공학석사 학위논문

Analysis on the downfall of rate of return for issuers of Equity Linked Securities and possible solutions via hedging with newly structured products

증권사 **ELS** 헷지 운용 순 마진율 악화 원인 분석 및 신구조화금융상품을 통한 해결방안 연구

2014년 2 월

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산업공학과

김 재 현

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Abstract

Analysis on the downfall of rate of return for issuers of Equity Linked Securities and possible solutions via hedging with newly structured products

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Equity Linked Securities products, also known as ELS, were and still are one of the most sold financial products in the Korean Market ever since the early 2000s. The issuers of ELS products, however, started to recognize lower rate of return than the heydays of ELS products due to the decrease of volatility levels of global markets. This paper focuses on how the downturn of global market's volatility level correlates with the rate of return's decrease for issuers of the ELS product and the solutions for hedging the already accrued risks of the ELS products that were issued with high volatility. Along with the fall of global market's volatility, the conventional hedging methods of ELS product's Vega risk such as Listed / OTC (Over the Counter) options hedge, issuing ELS products with notional protection, etc. are no longer as effective as it used to be during the mid-2000s when ELS products were at its most liquidity

In order to successfully hedge the Vega risk of ELS products, this paper suggests including "Short Vega" type of financial products to the hedge book, which differs from the common "Long Vega" products such as "Multi Asset Step-Down Auto-callable ELS" Products: "Reverse Accumulator" and "Worst of Call" products. With these "Short Vega" types of products it will effectively diminish the already high accrued Vega risks. By creating a virtual portfolio with different types of ELS products, when these new products were included the portfolio was able to drop off around 40~50% of the original Vega amount as well as 30% of the Delta amount. In addition, this paper suggests how it is important to know the overall volatility levels for different markets respectively, and therefore suggests building up the volatility surface in order to successfully hedge the overall "Greek Risks

Keywords: Equity Linked Securities, Hedging Greeks, Reverse Accumulator, Worst of Call, Volatility Surface
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Chapter 1. Introduction

Equity Linked Securities products, also known as ELS products were first issued in Feb, 2003 in the Korean Market, and they are still recognized as one of the main financial products of Korea. The reasons for the popularity for issuers of ELS products were due to the ability of profit creating nevertheless of direction betting of underlying assets but only needing to care about the level of volatility of the underlying assets during the back and forth period of financial crisis which encountered the entire global market around 2008. To reflect this tendency currently 23 Korean local financial companies are permitted to treat Over the Counter Derivative (OTC) products such as ELS products, whereas 6 foreign Investment Banks in Korea have given permission to issue these types of products as well in Korea. Ever since the early 2000s the ELS market has tremendously grew and have situated themselves as one of Korean markets' important asset class. Unfortunately, however, the rate of return for the issuers of ELS products has been decreasing for the past few years. The following Table shows the status quo of the real rate of returns for issuers of ELS.

Table 1 show that current issuing companies of Notional Protectable ELS products are scoring lower management profit levels compared to the payable interest rates and this leads to having negative rate of return when issuing ELS products. In practice, advanced research done by Y.S Kim(2010) shows that the issuing margin of Korean ELS market has been declining ever since the first issuing date of 2003. The already known reasons for the decrease of real rate of return for issuing ELS can be summarized into two big notions. First of all, local financial crisis like KIKO and unjustified Delta Hedge actions made by financial companies that had ELS products near the Knock -In barrier brought about the potential risk of ELS products to the Korean financial market.

<Table 1> Status Quo of Real Rate of Return for Notional Protectable ELS

Type	A Group	B Group	C Group	D Group	Average
Rate of Operating Profit for ELS	4.80%	3.87%	3.61%	3.86%	4.32%
Issuing Companies Payable Interest Rate	4.88%	5.06%	4.89%	4.84%	4.32%
Rate of Return	-0.08%	-1.19%	-1.28%	-0.97%	-0.60%

Source: Bank of Korea

In other words, due to these types of big and small events, various actions from financial authorities came into practice such as passing strict law orders relating to fair Delta Hedge and setting up specific standards in order to prevent issuers and managing parts of OTC products from easy profit taking by managing the hedge book according to their own profit making protocol schemes. In addition local financial companies and financial investigation institutes assert that the decrease of ELS rate of return is not only due to the financial authorities' intensification of market restriction which brought about the constraint of available of delta hedging amount but also due to the change of macro financial status provoked by intensifying competition of local issuing companies. In order to find out the fundamental reason for the decline of ELS products' rate of return, however, one must not overlook the decline of global markets' volatility level in which correlates in high possibility to the fair price of ELS products As a consequence, since the global market's volatility shows the tendency to keep on falling, the highest portion in handling the ELS product's risk management, Vega risk, has been ever more unpredictable to be hedged than it used to be such that it leads to the decline of rate of return of ELS products. Therefore, this paper focus on suggesting possible solutions to resolve decreasing the rate of returns for already issued and the upcoming issuable ELS products, with the approach of acknowledging the decline in global markets' volatility level as well as the absence of offsetting the already accrued "Greek risks" of these ELS products.

This paper emphasizes that the weakening of ELS products' rate of return is due to loss in unable to handle Vega Risk. To flesh out this assertion, the fall of volatility levels of underlying assets, imbalance in OTC option markets' bid offer levels and liquidity, diminishing market size of Korean Equity Linked Warrant (ELW) Market have been critical towards augmenting the ELS products' Vega Risk. Hence, this paper pinpoints that not only concentrates on handling the conventional Delta risks in which original financial products mainly focused on but also handling the Vega Sensitivity of structured products such as ELS is a very important issue. In order to solidify this assertion, trend of V-KOSPI (Volatility level of Kospi200 Index), status quo of bid and offer levels of KOSPI OTC option market and how ELW market is changing will be analyzed in further details, proving that traditional methods for Vega Risk handling are prone to expose limitations.

After this analysis is done, this paper will suggest two "Short Vega" types of financial products in which have opposite characteristics of the general "Multi Asset Step-Down Notional unprotected ELS" products. To be specific, products like "Reverse Accumulator" and "Worst of Call" have short maturity along with high yields and most of all having the "Short Vega" characteristics in which can both enhance the rate of return as well as drop off the Vega risk of the already accrued high Vega levels of current local financial companies. In practice issuing financial companies are most likely managing their issued products not product by product basis but rather as a portfolio oriented notion such that periodic P&L are estimated according to the overall Greek level change in the total portfolio as well as the sum in the change of each and every ELS products' fair prices. With products like "Reverse Accumulator" and "Worst of Call" included to the portfolio with large notional, the portfolio will be able to rebalance the overall "Greek Risk" by dropping off unwanted high level of Vega influence as well as other

Greek letter influences as well. This will be shown by running through a simulation of change of Greek levels of virtual portfolios of various ELS products of before and after the “Short Vega” financial products are put into the portfolio.

Along with the “Short Vega” products, as the Volatility level of underlying assets are acknowledged to be one of the critical components of issuing ELS products, it is essential to follow the trend and tendency to change in both long and short terms of Volatility levels of the respective underlying assets. This paper leads in emphasizing the importance of creating Volatility Surface for respective financial markets with trend and levels fully correlates with the global financial firms. Through these processes, this paper provides analysis upon finding out the reason for the decline in the rate of return of ELS products as well as the possible solution towards handling the problems invoked, assuring possible profit making chances in the near future.

The rest of the paper is constituted as follows: The Second section of the paper firstly talks about the already mostly sold ELS product: “Multi Asset Step-Down Notional unprotected ELS” in order to find out what kinds of risk have the virtually replicated Hedge book portfolio is prone to be exposed prior to the Short Vega products are included in the portfolio. Also, analysis on how to handle this Multi Asset Step-Down Notional unprotected ELS is the most optimal in “Greek Risk” handling from the issuer’s point of view. The third section will discuss about the factors that affect the fair price of ELS such as the trend of OTC option market and the ELW market. The fourth section will analyze function and the limitations of the conventional Vega risk hedging methods in order to emphasize the new needs of risk handling methodologies and products. The fifth section will discuss about the characteristics of “Short Vega” Products: “Reverse Accumulator” and “Worst of Call” and will go under the simulation of running the Greeks analysis of the virtual Hedge book replicated portfolio. In addition, the importance of building up the volatility surface will be indicated. Lastly on the sixth section the summary of this paper and the future workable direction will be provided.

