that so far, we assume that all agents have different expectations on the tipping point levels during the first simulation. Because of this, the decisions are dispersed depending on their personally different tipping point level and it can lower the volatility of the asset price. For helping understand how the ‘heterogeneous’ tipping point can reduce the volatility, let us assume that the current level of the asset price is 1.5 which has risen threefold than initial value. The price level is not enough to sell the assets for agent whose level is 1.8 because he believes that the price will have room to rise further. Understandably, the agent keeps their trading rules to buy additional assets if the price is on the rise. However, even if the same price level, 1.5 is given, the level is too high for the agent with 1.2 of tipping point level. For him it regarded as like a time boom nobody know when they are going to bust. For him it is reasonable to switch their decisions from buying to selling to avoid the capital loss. As a results, the decisions for trading asset is dispersed depending on the tipping point level which is different for all agents, and thus the price can keep the relatively stable level without surging or falling rapidly.

However, we may have totally different results if we allow some portion of agents to share the same level of tipping point and thus if it is possible that lots of agents can

move to one direction at the same time like a ‘herd’. Considering the ‘herding behavior’ is important in that they have some power to move the price to the same direction they choose.

In this section, we investigate the case where some agents share the same level of tipping point. Regarding of these topics, we did three simulations such as herding behavior with only one group, emergence of another group, and more than two groups with equal size. For each simulation, we show the dynamics of asset price and compare the volatility and average wealth for each group.

4.1. Impact of Herding Behavior with Only One Group

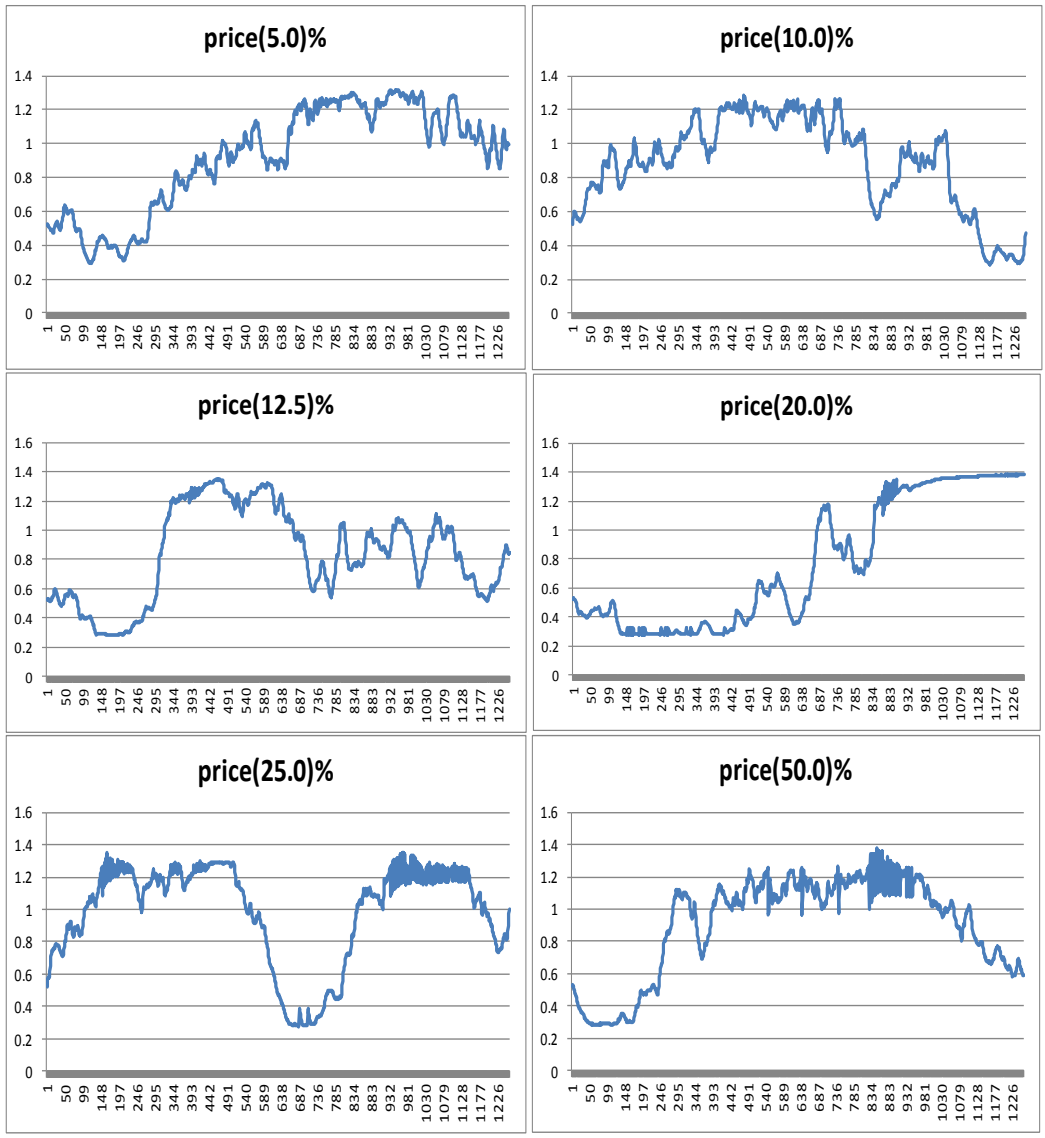
Now, extending the ‘tipping point’ experiment, we allow some agents to share the same expectations for the tipping point. For this simulation, we divide a population of agent into a single group and the rest, where we impose the same expectations about the tipping point to the agents belonging to this same group but different expectations among agents in the rest of the population. When a certain portion of agents share same beliefs about the tipping point, they make decisions about their trading strategies collectively and thus they move in the same direction simultaneously.

In this subsection, we do three simulations. First, we investigate the dynamics of the asset price imposing the same level of tipping point to some portion of agents. Second, as changing the portion of agents who did ‘herding behavior’ from 5% to 50%, we

explore how the size of agents who share the same tipping point can affect the dynamics of the asset price and volatility. Lastly, we compare the average wealth between agents in a herding behavioral group and individual agents outside the group.

The simulation results are shown in Figure 5. Each Figure represents the dynamics of asset price as the size of this group (in terms of its share in population) in which the affiliated agents share the same expectations about the threshold. As a result of this 'herding behaviors' of agents, we find that with the increasing size of the group, the interval with flat price tend to disappear, and that there are some periods (the part with very thick bands) where asset price fluctuates rapidly. By comparing the graphs with increased size of the group in Figure 5, we also confirm that this impact of herding behavior is larger when more agents share the same belief about the tipping point.

(a)



(b)

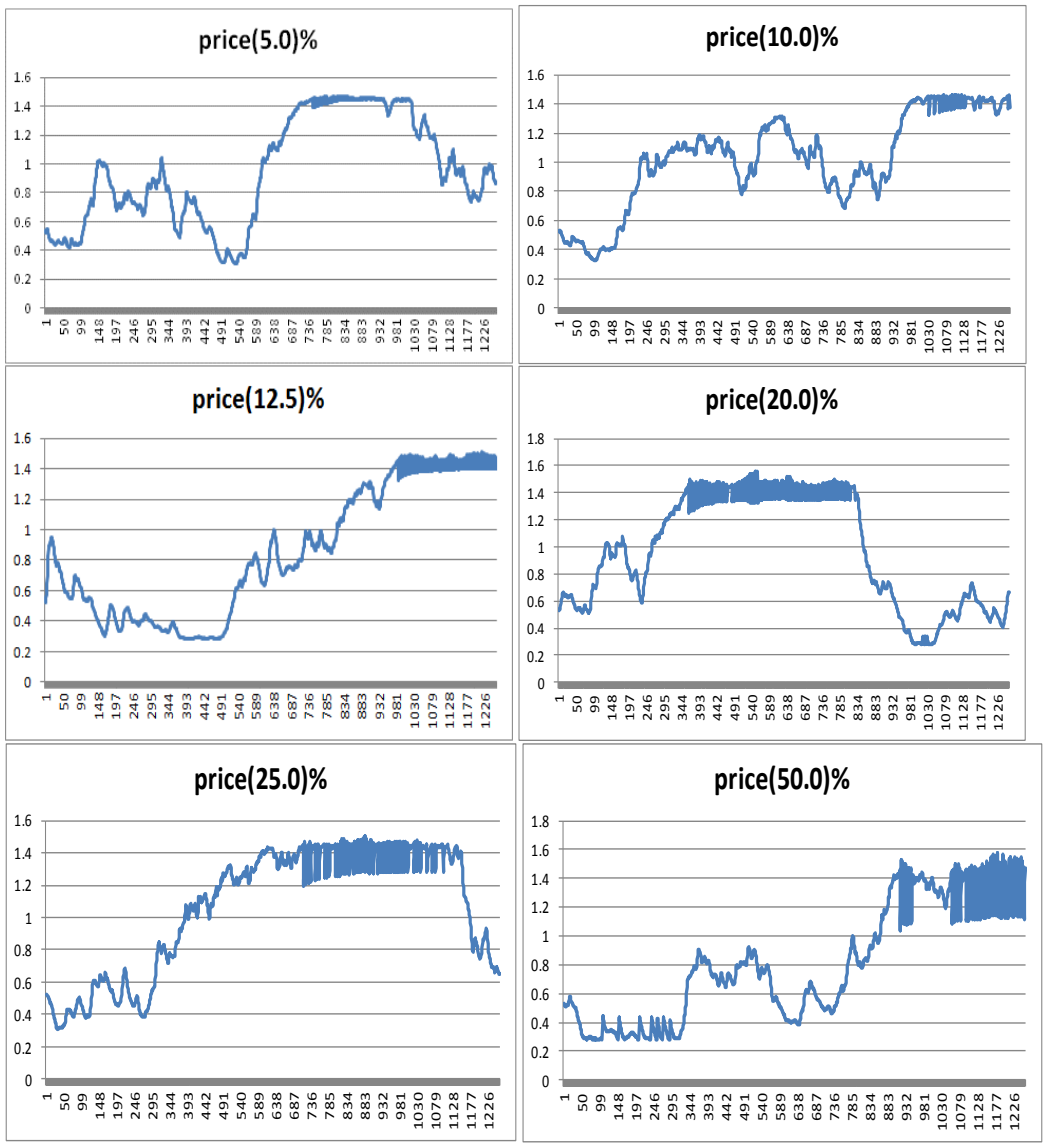


Figure 5 Time series of the asset price when we allow herding behavior with one group with following tipping point level (a) G_TP=(0.28, 1.25) and H_TP=(U[0,0.03], U[1,2]) (b) G_TP=(0.28, 1.45) and H_TP=(U[0,0.03], U[1,2])

To have more specific results of the impact of existence and the size of agents who behave as a group on the asset price, we report the maximum-minimum and standard deviation as a measure of volatility of the asset price.

Table 3 Compare the volatility as changing the portion of agent for herding behavior

Group size	5%	10%	12.5%	20%	25%	50%
Avg. max-min	0.08314	0.14702	0.18813	0.26762	0.31044	0.60881
Avg. std. dev	0.01746	0.03239	0.04210	0.06533	0.08130	0.16518

Note: average value of 10 simulations. G_TP: (0.28, 1.45), H_TP: (U[0,0.3], U[1,2])

Overall, the volatility increases as the size of the group increases. When only 5% of agents have same level of threshold, the max-min is just 0.083 and the average standard deviation is 0.017. However, the max-min and the standard deviation increase to 0.608 and 0.165, respectively, where 50% of agents share the same beliefs for the direction of the asset price.

