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공학석사학위논문

Effects on Thermo-Elastic Damping of Micro-Ring Structure with Arbitrarily Attached Masses

불규칙 질량 분포를 가진 마이크로 링 구조물의 열-탄성 댐핑 효과

2014 년 2 월

서울대학교 대학원

기계항공공학부

강 석 주

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이 논문을 공학석사 학위논문으로 제출함

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Abstract

In this paper, thermo-elastic damping of ring structure with random attached point masses is investigated. This work applies the thermo-elastic ring model with in-plane vibration of uniform rings. The model is assumed to have rectangular cross-section. And, attached point masses represent imperfections such as unbalanced mass distribution. Analysis is used to derive analytical expressions for natural frequency. Analytical model for circular ring is introduced using the vibration mode shapes. Extensional, bending and inextensional assumptions for the ring model are considered and discussed. In order to determine natural frequency of ring, equations of energy for the ring are used. As energy equations, Rayleigh-Ritz energy method is adopted. Also, equations of motion are derived for the rotating ring. For rotating model, extensional and inextensional assumptions are applied and discussed. Micro-ring with random point masses are represented by arbitrarily located point masses on the ring models in the formulation. Moreover, using heat conduction equations, effects of thermo-elastic damping are investigated. In order to obtain improved understanding of damping effects, concept of Quality factor(Q-factor) is used. Additionally, the influences of attached point masses are presented by the split of natural frequencies of the model. Q-factor and natural frequencies can be expressed as explicit functions of mode number for the ring. Moreover, various case studies are performed for further physical investigation of the models.

Key words : Ring, Thermo-elastic damping, Attached point mass, Quality factor(Q-factor), Frequency split

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List of Nomenclature

A	cross-sectional area of ring
a	mean radius of ring
b	radial thickness of ring
C_v	heat capacity per unit volume
E	Young's modulus
E_ω	frequency dependent Young's modulus
h	radial thickness of ring
$\mathrm{Im}(\omega)$	imaginary part of natural frequency
K	total kinetic energy
K_0	kinetic energy of ring
$K_{\max}$	maximum kinetic energy
K_P	kinetic energy with attached point masses
M	total mass of ring
M_θ	in-plane bending moment of ring
m_i	i-th point mass
n	mode number for eigenfrequencies
Q	quality factor
r	radius of ring
$\mathrm{Re}(\omega)$	real part of natural frequency
T	variation of temperature
T_a	ambient temperature
U	total strain energy
U_0	strain energy of ring
$U_{\max}$	maximum strain energy
U_T	strain energy with thermo-elastic damping

u	radial displacement of ring
v	tangential displacement of ring
α	coefficient of thermal expansion
α_n	amplitude ratio
β_i	ratio of attached point masses respect to total mass
ε	strain
ε_K	small parameter, $= K_P / K_0$
ε_θ	circumferential strain
ζ	shift angle for mode orientation
Θ	angular location of attached point mass
ν	Poisson's ratio
ξ	convenient notation, $= h\sqrt{\frac{\omega_{nS}}{2\chi}}$
ρ	density of ring material
σ	stress
σ_θ	circumferential stress
χ	thermal diffusivity
ω	natural frequency
ω_{0B}	natural frequency of bending ring
ω_{0E}	natural frequency of extensional ring
ω_{0I}	natural frequency of inextensional ring
ω_{nH}	n-th mode natural frequency of higher mode
ω_{nL}	n-th mode natural frequency of lower mode
ω^*	natural frequency with attached point masses
Ω	rotating speed of ring
Δ_E	relaxation strength in terms of Young's modulus
∇^2	Laplacian operator

1. Introduction

Ring structure is a common element among many vibrating structures such as rate sensors namely gyroscopes. Gyroscopes are used in wide range of engineering field, for example, aerospace, automotive and industrial applications. Because rings are major elements in engineering structures, many researchers have studied on the vibrations of rings. Suzuki [1] studied in-plane vibrations of circular rings. Charnley et al. [2] specified vibrations of thin rings of rectangular cross-section. Especially, for gyroscope, silicon is an excellent material which has ideal mechanical properties. So, silicon is investigated as a major topic. Beeby et al. [3] performed modeling and optimization of micromachined silicon resonators. Esmaeili et al. [4] studied ring microgyroscope modeling and performance evaluation. Zakrzhevskii et al. [5] analyzed Natural modes and frequencies of in-plane vibrations of a fixed elastic ring.

Rings are used in many applications such as automotive vehicle stability enhancement systems, traditional aerospace control and guidance systems. In recent years, interest of silicon micro gyroscope has greatly increased, while temperature is one of major factors that lower the performance of silicon micro gyro. As factors of low performance for gyroscopes, there are some different kinds of damping mechanisms. These can cause energy dissipation in micro-electro-mechanical systems(MEMS) resonators. Especially, the thermo-elastic damping is well known as the most important energy loss factors for micro- and nano-structures. Further, the damping effect is an essential factor to be estimated on the quality of device. Lifshitz et al. [6] investigated thermo-elastic damping in micro- and nano mechanical systems. Duwel et al. [7, 8] performed experimental study of thermo-elastic damping in MEMS gyros and researched engineering MEMS resonators with low thermo-elastic damping. Wong et al. [9, 10] performed a preliminary investigation of thermo-elastic damping in silicon rings and studied

thermo-elastic damping of the in-plane vibration of thin silicon rings. And, Metcalf et al. [11] analyzed thermo-elastic damping in micromechanical resonators. Rui et al. [12] researched on temperature dependent characteristics of quality factor of silicon MEMS gyroscope. Gologanu et al. [13] investigated damping effects in MEMS resonators.