Chapter 2. Multi-Asset Step down ELS and the management knowhow

1. Payoff of Step down ELS and the underlying asset's probability Process

The payoffs for the "Multi Asset Step-Down Notional unprotected ELS" is realized according to the underlying asset's movement until the end of the product's maturity. In order to check the fair price of products like ELS, the first and foremost thing to do is to assume the derivative's probability process of the underlying asset. The underlying asset's price process can be shown as the Stochastic Differential Equation (SDE) which is based upon the Geometric Brownian Motion (GBM), and it can be written as the following:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t^{P} \quad \ldots \quad (1)$$

From the above Equation 1, $S(t)$ represents the price of the underlying asset according to the time frame, μ and σ both respectively represents the drift term and the volatility term of the underlying asset. Thus, the underlying asset follows the SDE where the standard normal distribution (dW_t) with the average of 0 and the standard deviation of very short period of time frame (dt) in which determines the stock's rate of return. Furthermore, in order to perform Risk-Neutral Valuation by using the Girsanov Transformation, $(r + \sigma\lambda - d)$ is supplemented for μ of Equation 1.

$$dS(t) = (r + \sigma\varphi - d)S(t)dt + \sigma S(t)dW_t^{Q} \quad \ldots \quad (2)$$

Where r: risk free interest rate, σ: Volatility, φ: risky market price, d: Dividend

Rate

ELS products with more than one underlying assets need to consider not only the movement of the underlying assets but also the implied correlation. For example, 2 Stock Assets Step-Down Notional unprotected ELS is based upon the Stock SDE with the below correlation equation reflected:

$$dS_1(t) = (r + \sigma\lambda - d)S_1(t)dt + \sigma S_1(t)dW_{1,t}^Q \ldots \quad (3)$$
$$dS_2(t) = (r + \sigma\lambda - d)S_2(t)dt + \sigma S_2(t)dW_{2,t}^Q \ldots \quad (4)$$
$$dW_{1,t}^Q \, dW_{2,t}^Q = \rho dt \ldots \quad (5)$$

With the general Step Down ELS products in which follows the above stock process as a given buys the Digital Call Option on every early redemption observation dates. In the case of underlying assets which are at low probability to protect the starting notional amount can be replicated by selling the Down and In Put Option to be equivalent. These factors are important aspects in comprehending the relationship between ELS product hedge management with the methods in Vega Hedge.

2. Barrier option

Barrier options are well known for being in the state of going in or out of effect when the underlying asset's price level reaches the certain barrier strike level. These barrier options can be distinguished by Up-type and Down-Type according to the Strike price level as well as the In-type and Out-Type of options according to products' status of being in or not being in effect. In addition, by adding up the option characteristics of either Call or Put, the choice of barrier options can be enlarged in to the range of 8 possible barrier options. As mentioned above, Step down type of ELS products are equivalent to selling a Down and In Put option position when the worst performing underlying asset's price reaches the barrier exercise price during the observation dates. Here is the closed form equation of Down and In Put option.

1) Down and In Put ($X > B$) =

$$-\left[\mathrm{S(t)N}\left(-d_1\left(\frac{S(t)}{B}\right)\right) - \mathrm{P(t,T)XN}\left(-d_2\left(\frac{S(t)}{B}\right)\right)\right]$$

$$+\left[B\left(\frac{s(t)}{B}\right)^{-\frac{2r}{\sigma^2}}\mathrm{N}\left(d_1\left(\frac{B^2}{S(t)X}\right)\right) - P(t,T)X\left(\frac{s(t)}{B}\right)^{1-\frac{2r}{\sigma^2}}\mathrm{N}\left(d_2\left(\frac{B^2}{S(t)X}\right)\right)\right]$$

$$-\left[B\left(\frac{s(t)}{B}\right)^{-\frac{2r}{\sigma^2}}\mathrm{N}\left(d_1\left(\frac{B}{S(t)}\right)\right) - P(t,T)X\left(\frac{s(t)}{B}\right)^{1-\frac{2r}{\sigma^2}}\mathrm{N}\left(d_2\left(\frac{B}{S(t)}\right)\right)\right] \cdots \quad (6)$$

2) Down and In Put ($X < B$) =

$$-\left[\mathrm{S(t)N}\left(d_1\left(\frac{S(t)}{X}\right)\right) - \mathrm{P(t,T)XN}\left(d_2\left(\frac{S(t)}{X}\right)\right)\right] \cdots \quad (7)$$

$$\text{where B : Barrier Strike price,} \qquad \text{X : Put Option's strike price,}$$

$$\mathrm{P(t,T)} = e^{-r(T-t)}, d_1(x) = \frac{\ln(x) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, d_2(x) = d_1(x) - \sigma\sqrt{T-t}$$

The early knock out period is constituted with structured zero coupon bonds which gives out pre-designated yield at pre-designated periods, meaning that investors of Step Down ELS products are going on a long position for short term bonds such as 3 month-long singular bond or 6 month-long singular bond, which is equivalent to buying a Forward starting Digital Call Option.

3. Know-Hows on managing ELS hedge book: Handling profit and risk (ELS products' Greeks)

In order to manage the ELS hedge book, the overall Greeks' levels must be distinguished according to each Greek type while parameters that affect the ELS products' fair price are inclined to change accordingly. The change of ELS products' fair price can be explained through the equation of Taylor's expansion and the usage of Ito's lemma. The equation is shown as the following:

$$\mathrm{dV} = \sum_i \frac{\partial V}{\partial S_i} dS_i + \sum_i \frac{\partial^2 V}{\partial S_i^2} dS_i^2 + \frac{1}{2}\sum_i \sum_{j(i \neq j)} \frac{\partial^2 V}{\partial S_i \partial S_j} dS_i dS_j + \sum_i \frac{\partial V}{\partial \sigma_i} d\sigma_i + \frac{\partial V}{\partial t} + \cdots \quad \cdots \quad (8)$$

The primary premise of ELS Hedge Book management lies upon the possibility to discretely or by portfolio-wise neutralizing the ELS products' change of fair price according to the change of Greek values. In addition, since ELS portfolio managers investigate the fair price levels of Step Down ELS products by checking the fair price levels early knock out observation dates and the maturity observation dates, the Greek sensitivity level, calculated in the Continuous time notion, must be recalculated in the concept of Discrete time-wise. Issuers of ELS products run their hedge book with the mechanism shown in Table 2. ELS sellers and buyers can maintain the sensitivity positions by upholding the portfolio of "Down and In Put" options and Digital Call Option until the early knock out dates or the end of maturity date.

To explain further, as the volatility level decreases the fair price of Step down ELS will increase and the issuers will recognize this as loss on their Profit and Loss(P&L). This logic works the same for the buyer of the ELS products. During the ELS issuing period, ELS issuers gain the notional amount from the investors and buy the Down and In Put option and with the left over notional buy the Forward starting Digital Option.

<Table 2> Step down ELS Buyer/Seller's Sensitivity Position

Greeks	Seller(Issuer) Position	Buyer(Investor) Position
Delta	Short	Long
Gamma	Long	Short
Theta	Short	Long
Rho	Long	Short
Vega	Long	Short

After the issuing process is over, in order to maintain the Delta-Neutral position or a certain level of Greek position, underlying assets are frequently bought and sold in order to procure Delta profit or Gamma profit. Step down ELS products can be recalled as some sort of bond product with inherent function of option, having the property of both bond and options at the same time. Therefore, Step down ELS products have the characteristics of bond until it reaches the Knock-In barrier strike level, and if the underlying asset's price reaches the Knock-In barrier strike level it will have the same characteristics of a stock. Therefore when the ELS product's underling asset price does not breach the barrier strike level, ELS issuers will have their bond price increase as the knock out yield to maturity decreases such that Theta loss will occur every transaction days. Thus ELS issuing companies try to offset the Theta loss through Gamma profits lead by Delta Neutralization process.

Furthermore, ELS issuers try to decrease the ELS price level difference between the pricing volatility and the hedging volatility As mentioned before, ELS issuers manage the hedge book in the concept of running a portfolio such that the sensitivity distribution for overall Greek levels are always derived in order to successfully manage the Hedge Book risk. In the case of managing the Delta risk, amount of underlying assets that are needed to be hedged are calculated almost in real time manner during the transaction period. Gamma risk and Vega risk, however, have a different dimension compared to Delta risk as these risks cannot be solely broken down into solely hedge possible trades. Gamma risks and Vega risks are mostly correlated together and they can be dropped off through buying or selling OTC option trades with large notional in the concept of portfolio hedging. Another possible way to manage Gamma risk and Vega risk is to issue financial products that have opposite Greek risk compared to the Step down ELS products.