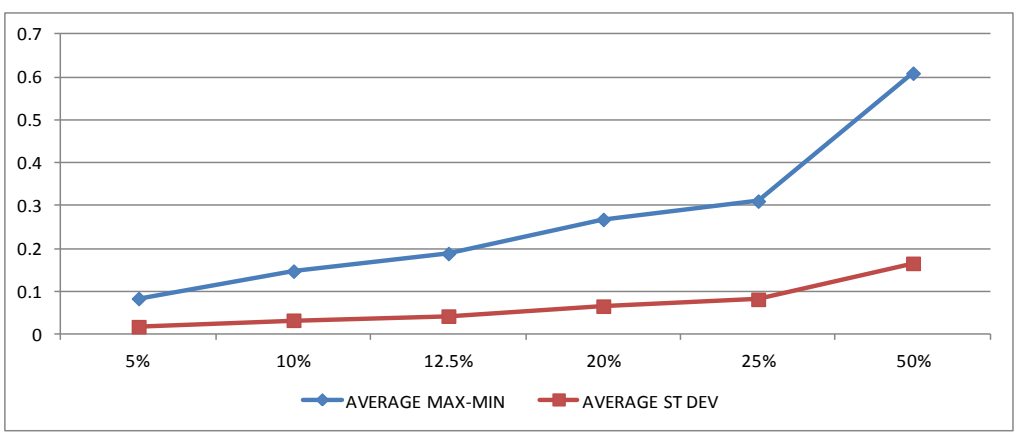


Figure 6 The volatility as increasing the number of agents who share the same threshold for the future asset price

Through the results of volatility test, we confirm again that as the more agents share the same belief, volatility of price becomes larger. We can also read the positive relationships between the volatility and the size of agents in the group through the Figure 6.

4.2. Impact of Herding Behavior on Wealth

So far, through the herding behavior simulation, we confirm that when some agents share the same belief about the expectations for the future asset price and move together to the one direction at the same, it can generate extreme instability where price fluctuates very rapidly in the short period of time. Especially, it is clear that the portion of agents doing herding behavior gets larger, the volatility also becomes larger. Let us then try to investigate the advantages of such herding behaviors in perspective of obtaining wealth. To answer this question, we compare the average wealth per agent between agent in the group and individual agents outside the group. Also, we calculate number of wins of 100 simulations to investigate who takes more advantage in wealth competitions.

For the first simulation, we set the 0.28 and 1.45 as a lower bound and upper bound for agents in the group, respectively. Lower bound and upper bound for individual agents are randomly drawn from uniform distribution with interval U[0, 0.3] and U[1, 2], respectively. We report average wealth of agents in the group and of heterogeneous

agents, the difference between them, and number of wins in a group of 100 simulations.

Table 4 Wealth for agents in the group and outside the group, difference between agents in the group and outside the group and number of wins for agents in the group (LB, UB)=(0.28, 1.45)

	Avg. wealth of agents in the group	Avg. wealth of heterogeneous agents	Difference	Number of wins
1%	3.07633	2.78613	0.29019	74
2%	3.07275	2.81871	0.25403	62
2.5%	3.06207	2.80779	0.25428	80
3.3%	3.01154	2.80965	0.20189	76
5%	2.92854	2.76048	0.16806	61
10%	2.91483	2.80386	0.11097	57
12.5%	2.81385	2.75751	0.05635	58
20%	2.81398	2.77328	0.04069	46
25%	2.79779	2.75144	0.04635	50
50%	2.78231	2.76587	0.01644	44

Note: Calculate the average of 100 runs.
(LB, UB)=(0.28, 1.45) is imposed to the agents in the group and the number which is randomly drawn from uniform distribution (LB, UB)=(U[0,0.3], U[1,2]) is imposed to individual agents.

Obviously, agents have some advantages by doing herding behavior both in the average wealth per agent and the number of wins. Especially, when the ratio of agent in a group is 2.5, the probability of wins is highest. However, the gap between agents belonging to the group and individuals get smaller as the portion of agents moving together get larger. Then, the result has completely changed when the ratio of agents who share the same belief for the tipping point exceeds 20 percent. Namely, there seems to be little difference between agents in a group and individual agents in both average

wealth per agents and the number of wins. Even when the portion of agents is 20 percent, average wealth of individual agents is higher than agents who are belonged to a group. That means, the benefit from herding behavior seems to disappear as more agents share the same belief.

This is because what we consider in this model is only financial market except the real economy. In this case, agents in the financial market play a zero sum game in that capital gains of some agents are came from the capital loss from other agents. At this time, agents whose expectation for the asset price is consistent with the real market price get some capital gains. In this respect, the herding behavior has some advantages in that they can have some power in that they can lead the asset price to the direction they want as they move together in the same directions. However, at the same time, since the number of agents who fail to read the future market price and lose their capitals also decrease, the benefits from herding behavior are reduced.

In the second simulation, we impose 1.25 as an upper bound which is relatively low comparing with it of the first simulation, while keeping lower bound is the same with the previous simulation as 0.28. We can understand this type of agents who have relatively lower upper bound and higher lower bound as a group which changes the trading rules at a reasonable price.

Table 5 Wealth for agents in the group and outside the group, difference between agents in the group and outside the group and number of wins for agents in the group where (LB, UB)=(0.28, 1.25)

	Avg. wealth of agents in the group	Avg. wealth of heterogeneous agents	Difference	Number of wins
1%	3.20401	2.87364	0.33037	76
2%	3.05896	2.79062	0.26833	80
2.5%	3.10313	2.83549	0.26764	85
3.3%	3.10646	2.74610	0.36036	85
5%	2.95904	2.76894	0.19011	65
10%	2.92870	2.78395	0.14475	61
12.5%	2.89705	2.73701	0.16004	64
20%	2.83608	2.76168	0.07440	53
25%	2.74501	2.67881	0.06620	53
50%	2.80913	2.79328	0.01585	51

Note: 100 runs and calculate the average.
(LB, UB)=(0.28, 1.25) is imposed to the agents in the group and the number which is randomly drawn from uniform distribution (LB, UB)=(U[0,0.3], U[1,2]) is imposed to individual agents.

As seen in Table 5, we have similar results with the first simulation in that when 2.5-3.3% of agents share the same expectations for the tipping point, the probability of winning is the highest whereas the portion of agents who share the same belief for the future asset price exceeds a certain level, 20% in this study, the advantage obtaining from herding behavior obviously is reduced.

In the third simulation, we impose 1.65 as an upper bound which is relatively high comparing with it of two previous simulations while keeping lower bound is the same with the previous simulations as 0.28.

Table 6 Wealth for agents in the group and outside the group, difference between agents in the group and outside the group and number of wins for agents in the group where (LB, UB)=(0.28, 1.65)

	Avg. wealth of agents in the group	Avg. wealth of heterogeneous agents	Difference	Number of wins
1%	2.98043	2.87052	0.10990	52
2%	2.93709	2.79667	0.14041	61
2.5%	2.99689	2.82960	0.16729	72
3.3%	2.95818	2.79896	0.15922	62
5%	2.91132	2.74850	0.16282	61
10%	2.83232	2.76315	0.06917	53
12.5%	2.84216	2.80221	0.03996	52
0%	2.79681	2.81611	-0.01929	48
25%	2.66116	2.66522	-0.00405	41
50%	2.83177	2.93680	-0.10503	35

Note: 100 runs and calculate the average.
(LB, UB)=(0.28, 1.65) is imposed to the agents in the group and the number which is randomly drawn from uniform distribution (LB, UB)=(U[0,0.3], U[1,2]) is imposed to individual agents.

As seen in Table 6, we have consistent results with earlier two simulations. When the ratio of agents who share the same belief for the future asset price is 2.5% of all agents, the advantages form the herding behavior is largest. However, if it exceeds 20% of total agents, the benefit from herding behavior obviously disappears.

The Role of Lower Bound

Now, we set lower bound for the group tipping point 0.3 which is even higher than

the level used for two previous simulations. What the lower bound is 0.3 can be understood that they tend to sell assets even at relatively higher price level considering the individuals is randomly drawn from uniform distribution with interval [0, 0.3]. Upper bound for expectations for the tipping point of agents in the group is the same level with the previous simulation as 1.25. In sum, agents in the group change their trading rules from selling to buying at relatively higher price level and buying to selling at relatively lower price level. The result is summarized at Table 7.

Table 7 Wealth for agents in the group and outside the group, difference between agents in the group and outside the group and number of wins for agents in the group where (LB, UB)=(0.3, 1.25)

	Avg. wealth of agents in the group	Avg. wealth of heterogeneous agents	Difference	Number of wins
1%	3.57109	2.80110	0.76999	84
2%	3.41277	2.82314	0.58963	80
2.5%	3.64055	2.77369	0.86687	85
3.3%	3.46463	2.83660	0.62802	83
5%	3.27394	2.83784	0.43611	71
10%	3.16757	2.69149	0.47608	82
12.5%	2.91931	2.69863	0.22068	70
20%	2.82760	2.69386	0.13374	64
25%	2.95690	2.72634	0.23056	80
50%	2.88806	2.77968	0.10838	78

Note: 100 runs and calculate the average.
(LB, UB)=(0.3, 1.25) is imposed to the agents in the group and the number which is randomly drawn from uniform distribution (LB, UB)=(U[0,0.3], U[1,2]) is imposed to individual agents.

To investigate the role of the lower bound, we compare the result of it with the case when we impose (0.28, 1.25) as a lower bound and upper bound to agents who share the same tipping point. The results are summarized in Table 8 at below.

Table 8 Compare the advantage from herding behavior depending on the level of tipping point between the cases of (0.28, 1.25) and (0.3, 1.25)

	(0.28, 1.25)		(0.3, 1.25)	
	Difference	Number of wins	Difference	Number of wins
1%	0.33037	76	0.76999	84
2%	0.26833	80	0.58963	80
2.5%	0.26764	85	0.86687	85
3.3%	0.36036	85	0.62802	83
5%	0.19011	65	0.43611	71
10%	0.14475	61	0.47608	82
12.5%	0.16004	64	0.22068	70
20%	0.07440	53	0.13374	64
25%	0.06620	53	0.23056	80
50%	0.01585	51	0.10838	78

Note: 100 runs and calculate the average.

It is clear that agents whose lower bound is relatively high have even higher probability to win in both the wealth difference with individual agents and the number of wins.

At this time, we change the upper bound for the group from 1.25 to 1.45 keeping the lower bound fixed at 0.3 and we compare this result with the previous simulation where (0.28, 1.45) is imposed as a tipping point for the group.

Table 9 Wealth for agents in the group and outside the group, difference between agents in the group and outside the group and number of wins for agents in the group where (LB, UB)=(0.3, 1.45)

	Avg. wealth of agents in the group	Avg. wealth of heterogeneous agents	Difference	Number of wins
1%	3.31527	2.79589	0.51939	73
2%	3.42253	2.76935	0.65317	81
2.5%	3.55038	2.81349	0.73689	90
3.3%	3.42380	2.76158	0.66222	86
5%	3.25723	2.72305	0.53417	73
10%	3.15612	2.73312	0.42300	76
12.5%	2.91276	2.72131	0.19146	64
20%	2.83866	2.69915	0.13951	67
25%	2.82431	2.71701	0.10730	63
50%	2.76842	2.71954	0.04888	59

Note: 100 runs and calculate the average.
(LB, UB)=(0.3, 1.45) is imposed to the agents in the group and the number which is randomly drawn from uniform distribution (LB, UB)=(U[0,0.3], U[1,2]) is imposed to individual agents.

Table 10 Compare the advantage from herding behavior depending on the level of tipping point between the cases of (0.28, 1.45) and (0.3, 1.45)

	(0.28, 1.45)		(0.3, 1.45)	
	Difference	Number of wins	Difference	Number of wins
1%	0.29019	74	0.51939	73
2%	0.25403	62	0.65317	81
2.5%	0.25428	80	0.73689	90
3.3%	0.20189	76	0.66222	86
5%	0.16806	61	0.53417	73
10%	0.11097	57	0.42300	76
12.5%	0.05635	58	0.19146	64
20%	0.04069	46	0.13951	67
25%	0.04635	50	0.10730	63
50%	0.01644	44	0.04888	59

Note: 100 runs and calculate the average.

Through the result summarized in Table 10, we again confirm that selling earlier than others can be a good trading strategy to enhance the winning probability in wealth competition.

The Role of Upper Bound

So far, through several experiments we show that agents having a relatively higher lower bound and thus selling assets earlier have more chance to win in the wealth competition. Next, we do the similar experiment to investigate the role of upper bound as changing the upper bound of the group tipping point level from 1.25, to 1.45, to 1.65 and to 2.0 while keeping the lower bound fixed at 0.3.

Since we already have the results of the case when we consider 1.25 and 1.45 as an upper bound respectively, we performed two additional cases of 1.65 and 2.0 as an upper bound for a group.