In practical point of view, imperfections of structure are inevitable, such as irregularity of mass distribution. This work expresses imperfections as arbitrarily attached masses or random point masses. These masses results in the deviation of accuracy of the MEMS and NEMS(nano-electro-mechanical systems). For resonators, mass imperfection is very important factor to determine the performance of device. Thus, many studies have been performed by researchers. Allaei et al. [14] investigated natural frequencies and modes of rings that deviate from perfect axisymmetry. Fox [15] analyzed a simple theory for the analysis and correction of frequency splitting in slightly imperfect rings. McWilliam et al. [16] studied on the statistics of natural frequency splitting for rings with random mass imperfections. Bisegna et al. [17] investigated frequency split and vibration localization in imperfect rings. Prabhakar et al. [18] researched thermo-elastic damping in hollow and slotted microresonators. Leizerovich et al. [19] analyzed some features of the natural flexural vibrations of circular rings with initial imperfections. Kang et al. [20] studied oscillation of microbeam structure with irregular mass distribution.

For more practical analysis of ring structures, effects of rotating are adopted to the model. Effects according to the rotating speed of ring also have been studied in engineering fields. Endo et al. [21] investigated flexural vibration of a thin rotating ring. Bickford et al. [22] studied on the in-plane vibrations of rotating rings. Lin et al. [23] researched on general in-plane vibrations of rotating thick and thin rings. Eley et al. [24] considered coriolis coupling effects on the vibration of rotating

rings. Kim et al. [25] investigated thermo-elastic damping effect on in-extensional vibration of rotating thin ring. Kang et al. [26] researched vibration of thermo-elastically damped imperfect ring gyro.

In this paper, mass imperfections of circular ring with thermo-elastic damping are considered, and Q-factors can be obtained systematically. As a model, a silicon ring is adopted with rectangular cross-section. For systematic analysis, extensional, bending and inextensional cases of ring model are investigated. Additionally, effects of rotation for the ring are adopted and the rotating speed is considered. Also, one-dimensional heat conduction is considered, and analyzed frequency split due to the random point masses. In order to determine the split, Rayleigh-Ritz method is adopted to obtain analytic solutions. To check the validity of this work, the results are compared with the data of previous studies. Also, natural frequency and Q-factor are presented for various parameters of the imperfect micro-ring model.

2. Micro-ring model including attached masses

In this chapter, mass imperfections in circular ring of thermo-elastic damping are considered in order to estimate the Q-factor of the model. Figure 1 shows a circular ring model with point masses. The model represents a thin ring of rectangular cross-section. The figure shows a global polar coordinate system (r, θ, Z) and a local coordinate system (x, y, z) .

The physical model of ring is assumed to be isotropic and geometrically axisymmetric with free boundary conditions as shown in the figure, and imperfection is expressed in terms of arbitrarily random point masses.

In the ring model, a is mean radius and b is radial thickness. The local displacement components are expressed by u and v with respect to the directions of x and y, respectively. Also, θ and ζ mean the angular location of random point masses and the shift angle for the orientation of corresponding mode shape, respectively.

As the value of a is sufficiently larger than b, it is possible to employ the assumptions of Euler-Bernoulli beam that plane cross-section remain plane and perpendicular to the neutral surface during bending, and that shear deformation and rotary inertia can be neglected. In this paper, only in-plane flexural vibrations are considered for the ring.

2.1. Displacement fields with thermo-elastic damping

Euler-Bernoulli assumptions are applied to derive equations of frequency for imperfect ring model with thermo-elastic damping effect. For in-plane vibrations, the strain ε_θ varies the temperature fields (x, θ, t). Then, the total strain energy of

the imperfect ring with thermo-elastic effects consists of bending stress and thermal expansion, otherwise strain energy is negligible for the assumption of point masses.

Stresses σ and strains ε relation can be expressed as

$$\varepsilon_\theta = \frac{1}{E}\sigma_\theta + \alpha T \tag{1.a}$$

$$\varepsilon_r = \varepsilon_z = -\frac{\nu}{E}\sigma_\theta + \alpha T \tag{1.b}$$

Then, the circumferential stress is

$$\sigma_\theta = E\left(\varepsilon_\theta - \alpha T\right) \tag{2}$$

In this study, thermo-elastic damping term is added to bending strain energy. Note that the terms with respect to radial direction are assumed to be negligible. In-plane flexural vibrations of the ring are described in terms of the radial and tangential displacements of the mid-point of the ring cross-section at circumferential location θ.