One thing to bear in mind at this point is that Vega value is the core Greek values that affect the ELS price P&L. Unlike Vega values, Delta, Gamma and Theta Values can be at least calculated before the market opens in a scenario-based formula, and thus can be managed when efforts are put through during the trading hours. Delta and Gamma risks can be managed through scenario predictions before the maturity is met according to maximum upside and downside potentials due to law orders authorized by Korea Exchange (KRX). Theta loss can be calculated by the ELS product's bond position's as yield to maturity tend to decrease throughout the period in between the early Knock Out period and the maturity period. Vega risks, however, has limitations upon managing with expected scenarios since expecting volatility level tend to have so many parameters needed to put into consideration such that accurate prediction is a very difficult thing to do in ELS hedge book managing. There are indeed many volatility prediction methods based on different types of statistical models such as ARCH, GARCH or EWMA. Unfortunately statistical models only assign the statistical meaning of the numbers, still unable to critically pinpoint the reason for the change in the volatility level. Therefore, the successful ELS hedge book management lies upon the efficient management of Vega risks.

Chapter 3. Original Vega hedge method and its limitation

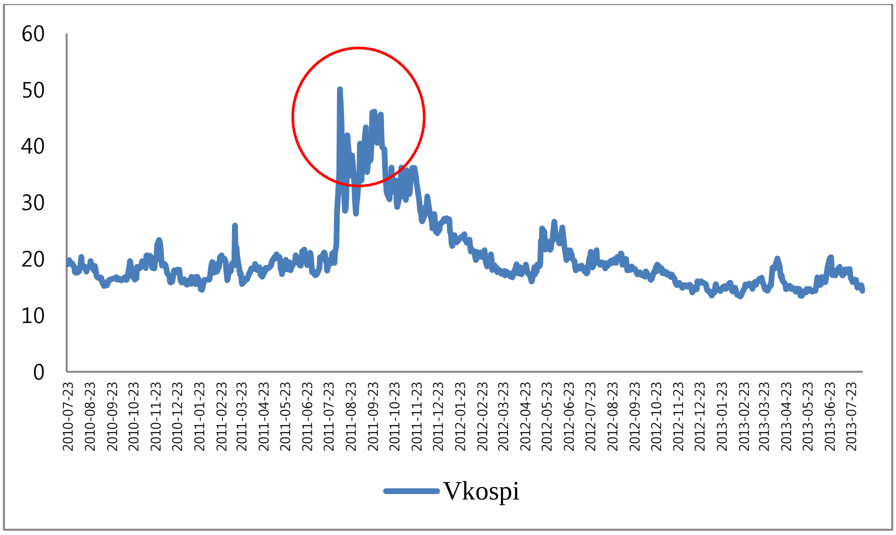
On this chapter ELS issuer's Vega hedging methods will be introduced and upon these methods the purpose and limitations will be mentioned. Regarding to the limitations of the original Vega hedge methods, the global underlying asset's volatility levels will be checked. Through this analysis the needs and reasons for a new type of Vega hedging method will be invoked.

As mentioned before, ELS values highly correlates with the change of Volatility levels. Therefore, hedge book owners put a lot of effort in hedging the Vega risks. Since this paper is stating that the decline in ELS products' rate of return is mainly due to the overall shrinkage of volatility levels in all sorts of underlying assets which invokes the loss at the long Vega position. In order to flesh out this assertion, the trend of V-Kospi level is to be investigated as well as the individual stock subjects. In addition, the liquidity of ELW market is to be checked so that it is possible to see how these investigations can give influence in handling ELS product's Vega risk.

1. The trend of V-Kospi level and the Individual stock Realized volatility level

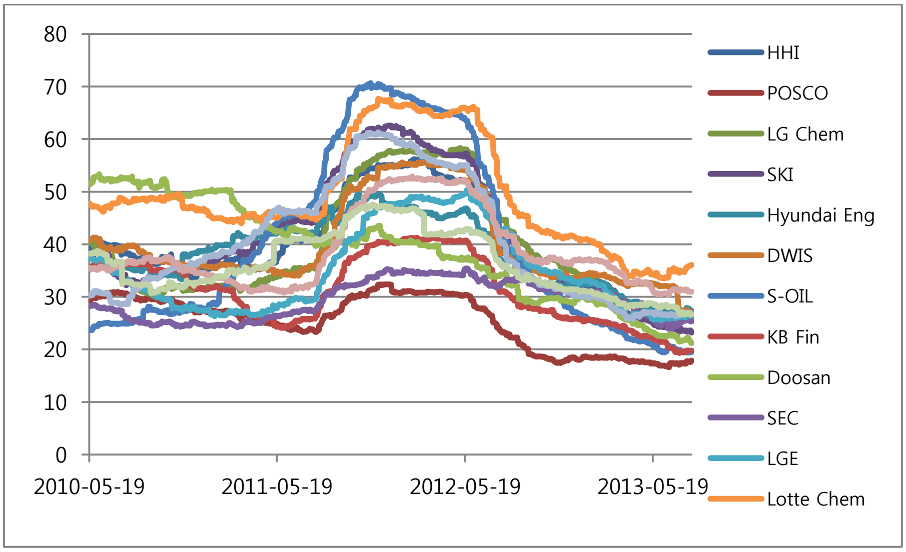
Underlying assets for ELS products can be distinguished by Individual stock types and Index types. <Figure 1> shows the trend of V-Kospi ever since the first presented Jul, 2010 till Jul, 2013, and the trend seems to show a continuation of decline its' level after experiencing Europe's financial crisis. <Figure 2> shows the trend of 250 days realized volatility change of individual stocks in the Kospi200 Market. It is apparent that the volatility levels are tending to follow a declining tendency for most of the individual stocks that are mentioned in Figure 2. Acknowledging the fact that these underlying assets are the ten most liquid stocks in the Kospi200 Market, it is most likely that the falling trend of volatility of V-Kospi and these individual stocks are in high correlation. This overall trend of volatility decrease indicates that the expected loss in ELS pricing will increase if this downward trend continues.

<Figure 1> V-KOSPI Trend Change



Source: KRX

<Figure 2> Individual stock 250 Day Realized Volatility



Source: Bloomberg

2. Equity Linked Warrant (ELW) market trend

Equity Linked Warrant products, also known as ELW products were first issued in Dec, 2005 and increased its popularity until 2011. ELW product structure is similar to BW (Bond with Warrant), just like vanilla Call option, where investors are able to gain certain amount of profit when the underlying asset's convertible price is higher than the strike price. ELW products are straight forward in mostly buying Call and Put Options with the main focus on underlying asset prices' movement.

<Table 3> Instigations for stabilizing ELW market

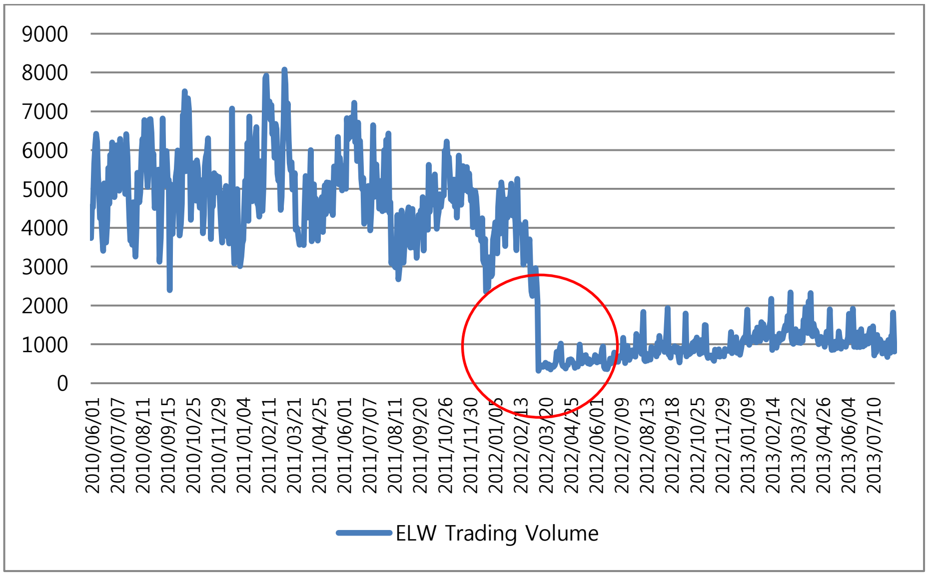
Measures	Dates	Important Factors
First measure to stabilize ELW market	2010- 11- 11	-Compulsory tutorial sessions for private investors - Compulsory LP price offering with frequent time period
Second measure to stabilize ELW market	2011- 05- 20	- Introduction of compulsory balance deposit - Order speed related standard given
Third measure to stabilize ELW market	2012- 03- 12	- Limitation on LP price offering spread