Table 11 Wealth for agents in the group and outside the group, difference between agents in the group and outside the group and number of wins for agents in the group where (a) (LB, UB)=(0.3, 1.65) and (b) (LB, UB)=(0.3, 2.0)

(a)

	Avg. wealth of agents in the group	Avg. wealth of heterogeneous agents	Difference	Number of wins
1%	3.25710	2.78272	0.47438	74
2%	3.31520	2.83367	0.48153	73
2.5%	3.22470	2.75581	0.46889	68
3.3%	3.36303	2.83736	0.52567	70
5%	3.11485	2.78661	0.32824	68
10%	3.08368	2.73269	0.35098	68
12.5%	2.99917	2.73992	0.25925	69
20%	2.85776	2.71297	0.14479	59
25%	2.97347	2.84010	0.13337	65
50%	2.87420	2.81513	0.05906	59

(b)

	Avg. wealth of agents in the group	Avg. wealth of heterogeneous agents	Difference	Number of wins
1%	3.43683	2.78272	0.65411	65
2%	3.39983	2.81326	0.58657	73
2.5%	3.24137	2.74909	0.49227	75
3.3%	3.38452	2.82664	0.55788	77
5%	3.28246	2.75753	0.52494	75
10%	3.15915	2.81373	0.34542	65
12.5%	3.09845	2.82321	0.27524	71
20%	2.89899	2.74615	0.15283	68
25%	2.91146	2.81005	0.10141	61
50%	3.01908	3.02824	-0.00916	61

Note: 100 runs and calculate the average.
For (a) (LB, UB)=(0.3, 1.65) and for (b) (LB, UB)=(0.3, 2.0) is imposed to the agents in the group and the number which is randomly drawn from uniform distribution (LB, UB)=(U[0,0.3], U[1,2]) is imposed to individual agents.

We summarize the results investigating the effect of level of upper bound on the average wealth in Table 12. This is the result calculating the difference of average wealth between agents behaving as a group and individual agents. Except one case where 50 percent of agents are supposed to be a group and (0.3, 2.0) is imposed as a tipping point, agents in the group have some advantages over individuals in wealth competition. Also, in the results comparing the number of wins, we find that agents in a group have some advantages to take a higher wealth than individual agents.

Table 12 Summary of results investigating the effect of level of upper bound on the difference of average wealth with individuals(above) and number of wins(bottom)

(LB, UP)	(0.3, 1.25)	(0.3, 1.45)	(0.3, 1.65)	(0.3, 2.0)
1%	0.76999	0.51939	0.47438	0.65411
2%	0.58963	0.65317	0.48153	0.58657
2.5%	0.86687	0.73689	0.46889	0.49227
3.3%	0.62802	0.66222	0.52567	0.55788
5%	0.43611	0.53417	0.32824	0.52494
10%	0.47608	0.42300	0.35098	0.34542
12.5%	0.22068	0.19146	0.25925	0.27524
20%	0.13374	0.13951	0.14479	0.15283
25%	0.23056	0.10730	0.13337	0.10141
50%	0.10838	0.04888	0.05906	-0.00916

(LB, UP)	(0.3, 1.25)	(0.3, 1.45)	(0.3, 1.65)	(0.3, 2.0)
1%	84	73	74	65
2%	80	81	73	73
2.5%	85	90	68	75
3.3%	83	86	70	77
5%	71	73	68	75
10%	82	76	68	65
12.5%	70	64	69	71
20%	64	67	59	68
25%	80	63	65	61
50%	78	59	59	61

Although it is clear that agents can absolutely enhance their average wealth from having a relatively higher lower bound and thus selling asset earlier than other agents, the impact of the size of upper bound on the average wealth is quietly ambiguous. The results vary depending on the portion of agents. For example, when we suppose that 5%

of agents share the same belief, a relatively low upper bound, 1.25 is superior in wealth competition. However, when 20% of agents are supposed to have the same tipping point, a relatively high upper bound, 2.0, is superior in average wealth.

4.3. Emergence of another Group

So far, we have explored the impact of herding behavior with only one group on the dynamics of asset price, volatility and obtaining average wealth. Let us further explore effects of emergence of another group with equal size on price dynamics and average of wealth and the probability of wins. We begin with assuming that the portion agent in each group is 10 percent. 80 percent of agents are heterogeneous.

In the first simulation, we allow for group 1 and group 2 (0.28, 1.65) and (0.3, 1.25) of lower bound and upper bound as a tipping point. The lower bound and upper bound for the tipping point for the rests are drawn from uniform distribution with interval U[0,0.3] and U[1,2], respectively. Since the agents of the first group have a relatively low lower bound and high upper bound, they wait until the price falls or rise further more. On the other hand, agents of the second group whose lower bound is relatively high and upper bound is relatively low tends to change their trading rules earlier than agents of the first group and individual agents. The results are as follows.

Table 13 Average wealth and number of wins of two groups with G1_TP:(0.28, 1.65) and G2_TP:(0.3, 1.25), and heterogeneous agents

	Ratio of agents	Avg. wealth	Number of wins
G1_TP:(0.28, 1.65)	10%	2.6700	12
G2_TP:(0.3, 1.25)	10%	3.0471	76
H_TP:(U[0,0.3], U[1,2])	80%	2.6846	12

Note: 100 runs

As a result, agents of second group beat the rests, both agents of the first group and heterogeneous agents, in average wealth per agent and the probability of wins. That means trading assets earlier than others can be a good strategy to obtain gains from trading. Especially as already shown in the previous results, buying assets earlier than others and selling it at a rising price in the future is favorable to obtain gains.

In the second simulation, we impose (0.28, 1.25) as a tipping point for agents in the first group and (0.3, 1.65) for agents in the second group. The tipping point for the individual agents is the same with the previous simulation. Since agents in the first group have relatively low both of lower bound and upper bound, they can think of that they take a negative view of financial market. On the other hand, agents in the second group take a positive view on the future asset market. They believe that the price will rise until relatively high and not fall too much.

Table 14 Average wealth and number of wins of two groups with G1_TP:(0.28, 1.25) and G2_TP:(0.3, 1.65), and heterogeneous agents

	Ratio of agents	Avg. wealth	Number of wins
G1_TP:(0.28, 1.25)	10%	2.7284	12
G2_TP:(0.3, 1.65)	10%	3.0126	77
H_TP:(U[0,0.3], U[1,2])	80%	2.7143	11

Note: 100 runs

As a result of 100 simulations, we confirm that second group who has a positive view on the future asset market has more probability to win in competition of wealth with others. What we can infer from this and previous simulation is that having a higher lower bound and selling it at a rising price is superior to gain wealth.

4.4. Herding Behavior with more than One Group with the Equal Size

Simulations we did so far heterogeneous agents who move independently as their heterogeneous tipping point exist regardless of the number of groups which move together at the same time. Now, we divide agents into several groups and impose the same belief about the tipping point to agents belonging to the same groups. Figure 7 reports the result of this simulation. As a result, similar to the herding effect test in the previous section, we could find some intervals where the asset price moves rapidly

within a short period of time. In addition, by reducing the number of groups, the more agents share the same level, the higher price volatility emerges.

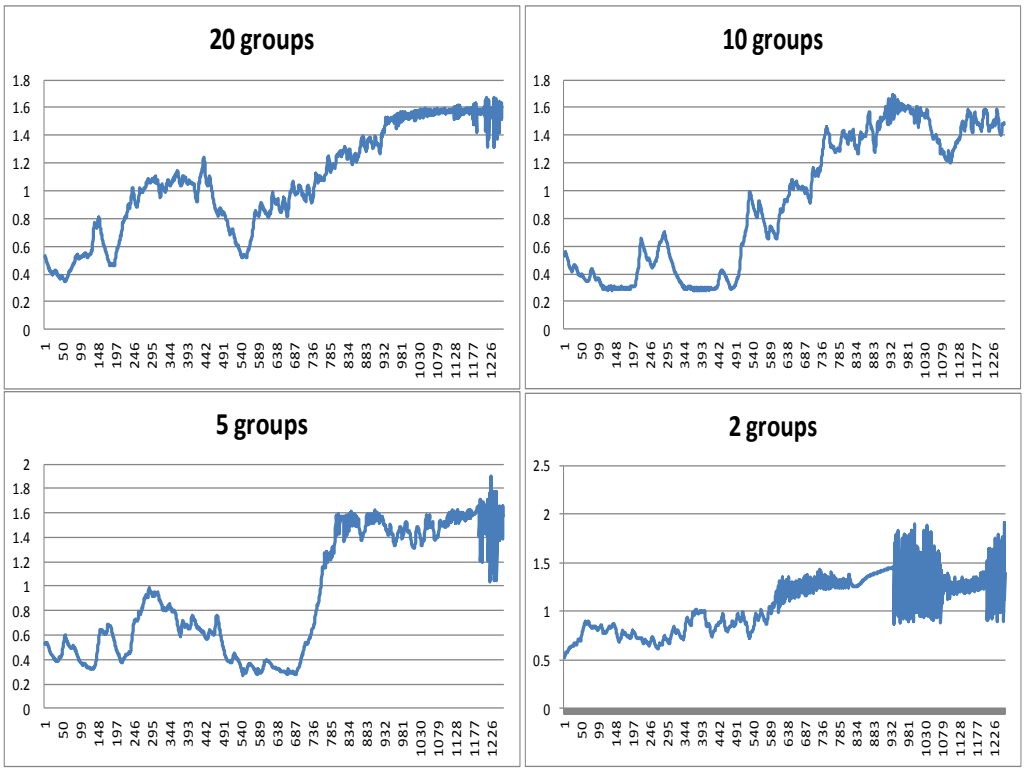


Figure 7 Groups from 20, 10, 5, and 1

The thresholds we impose on each group are randomly drawn from uniform distribution [1, 2]. Thresholds used for the simulation are the following.

For 20 groups: g1=(0.167,1.912), g2=(0.211,1.057), g3=(0.016,1.095), g4=(0.205,1.653), g5=(0.065,1.364), g6=(0.238,1.597), g7=(0.215,1.083), g8=(0.001,1.632),g9=(0.002,1.926), g10=(0.178,1.879), g11=(0.285,1.708), g12=(0.105,1.573), g13=(0.271,1.547), g14=(0.193,1.537), g15=(0.247,1.802), g16=(0.283,1.619), g17=(0.153,1.970), g18=(0.123,1.503), g19=(0.196,1.420), g20=(0.061,1.663)

For 10 groups: g1=(0.014,1.751), g2=(0.108,1.582), g3=(0.191,1.864), g4=(0.190,1.025), g5=(0.272,1.903), g6=(0.00078,1.948), g7=(0.224,1.631), g8=(0.290,1.756), g9=(0.190,1.604), g10=(0.184,1.149)

For 5 groups: g1=(0.106,1.245), g2=(0.279,1.685), g3=(0.102,1.573), g4=(0.012,1.726), g5=(0.199,1.655)

For 2 groups: g1=(0.28, 1.45), g2=(0.3, 1.25)

Comparing Figure 7 with the previous one, Figure 5, we can see that even if a group of agents has the power to change the asset price in the market as they move at the same time, the pattern of the market price will be different depending on whether the rest of the agents maintain heterogeneity regarding the threshold or not.

Let us take a closer look at the results; some small fluctuations are detected around the tipping point level of agents moving together. However the volatility of each fluctuation is not that much large before combined with the level of tipping point of other group. Especially, the asset price reaches the level of the majority of the tipping point of each group, the fluctuation become larger. For example, when 5% of agents move together, the interval with large volatility is detected around 1.62 of the price level. The total number of groups whose upper bound for the tipping point is below 1.62 are 11 which is the majority of the whole number of groups. This is valid for other cases where each 10, 5, and 2 group exists.

However, there is some similarity in that as the size of the group becomes larger, its effect on the market price also becomes more powerful. So, in this case, we can find some unstable intervals during which the asset price changes dramatically within a very short period of time. Especially, as shown in the last two graph of Figure 7 (with only 5 or 2 group consisting of the 20% or the half population with a high degree of herding behavior), the price fluctuates very fast within a short period of time, and the collapse of the system is inevitable. Such ‘system failure’ interval also emerges with the case of 5 groups in Figure 7 but with smaller vertical range of price fluctuations.

5. Conclusions

In this paper, we investigate conditions of instability such as bubbles and crashes in asset markets that created as a consequence of the behaviors of heterogeneous agents and their interactions, based on an ABM. In contrast to the previous literatures which deal with financial market with mainly heterogeneous agents and their interactions based on ABM, we additionally considered the different expectations of agents about the ‘tipping point’ where price are expected to fall or rise and investigate how the existence of ‘tipping point’ and its level affects the dynamics of asset price.