Then, the radial and tangential displacements for harmonic vibration at frequency ω in the relevant modes can be expressed as

$$u(\theta,t) = \mathrm{U}_0(\theta)e^{\mathrm{i}\omega t}, \quad v(\theta,t) = \mathrm{V}_0(\theta)e^{\mathrm{i}\omega t} \tag{3}$$

where

$$\mathrm{U}_0(\theta) = U_0 \sin(n\theta), \quad \mathrm{V}_0(\theta) = V_0 \cos(n\theta) \qquad (n=2, 3, 4, \ldots)$$

The natural frequency ω and amplitude ratio $\alpha_n = U_0/V_0$ for the mode can be calculated using the Rayleigh-Ritz method once the maximum kinetic energy and strain energy have been determined.

2.2. Heat conduction equation

In order to determine temperature profile in the ring, heat conduction equation is

required to obtain the temperature profile for the structure as in Ref. [6]

$$\frac{\partial T}{\partial t} - \chi \nabla^2 T = -\frac{E\alpha T_a}{C_v(1-2\zeta)}\frac{\partial \varepsilon}{\partial t} \tag{4}$$

where ∇^2, χ, C_v and T_a denote the Laplacian operator, the thermal diffusivity of the material, the heat capacity per unit volume and the ambient temperature in Kelvin, respectively.

Also, the above equation can be simplified as

$$\frac{\partial^2 T}{\partial r^2} - \frac{1}{\chi}\frac{\partial T}{\partial t} = -\frac{1}{\chi}\frac{\Delta_E}{\alpha}\frac{\partial \varepsilon}{\partial t} \tag{5}$$

where Δ_E is the relaxation strength of the Young's modulus defined as $\Delta_E = \dfrac{E\alpha^2 T_a}{C_v}$

The solution of Eq. (4) can be assumed as Ref. [6]

$$T(x,\theta,t) = T_0(x,\theta)e^{i\omega t} \tag{6}$$

where

$$T = \frac{\Delta_E}{\alpha a^2}\left[a\left(u + \frac{\partial v}{\partial \theta}\right) + \left(\frac{\partial^2 u}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right)\left(x - \frac{\sin kx}{k\cos(kh/2)}\right)\right] \tag{7}$$

In here, k is complex value as $k = (1+i)\sqrt{\dfrac{\omega}{2\chi}}$.

According to the moment equation such as

$$M_\theta = \int x\sigma_\theta dA \tag{8}$$

And substitute Eq. (2) and Eq. (7) into Eq. (8), then

$$M_\theta = -E\int_A \frac{x^2}{a^2}\left(\frac{\partial^2 u}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right)dA - E\alpha\int_A \frac{\Delta_E}{\alpha a^2}\left[a\left(u + \frac{\partial v}{\partial \theta}\right) + \left(\frac{\partial^2 u}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right)\left(x - \frac{\sin kx}{k\cos(kh/2)}\right)\right]xdA \tag{9}$$

In here, the first term of the integration $\int_{-h/2}^{h/2}\frac{x}{a}\left(u + \frac{\partial v}{\partial \theta}\right)dx$ is zero because the

coordinate is chosen as symmetry of the ring in the thickness direction.

Then the bending moment is expressed as

$$M_\theta = -\frac{E\left[1+\Delta_E\left\{1+f\left(\omega\right)\right\}\right]I}{a^2}\left(\frac{\partial^2 u}{\partial\theta^2}-\frac{\partial v}{\partial\theta}\right) = -\frac{E_\omega I}{a^2}\left(\frac{\partial^2 u}{\partial\theta^2}-\frac{\partial v}{\partial\theta}\right) \tag{10}$$

where $f\left(\omega\right) = \frac{24}{h^3k^3}\left(\frac{kh}{2}-\tan\left(\frac{kh}{2}\right)\right)$ and $E_\omega = E\left[1+\Delta_E\left\{1+f\left(\omega\right)\right\}\right]$

On the other hand, the circumferential strain ε_θ due to in-plane bending is

$$\varepsilon_\theta = -\frac{1}{a}\left(u+\frac{\partial v}{\partial\theta}\right)-\frac{x}{a^2}\left(\frac{\partial^2 u}{\partial\theta^2}-\frac{\partial v}{\partial\theta}\right) \tag{11}$$

Insert Eq. (11) into Eq. (2), it follows that the circumferential stress is

$$\sigma_\theta = -\frac{E}{a}\left(u+\frac{\partial v}{\partial\theta}\right)-\frac{Ex}{a^2}\left(\frac{\partial^2 u}{\partial\theta^2}-\frac{\partial v}{\partial\theta}\right)-\alpha TE \tag{12}$$

Once again, shape functions of the ring can be written as

$$u(\theta,t) = U_0\sin\left(n\theta\right)e^{i\omega t},\qquad v(\theta,t) = V_0\cos\left(n\theta\right)e^{i\omega t} = \frac{U_0}{\alpha_n}\cos\left(n\theta\right)e^{i\omega t} \tag{13}$$

where amplitude ratio is defined as $\alpha_n = U_0/V_0$. Similarly, the displacement functions for i-th point mass m_i can be written as in Ref. [27].

$$u_{m_i}(\theta,t) = U_0\sin\left(n\left(\theta_i-\zeta_S\right)\right)e^{i\omega t},\qquad v_{m_i}(\theta,t) = \frac{U_0}{\alpha_n}\cos\left(n\left(\theta_i-\zeta_S\right)\right)e^{i\omega t} \tag{14}$$

where subscript s denotes the higher and lower frequency split angle from original frequencies.