Source: Financial Supervisory Service, Korea

With the high leverage effect, ELW products were recognized as high yield products. This resulted in high liquidity, as in 2005 when ELW was first issued it recorded daily average liquidity of 21 billion KRW notional traded whereas at the end of 2010 it recorded daily average liquidity of 2.1 trillion KRW notional. This overheated phenomena, however, instigated Liquidity providers (LP) to give out high spread between bid and offer prices, influx of Scalpers with high trading volumes, and many more violating integrity issues, and as a consequence three authorizing measures were taken which tremendously downsized the trading volume of ELW products. Table 3 is the summary of the three major authorizing measures taken to calm down the ELS market:

Throughout the contents on <Table 3> authorizers were successful to decrease the ELW LP from lowering the participation rate to this market. On the other hand, this enabled the foreign investors and the private investors to increase the volume in which cause the imbalance of supply and demand of the ELW market. Figure 3 shows the trading volume of ELW market after the market authorization has been instigated:

<Figure 3> Equity Linked Warrants (ELW) Trading Volume (Unit: # of products)



Source: KRX

ELW market had a huge role of maintaining the Volatility level of KOSPI market as ELW products balanced out the volatility level with the ELS products as in the investor's point of view ELW products have the characteristic of buying the Gamma and Vega in the investors' point of view, which is the exact opposite of ELS. Therefore, issuers of ELW can manage both ELW and ELS products in the concept of bigger portfolio so that these two products can maintain the balance of the volatility level as bid and offer levels simultaneously remain strong. This balance is no longer in practice since the three authorization acts are passed through the bar, liquidity of ELW products have dropped down tremendously which also caused the long Vega type of ELS products to drop down in the volatility level as well.

3. Listed / OTC option market

Individual stock Option markets recorded around 60,000 trading amount in 2002 when the first traded was done, but it started to lose interest and the trading volume of the last 5 years can be shown in the following Table:

<Table 4> Trading Volume for Individual stock Options Source: KRX

Year	CALL	PUT	Total
2007	2	101	103
2008	7	14	21
2009	526	456	982
2010	11,542	60	11,602
2011	0	2	2
2012	0	0	0

As mentioned above, in order to hedge ELS products' Greeks exposure, using options to go on a long or short position of options according to the exposed Greek amount can be sought of. Listed options, however, only provide the maturity of 3 months unit such that every case of position structuring must have in mind the concept of rolling over to the next maturity. To wrap up, using listed individual stock options to perfectly hedge the ELS products' Greek exposure due to roll-over risk and low liquidity. Listed individual stock options may have another side effect to ELS issuers when it comes into the realm of transaction cost and information leakage to other ELS issuers upon the risk exposure of their own. With these types of matters in mind, it is less intriguing for ELS issuers to join the Individual stock Listed Option markets.

Individual stock OTC option markets, on the other hand, is still one of the most actively traded markets via OTC brokers acting as a market maker in between local and foreign banks. Throughout individual stock OTC option markets, ELS issuers try to hedge their exposed Greek positions as much as they can, but as the ELW market, which worked as a supplementary for providing a solid volatility level for ELS pricing, is suffering from may authorizing issues and downsizing its liquidity ever since the authorizing issues popped up, the imbalance of supply and demand caused asymmetry on the option's bid and offer spread. This widened bid offer spread makes it harder for options to be made as a deal. This phenomenon leads to the fall of the individual stock OTC option market, and it will increase the ELS issuer's expected Vega loss

4. Issuing ELS with Notional protection

ELS with notional protection is exactly opposite of the well-known Step down ELS that this paper has been dealing with from the very beginning. ELS issuers may issue this type of ELS products not only to realize profit but also to handling Greek risks when managing a portfolio o f ELS products. The Notional protection ELS is still less issued compared to the notional unprotected Step down ELS products, meaning the needs are not high rocketing compared to the original Step down ELS. This is due to the low coupon rates that the Notional Protection ELS can offer to the investors due to the structure of the ELS that cannot give out high rate of return in exchange of protecting the notional. This implicates that in order to increase the issuing amount of ELS, ELS marketers need to figure out the right needs of the customers, notwithstanding how favorable the product is to the customers.

As the global market depression led to the downfall of various financial market products rate of return, investors tend to pursue financial products with high rate of return such as Step down ELS products. Therefore, it is actually mind-boggling matter for issuers to push the sales of ELS products like Notional Protection ELS. It is clear that controlling ELS Greek exposure cannot be solely hedge through the ELS products with the exact opposite risk positions.

Chapter 4. Portfolio risk handling via issuing new structured products

Throughout the analysis of Chapter 3, this paper has shown that ELS Vega loss is not just an incidental phenomenon, and when the current trend of Volatility level is considered, it is obvious to find out that the original hedging method is not efficient enough to control the whole Vega risk of the hedge books that the issuers usually hold. In order to find solutions for solving this problem, issuing newly structured products that can both hedge the Greek risks of already accrued positions and meet the investors' needs in providing high yield can be one solution. Hence, in order to offset the original "Long Vega" type of products, "Short Vega" type of products must be included into the portfolio for the overall Greek level rebalancing. Simulation will be shown in order to prove that newly structured products will be efficient in diminishing the total Vega levels. In addition, the importance of tracking the overall volatility level of the markets that the issuer is participating in will be shown and the reason for building up a volatility surface will be emphasized.

1. New Product Suggestion: Reverse Accumulator, Worst of Call Option

In order to understand the product called Reverse Accumulator, Accumulator must be understood in advance. Accumulator is a product which was famous in Hong Kong and Singapore in the early 2000s, aimed to make profit during the bullish market with straightforward investment scheme. Accumulators are also known as KODA (Knock out Discount Accumulator), and it is equivalent to long position in Up and Out Call Option, and two short position of Up and out Put Options. The following equation shows how to calculate the fair price of the Accumulator:

$$V = \sum_{i=1}^{n}(C_{u\&o}(t_i, K, H) - 2P_{u\&o}(t_i, K, H)) \ldots (9)$$

where $C_{u\&o}$: Up & *Out Call*, $P_{u\&o}$: Up & *Out Put*, t_i : transaction date,

K : Strike Price, H : Barrier Touch price

The payoff of Accumulator products can be broken down into the following equation as well:

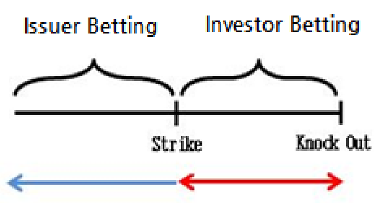
$$\begin{cases} 0 \ \text{if} \ \max_{0\le\tau\le t_i}(S_\tau \ge H) \\ S_{t_i} - K \ \text{if} \ \max_{0\le\tau\le t_i}(S_\tau < H, S_{t_i} \ge K) \\ 2(S_{t_i} - K) \ \text{if} \ \max_{0\le\tau\le t_i}(S_\tau < H, S_{t_i} < K) \end{cases} \ldots (10)$$

According to the above equations, when the underlying asset price does not go above the strike price, then the investor will get the profit of the difference between the call option's strike price and the underlying asset's price. If the underlying asset's price is higher than the Up and Out Call barrier strike price than the contract meets an end and the investors will be able to take the accrual profit before the date when the barrier strike price is breached. When the Up and out Call option goes below the strike price, however, the investors will realize loss with the standard of strike price. To sum up, investors of this product will make a bet upon the future strike price to be in between the Call option's strike price and the barrier's strike price. Since Accumulator is a product with a short maturity and the profit pricing period can be sliced down to even in weekly base such that investors can realize or unwind the position without having too much burden on waiting for a long period of time to find out the expected payoff.

The product that this paper is suggesting is a modified version of the above introduced Accumulator, the Reverse Accumulator. Reverse Accumulator is the seller version of Accumulator position, so the replicating portfolio scheme and the payoff structure is the exact opposite of the Accumulator. Reverse Accumulator has a lower strike than the Knock Out

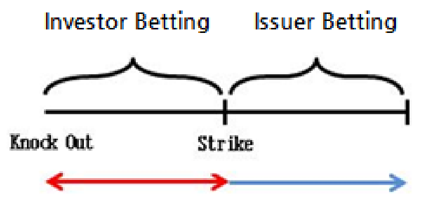
Barrier Strike such that it will only be able to earn profit at a bearish market. <Figure 4> and <Figure 5> shows the payoff section for both Accumulator and Reverse Accumulator.