Our model makes contributions to demonstrating the main findings. First, before introducing the ‘tipping point’ to the model, we simulate the dynamics of asset price as a result of behaviors of heterogeneous agents in anticipating the future asset price based on ABM. We present the realistic looking dynamics of asset price as a result of the heterogeneous expectations for the future price of agents and their interactions through the market price.

Second, for a more realistic analysis, we consider the different expectations about the ‘tipping point’ of agents in the model. As already mentioned, agents in this model choose the best actions to maximize their capital gains and such behaviors in asset markets push the asset price to a higher level. However, during the process, some agents might change their trading strategies from buying to selling(selling to buying) due to concern about a possible collapse(rise) of the asset price if the price level surges above(fall below) a certain level. To reflect this aspect, we impose a threshold which

represents the agent's expectation about the tipping point. The level is different for all agents. The simulation results confirm that the existence of heterogeneous expectations about the tipping point and the level affect price dynamics and the maximum level that the asset price can attain. Some seemingly flat intervals appear instead of collapse in asset price because the decisions for trading assets will diverge depending on the tipping point level that each agent expects. Also, the maximum level the price can reach becomes higher as the upper limit of expectations about tipping point get larger. With the same logic, the minimum level the price can fall becomes lower as the lower limit of expectations about tipping point get smaller. We also confirm that the existence of tipping point can reduce the volatility of the asset price.

However, the results of the impact of heterogeneous tipping point on volatility of the asset price can be completely changed when we allow some agents to share the same belief and thus move together at the same time. In the last of the section of this paper, we investigate how the dynamics of asset price could be changed when some portion of agents share the same expectations for the tipping point. To analyze the impact of 'herding effect' on the pattern of dynamics for the asset price and compare the average wealth per agents with individual agents, we conduct several simulations. By changing the portion of agents allowing herding behavior, we generate a case of extreme instability (high volatility) where the asset price fluctuates rapidly in large amount in the short period of time. That means that instability in the asset market is more like to emerge when heterogeneous expectations about prices are combined with herding behavior among agents. Such behavior, which leads lots of agents to move together,

simultaneously can exacerbate the financial stability as increasing the volatility of the price in the financial market.

In the perspective of advantage to obtain the wealth from trading, the herding behavior does not always guarantee the higher wealth than individuals but it is also true that they have some advantages to win in the wealth competition. One interesting thing is that when the ratio of agents of herding behavior is 2.5-3.3% of the population, the advantages obtaining from herding behavior is highest. However, if the level of agents who share the same belief for the future asset movement exceeds a certain level, 20% in this paper, the benefits from herding behavior seems to disappear. This is came from that agents can enhance the power to predict future asset price through herding behavior, but at the same time, they can lose gains from distinction with others.

Also, through the additional simulations which changes upper bound or lower bound, we find that trading assets earlier than others can be a good strategy to obtain gains from trading. Especially buying assets earlier than others and selling it at a rising price in the future is favorable to obtain gains.

Lastly, we divide agents into groups and imposed the same belief about the tipping point for agents belonging to the same group. In both cases, because agents in the same group change their trading behavior at the same time, they have a power to move the market price. As a final outcome, we discover some unstable intervals where asset price fluctuated rapidly in the short term. In addition, by reducing the number of groups, as more agents share the same expectations for the tipping point, higher price volatility emerges.

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Ⅲ. The Incentive to the Risky Asset and the Probability of Bankruptcy

1. Introduction

In September 2008, the Reserve Primary Fund calculated its net assets at 97 cents per share known as “break the buck” and suspended payment of redemption proceeds to fund investors for seven days after writing off $785 million of debt. This was caused by the bankruptcy of Lehman Brothers Holdings Inc. and assets of the fund plunged more than 60% in two days following this. The financial market was shocked because money market funds (MMFs) are regarded as relatively safe investments. A remarkable part of this event was that although the commercial paper issued by Lehman Brothers Holdings Inc. was only 1.5% of total assets of the Reserve Primary Fund, sudden requests for redemptions by many investors made the net asset value of the fund per share fall below $1.00. Namely, it was the fear of future insolvency rather than the financial condition of the fund that caused its bankruptcy. This phenomenon was similar to a bank run.

Fund runs or bank runs have been observed for a long time in the financial market. So far, bank runs (fund runs) can be explained by the behaviors of investors. The panic-based runs of Diamond and Dybvig (1983) and the information-based runs of Jacklin and Bhattacharya (1998) are representative of this. Namely, these studies focus on

whether the decisions of investors for contributing and withdrawing funds are based on pure panic regardless of the financial condition of the bank or on information about it.

Based on the panic or fundamental-based analysis approach, we built a competitive artificial capital market consisting of two types of agents as follows: investors and a fund manager. We assumed that the capital market is highly competitive in that the investors are greatly affected by the performance of the fund (Jank and Wedow, 2010) when they make decisions for investment in the fund, and thus, the fund manager has some incentives to increase the proportion of risky asset to attract more investors. We focused on how such incentives for a fund manager to increase the proportion of the risky assets can affect the risk of the fund, i.e., the probability of bankruptcy based on an ABM.

As seen in the above example of the Reserve Primary Fund, a relatively small default in comparison to the total size of the fund can trigger the bankruptcy of the fund. The mechanisms by which a small default and loss can lead to the bankruptcy of a fund can be understood by a positive feedback mechanism. In such a situation, a small redemption from the fund because of the negative performance of the fund can stir other investors. Then, more investors join the redemptions based on the fear of future insolvency of the fund and thus more investors are affected. With this process, redemptions by several investors can occur in a short period of time. However, it is hard to explain this process of runs with traditional economic theory.

The traditional economic models, which were widely-accepted, are increasingly perceived as weak in explaining phenomena such as herd behavior, information

asymmetries, and externalities (Thurner, 2011). Especially after the global financial crisis in 2008, there is some consensus that we need an alternative approach for better understanding the complex dynamics of financial markets, and ABM is getting attention as a good alternative. ABM—unlike traditional economics, which is based on somewhat unrealistic assumptions such as representative agents with perfect rationality and steady state equilibrium assumptions—takes a bottom-up approach that means it gives a set of behavior rules for each agent and analyzes the results from interactions among agents. Thus, it is a useful tool to explore the dynamics of complex events such as herd behavior, bubbles and crashes, and financial crises, which are difficult to explain with traditional economic theories but occur with frequency in the financial market. This study develops an ABM to explain the mechanisms behind fund runs based on the interactions of the behavior of investors and the incentives to increase the proportion of risky assets in a fund.

This study addresses three main questions. First, we investigate the mechanisms of bankruptcy of a fund by focusing on the behavior of agents. In the panic-based simulation, we assume that investors make decisions for withdrawing money from the fund based on the number of withdrawals over previous period of time. On the other hand, in the fundamental-based simulations, we assume that investors make decisions for the investment in the fund based on the liquidity level of the fund. We expect that this study will show the different mechanisms of the bankruptcy when two types of investors coexist and will allow us to draw a conclusion to identify the component that can trigger the bankruptcy of the fund. Here, the bankruptcy of the fund means that the

amount of withdrawal is larger than the total amount of wealth of the fund so the fund manager cannot cover all withdrawals requested by investors.

Second, in the competitive financial environment where investors change their decisions for the investment in the fund depending on the performance of the fund, the fund manager has incentives to increase the proportion of risky assets to attract more investors. We focus on how this incentive for the fund manager can affect the bankruptcy risk of the fund. According to a number of previous studies, the return of the fund is determined by the portfolio of the fund, and managers can increase the performance of the fund by increasing the proportion of the risky assets. This is especially true in a competitive environment where net inflow to the fund through investment for investors is determined in accordance with the performance of the fund. Fund managers then have strong incentives to increase the proportion of risky assets. We reflect this idea in the model to investigate the effect of a manager's incentives to increase the proportion of risky assets on the probability of bankruptcy of the fund.

Finally, we investigate the relationships between the fund manager's payoff scheme, which consists of management fees and performance fees, and the bankruptcy risk of the fund. Through this simulation, we can suggest the payoff scheme, which can reduce the probability of the bankruptcy of the fund.

This study is strongly rooted in the bank run literature. As already explained, panic-based runs (Diamond and Dybvig, 1983) and information-based runs (Jacklin and Bhattacharya, 1998) are representative of such cases. Diamond and Dybvig (1983) wrote a pioneering paper on studies of bank runs. They explain the transformation

service of the bank and show the following two possible equilibria: bank run equilibrium—where agents do not want to withdraw when they do not have concerns about the failure of the bank—and no bank run equilibrium. Also, they show that the bank can cause problems in the real economy because banks in a good financial state can fail to survive under panic situations. Extending the Diamond and Dybvig (1983) findings, which does not provide the bank-run equilibrium and cannot determine the overall welfare, Goldstein and Pauzner (2005) show a unique equilibrium where a run occurs if and only if the fundamentals of the economy are below some threshold level. Also, they provide the conditions where banks increase welfare overall. Other notable papers on bank runs, including the one by Jacklin and Bhattacharya (1998) focus on information-based runs. Distinguishing themselves from the pure panic-based runs by Diamond and Dybvig (1983), they show that information-based runs are based on two sides of information asymmetries. One is that banks cannot observe the needs for liquidity by the depositors. The second is that depositors do not know the quality of the bank asset. Agents in this model can seek to discern which banks are solvent and which are not. According to this perspective, only insolvent banks face bank runs.

As a simulation paper studying bank runs, Lin (2006) investigates the emergence of bank runs resulting from agents' learning. In this study, agents have the following two options for resource allocations: one is simple storage that does not give any returns and the other is deposit in a bank. In this case, agents can get returns on their investment, but there is a default risk on the loan. The actual risk is unknown to agents. Therefore, depositors have different beliefs for this unknown parameter and update their beliefs

about this risk. They model this update process through a genetic algorithm. As an experimental study on the causes of bank runs, Chakravarty, Fonseca, and Kaplan (2014) conducted bank run experiments in a modified Diamond–Dybvig model with two banks and showed the mechanisms of bank run contagions.

Another related study for this paper is known as a negative externality and self-fulfilling concept. Chordia (1996); Nanda, Narayanan, and Warther (2000); and Edelen (1999) show that outflow from the fund can generate a negative externality to investors who still remain invested in the fund. Goldstein and Pauzner (2005) show that the suspect of other agents can withdraw money from the fund and can result in more withdrawals, and can thus generate a so-called "self-fulfilling run."

Also, this study is related to the literature that focuses on the portfolios of the fund, its rate of return, and the performance of the fund and net inflow to the fund. Jank and Wedow (2010) analyze the relationship between the return of the fund and flows to the fund before and after the US subprime crisis by using a panel of German retail MMF. They show that the return of MMF is generally persistent, and the performance persistence is affected not only by expenses, but also by the portfolio structures. Also, they show a significant performance-flow relationship that means investors withdraw their money when the funds they invest in underperform, whereas they invest in funds that outperform. Koppenhaver (1999) shows that the portfolio affects the return of the fund by cross-sectional regression. They use the ratio of the agency securities and commercial paper as a proxy of credit risk and show that these variables have positive relationships with the returns of the fund. Also, a high weight on average maturity can

generate a high rate of return of the fund. In this perspective, fund managers can increase the return of the fund by increasing the credit or interest rate of risk. Warther (1995) finds that there is a strong relationship between security return and concurrent unexpected cash flow into the funds but no relationship with concurrent expected flow into the funds. Also, it is confirmed that fund flows have some correlation with the returns of the securities invested by the funds, whereas there are no correlations with the returns of other types of securities.

To summarize, there is much literature on the theory and empirical study which deals with issues on runs in financial institutions, but they have some limitations in showing the mechanisms of the complex dynamics of fund runs in that they rely on the past data or try to find the static equilibrium. This study focuses on the results from interactions between the following financial agents: investors who seek more returns from the investment of the fund and a fund manager who has an incentive to increase the proportion of the risky assets to attract more investors. We try to show the dynamics of the mechanisms under which fund runs and bankruptcy occur. Also, at the end of the study, we suggest a manager payoff scheme, which can reduce the probability of runs and bankruptcy of the fund.

The study is organized as follows. Section 2 describes a basic model. In this section, we introduce the artificial financial market, which consists of the following two types of agents: investors and a fund manager. In section 3, a basic simulation looks at the mechanisms of bankruptcy, and we discuss the results. In section 4, we show the results by changing the weight of net outflow representing the fund manager’s incentive for the

investment in risky assets. In section 5, we report the results of the relationships between the fund manager's payoff scheme and the probability of bankruptcy, and suggest an effective payoff scheme, which is helpful in reducing the risk of the bankruptcy of the fund.