Especially, the circumferential centerline of the ring is in-extensible, the relation between u and v can be expressed as

$$u(\theta,t) = -\frac{\partial v(\theta,t)}{\partial \theta} \tag{15}$$

Then, $\alpha_n = n$ and the mode shape can be expressed in terms of radial displacement only. While, the ring is extensible without in-plane term, $\alpha_n = -\frac{1}{n}$.

2.3. Kinetic energy and strain energy

In order to obtain the natural frequency of the imperfect ring with thermo-elastic damping, kinetic and strain energy relation is applied.

The imperfection effect in the stain energy is assumed to be negligible, thus, only kinetic energy can be expressed as

$$K = K_0 + K_P \tag{16}$$

In here, subscript *P* denotes the energy of point masses.

At first, the kinetic energy of the ring can be written as

$$K_0 = \frac{1}{2} ab\rho h \int_0^{2\pi} \left(\dot{u}^2 + \dot{v}^2 \right) d\theta = \frac{1}{2} M \int_0^{2\pi} \left(\dot{u}^2 + \dot{v}^2 \right) d\theta \tag{17}$$

where ρ denotes the density of ring. And *M* is the total mass of the ring.

The kinetic energy of the point masses for the ring is expressed as

$$K_P = \sum_{i=1}^{n} \frac{1}{2} m_i \left(\dot{u}_{m_i}{}^2 + \dot{v}_{m_i}{}^2 \right) \tag{18}$$

The maximum kinetic energy with point masses is

$$K_{\max} = \pi a \rho b h \frac{\left(\omega \mathrm{V}_0\right)^2}{2} \left(\alpha_n^2 + 1\right) \left[1 + \sum_{i=1}^{p} \frac{m_i}{2\pi a \rho b h} \left\{ 1 \pm \frac{\left(\alpha_n^2 - 1\right)}{\left(\alpha_n^2 + 1\right)} \cos\left(2n\left(\theta_i - \zeta_S\right)\right) \right\} \right] \tag{19}$$

where m_i stands for the additional masses.

Next, because the thermo-elastic damping effect exist only for the strain energy, the total strain energy can be expressed as

$$U = U_0 + U_T \tag{20}$$

In here, subscript *T* denotes the strain energy of thermo-elastic damping.

Considering of in-plane flexural vibration only, the total strain energy of the ring with thermo-elastic damping can be expressed as

$$\begin{aligned} U &= \frac{1}{2}ab\int_0^{2\pi}\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_\theta\left(\varepsilon_\theta-\alpha T\right)drd\theta \\ &= \frac{1}{2}ab\int_0^{2\pi}\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_\theta\varepsilon_\theta drd\theta-\frac{1}{2}ab\int_0^{2\pi}\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_\theta\alpha T drd\theta \end{aligned} \tag{21}$$

where, σ_θ and ε_θ denote bending stresses and strains, respectively. In here, for the total strain energy, thermo-elastic damping term is added to bending strain energy. Note that the terms with respect to radial direction are assumed to be negligible.

To get the analytical expressions for the kinetic and potential energies, the displacement terms need to be expressed by corresponding assumed mode shape functions.

Maximum strain energy is expressed as

$$U_{\max}=\frac{E^2bh\pi \mathrm{V}_0^2}{2a^3\left(1-\nu^2\right)E_\omega}\left[\left(n-\alpha_n\right)^2\left(1+2\Delta_E\right)+\frac{h^2}{12}\left(n^2-1\right)^2\alpha_n{}^2\left\{1+2\Delta_E\left(1+f(\omega)\right)\right\}\right] \tag{22}$$

Using Rayleigh's method, $K_{\max}=U_{\max}$ and thus, the natural frequency can be similarly obtained as [15]

$$\omega_{0E}^2=\frac{E}{\rho a^4\left(1-\nu^2\right)}\frac{a^2A\left(n-\alpha_n\right)^2+I\left(n^2-1\right)^2\alpha_n{}^2}{A\left(1+\alpha_n{}^2\right)} \tag{23}$$

In here, subscript 0*E* denotes the natural frequency of extensional case.

Especially, bending case is $\left(n-\alpha_n\right)^2=0$, then Eq. (23) can be written as

$$\omega_{0B}^2=\frac{E_\omega}{\rho a^2\left(1-\nu^2\right)}\frac{I\left(n^2-1\right)^2\alpha_n{}^2}{A\left(1+\alpha_n{}^2\right)} \tag{24}$$

In here, subscript 0*B* denotes the natural frequency of bending case.

Also, in-extensional case is $\alpha_n = n$, then Eq. (23) can be written as

$$\omega_{0I}^2 = \frac{n^2\left(n^2-1\right)^2}{a^4\left(1+n^2\right)}\frac{E_\omega I}{\rho\left(1-\nu^2\right)A} \tag{25}$$

In here, subscript 0*I* denotes the natural frequency of inextensional case.