<Figure 4> Payoff section for Accumulator



Strike < Knock Out

<Figure 5> Payoff section for Reverse Accumulator

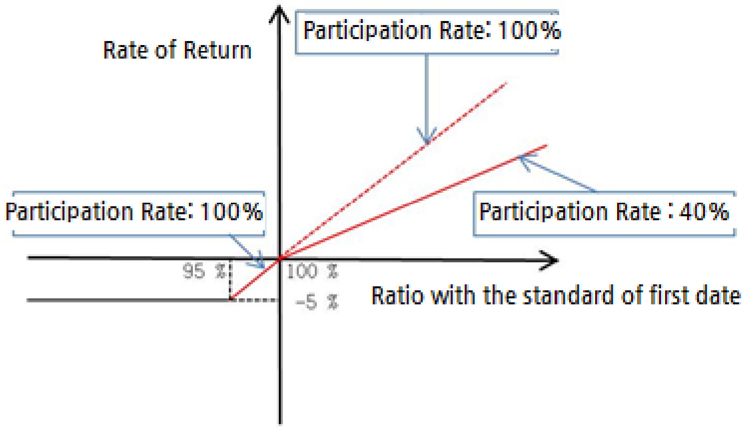


Strike > Knock Out

The next product to be introduced is the Worst of call option. Worst of call option is a call option in which constitutes its underlying assets as a basket. Most of the Worst of Call option's payoff is decided by the worst performing of the two underlying assets. Worst of call option has a variety of advantage compared to the vanilla call option. The most benefit that an investor can get from this is that participation rate can be set according to the underlying asset level. By setting up the underlying asset to a low volatility underlying, profit making will be an easy task. In addition, the Worst Of Call Option's price is usually valued less than the vanilla options such that it can be known for its price competitive edge. Below <Figure 6> shows the Worst of Call Option's Payoff scheme.

The two newly introduced products not only have creativity on the payoff structure but also have unique characteristics in the scope of Greek sensitivity. In the concept of Delta hedge, these two products are free from the Kinked Point risk. Since barrier options and digital options constitute the payoff of the Stepdown types of ELS, whenever the maturity is near or observation date is breached, not only the model delta increase exponentially but also the sensitivity position, especially Gamma, will have an opposite position whenever option sensitivity's point of inflection is passed (breaching the pin). Especially for the Digital option, kinked point may hit the highest point of Dirac delta, which can even be recorded as infinity, in which makes it very difficult to be Delta hedged

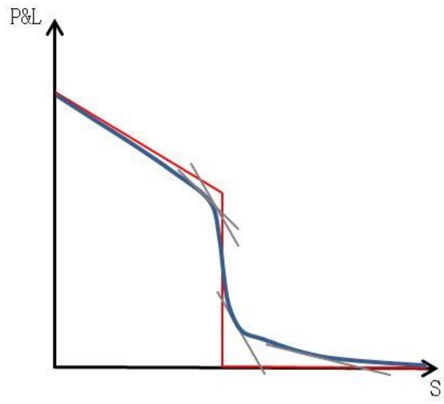
<Figure 6> Payoff section for the Worst of Call option.



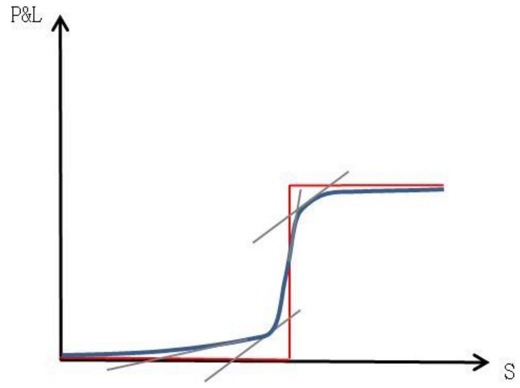
Delta Hedging Issue has always caused legitimate controversies between the investors and the financial institutes due to the ELS product's structural uniqueness. The next two Figures show how the delta position changes at a barrier option and a digital option throughout the change of underlying price. Unlike ELS, Reverse Accumulator and Worst of Call option has less sensitivity in the Delta which can give out new paradigm on controlling the Greek positions as well as create revenue to the portfolio.

In the Vega Hedging Scheme, issuing the so called "Short Vega" type of products, Reverse Accumulator and Worst of Call options is an effec tive way to drop off some unwanted Vega position from the portfolio. In other words, Long Vega ELS portfolio needs Short Vega types of products with equally attractive coupon yields to investors but just having offsetting Greek positions to the Long Vega Portfolio. Since Long Vega Portfolio is exposure to the loss in volatility when volatility shows a declining tendency such that when Short Vega type of products such as Reverse Accumulator and Worst of Call options are included then the Vega sensitivity can be less sensitivity to the overall fall of volatility of the underlying assets of the ELS portfolio.

<Figure 7> Change of Delta Position in Down & In PUT Option



<Figure 8> Change of Delta Position in Digital Call Option



2. Verification of the possibility to manage Portfolio Delta / Vega sensitivity through simulation

Throughout this chapter, the virtual ELS replicated portfolio will see how the Greeks change upon linking the notional to each and every ELS products included in the portfolio. Also, the change in Greek positions will be focused on when products such as Reverse Accumulator and Worst of Call options come into play in the portfolio. First of all, Table 5 will be given to check the information upon the ELS products will be given after the calculations of 2 Stock Multi Chance Step down ELS via the method of Finite Difference methods.

<Table 5> shows us that from ELS product #1 to ELS product #4 are all 2 Stock Step down ELS products. The Greek characteristics of Step down ELS products can be seen as having Long Gamma, Vega positions along with Short Theta positions when issued and included into the ELS portfolio.

By looking through Table 6 one is able to find out how each and every Greek values cause the P&L according to the issued Notional, which is broken down into by each and every underlying assets' Greek exposure:

By looking at Table 6 it is possible to realize the total Vega exposure is summed up to about 7.5 million KRW which is very pronounced that it will be very susceptible to the volatility level change. By including the Worst of Call option to the portfolio Vega risk shall be tried to be reduced. Every other parameter is reluctant to change except the Vega sensitivity. The Wo rst of Call option's product by product sensitivity is based on the prici ng of 1 year maturity, 100% and 60% participation rate per options w hile the strike prices are 95% and 100% each.

<Table 5> 2D Multi-Chance Step Down ELS Portfolio

	ELS Product #1	ELS Product #2	ELS Product #3	ELS Product #4	ELS Product #5
Notional	3,000,000,000	1,500,000,000	900,000,000	700,000,000	500,000,000
Stock 1	KOSPI200	Samsung Life	KOSPI200	KOSPI200	LGE
Stock 2	KT	KT	KEPCO	LGE	KEPCO
Coupon	8%	10%	8.40%	7%	10%
Fair Price	97.46	96.24	95.94	97.39	96.11
Delta	10.90%	17.58%	12.94%	10.43%	26.67%
Delta2	34.16%	44.44%	46.05%	32.33%	28.60%
Gamma	-1.11%	-1.73%	-1.62%	-1.12%	-1.76%
Gamma2	-1.90%	-2.57%	-2.63%	-1.96%	-1.77%
Cross-Gamma	0.03%	0.28%	0.24%	0.12%	0.36%
Vega1(1%)	-32.12%	-29.88%	-22.77%	-28.05%	-47.35%
Vega2(1%)	-61.76%	-50.71%	-50.85%	-60.20%	-49.23%
Theta(1d)	2.36%	3.71%	3.57%	2.23%	2.83%
Vol 1	16	19	16	16	24
Vol 2	25	25	25	24	25
Series of Strikes	95-95-90-90-85-85	90-90-85-85-80-80	90-90-85-85-80-80	95-95-90-90-85-85	95-95-90-90-85-85
Frequency of observation	6M	6M	6M	6M	6M
Maturity	3 Y	3 Y	3 Y	3 Y	3 Y
Dividend 1	-	-	-	-	-
Dividend 2	-	5	5	-	-
LB	60	60	60	60	60

<Table 6> Greek sensitivity of ELS Portfolio

	Delta	Gamma	Vega	Theta
KOSPI200	- 516,470,000	55,720,000	- 3,900,000	- 18,540,000
Samsung Life	- 263,700,000	25,950,000	- 4,200,000	- 55,650,000
LGE	- 359,660,000	22,520,000	194,550,000	- 29,760,000
KEPCO	- 557,450,000	32,520,000	441,680,000	- 46,280,000
KT	- 1,691,400,000	95,550,000	95,550,000	- 126,450,000
Sum	**- 3,388,680,000**	**232,260,000**	**723,680,000**	**- 376,680,000**