2. The Model

We construct a simple model of the capital market to investigate the mechanisms by which a small default can trigger fund runs and the bankruptcy of a fund. The model assumes the following two types of agents: investors and a fund manager. A fund manager and investors interact through decisions for investment in that each agent makes a decision reacting on the decisions of other types of agent. In the following section, we explore the mechanisms of bankruptcy of a fund and discuss the main factors, which can increase the risk of bankruptcy. Also, we focus on how the incentives to increase the proportion of risky assets are based on the decisions of investors and can affect the probability of bankruptcy of the fund based on a simple ABM.

2.1. Description of the Model

There are two types of agents considered in this study as follows: a fund manager and investors in the financial markets. At period 0, all investors have two units of cash, which can be invested in either a fund or a risk-free asset giving a zero rate of return. However, no resources are given to the fund manager. Instead, the fund manager has the ability to invest money in risky assets and earn positive rates of return in excess of the risk-free assets (Nanda et al., 2000). However, since they do not have any initial endowments, it is necessary to attract investors for a fund to invest somewhere. An additional premise of the model is that a fund manager has informational superiority over individual investors. Thus, individual investors prefer buying some funds and receiving some payoffs to invest directly in the risky asset.

The payoff for the fund manager is composed of a management fee and performance fee. The management fee is based on the size of the total wealth of the fund managed by a fund manager, and the performance fee is based on the performance (value increase of the fund over a period of time) of the fund. To summarize, we can say that the payoff for the fund manager is determined by the size of the fund and their performance. Thus, a fund manager has an incentive to increase the ratio of the risky asset to attract more investors. This incentive is supported by previous literature, which studies relationships between the portfolio of the investment and the rate of return.

In this model, each investor makes a decision for either investing or withdrawing money from the fund based on the moving average of its recent performance (Del Guercio and Tka, 2002; Thurner, 2011). Also, since the fund manger already knows that each investor reacts to the performance of the fund when they decide to either remain in

the fund or move to another fund, there is an incentive to increase the portion of the risky asset to enhance the rate of return. This is especially true if they detect some net outflows from the fund that they manage; they will take a high-risk and increase the proportion of the risky asset. Through increasing the risk of their portfolio, they may increase its performance.

2.2. Behaviors of Agents

2.2.1. Investor's Decision

At an initial period, investors are endowed with two units of cash such that$A_{i0}(i = 1, \dots, N) = 2$. Investors allocate their wealth between the fund and cash as follows:

$$A_{it} = F_{it} + M_{it} \tag{1}$$

where A_{it} is total amount of asset, F_{it} is amount of fund, M_{it} is amount of cash.

The investors make decisions for the investment in the fund based on the expected rate of return of the fund, which is determined by the performance of the previous period of time as follows:

$$\bar{r}_{it}^{F} = r_{t-1}^{F}\left(1 + \psi_i\left(\frac{r_{t-1}^{F} - r_{t-2}^{F}}{r_{t-2}^{F}}\right)\right) \quad (2)$$

where $\psi_i \sim U[0, 1]$

Here, ψ_i represents the weight for the rate of change on expectations of the rate of return from the investment to the fund, and we assume that it is different for all investors.

Based on the expectations of the rate of return of the fund, investors make decisions for their investment behaviors for either additional investment in the fund, withdrawing some portion of their investment in the fund, or withdrawing all money from the fund.

The decision rule for investors is simple. For every period of the time, investors calculate the expected rate of return of the fund based on the performance of the fund in the past. If investors expect positive rates of return from investment in the fund, they will increase the proportion of the fund in their investment portfolio, whereas if losses from the fund are expected, then they will reduce their investment in the fund. The total wealth of the fund (the size of the fund the fund manager can manage) changes depending on the investors' decisions for the investment. Increasing or decreasing the total amount of assets as a result of new inflow into the fund or withdrawals is routine, and thus, it is not a serious problem. The fund manager can repay the request for redemptions of some investors with their available funds.

However, runs on the fund can be a serious problem in that several investors withdraw their money from the fund at the same time, and the size of the fund is not enough to cover all requests for redemptions. For the criterion that determines money flows from the fund, we can think of the following two scenarios as follows: a panic-based run and a fundamental-based run based on the previous studies of the bank run or fund runs. Let us explore both cases and investigate the mechanisms of fund runs and bankruptcy.

(1) Panic-based Perspective

First of all, we investigate the panic-based simulation, which is similar to the panic-based run of Diamond and Dybvig (1983). We assume that investors are sensitive to the relative net outflow rather than fundamental or financial condition such as the liquidity level of the fund. Usually, investors make a decision for the investment to the fund depending on the expected rate of return, which is calculated based on the current performance of the fund. This trading rule keeps going before the relative outflow from the fund exceeds a certain level that will be the starting point for withdrawing money from the fund because of the panic. If the net outflow from the fund exceeds a certain level, investors fear that other agents will cause a run on the bank and will withdraw money from the fund. Thus, they may fear a possibly bigger drop or insolvency in the near future. Namely, when the net outflow of the fund exceeds a certain level, the investors may consider the funds dangerous aside from actual solvency of the fund;

investors may panic because of the fear of insolvency in the future, and they quickly sell all of the funds they currently have. These rules can be summarized as follows:

$$\frac{-\sum_i n_{it-1}}{W_{t-1}} \leq \lambda_i \Longrightarrow n_{it}^C = \eta(\bar{r}_{it}^F - r_{t-1}^F)M_{it}$$

$$\frac{-\sum_i n_{it-1}}{W_{t-1}} > \lambda_i \Longrightarrow n_{it}^C = -\max(0, F_{it})$$

(3)

Here, $\frac{-\sum_i n_{it-1}}{W_{t-1}}$ means the relative net outflow at previous period of t, and λ_i, which is randomly drawn from the uniform distribution with interval $[\underline{\omega}, \overline{\omega}]$, indicates the safe level of withdrawals each investor believes. Also, n_{it}^C signifies the number of claims by agent i.

(2) Fundamental-based Perspective

Second, in fundamental-based simulation, we assume that investors make decisions to withdraw all money from the fund they currently have, depending on the liquidity level. The liquidity level is related to the soundness of the financial structures of the fund. To summarize, if the liquidity level is high enough that the investors believe it to be safe, investors decide to buy some of the funds if they expect additional profits from the fund and to sell some of the funds if they expect some loss from the fund. However,

if the liquidity level falls below a certain level each investor believes to be the safe, the investor chooses to withdraw all money from the fund to avoid loss from possible insolvency.

To summarize, the claim of the fund is determined as follows. The investors make their decisions about whether to invest or withdraw from the fund based on the degree of risk aversion of each agent. Here, λ_i means the safe level of liquidity each investor believes, and it measures the degree of risk aversion of each investor i.

$$\frac{C_{t-1}}{W_{t-1}} > \lambda_i \Longrightarrow n_{it}^{C} = \eta(\bar{r}_{it}^{F} - r_{t-1}^{F})M_{it}$$

$$\frac{C_{t-1}}{W_{t-1}} \leq \lambda_i \Longrightarrow n_{it}^{C} = -\max(0, F_{it}) \quad (4)$$

(3) Combination of Both Types of Investors

Lastly, we consider the case where some investors consider the net outflow from the fund whereas others consider the liquidity level as a criterion for withdrawing all money from the fund. This means that two types of agents coexist in the financial market. By changing the ratio of each type of investor, we can investigate how the types of agents can affect the probability of bankruptcy.

For the simulation, we consider various cases by changing the ratio of each investor, (panic-based, fundamental-based) such that (0,1), (0.1, 0.9), (0.25, 0.75), (0.5, 0.5),

(0.75, 0.25), and (1, 0). We calculate the number of bankruptcies, and time it takes for bankruptcy, and compare them.

2.2.2. Redemptions and Bankruptcy

After the investors make a decision, the fund manager complies with the request for each investor. The fund manager may raise more money for investment or may need more liquidity if the requests for withdrawing money from the fund exceeds new investments in the fund.

However, the fund manager does not always comply with the requests of investors. After the end of the decision for the investment to the fund by investors, the fund manager calculates the net inflow (or outflow) to the fund. If the net inflow is positive, then the total wealth of the fund, which can be invested in risky assets or risk-free assets increases. On the other hand, if the net inflow is negative, i.e., net outflow is detected, the fund manager compares the amount of assets of the fund, which can be used for repaying requests for withdrawals and the amount of total requests by investors. If the amount of assets, W_t^D, which is calculated by total value of the fund, minus fees for fund manager, is higher than the amount of requests for withdrawals, the fund manager can satisfy the requests.

$$-\sum_{i} n_{it}^{C} \leq W_{t-1}^{D} \Longrightarrow n_{it} = n_{it}^{C}$$

$$\text{where } W_{t-1}^{D} = (1-\gamma)W_{t-1} + (1-\beta)H_{t-1}\left(P_{t-1}^{H} - P_{t-2}^{H}\right)$$

(5)

However, if the value of the assets of the fund is less than the total amount of withdrawals, then the fund may need to go through a bankruptcy process. That means it cannot repay all redemptions for the investors.

$$-\sum_{i} n_{it}^{C} > W_{t-1}^{D} \Longrightarrow n_{it} = 0 \text{ for } n_{it}^{C} \geq 0 \text{ and } n_{it} = \frac{n_{it}^{C}}{\sum_{i} n_{it}^{C}} W_{t-1}^{D} \text{ for } n_{it}^{C} < 0$$

(6)

2.2.3. Fund Manager's Decision

Unlike investors who have initial endowments, no resources are given to the fund managers. Thus, a fund manager needs to raise funds for investment. Since the payoff of the fund manager, which consists of the management fee and performance fee, is determined by the size of the fund and the performance of the fund at one period of time, a fund manager has to make a significant profit in order to attract more investors whose decisions for the investment are affected by the performance of the fund.

At each period of time, a fund manager allocates its wealth between risk free assets and risky assets. Investment to the risky assets can give positive returns but simultaneously there is a greater risk of loss. Also, a fund manager needs to leave some assets as a cash in preparation for possible redemptions. Thus, the wealth of the fund at t is the value of the risky asset plus the amount of cash.

$$W_t = W_{t-1}^D + \sum_i n_{it} \tag{7}$$

where P_t^H is value of the risky asset, H_t is amount of the risky asset, $C_t$is amount of cash, $\sum_i n_{it}$ is net inflow to the fund

At every period of time, the fund manager makes a decision for the proportion of the risky assets as part of the total assets. To prepare the redemptions, a fund manager leaves some assets as risk free assets. The proportion of the risky assets among total amount of assets changes at every period of time based on both of expected rate of return of risky assets and the net inflow (or outflow) to the fund. The proportion of risky assets is increasing in the expected rate of return of the risky assets, and at the same time it also reflects the net inflow or outflow to the fund. That means that when some net outflows are detected, the fund managers increase the proportion of risky assets to attract more investors by increasing the possibility for increased return of the fund. On the other hand,

when net inflow to the fund increases, the fund manager does not need to increase the proportion of the risky assets.

Based on this logic, the change rate of risky assets as a proportion of the total amount of fund wealth is determined by the expected rate of return and net inflow to the fund as follows:

$$\frac{\frac{H_t P_{t-1}^H}{W_t} - \frac{H_{t-1} P_{t-2}^H}{W_{t-1}}}{\frac{H_{t-1} P_{t-2}^H}{W_{t-1}}} = \kappa^H \left(\frac{\bar{r}_t^H - r_{t-1}^H}{r_{t-1}^H} \right) - \theta^H \left(\frac{\sum_i n_{it}}{W_t} \right) \tag{8}$$

where $\sum_i n_{it}$ is net inflow to the fund at the beginning of time t, κ^H is weight for the expected rate of return, θ^H is weight for net inflow to the fund, $\kappa^H > 0$ and $\theta^H > 0$

$$\frac{H_t P_{t-1}^H}{W_t} = (1 + K) \frac{H_{t-1} P_{t-2}^H}{W_{t-1}} \tag{9}$$

$$\text{where } K = \kappa^H \left(\frac{\bar{r}_t^H - r_{t-1}^H}{r_{t-1}^H} \right) - \theta^H \left(\frac{\sum_i n_{it}}{W_t} \right)$$

We adopt the assumption that the fund manager expects the performance of the risky asset for the next period to be based on an exponential moving average of its recent performance.

$$\bar{r}_t^H = (1 - a)\bar{r}_{t-1}^H + ar_{t-1}^H \tag{10}$$

The value of the risk asset is determined by the rate of return of the risky asset as follows:

$$P_t^H = (1 + r_t^H)P_{t-1}^H \tag{11}$$

where r_t^H is the rate of return of the risky asset.