Applying Rayleigh-Ritz method, natural frequencies of the *n*-th mode for imperfect ring with thermo-elastic damping as

$$\omega_{nS}^2 = \frac{E}{\rho a^4\left(1-\nu^2\right)}\frac{\left[a^2A\left(n-\alpha_n\right)^2\left(1+2\Delta_E\right)+I\left(n^2-1\right)^2\alpha_n{}^2\left\{1+\Delta_E\left(1+f(\omega)\right)\right\}\right]}{A\left(1+\alpha_n{}^2\right)\left[1+\sum_{i=1}^{p}\beta_i\left(1\pm\frac{\left(n^2-1\right)}{\left(n^2+1\right)}\cos\left(2n\left(\theta_i-\zeta_S\right)\right)\right)\right]} \tag{26}$$

$(S=H,L)$

where $E_{\omega_{nS}} = E\left\{1+\Delta_E\left[1+f(\omega_{nS})\right]\right\}$. Positive and negative sign means lower and higher mode, respectively. In here, ω_0 is natural frequency of perfect ring. The β_i denotes $\frac{m_i}{M}$. And, ω_{nH} and ω_{nL} denote n-th mode natural frequencies of higher mode *H* and lower mode *L*, respectively. These mean the split of natural frequency for the ring due to attached masses.

And, for the bending case, Eq. (24) can be expressed as

$$\begin{aligned}\omega_{nS}^2 &= \frac{E_\omega}{\rho a^2\left(1-\nu^2\right)}\frac{\left[I\left(n^2-1\right)^2\alpha_n{}^2\right]/A\left(1+\alpha_n{}^2\right)}{\left[1+\sum_{i=1}^{p}\beta_i\left(1\pm\frac{\left(n^2-1\right)}{\left(n^2+1\right)}\cos\left(2n\left(\theta_i-\zeta_S\right)\right)\right)\right]} \\ &= \omega_{0B}^2\frac{1+\Delta_E\left(1+f(\omega_{nS})\right)}{\left[1+\sum_{i=1}^{p}\beta_i\left(1\pm\frac{\left(n^2-1\right)}{\left(n^2+1\right)}\cos\left(2n\left(\theta_i-\zeta_S\right)\right)\right)\right]} \qquad (S=H,L)\end{aligned} \tag{27}$$

Also, for the inextensional case, Eq. (25) can be expressed as

$$\omega_{nS}^2 = \frac{n^2\left(n^2-1\right)^2}{a^4\left(1+n^2\right)} \frac{E_{\omega_{nS}} I / \left(\rho\left(1-\nu^2\right)A\right)}{\left[1+\sum_{i=1}^{p}\beta_i\left(1\pm\frac{\left(n^2-1\right)}{\left(n^2+1\right)}\cos\left(2n\left(\theta_i-\zeta_S\right)\right)\right)\right]}$$
$$= \omega_{0I}^2 \frac{1+\Delta_E\left(1+f(\omega_{nS})\right)}{\left[1+\sum_{i=1}^{p}\beta_i\left(1\pm\frac{\left(n^2-1\right)}{\left(n^2+1\right)}\cos\left(2n\left(\theta_i-\zeta_S\right)\right)\right)\right]} \qquad (S=H,L) \tag{28}$$

Frequency with thermo-elastic damping effect of the ring with attached masses is simplified as

$$\omega_{nS}^2 = \omega_{nS}^{*2}\left[1+\Delta_E\left(1+f(\omega_{nS})\right)\right] \tag{29}$$

where

$$\omega_{nS}^{*2} = \omega_0^2 \frac{1}{1+\sum_{i=1}^{p}\beta_i\left(1\pm\frac{\left(n^2-1\right)}{\left(n^2+1\right)}\cos\left(2n\left(\theta_i-\zeta_j\right)\right)\right)}$$

In here, ω_{nS}^* is the natural frequency with attached masses. Without attached masses, the equation is same as Ref. [6].

2.4. Quality factor of the model

In order to determine the Q-factor of the ring for selected frequency mode of vibration, using the convenient notation as

$$\xi_{nS} = h\sqrt{\frac{\omega_{nS}}{2\chi}} \tag{30}$$

And then, Δ_E is small enough, Eq. (29) is transformed the linearized form as

$$\omega_{nS}^2 = \omega_{nS}^{*2}\left[1+\Delta_E\left(1+f(\omega_{nS}^*)\right)\right] \tag{31}$$

Using Taylor series, Eq. (31) becomes

$$\omega_{nS} = \omega_{nS}^*\left[1+\frac{\Delta_E}{2}\left(1+f(\omega_{nS}^*)\right)\right] \tag{32}$$

Based on the definition of Q-factor in the work of Lifshitz and Roukes [6], present

results are expressed as

$$Q_{nS}^{-1} = 2\left|\frac{\mathrm{Imag}(\omega_{nS})}{\mathrm{Real}(\omega_{nS})}\right| \tag{33}$$