<Table 7> Worst of Call Option Pricing Information

	ELS Product #6	ELS Product #7		ELS Product #6	ELS Product #7
Stock 1	LGE	Samsung Life	Vega (1%)	0.0345	0.0692
Stock 2	Hyundai Heavy	Hyundai Heavy	Vega2 (1%)	0.0345	0.004
Notional	2,530,000,000	3,000,000,000	Theta1(1d)	-0.0045	-0.0038
Fair price	5.3	4.75	Theta2(1d)	-0.0045	-0.0038
Delta1	0.141	0.1918	Vol 1	30	19
Delta2	0.141	0.1138	Vol 2	30	30
Gamma1	-0.0017	-0.0017	Dividend 1	0	0
Gamma2	-0.0017	-0.0021	Dividend 2	0	0

After including the Worst of Call option products in the ELS replicating portfolio as it is shown in the above <Table 7>, the replicating ELS portfolio will become like Table 8 and the Greek exposures will be changed as the following:

<Table 8> ELS portfolio's Greek sensitivity after including Worst of Call Options

	Delta	Gamma	Vega	Theta
KOSPI	- 516,470,000	55,720,000	- 3,900,000	- 118,540,000
Samsung Life	311,700,000	31,050,000	- 211,800,000	- 44,250,000
LGE	- 2,930,000	26,821,000	107,265,000	- 18,375,000
KEPCO	- 557,450,000	32,520,000	441,680,000	- 46,280,000
KT	- 1,691,400,000	95,550,000	95,550,000	- 126,450,000
Hyundai Heavy	- 698,130,000	10,601,000	- 99,285,000	22,785,000
Sum	**- 3,154,680,000**	**252,262,000**	**329,510,000**	**- 331,110,000**

By looking through <Table 8>, it is possible to find out that Vega amount has fallen to the level of its one and a half level from the beginning. Delta amount increased a little and Theta position along with Gamma position is pretty much the same before the Worst of Call option is included. In <Table 8> the underlying asset which had the most Vega sensitivity was KEPCO, and in order to reduce the Vega amount of this underlying, the portfolio will include a Reverse Accumulator product with KEPCO as the underlying asset. <Table 9> tell us the Reverse Accumulator's Greek Parameters. Using <Table 9>, <Table 10> can be deduced and here are the changed Greek Sensitivity after this product is included in the Portfolio:

<Table 9> Reverse Accumulator Pricing Information

ELS Product #8: KEPCO Reverse Accumulator					
Underlying Asset	KEPCO	Lower Barrier	25830	Delta (%)	-26.47
Underlying Price	Buy - 28700 KRW	Frequency	Weekly	Gamma (%)	0.4905
Trade Date	08/24/2013	Notional	1,000,000,000	Vega	2.34
Issue Date	08/26/2013	Start Date	08/24/2013	Theta	-1.21
Style	Reverse Accumulator	End Date	11/22/2013	Rho	-0.3
Type	Within range	Maturity	11/22/2013 - 15:15	Gearing	-1332.04
Position	Sell- 1.00	U	89 - 19:26	Price(Total)	-21.55
Strike	Money (ATM) 28700	Model	Continuous BS model	price(Stock)	-21.5458
Higher Barrier	31570	Mid Vol	-30.181	Others (%)	-0.0751

<Table 10> Portfolio Greek Change after including Reverse Accumulator

	Delta	Gamma	Vega	Theta
KOSPI200	-516,470,000	55,720,000	- 3,900,000	- 118,540,000
Samsung Life	311,700,000	31,050,000	- 211,800,000	- 44,250,000
LGE	-2,930,000	26,821,000	107,265,000	- 18,375,000
KEPCO	-292,750,000	27,615,000	418,280,000	- 34,180,000
KT	-1,691,400,000	95,550,000	95,550,000	- 126,450,000
Hyundai Heavy	-698,130,000	10,601,000	- 99,285,000	22,785,000
Sum	**-2,889,980,000**	**247,357,000**	**306,110,000**	**- 319,010,000**

The overall change in Greek sensitivity can be summarized into two big notions after the inclusion experience of Short Vega products: First of all, although the issuing amount of these Short Vega products was less than the original Step down ELS products, they were able to reduce the Portfolio Vega. The issuing amount of Worst of Call Option is 5.3 times less the size of the issuing amount of the reverse accumulator and when linking the size effect to the Vega reduction effect, the Worst of Call option reduces the total Vega exposure to about 54% while the Reverse Accumulator reduces the total Vega exposure to about 40%. This result substantiates the assertion that by issuing fully-investor attractable newly structured products is actually available to effectively reduce the Vega exposure. Therefore, issuers of ELS products can possibly manage to handle the Greek levels even in the volatility dropping market by case by case maturity checking and issuing these Short Vega products considering the size of ELS portfolio that could possible drop off the Vega exposure.

Secondly, by issuing Reverse Accumulator the Delta exposure of the portfolio has been reduced as well. Considering the issued notional of Worst of Call option, Reverse Accumulator has dropped off about 30% of the total Delta exposure. This effect can be making the book less burdensome on the amount of stocks that should be Delta Hedged, enabling traders to be less caring about the Delta exposure and to focus on Gamma and Vega Hedge itself.

Chapter 5. Portfolio risk handling via utilizing Volatility Surface

1. Importance of Volatility Surface construction

Along with issuing newly structured products that could lighten up the overall unwanted Greek positions of ELS portfolio, one thing financial services must have in mind is that trading options has always been the easiest way to increase and decrease the exposure of Greeks. As mentioned above, trading through the OTC market or the listed market, options play an important role in buying and selling the volatility. Since the global market has been showing the tendency to decrease in the overall volatility levels, financial services must see the long term volatility and short term volatility levels for both risk management point of view and trading point of view. In the risk management point of view, financial institutions should have the idea of the status quo of how much skew there is upon the volatility of the underlying assets that is traded such that this will help to confirm the volatility level when it comes to deciding the volatility parameter when issuing the ELS products. If the volatility level is very different from the already issued ELS products then the re-balancing of volatility levels should be needed. In order to reconcile the volatility level with the already booked volatility level the information for volatility term structure is needed. The requirement for volatility term structure, also known as the volatility surface, is a mandatory item for trading point of view as this volatility term structure. Traders must know whether the volatility is undervalued or overvalued compared with the daily, weekly, monthly, and yearly basis. Trading ideas can be derived from this, and therefore information upon short and long term maturity is to be gathered.

In order to construct the volatility surface, mostly information for front month volatilities are gathered from the listed options whereas long term volatilities are gathered from OTC markets. Throughout the construction of volatility surface, it is possible to compare the book's risk term structure in which enables the management of the unwanted Greek exposures, especially for unwinding the Vega risks Hedging Vega risks through matching the risk term structure of the ELS portfolio along with Volatility term structure. Whenever the Vega risk seems to be too much exposed under the certain underlying maturity with the corresponding maturity then it can be hedged via the OTC market. This process can be dynamically managed in daily, weekly, monthly, and yearly basis.

Not many studies upon the emphasizing the importance of constructing the volatility surface in the concept of hedging Greek exposures are done with the correspondence of hedging ELS portfolio. Advanced studies upon constructing Volatility surface, however, are mostly focused upon methodologies of smoothing out the 3-Dimensional volatility surface. Former study that tried to construct implied volatility surface used simple interpolation methods whereas Fengler (2009) suggested implementing cubic spline method in order to smoothen the surface. Simple steps upon replicating Fengler's construction method can give some implication upon estimating the volatilities in discrete time steps can be made prior to constructing the volatility surface with the given information. The Black-Scholes implied volatility and its derivatives are interpolated as $T_j \leq T \leq T_{j+1}$

$$\Sigma(K,\ T) = \sqrt{\frac{T_{j+1}(T - T_j)\Sigma(K, T_{j+1})^2 + T_j(T_{j+1} - T)\Sigma(K, T_j)^2}{T(T_{j+1} - T_j)}} \ \ldots (11)$$

$$\frac{\partial\Sigma(K,\ T)}{\partial \mathrm{T}} = \frac{T_{j+1}T_j\left(\Sigma(K,T_{j+1})^2 - \Sigma(K,T_j)^2\right)}{2T^2(T_{j+1}-T_j)\sqrt{\frac{T_{j+1}(T-T_j)\Sigma(K,T_{j+1})^2 + T_j(T_{j+1}-T)\Sigma(K,T_j)^2}{T(T_{j+1}-T_j)}}} \ldots (12)$$

$$\frac{\partial\Sigma(K,\ T)}{\partial \mathrm{K}} = \sqrt{\frac{T_{j+1}(T-T_j)\left(\frac{\partial\Sigma(K,T_{j+1})}{\partial \mathrm{K}}\right)^2 + T_j(T_{j+1}-T)\left(\frac{\partial\Sigma(K,T_{j+1})}{\partial \mathrm{K}}\right)^2}{T(T_{j+1}-T_j)}} \ldots (13)$$

$$\frac{\partial^2\Sigma(K,\ T)}{\partial \mathrm{K}^2} = \sqrt{\frac{T_{j+1}(T-T_j)\left(\frac{\partial^2\Sigma(K,\ T)}{\partial \mathrm{K}^2}\right)^2 + T_j(T_{j+1}-T)\left(\frac{\partial^2\Sigma(K,\ T)}{\partial \mathrm{K}^2}\right)^2}{T(T_{j+1}-T_j)}} \ldots (14)$$