We assume that the rate of return of the risky asset follows AR (1) process.

$$r_t^H = \rho r_{t-1}^H + (1 - \rho)r^H + \varepsilon_t \tag{12}$$

where $\rho < 1$, ε_t is white noise.

2.2.4. Payoff for the Fund Manager and Investors

After the performance of the risky asset the fund manager invests in is determined, both payoffs for the fund manager and investors are determined. A fund manager's income is generated from fees from investors. In the model, we assume that fees consist of a performance fee and a management fee. While the management fee is determined by the amount of assets under management, W_t, the performance fee can be generated

by the amount of return (performance) they made at that period (Thurner, 2011). Since their income depends on the amount of wealth, and the performance of the fund, they have incentives to increase the proportion of the asset, which can be expected to result in a higher rate of return despite its high risk. To summarize, the equations of payoff for the fund manager, which depends on both the management fee, γ and performance fee, β is as follows. If the performance of the fund is negative, then the performance fee becomes zero.

$$\mathrm{rev}_t = \max\left(\gamma W_t + \beta H_t\left(P_t^H - P_{t-1}^H\right), \gamma W_t\right) \tag{12}$$

The rate of the return of the fund, which can be distributed to the investors, is calculated by determining the increase of the fund wealth, minus the fees for the fund manager.

$$r_t^F = \left(\frac{(1-\gamma)W_t + (1-\beta)H_t\left(P_t^H - P_{t-1}^H\right)}{W_t}\right) - 1 \tag{13}$$

The value of the fund for each agent is updated based on the performance of the fund (payoff from the fund).

2.3. Timeline of Events

The numerical simulation follows at the protocol outlined below.

1) Investors make decisions to contribute additional money to the fund, or withdraw money from it, based on the risk of the fund and the performance of the fund in the past.

2) The fund manager repays claims of each investor if the wealth of the fund is higher than total amount of redemptions over a period of time. If the wealth of the fund falls below zero, it goes bankrupt and the simulation ends. If the fund does not go bankrupt, the fund manager divides the total amount of the fund into the risky asset and risk-free asset. The ratio of risky assets as a proportion of the total amount of the fund is based on the expected performance of the risky asset and the net inflow into the fund.

3) The performance of the fund is determined by the rate of return of the risky asset invested by the fund manager.

4) The payoff of each agent, investors, and the fund manager is determined. The fund manager makes a profit from the fee, which consists of the management fee and the performance fee. The payoff for the investors is determined by the total increase in the value of the fund minus fees for the fund manager.

5) A new period begins, and the process is repeated.

3. Simulation Analysis

3.1. Summary of Parameter Values Used in the Model

These are the parameter values we used for the basic simulation.

Table 15 Parameter values used in the simulation

Variable	Description	Value
Fund managers		
W_t	Total amount of fund	
period	Number of time steps	300
H_t	Amount of risky asset	
C_t	Amount of risk free asset	
r^H	Parameter for AR(1)	0.05
ρ	Parameter for AR(1)	0.90
σ^2	Variance for white noise	0.001
$\bar{r}_0^H$	Expected rate of return of the risky assets	0.05
a	Moving average parameter	0.1
p_0^H	The value of the fund at time 0	1
κ^H	Weight on the expected rate of return	0.001
θ^H	Weight on the relative net inflow	0.01
β	Performance fee	0.05(5%)
γ	Management fee	0.015(1.5%)
rev_t	Payoff for the fund manager at time t	
W_t^D	Distributable amount of wealth of the fund	
$\frac{H_0 P_0^H}{W_0}$	Ratio of risky asset to the amount of wealth of the fund at time 0	0.7

Investors		
N	Number of investors	450
e_{i0}	Initial endowment	2
A_{it}	Total amount of asset	
F_{it}	Value of the fund	
M_t	Total amount of cash	
r_t^F	Rate of return(payoff) of the fund	
$\bar{r}_0^F$	expected rate of return at time 0	0.04
λ_i	Safe level the investor believes	U[0.2,0.25] for P-Test U[0.1, 0.3] for F-Test
n_{it}^C	claims	
n_{it}	Net inflow	
η	Portion of trading	0.15

3.2. Simulation Results

We show the process of how a small negative loss in the fund can trigger bankruptcy and what mechanism works during the process. We report net inflow, wealth of the fund, net inflow relative to the total wealth of the fund, expected rate of return of the risky asset, weight on the high risky asset, and payoff of the fund for investors.

3.2.1. The Trigger of Bankruptcy

We find some positive relationships between payoff (performance of the fund) and the net inflow. Also, as seen in Figure 8, we find that the net outflow due to the negative

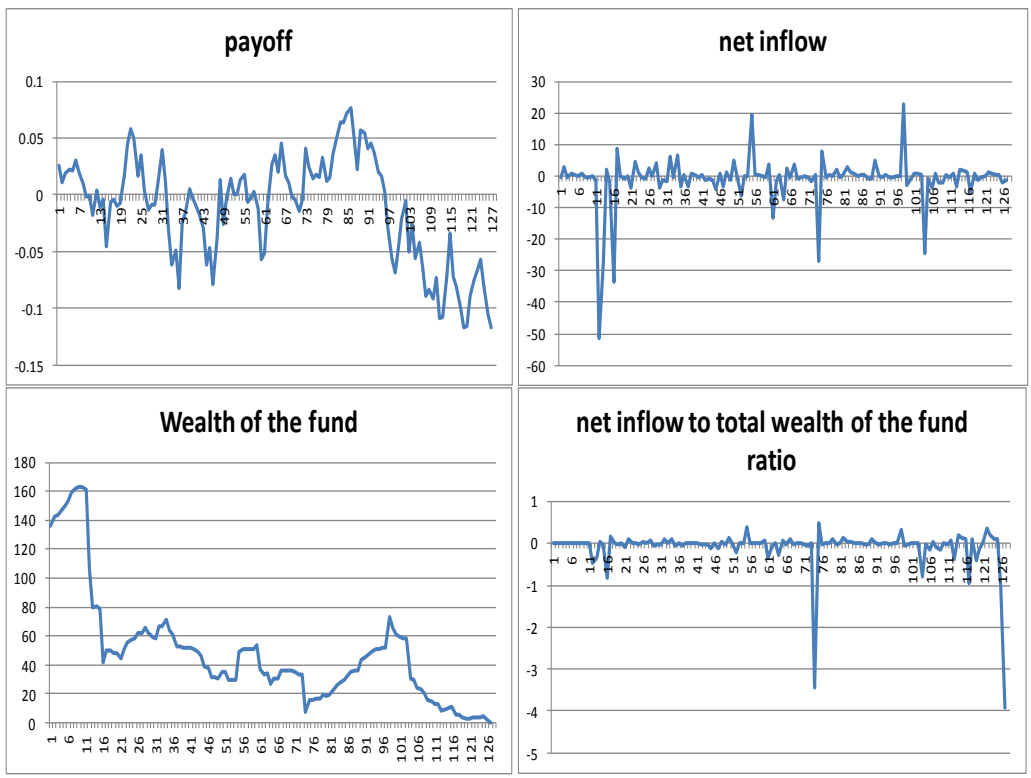
performance of the fund can trigger the fund to go bankrupt.

The process of bankruptcy for the fund is the following. If the negative rate of return of investment occurs, the loss is evenly allocated to investors who invest in the fund, and it results in a negative payoff for investors. Since the decision for future investment in the fund depends on the expected rate of return, which is calculated based on the recent performance of the fund, investors adjust their decisions for the investment by reducing the proportion in the fund and holding more assets in cash. Then some agents will withdraw some money from the fund as reacting on the negative performance of the fund. Then this withdrawal causes other investors to withdraw and may lead to the decrease of the total wealth of the fund. If an additional loss from the fund results, it is allocated to the remaining investors who did not withdraw their money from the fund. Then again, more investors withdraw money from the fund in reaction to the successive negative performance of the fund.

During the process, if the level of net outflow exceeds a certain level, which is randomly determined by a uniform distribution with an interval of [0.2, 0.25], the fear of possible insolvency begins. Then, starting from more risk aversion whose λ_i is relatively low, a massive number of redemptions would begin. Then, the total amount of outflow from the fund increases very sharply, and this can incur successive numbers of redemptions from the fund. However, if the liquidity level is maintained above a certain level, investors can make decisions for the redemptions based on the liquidity level and do not need to run to the fund manager.

However, large numbers of redemptions may panic investors and result in huge net

outflows of the wealth of the fund. This can cause the fund manager to allocate more funds to risky assets in order to attract more investors. The liquidity level then falls below certain level, and this can cause another run on the fund. Bankruptcy will eventually occur. These processes are shown in Figure 8.



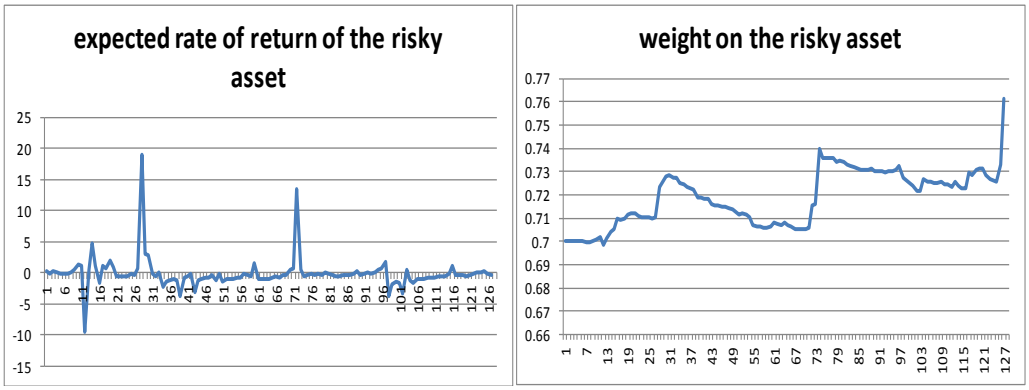


Figure 8 Time series of a fund for dynamics of Payoff for the Investors, Net Inflow, Wealth of the Fund, Net Inflow to the Total Wealth of the Fund Ratio, Expected Rate of Return of the Risky Asset, and Weight on the High Risky Asset, (P, F)=(0.25, 0.75)

3.2.2. The Ratio of Each Type of Agent and the Probability of Bankruptcy

We report the number of bankruptcies and time to reach the bankruptcy of 100 runs. We impose the threshold level, which is a criterion for withdrawing all money from the fund, to a number, which is randomly drawn from uniform distribution with interval [0.1, 0.3], which is relatively dispersed compared to [0.2, 0.25] of panic-based investors.

Table 16 Number of bankruptcies and time to bankruptcy as changing the ratio of each type of investor

(P, F)	(0,1)	(0.1,0.9)	(0.25,0.75)	(0.5,0.5)	(0.75,0.25)	(0.9,0.1)	(0,1)
# of Bankruptcies	27	35	37	37	46	43	51
Time to Bankruptcy	0.8347	0.7575	0.7287	0.7374	0.6615	0.6744	0.5854

When all agents make decisions for withdrawing all money from the fund based on the liquidity level, the number of bankruptcies is 27. However, by increasing the number of investors who make decisions for the withdrawal based on the number of withdrawals for a previous period, the probability of bankruptcy, as measured by the number of bankruptcies and time taken to bankruptcy, also increases.

4. The Relationship between the Incentives to Invest in the Risky Asset and the Probability of Bankruptcy of the Fund

4.1. Fund Manager's Incentive to Increase the Risk in Their Portfolio

We performed a simulation where two types of investors, panic-based and fundamental-based, are mixed. We find that the performance of the fund and the net

inflow to the fund are positively related. Also, a negative performance of the fund can trigger fund runs and the bankruptcy of the fund.