Next, re-analysis of Zener's approximation by Lifshitz and Roukes [6] is presented. From Eq. (26), the real and imaginary parts can be extracted as

$$\mathrm{Re}(\omega_{nS}) = \omega_{nS}^{*}\left[1+\frac{\Delta_E}{2}\left(1-\frac{6}{\xi_{nS}^3}\frac{\sinh\xi_{nS}-\sin\xi_{nS}}{\cosh\xi_{nS}+\cos\xi_{nS}}\right)\right] \approx \omega_{nS}^{*} \tag{34.a}$$

$$\mathrm{Im}(\omega_{nS}) = \omega_{nS}^{*}\left[\frac{\Delta_E}{2}\left(\frac{6}{\xi_{nS}^3}\frac{\sinh\xi_{nS}+\sin\xi_{nS}}{\cosh\xi_{nS}+\cos\xi_{nS}}-\frac{6}{\xi_{nS}^2}\right)\right] \tag{34.a}$$

Substituting Eq. (34) into Eq. (33), inverse Q-factor of the imperfect ring is obtained as

$$Q_{nS}^{-1} = \frac{E\alpha^2 T_0}{C}\left(\frac{6}{\xi_{nS}^2}-\frac{6}{\xi_{nS}^3}\frac{\sinh\xi_{nS}+\sin\xi_{nS}}{\cosh\xi_{nS}+\cos\xi_{nS}}\right) \tag{35}$$

Inverse Q-factor means the amount of thermo-elastic damping. In case of the Q-factor for in-plane flexural vibration, it is suitable for using the LR's methodology [10].

Additionaly, the shift angle can be obtained as in Ref [15],

$$\frac{\partial\omega^2}{\partial\zeta_i} = 0 \tag{36}$$

Substituting Eq. (26) into Eq. (32) yields:

$$-\frac{1}{\left(1+\varepsilon_K\right)^2}\frac{\partial\varepsilon_K}{\partial\zeta_i} = 0 \tag{37}$$

where $\varepsilon_K = T_{mn} / T_{0n}$ and subscript *m* denotes the attached masses.

In here, $\varepsilon_K \ll 1$ is reasonable for point masses, mode orientation angle can be determined as

$$\tan\left(2n\zeta_j\right)=\frac{\sum_{i=1}^{p} m_i \sin\left(2n\theta_i\right)}{\sum_{i=1}^{p} m_i \cos\left(2n\theta_i\right)} \tag{38}$$

where the equation can be solved to give two values of ζ_j, separated by $\pi/2n$, which specify the angular orientation of the two modes of the imperfect ring.

2.5. Rotating effect

Up to now, thermo-elastic damping of ring with arbitrary attached ring is investigated. In this chapter, effects of rotation are considered. Figure 2 shows coordinate system of rotating of ring. Considering rotation, differences of effects are compared with non-rotating model. For rotating model, extensional and inextensional assumptions are applied and discussed. In order to obtain Q-factor of rotating case, natural frequencies of rotation for extensional and inextensional model are derived, and then, they are applied.

First, equation of natural frequency for extensional ring is expressed as in Ref [21],

$$\omega_{rt,nS}=\frac{2n\Omega}{n^2+1}\pm\sqrt{\omega_{nS}^2-\frac{\left(n^4-n^2+2\right)}{\left(n^2+1\right)^2}\Omega^2} \tag{39}$$

where

$$\omega_{nS}^2=\omega_{0E}^2\frac{1+\Delta_E\left(1+f(\omega_{nS})\right)}{\left[1+\sum_{i=1}^{p}\beta_i\left(1\pm\frac{\left(n^2-1\right)}{\left(n^2+1\right)}\cos\left(2n\left(\theta_i-\zeta_S\right)\right)\right)\right]} \qquad (S=H,L)$$

In here, subscript *rt* denotes the natural frequency of rotating case.

The real and imaginary parts of natural frequency can be obtained as

$$\mathrm{Re}(\omega_{rt,nS}) = \left(\frac{2n}{n^2+1}\right)\Omega \pm \omega_{0E}\left[1+\frac{\Delta_E}{2}\left(1-\frac{6}{\xi_{rt,nS}^3}\frac{\sinh\xi_{rt,nS}-\sin\xi_{rt,nS}}{\cosh\xi_{rt,nS}+\cos\xi_{rt,nS}}\right)-\frac{\left(n^4-n^2+2\right)}{2\left(n^2+1\right)^2}\Omega^2\right] \quad (40.a)$$

$$\mathrm{Im}(\omega_{rt,nS}) = \pm\frac{\Delta_E}{2}\omega_{0E}\left(\frac{6}{\xi_{rt,nS}^3}\frac{\sinh\xi_{rt,nS}+\sin\xi_{rt,nS}}{\cosh\xi_{rt,nS}+\cos\xi_{rt,nS}}-\frac{6}{\xi_{rt,nS}^2}\right) \quad (40.b)$$