These equations will make the local volatility equation reset in discretized forms, allowing us to determine the Flat Forward Local Volatility (between $\mathrm{T_j}$ and $\mathrm{T_{j+1}}$)

$$\sigma(\mathrm{K}, T_j, T_{j+1}) = \frac{\int_{T_j}^{T_{j+1}} \sigma(\mathrm{K},\mathrm{T})dT}{T_{j+1} - T_j} \ldots (15)$$

The flat forward local volatility could not be derived explicitly, so we use Gauss - Legendre Quadrature to be solved.

In general, a pricing model function does not have a closed form solution for its inverse. Instead, a root finding technique is used to solve the equation. Using fixed point iteration, set $\Sigma = \hat{\Sigma}$, then

$$C_{BS}\ (S_0\ K, \Sigma(K,T), T)\ \text{-}\ \hat{C}_{BS}\left(S_0, K, \hat{\Sigma}(K,T), T\right) < \varepsilon \ldots (16)$$

To use cubic spline smoothing, set $\Sigma(\mathrm{K}, \cdot) = \hat{\Sigma}(\mathrm{K}, \cdot)$,

$$\min\ \sum_{i=1}^{n}\{\Sigma(K_i,\ \cdot) - \hat{\Sigma}(K_i,\ \cdot)\}^2 + \lambda \int_{K_1}^{K_n} \frac{\partial^2\Sigma}{\partial dK^2} dK \ldots (17)$$

Along with $\Sigma(\mathrm{K},\mathrm{T}) \in \mathrm{C}^2(\mathrm{R} \times [0,\infty))$, interpolation among strikes is possible when smile interpolation among expiries finishes up implied volatility term structure. Assume flat forward volatility among expiries and set $\Sigma(\cdot, \mathrm{T}) = \hat{\Sigma}(\cdot, \mathrm{T})$, and while $\mathrm{T_j} \le \mathrm{T} \le \mathrm{T_{j+1}}$

$$\hat{\Sigma}(\cdot, T) = \sqrt{\frac{T_{j+1}(T-T_j)}{T(T_{j+1}-T_j)} \hat{\Sigma}(\cdot, T_{j+1})^2 + \frac{T_j(T_{j+1}-T)}{T(T_{j+1}-T_j)} \hat{\Sigma}(\cdot, T_j)^2} \quad \ldots(18)$$

In order to complete the arbitrage option price surface, we can set the no-arbitrage constraints as firstly strike-arbitrage free conditions for $i = 2, \cdots, n-1$,

$$\frac{\partial^2 C(K_i, t_m)}{\partial K_i^2} \geq 0, \quad \frac{\partial C(K_1, t_m)}{\partial K_1} \geq -e^{-\int_{t_m}^{T} r_s ds}, \quad \frac{\partial C(K_n, t_m)}{\partial K_n} \leq 0,$$
$$C(K_1, t_m) \geq e^{-\int_{t_m}^{T} \delta_s ds} S_t - e^{-\int_{t_m}^{T} r_s ds} K_1, \; C(K_n, t_m) \geq 0 \quad \ldots(19)$$

and Calendar Spread Arbitrage Free

$$C(K_1, t_m) \leq e^{-\int_{t_m}^{T} \delta_s ds} S_t, \; C(K_i, t_j) \leq e^{\int_{t_m}^{T} \delta_s ds} C(K_i, t_{j+1}) \quad \ldots(20)$$

As shown above methods for constructing Volatility Surface is quite straight forward. What matters the most is to gather reliable information. Using the OTC market Volatility quotes along with the upfront month Volatility levels, a volatility surface can be made

2. Utilizing Volatility Surface in issuing ELS & Hedging Greeks

When the issue of data collecting is resolved, by translating the list of equations noted above into programming language, one can construct a solid volatility surface. The issue after constructing a volatility surface is utilizing it. Volatility surfaces can be used efficiently in both issuing ELS products as well as hedging certain Greek parameters. First of all, when it comes to issuing ELS products, one must know the current volatility level of the specific underlying asset. Not all underlying asset’s volatility remains the same in yearly, quarterly, monthly or even daily basis.

Hence, if one is aiming for issuing ELS products, it is essential to protect oneself from offering higher volatility levels than the market level so that stable profit from issue margin can be provided. Therefore, one must always update the volatility levels of the market to realize how the levels are different from time to time. Using the whole sequence of equations mentioned above, it is possible to construct local volatility surface for the underlying such as Kospi200 index, S&P500 Index, LBMA Gold Index, etc.

<Table 11> Local Volatility Surface for Kospi200 Index

Maturity Level (Year)	Strike Level (K) (KOSPI200) (%)								
0	70%	80%	90%	95%	100%	105%	110%	115%	120%
0.1	71%	47%	26%	18%	13%	15%	22%	29%	36%
0.2	48%	33%	20%	15%	12%	13%	16%	19%	24%
0.3	47%	33%	21%	17%	14%	14%	17%	21%	25%
0.5	42%	31%	22%	18%	16%	15%	16%	17%	18%
0.8	32%	25%	20%	18%	17%	16%	17%	18%	20%
1.0	31%	25%	20%	18%	17%	16%	16%	17%	18%
1.5	28%	23%	19%	18%	17%	17%	16%	17%	17%
2.0	27%	22%	19%	18%	17%	17%	16%	17%	17%

By looking through Table 11, 12, 13, one can find the change of volatility level according to the strike levels and maturity levels. Along with this volatility surface, it is also essential to have the grasp of the summary of the change of volatility levels compared in short term, and that would be in daily and weekly basis. Table 14 and 15 shows the ATM Volatility level change by weekly basis and the skew levels according to the maturity levels.

When this process is completed, the hedging of Greek parameters can be done accordingly by either issuing newly structured products or through selling or buying OTC options.

<Table 12> Local Volatility Surface for S&P500 Index

Maturity Level (Year)	Strike Level (K) (S&P500) (%)								
0	70%	80%	90%	95%	100%	105%	110%	115%	120%
0.08	83%	57%	33%	22%	13%	9%	15%	23%	30%
0.17	60%	42%	27%	20%	14%	10%	12%	17%	23%
0.25	55%	40%	26%	20%	14%	11%	11%	14%	18%
0.50	44%	33%	24%	20%	16%	13%	11%	11%	12%
0.75	39%	31%	23%	20%	17%	14%	13%	12%	12%
1.00	37%	30%	23%	21%	18%	16%	14%	12%	12%
1.50	34%	29%	24%	21%	19%	17%	16%	15%	14%
2.00	33%	28%	23%	21%	20%	18%	17%	15%	15%

<Table 13> Local Volatility Surface for LBMA Gold Index

Maturity Level (Year)	Strike Level (K) (LBMA Gold Index) (%)								
0	70%	80%	90%	95%	100%	105%	110%	115%	120%
0.08	85%	57%	32%	23%	18%	20%	27%	34%	42%
0.17	63%	44%	28%	22%	19%	19%	21%	25%	29%
0.25	55%	39%	27%	22%	19%	18%	20%	23%	26%
0.50	40%	31%	24%	21%	19%	19%	19%	20%	22%
0.75	36%	28%	23%	21%	19%	19%	19%	19%	20%
1.00	32%	26%	22%	21%	20%	19%	19%	20%	21%
1.50	27%	23%	21%	20%	20%	20%	20%	21%	21%
2.00	27%	23%	21%	20%	20%	20%	20%	20%	21%