Now, we investigate the relationships between the manager's incentive to seek risky assets and the probability of bankruptcy. According to the literature, the portfolio of the fund determines the return of the fund, and a fund manager has an incentive to increase the ratio of the risky asset in order to increase their rate of return of the funds they manage. This is especially true when the decisions for the investment by the fund by investors are affected by its rate of return. In this case, the fund manager has to perform well to attract investors, and thus the fund manager has some incentives to increase the ratio of risky assets in their portfolio. We reflect this idea in the model and investigate how the incentive to increase the weight on the net outflow of the fund can affect the risk of the fund run.

4.2. Probability of Bankruptcy

As already explained, the fund manager decides the proportion of the risky asset depending on the expected rate of return of the risky asset and net outflow to the fund. That means that the proportion of the risky asset is increasing function in expected rate of return of the risk asset and decreasing function in net inflow to the fund. We investigate the effect of the proportion of risky assets by changing the weight for the net inflow, θ^H on the probability of bankruptcy of the fund. For this study, we did

simulations of three cases by changing each type of investor as follows: panic-based and fundamental-based = (0.1, 0.9), (0.5, 0.5), (0.9, 0.1). We report both the number of bankruptcies and the time taking to the bankruptcy of 100 simulations.

First, we summarize the number of bankruptcies of 100 runs. Compared with the case where $\theta^H = 0.0$, the overall number of bankruptcies increases in every case regardless of the ratio of types of investors.

Table 17 Number of bankruptcy as the ratio of each type of investor

The size of θ^H	0.00	0.005	0.01	0.015	0.02	0.05
(P, F)=(0.1, 0.9)	25	32	35	46	45	50
(P, F)=(0.5, 0.5)	30	35	37	45	45	48
(P, F)=(0.9, 0.1)	34	39	43	40	44	44

We find that when the ratio of investors whose criterion for withdrawing all money from the fund is liquidity level is relatively high at 0.9, the impact of the existence of θ^H on the bankruptcy is greater than other two cases. The number of bankruptcies is only 25 of 100 when $\theta^H = 0.0$, but it increases to 50 when $\theta^H = 0.05$. On the other hand, when the ratio of investors, whose criterion for the redemptions is the number of withdrawals is relatively high, the impact of θ^H on the probability of bankruptcy is relatively low. It only increases from 34 to 44 of 100 runs.

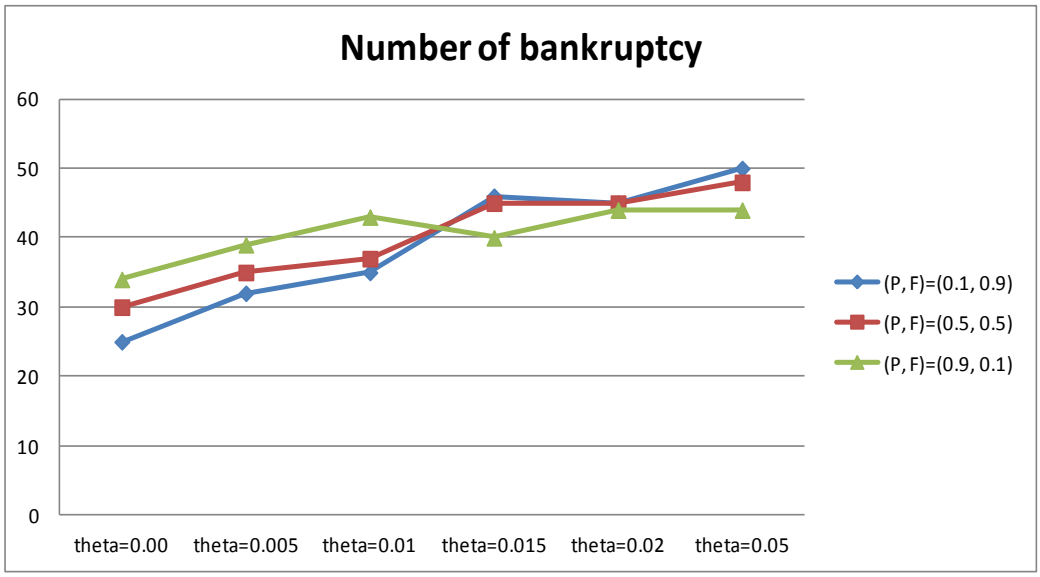


Figure 9 Number of bankruptcies

We have consistent results in the simulation when comparing the time to reach bankruptcy.

Table 18 Time to bankruptcy as the ratio of each type of investor

The size of θ^H	0.00	0.005	0.01	0.015	0.02	0.05
(P, F) =(0.1, 0.9)	0.8314	0.8011	0.7575	0.7062	0.6761	0.6584
(P, F) =(0.5, 0.5)	0.8001	0.7527	0.7374	0.6974	0.6804	0.6585
(P, F) =(0.9, 0.1)	0.7390	0.7247	0.6744	0.6938	0.6402	0.6462

We find that the existence of θ^H, and its size, have an effect on the probability of bankruptcy. It increases the number of bankruptcies and shortens the time taken to bankruptcy. The degree of influence of θ^H on the bankruptcy is different from the ratio of each type of investor. Similar to the previous simulations, when the ratio of agents

whose criterion for the redemption is liquidity level is high, its impact on the probability of bankruptcy is also high.

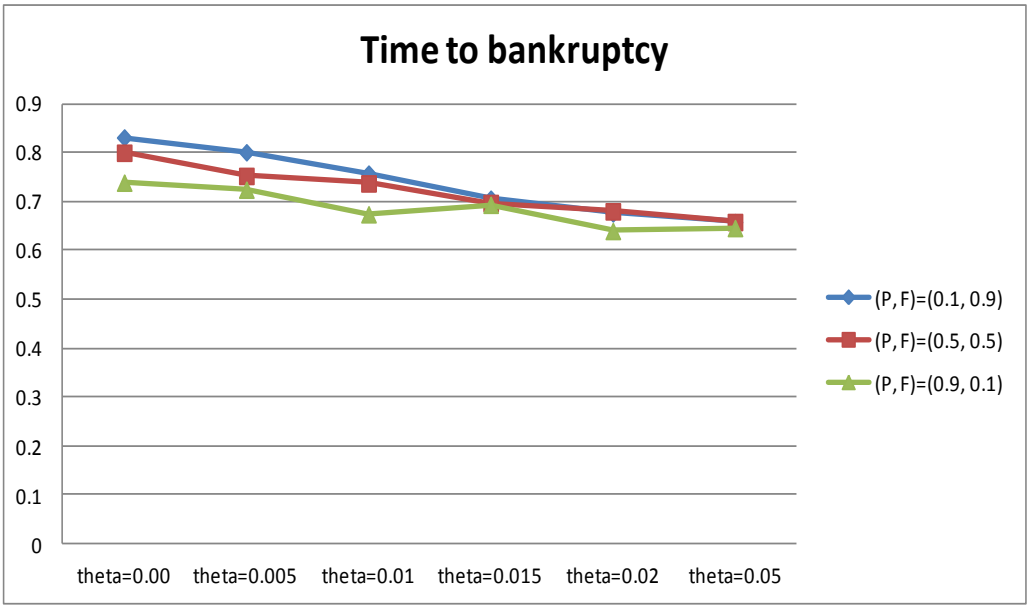


Figure 10 Time it takes to reach bankruptcy

Overall, we find that the increase in the proportion of risky assets imposes a higher weight on the net inflow (or outflow) and can increase the probability of bankruptcy in that it shortens the time it takes to bankruptcy and increases the number of bankruptcies out of 100 simulations. This means that the higher proportion of risky assets can be a potential risk for fund runs and bankruptcy.

To be more specific, we compare the impact of increases of the proportion of high-risk assets on the probability of bankruptcy by the ratio of each type of investor. The number of bankruptcies is higher, and a shorter time to reach bankruptcy, when more investors regard the number of withdrawals as a criterion for redemption. However,

when we compare the change rate before and after increasing the weight for the net inflow, the impact of increasing weight for the net inflow in determining the ratio of the risky assets is higher when more investors consider liquidity levels as a criterion of the redemption. This is because the increase of risky assets is directly connected to the decrease of liquidity. Also, it means that the higher incentive for the risk asset does not simply arouse the fear on bankruptcy but also weakens the soundness of the financial condition.

5. Relationship between a Manager's Payoff Scheme and Bankruptcy Risk of the Fund

We now explore the relationship between payoff schemes of fund managers and the probability of bankruptcy. As already mentioned in section 2, payoff for the fund manager is determined by the size of management fee, γ and performance fee, β.

In this section, we report the results of two simulations regarding the manager's payoff scheme and the bankruptcy risk of the fund. In the first simulation, we investigate how the size of the performance fee affects the risk of bankruptcy by changing the size of performance fee from 0.0, to 0.025, to 0.05, to 0.075, and to 0.1 while keeping the size of the management fee fixed at 0.015. In the second case, by contrast, we change the size of management fee from 0.00, to 0.005, to 0.01, to 0.015, and to 0.02 while keeping the size of performance fee at 0.05. Two simulations are performed where the

ratio of each type of investor is the same at 0.5. Half of the investors considered the number of withdrawals relative to wealth of the fund as a criterion for withdrawing all money from the fund. The other 50% of the agents used the liquidity level as a criterion for redemptions.

(1) Management Fee Test

In the simulation, we changed the size of management fee from 0.0, to 0.005, to 0.01, to 0.015, and to 0.02 while the size of performance fee was fixed at 0.05.

Table 19 Risk of bankruptcy as increasing the size of management fee

Management Fee	0.0	0.005	0.01	0.015	0.02
# of Bankruptcies	18	32	39	38	51
Time to Bankruptcy	0.9046	0.7990	0.7620	0.7374	0.6467

Note: the size of performance fee is fixed at 0.05
We impose 300 to the case of no bankruptcy

We found that as the size of the management fee increases from 0.0 to 0.02, the number of bankruptcies also increases from 18 to 51. The time to bankruptcy also shortened from 0.9046, when no management fee existed, to 0.6464 when the size of the management fee increased 0.02.

(2) Performance Fee Test

Now, let us look at the results for the performance fee test.

Table 20 Risk of bankruptcy as increasing the size of performance fee

Performance fee	0.00	0.025	0.05	0.075	0.1
# of Bankruptcy	37	38	38	44	50
Time to Bankruptcy	0.7395	0.7528	0.7374	0.7397	0.6784

Note: the size of management fee is fixed at 0.015
We impose 300 to the case of no bankruptcy

As seen in Table 20, by increasing the size of performance fee from 0.0 to 0.1, the number of bankruptcies increased from 37, to 50 of 100 simulations. Also, the time to reach the bankruptcy divided by time step shortened from 0.7395 to 0.6784.

(3) Management Fee vs. Performance Fee

So far, we have explored the impact of the size of each management fee and performance fee on the bankruptcy risk of the fund where half of the investors make decisions for the redemptions of money from the fund based on the liquidity level. The rest of 50% of agents act based on the number of withdrawals from the fund during a previous period.

Finally, we compare the degree of influence of each fee for the fund manager on the risk of bankruptcy. The absolute level is not important because it can vary as the value of the parameters imposed during the simulation. Thus, we focus on the degree of change rate as changing the size of fees. As a result, we find that the impact of the size of the management fee has more effect on the risk of the bankruptcy.