Inverse Q-factor of the imperfect ring is obtained as

$$Q_{rt,nS}^{-1} = 2\left|\frac{\mathrm{Im}\left(\omega_{rt,ns}\right)}{\mathrm{Re}\left(\omega_{rt,ns}\right)}\right| = \left|\frac{\Delta_E\left[\frac{6}{\xi_{rt,nS}^3}\frac{\sinh\xi_{rt,nS}+\sin\xi_{rt,nS}}{\cosh\xi_{rt,nS}+\cos\xi_{rt,nS}}-\frac{6}{\xi_{rt,nS}^2}\right]}{\left(\frac{2n}{n^2+1}\right)\frac{\Omega}{\omega_{0E}}\pm\left[1-\frac{\left(n^4-n^2+2\right)}{2\left(n^2+1\right)^2\left(\frac{\Omega}{\omega_{0E}}\right)^2}\right]}\right| \quad (41)$$

Next, equation of natural frequency for inextensional ring is expressed as in Ref [21],

$$\omega_{rt,nS} = \frac{2n\Omega}{n^2+1} \pm \sqrt{\omega_{nS}^2 - \frac{\left(n^2-1\right)^2}{\left(n^2+1\right)^2}\Omega^2} \quad (42)$$

where

$$\omega_{nS}^2 = \omega_{0I}^2\frac{1+\Delta_E\left(1+f(\omega_{nS})\right)}{\left[1+\sum_{i=1}^{p}\beta_i\left(1\pm\frac{\left(n^2-1\right)}{\left(n^2+1\right)}\cos\left(2n\left(\theta_i-\zeta_S\right)\right)\right)\right]} \qquad (S=H,L)$$

The real and imaginary parts of natural frequency can be obtained as

$$\mathrm{Re}(\omega_{rt,nS}) = \left(\frac{2n}{n^2+1}\right)\Omega \pm \omega_{0I}\left[1+\frac{\Delta_E}{2}\left(1-\frac{6}{\xi_{rt,nS}^3}\frac{\sinh\xi_{rt,nS}-\sin\xi_{rt,nS}}{\cosh\xi_{rt,nS}+\cos\xi_{rt,nS}}\right)-\frac{\left(n^2-1\right)^2}{2\left(n^2+1\right)^2}\Omega^2\right] \tag{43.a}$$

$$\mathrm{Im}(\omega_{rt,nS}) = \pm\frac{\Delta_E}{2}\omega_{0I}\left(\frac{6}{\xi_{rt,nS}^3}\frac{\sinh\xi_{rt,nS}+\sin\xi_{rt,nS}}{\cosh\xi_{rt,nS}+\cos\xi_{rt,nS}}-\frac{6}{\xi_{rt,nS}^2}\right) \tag{43.b}$$

Inverse Q-factor of the imperfect ring is obtained as

$$Q_{rt,nS}^{-1} = 2\left|\frac{\mathrm{Im}\left(\omega_{rt,ns}\right)}{\mathrm{Re}\left(\omega_{rt,ns}\right)}\right| = \left|\frac{\Delta_E\left[\frac{6}{\xi_{rt,nS}^3}\frac{\sinh\xi_{rt,nS}+\sin\xi_{rt,nS}}{\cosh\xi_{rt,nS}+\cos\xi_{rt,nS}}-\frac{6}{\xi_{rt,nS}^2}\right]}{\left(\frac{2n}{n^2+1}\right)\frac{\Omega}{\omega_{0I}} \pm \left[1-\frac{\left(n^2-1\right)^2}{2\left(n^2+1\right)^2\left(\frac{\Omega}{\omega_{0I}}\right)^2}\right]}\right| \tag{44}$$

3. Results and Discussion

In this chapter, natural frequencies and Q-factor of imperfect ring with thermo-elastic damping effect are calculated for extensional, bending and inextensional cases. And then, for extensional and inextensional cases, condition of rotation is applied. Comparing non-rotating ring with rotating model, differences of each case are confirmed and discussed. Through the verification of formulations, it is proved that results of this work are similar to previous studies. Also, numerical works show the data for various dimensions and imperfect masses. As parameters for cases, effects of mass weight and locations of random masses are adopted, and then effects due to parametric factors are discussed.

3.1. Verifications

Primarily, the frequency formula is exactly the same for the vibration for a ring without considering the thermo-elastic damping and attached masses. Also, the thermo-elastic damping effect for the silicon ring is considered as the results in Ref. [9]. Table 1 shows the results compared with the previous data for the perfect ring to check the validity of the natural frequencies for the n=2 mode. For inextensional and extensional cases, errors of results between previous study and this work are less than 0.01%. For each mode of the ring models, relative difference with respect to Ref. [15] is sufficiently small as indicated. Table 2 indicates Q-factors as compared with Ref. [10], and shows very small difference.

3.2. Numerical results

Masses are assumed to be very small relative to total mass of structures, so they

are called “point masses”. Extensional, bending and in-extensional imperfect ring models of thermo-elastic damping with attached masses are investigated. After then, considering the effect of rotating, results of extensional and in-extensional modes for the structure are discussed. In this work, the total weight of attached point mass is assumed as 1% of total mass of the ring.

3.2.1. Non-rotating ring

Figure 3 shows Q-factors for perfect ring. Extensional case has much larger value of Q-factor than the values of inextensional case and bending case. This means that extensional case is little energy dissipation. And, bending case has the lowest value of Q-factor.