<Table 14> Weekly Volatility Term Structure for Global Indices

Maturity	ATM-fwd Implied Volatility									
	NKY		KOSPI2		HSCEI		NIFTY		SPX	
Dec13	18.8	-1.4	13.6	-0.3	23.3	-1.4	16.1	-7.8	11.6	+0.7
Jan14	25.3	+1.3	14	-0.7	23	-1.1	16.7	-3.2	12	unch
Mar14	25.4	+1.0	15.2	-0.4	23	-0.4	19.9	-0.7	13.3	-0.3
Jun14	24.6	+0.5	16.2	-0.2	23.1	-0.4	22	+0.1	14.3	-0.4
Sep14	24.1	+0.2	17.1	-0.1	23.2	-0.4	22.4	unch	15.2	-0.4
Dec14	23.7	+0.1	17.2	-0.2	23.2	-0.5	22.5	unch	15.9	-0.4
Jan15	23.6	+0.1	17.3	-0.1	23.2	-0.5	22.5	unch	16.1	-0.4
Jun15	23.3	-0.1	17.7	unch	23.3	-0.5	22.8	unch	17.1	-0.4

Maturity	ATM-fwd 90%-110% Skew									
	NKY		KOSPI2		HSCEI		NIFTY		SPX	
Dec13	38	+28.2	10.4	+3.0	2	+0.2	11.1	+8.3	21.3	+5.8
Jan14	6	+1.2	4.2	+1.4	0.8	-0.9	4.8	+1.7	12.2	+1.5
Mar14	2.8	+0.2	2.8	+0.2	1.2	-0.4	4.2	-0.6	8.3	+0.6
Jun14	1.7	+0.3	2	+0.2	1.3	-0.4	4	+0.5	6.9	+0.1
Sep14	1.3	+0.3	1.6	+0.1	1.3	-0.2	3.5	+0.4	6	unch
Dec14	1	+0.1	1.5	unch	1.2	-0.1	3	+0.3	5.3	+0.1
Jan15	1	+0.1	1.4	unch	1.2	-0.1	2.8	+0.3	5.1	+0.1
Jun15	0.8	+0.1	1.2	unch	1.1	-0.1	2.4	+0.2	4.3	+0.1

Chapter 6. Conclusion and further possible studies.

Korean ELS market has grown up to take the high portion of the financial realm, however, the real rate of return is tending to decline every year due to few key reasons. Products like ELS are structured products of the original securities characteristic along with the function of optionality inhered inside the product. Due to this reason, ELS margin has a very keen relationship with the parameters that works as an input to the pricing of ELS products. In order to have the idea of the distribution of the real rate of return of ELS, it is essential to analyze the trend and the relative level of the most important factor in pricing the ELS product, Volatility.

Throughout this paper, the decline of volatility in the overall worldwide financial market have made the original hedging methodology to be less effective and therefore as a solution issuing “Short Vega” products such as Reverse Accumulator and Worst of Call options, have been suggested, which have the exact opposite Greek characteristics compared to the already well sold “Long Vega” products. With all the obstacles that are happening around the financial market ever since the financial crisis occurred in 2008 such as the shrinkage of ELW market in Korea due to financial authorities trying to decrease the liquidity of structured products, have led the Korean market less attractive to the other volatile overseas market. Issuing the two suggested “Short Vega” type of products are able to work as reasonable methods in handling the unwanted Greek exposures in the relatively less-liquid market.

In addition to issuing the “Short Vega” type of products, to effectively hedge the Vega risks, Volatility term structure is suggested to be constructed via gathering the front month volatility levels from the listed market while volatilities with longer maturities are gathered from the OTC markets. With the verified volatility levels to be calculated and constructed as volatility term structures, it will be convenient to match the Volatility term structure with the book’s risk term structure. High concentration will be needed in elaborating the volatility surface. Furthermore, since Reverse Accumulator and Worst of Call options are not actively or not even well known to the Korean financial market, further studies must be done in order to come into practice without having pessimistic signal towards the market.

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초 록

증권사 ELS 헷지 운용 순 마진율 악화 원인 분석 및 신구조화금융상품을 통한 해결방안 연구

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본 연구는 한국시장의 주가연계증권(Equity Linked Security)의 순 마진율 감소가 변동성 하락에 따른 발행사의 ELS 베가 손실로 인한 운용 수익률 악화에서 비롯되었다고 생각하며, 이에 따라 ELS 순 마진율 감소와 관련하여 발행사의 운용수익률 악화 원인을 금융 시장 변동성 국면 전환과 ELS 베가 헤지를 토대로 분석하고 대안 책을 제시하는 것을 목적으로 한다. 일반적으로 구조화 상품에 대한 선행 연구 결과에 의하면 구조화 상품 시장이 성장할수록 마진이 증가한다는 공통된 주장을 펼치고 있다. 그러나 한국 ELS 시장은 이러한 선행연구 결과와 다른 방향으로 흘러가고 있다. 이에 본 논문은 ELS 순 마진율 감소가 일시적인 시장 이례현상(Anomaly)인지 아니면 시장의 구조적 변화인지를 분석하기 위해, 발행사의 운용수익률 악화 원인을 금융 시장 변동성 국면 전환과 ELS 베가 헤지를 토대로 분석하고 대안 책을 제시하는 것으로 연구 방향을 설정했다. 구체적인 연구를 진행하기 위해 우선적으로 스텝다운 ELS의 헤지 운용원리를 민감도 측면에서 설명했고, 스텝다운 ELS 운용에 있어 델타 헤지뿐만 아니라 베가 헤지 또한 중요한 관리 민감도임을 확인했다. 그리고 VKOSPI 및 장외옵션시장 변동성 추세, ELW 시장의 수급 상황이 최근 몇 년 사이에 바뀜에 따라 ELS 운용 수익률 악화가 롱 베가(Long Vega) 손실에서 비롯되었음을 확인했다. 금융 시장의 변동성 국면이 하락 추세로 전환됨에 따라 기존의 베가 헤지 수단(장내/외옵션, 원금보장형 ELS 발행, 분산 스왑)의 한계가 있음을 확인하였으며, 새로운 베가 헤지 수단의 필요성을 역설하였다. 이에 따라 본 논문은 새로운 베가 헤지 수단으로 숏 베가(Short Vega) 성격을 지닌 ‘역 어큐뮬레이터(Reverse Acummulator)’와 ‘워스트 콜(Worst of CALL)’과 같은 새로운 상품 발행을 통

해 스텝다운 ELS 베가 손실을 줄일 수 있다고 제시했다. 새로운 헤지 수단의 유효성을 확인하기 위해 스텝다운 ELS와 숏 베가(Short Vega) 상품 결합한 포트폴리오를 시뮬레이션(Simulation)으로 평가하였고, 그 결과 ‘어큐뮬레이터(Reverse Acummulator)’와 ‘워스트 콜(Worst of CALL)’과 같은 숏 베가(Short Vega) 상품이 스텝다운 ELS 포트폴리오의 베가 민감도 비율을 종전보다 40~50%를 감소시키며 델타 민감도 또한 약 30%를 감소시키는 것으로 확인되었다. 이런 사실은 ‘역 어큐뮬레이터(Reverse Acummulator)’와 ‘워스트 콜(Worst of CALL)’과 같은 상품이 변동성이 하락하는 시장에서 스텝다운 ELS의 효과적인 베가 헤지 수단이 될 뿐만 아니라, 핀 리스크(Pin Risk)에 의한 스텝다운 ELS 불완전 델타 헤지 문제를 해결해 줄 수 있는 획기적인 상품이 될 수 있음을 제시했다. 또한 이런 신상품의 발행가능성을 확인하기 위해, 향후 글로벌 경제 상황과 기관 투자자의 포트폴리오 구성비율을 분석을 통해 연기금이나 운용사와 같은 기관 투자자(Buy-Side)에게 충분히 수요가 있음을 확인하였다. 결국 ‘역 어큐뮬레이터(Reverse Acummulator)’와 ‘워스트 콜(Worst of CALL)’과 같은 유형의 상품이 발행된다면 발행사는 ELS 포트폴리오의 베가 손실 문제를 해결할 수 있을 뿐만 아니라 새로운 수입원(原)을 창출할 수 있다. 이와 더불어 투자자는 새로운 투자 기회를 포착하여 포트폴리오의 자산 계층(Class)를 다변화하여 분산효과를 누릴 수 있는 기회를 얻게 된다. 즉, 신상품 발행은 발행사와 투자자가 모두 성장할 수 있는 좋은 기회이며, 한국 금융 시장의 균형 있는 성장을 이룩할 수 있는 좋은 기회가 될 것이다.

주요 단어: 주가연계증권, 베가 헤지, Reverse Accumulator, Worst of Call

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