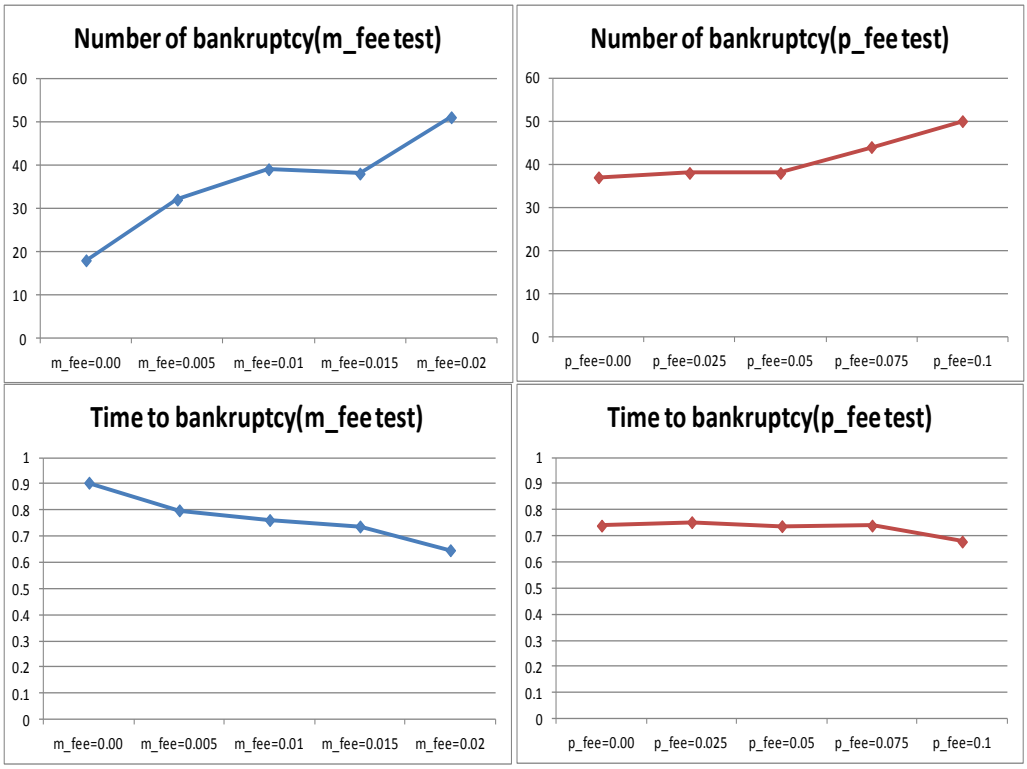


Figure 11 Results of number of bankruptcies and time to bankruptcy of management fee and performance fee test where (P, F) = (0.5, 0.5)

In the simulation for the time to reach the bankruptcy, we have a similar result. When comparing the effect of each fee on the time it takes to bankruptcy, we find that the increase of management fees has been shown to further reduce the time it takes to cause a bankruptcy when compared to an increase the performance fee. This is because the management fee is determined by the amount of the total size of the fund raised at the beginning of every period time; it does not reflect the loss of investment by the fund manager. However, because the performance fee is based on the profit from investment by the fund manager, the size of the performance fee is adjusted to reflect the result of

the investment. Thus, we can suggest that reducing the management fee and increasing the performance fee can reduce the probability of the bankruptcy of the fund.

6. Conclusions

This paper investigates how small losses occurred in a fund can trigger fund runs and the bankruptcy of the fund based on ABM. While lots of previous literature studies bank runs or fund runs by focusing on the behaviors of investors under panic-based runs and information based runs, this paper focus on the fund manager's incentives to increase the ratio of risky assets in their portfolios where investors react sensitively to the rate of return of the fund when they make decisions for the investment in financial assets. Also, we explore the relationships the manager's payoff scheme, which consists of performance fees and management fees, and the probability of bankruptcy of the fund, and suggest the payoff scheme for the fund manager, which can reduce the risk of the bankruptcy of the fund.

The main finding of the paper is as follows. First, we investigate the mechanisms of bankruptcy of a fund based on ABM and draw the conclusion that the negative performance of the fund can trigger fund runs and the bankruptcy of the fund. The mechanism is that after the negative performance occurs in the fund, the loss is evenly allocated to all investors who invest in that fund. Then, since the decisions of investors for the future investment to the fund is determined by the performance of the fund in the

past, agents begin withdrawing money from the fund. That means the size of the fund decreases and the payoff to the fund manager decreases. Then the fund manager chooses to increase the ratio of risky assets to enhance their performance of the fund, and this can increase the potential risk of the fund.

Also, we find that when the fund manager increases the weight of risky assets by reacting to the net outflow, the probability of bankruptcy also increases. We calculate both the number of bankruptcies for 100 runs and the time it takes for bankruptcy to occur. The effect of the fund manager's incentive on the probability of bankruptcy is especially large when more investors make decisions for redemptions based on the liquidity level. That means the manager's incentive to increase the ratio of risky assets can weaken the financial conditions of the fund and increase the risks inherent to the fund.

Lastly, we simulate the relationships between the fund manager's payoff scheme, which consists of the performance fee and management fee, and the probability of bankruptcy. For this simulation, we allowed half of the investors to make decisions for redemptions based on the number of withdrawals of the total value of the fund. The other 50% of investors acted based on the liquidity level as a criterion for redemption. We found that both the increase of the size of management fee and performance fee increases the probability of bankruptcy. However, the degree of influence of each of them on the probability of bankruptcy is different; the increase of the management fee had more of an effect on the bankruptcy. This result is because the loss of the fund is reflected in performance fee. However, the size of the management fee is determined by

the total amount of wealth of the fund, which rises at the beginning of every period of time; the loss during the investment is not reflected. Also, the absolute size of the management fee is larger than that of performance fee. Taken altogether, we suggest that decreasing the weight of the management fee, and increasing the weight for the performance fee, can help to reduce the probability of bankruptcy.

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Ⅳ. Concluding Remarks and Discussion

This study investigates capital market instability based on an ABM. In the first part, we pointed out the herding behavior as a main cause in generating instability in asset prices in capital markets. Also, it turns out that the effect of herding behavior on capital market instability increases as more agents share the same belief for the price of the future capital market.

The examples of how such herding behavior disturbs the stability of the market price can be easily found. As an example, we can think of institutional investors in stock market. In most of the cases, because the amount of resources that institutional investors have is much larger than that of individual investors, and because they tend to be superior in informational awareness and/or investment techniques, their effect on the market price is also huge. Thus, their attitudes and decision rules for trading assets are important for the stability of the capital market.

This paper describes such a point. Compared with other simulations, which do not allow for herding behaviors, extremely unstable intervals where asset price fluctuated very rapidly in the short time were detected. Also, in the results of reporting the volatility, measured by maximum–minimums and standard deviations, we found that herding behavior can exacerbate the instability of the capital market by increasing the volatility of the asset price. This means that herding behavior can undermine the stability of the capital market. Also, we found that agents included in a group are placed

in a strong position to obtain more wealth from the herding behavior. This is especially true when the ratio of agents in a group is between 2.5–3.3% of total number of agents because the probability of winning in the competition of obtaining wealth is highest. However, when the ratio of agents doing herding behavior exceeds 20% of total agents, the advantages from herding behavior almost disappears. In addition, as shown in the simulation where we divided agents into several groups with equal size, as more agents are included in a group and heterogeneity is reduced, higher volatility emerges. This is especially true when the price exceeds the thresholds of the majority of the number of groups as the fluctuation become larger.

Through these simulations results, we found that the existence of herding behavior can be a significant factor for disturbing capital market stability. Thus, caution against sudden changes in decisions for trading assets or investments by herding behavior is needed. We need to consider policies to prevent agents from becoming short-term speculators. This would help induce them to take a role as a buffer against capital market instability.

In the second part, we investigated how the incentives to increase the ratio of risky assets by the fund manager affect the probability of bankruptcy of the fund. In an environment where the decisions of investors are greatly affected by the recent rate of return of the fund, the fund manager has some incentives to increase the ratio of risky assets in order to attract more investors. As a result, we find that the incentives to increase the ratio of the risky assets, by reacting to the inflow or outflow of the fund, can exacerbate the bankruptcy risk of the fund.

This is related to the payoff scheme of the fund manager, which consists of management fees determined by the size of fund the fund manager manages and the performance fee determined by the rate of return of that period. Under this payoff scheme, the fund manager always tries to attract more investors in order to enhance their payoff. Namely, the fund manager has some incentives to increase the proportion of risky assets to increase their revenue by attracting more investors with high rates of return. As a result, we find that this incentive increases the likeliness of bankruptcy of the fund and shortens the time to reach the bankruptcy by raising the risks in the fund.

In an additional simulation, we investigates the relationships between the bankruptcy risk of the fund and the payoff scheme of the fund manager by increasing the size of management fee while keeping the performance fee fixed and increasing the size of performance fee while keeping the size of performance fee fixed. Increases in the size of both fees increase the probability of bankruptcy. However, the degree of influence of each of them on the probability of bankruptcy is higher when the management fee increases. We associate this result with the lack of a proper payoff scheme that puts a fund manager in charge of losses in the fund. In the some papers, it is pointed out that the payoff scheme based on the short-term performance of the fund, and lack of responsibility for the loss of the fund, can make the fund manager increase the risk of the fund regardless of its safety. Under the current payoff scheme, the fund manager receives a payoff based on the positive performance of the fund whereas the only penalty they receive for the loss of the fund is no performance fee. In this study, the problem of a lack of responsibility for the loss of the fund is revealed. Since the

management fee in this paper is based on the amount of wealth they generate at the beginning of each period, it is even less conducive to personal responsibility than the performance fee. The only penalty the fund manager may have is a reduction of their fee from a decreasing number of investors. Thus, we find that the lack of an appropriate payoff scheme, which can share loss properly, can be a main factor in increasing the risk of the bankruptcy of the fund. We need an improved payoff scheme, which can induce the prudent investment of the fund manager and thus reduce risk in the fund. Also, the payoff scheme should be based on long-term performance, not based on the short-term rate of return. This might be helpful for improving the stability of the capital market.

국문초록

Agent-based model 에 기반한 자본시장 불안정성 연구

본 논문은 ABM 에 기반해 자산시장에서 발생하는 불안정성을 설명하였다. 첫 번째 논문에서는 군집행동(herding behavior)이 자산시장의 가격변동성을 높여 자산시장의 불안정성을 높일 수 있음을 보였다. 지금까지 ABM 에 기반해 자산시장의 동태를 설명한 대부분의 모델의 경우 주로 chartist 의 존재로 버블과 같은 자산시장의 불안정성을 설명하고 있는 반면, 본 논문은 여기에 추가적으로 가격의 상한과 하한에 대한 한계선이자 의사결정을 바꾸게 하는 지점(threshold)을 고려하였다. 이는 agent 마다 다를 수 있으며, 경우에 따라 다수의 agent 들이 동일한 threshold 를 가지고 일종의 군집행동을 할 수도 있다. 결과적으로 서로 이질적인 threshold 의 존재는 자산가격의 변동성을 낮춰 자산시장의 불안정성을 줄이는데 도움이 되지만 군집행동이 나타날 경우 단기간에 자산 가격이 매우 빠르게 변하는 구간을 발생시켜 자산시장의 불안정성을 높이는 것으로 나타났다. 또한, 이러한 군집행동은 평균적으로 집단에 속한 투자자들이 집단에 속하지 않은

개별 투자자들과 비교해 상대적으로 높은 수익률을 획득하는데 유리한 위치에 놓이게 하는 것으로 나타났다. 특히, 군집행동을 하는 투자자의 비율이 2.5-3.3%일때 수익률 획득에 가장 유리한 반면, 집단의 크기가 20% 이상으로 커지면 군집행동으로 인한 이득이 거의 사라지는 것으로 나타났다. 두번째 논문은 투자자들이 펀드의 단기적인 성과에 민감하게 반응해 투자와 환매를 결정하는 펀드시장을 가정하고, 이러한 환경에서 보다 많은 투자자 유치를 위한 펀드매니저가 처한 고수익-고위험 자산으로의 투자유인이 펀드의 파산확률에 주는 영향을 ABM 에 기반해 분석하였다. 그 결과 펀드매니저의 고수익-고위험 추구 유인이 증가할수록 펀드의 파산확률이 증가하고, 파산까지 걸리는 시간이 단축 되는 등 펀드의 파산 리스크는 커지는 것으로 나타났다. 또한, 펀드 매니저의 보수체계와 펀드의 파산확률 사이의 관계를 알아본 연구 결과 관리보수의 증가가 성과보수에 비해 펀드의 파산확률에 더 크게 영향을 미치는 것으로 나타났다. 이는 펀드의 손실에 대해 상대적으로 펀드매니저에게 책임을 덜 요구하는 펀드매니저의 보수체계의 불합리성과 연관 지어 생각할 수 있다. 본 연구의 결과 역시 상대적으로 펀드의 손실이 더 적게 전가되는 관리보수의 증가가 더 큰 펀드의 파산 리스크의 원인이 될 수 있음이 나타났다. 위 결과를 종합해보면, 투자자들이 펀드의 단기 성과에 기반해 펀드의 투자를 결정하는 환경과 상대적으로 손실에 대한 책임이 가벼운 펀드매니저의 보수체계는 펀드매니저로 하여금 고수익-고위험 추구 유인을 강화시키는 원인이 되며, 이는 결과적으로 자산시장을 불안정하게 하는 원인이 됨을 알 수 있다.

주요어: 군집행동, 자산가격 변동성, 고수익-고위험 추구 유인, 펀드의 파산리스크, 펀드매니저의 보수체계, 자산시장 불안정성