Figure 4 shows Q-factor for inextensional, bending and extensional model with attached mass. For inextensional and bending cases, tendency of Q-factor according to the variation of radius and thickness is almost same. In here, as radius and thickness of the ring is decreased, the value of Q-factor is increased. Meanwhile, for extensional case, tendency of Q-factor is different from previous two cases. This case depends on the variation of thickness for the ring. As radius of the ring is decreased, the value of Q-factor is increased. In small and thick ring, extensional term is dominant, so Q-factor is efficiently larger. And large and thin ring, bending term is more influent, but not more than extensional term.

Figure 5 shows Q-factor split with respect to imperfect mass ratio. Split of Q-factor for bending case is larger than the value of inextensional case, and extensional case is the lowest value. This result shows that the magnitude of Q-factor split is inversely proportional to Q-factor for each case. Though the values of Q-factor are different, Q-factor splits according to mass ratio have similar tendency.

Figure 6 shows decreasing of amplitude according to time for perfect and imperfect rings. For the case of imperfect ring, decreasing speed of amplitude is faster than the speed of perfect model. This result means that the ring without random point

masses is more stable than the ring with imperfections. For the ring structure with attached masses, the energy of model is more dissipated than the case of perfect model. And, case of extensional case is more stable than inextensional case.

Figure 7 shows Q-factor according to the variation of mass number and location. Total mass ratio is fixed as 1%, and number of masses is applied as one or two. Also, location of masses is/are 0 degree or/and 30 degree. Each figure shows single mass of 0 degree, single mass of 30 degree, same double masses of 0/30 degree and different double masses of 0/30 degree. The values of Q-factor for each case are almost same and the tendency of Q-factors according to the variation of mass number and location is also same. Table 3 shows the Split of Q-factor according to variation of radius (thickness 40μm). From this table, results show that the values of Q-factor for each case are almost same.

3.2.1. Rotating ring

Table 4 shows Q-factor with respect to various temperatures. In all models, Q-factors of extensional case are larger than the case of inextensional. And, as temperature is increased, value of Q-factor is decreased. That is to say, quality of performance is better in lower temperature. Also, Q-factors of rotating case are larger than the case of non-rotating.

Figure 8 shows Q-factors for perfect ring adopting rotating effect. Like the preceding, extensional case also has much larger value of Q-factor than the values of inextensional case. And, as speed of rotating is increased, value of Q-factor is also increased. Due to the effect of rotation, little energy dissipation is occurred, and then, structures can have higher quality.

Figure 9 shows Q-factor of rotating imperfect ring for extensional and inextensional model with attached mass. For this case, tendency of Q-factors between extensional and inextensional modes are similar according to the variation of radius and thickness. In here, as radius of the ring is decreased, the value of Q-

factor is increased. This is similar to non-rotating case. Like the preceding, as speed of rotating is increased, value of Q-factor is also increased.

4. Conclusions and future work

In practical case, manufacturing processes are not perfect, there always have been imperfect mass distributions resulting in frequency split for structures. The random point masses of structures have reducing the qualities and capabilities for the devices, such as gyroscopes. In this work, analytical model for imperfect circular ring with thermo-elastic damping is investigated. The ring is assumed to resonate with extensional, bending and in-extensional modes. Attached masses are assumed to be very small relative to total mass of structures, so they are called “point masses”. Additionally, the flexural vibration of rotating ring is considered for the cases of extensional and in-extensional modes. Due to the damping, the total energy of the ring dissipates. An analytical expression for the Q-factor due to thermo-elastic damping is derived for the thin ring in the present work. It can be shown that the Q-factor of the circular silicon ring is in inverse proportion to the surrounding temperatures. Moreover, as the dimensions of the thin ring become smaller, thermo-elastic damping considerably increases. Moreover, the irregular masses can represent uneven mass distribution of practical ring. This imbalance affects to whole system and results in split of natural frequencies and increase of the damping effect. The intensity of imperfection effect varies through the geometry of the ring. The results of this work show good agreement with previous studies which deal with thermo-elastic damping of ring and ring with point masses, respectively. Generally, the gyroscopes will have different mode shape orientations and split frequencies due to imperfection. Also, the thermo-elastic damping changes the amplitude of split frequencies. Thus, these factors should be considered when designing resonator gyroscopes. As future works, studies on trimming methods for eliminating or reducing the effects of imperfections will be investigated.

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Table 1. Code verification for natural frequencies with imperfect mass

$\omega = 2\pi f$	Inextensional ring			Extensional ring		
n=2	Experimental result [15]	Present	error(%)	Experimental result [15]	Present	error(%)
L-mode	222.3619	222.3648	0.0012	38514.67	38514.86	0.0005
H-mode	228.8336	228.8128	0.0091	39631.82	39631.69	0.0003
Perfect	231.0893	231.09	0.0003	40023.89	40026.13	0.0056

Table 2. Code verification for Q-factors($\times 1000$) with thermo-elastic damping ($n=2$, $h=100\mu m$)

Radius [mm]	Experimental result [9]	Present	error(%)
1	139	138.816	0.1324
3	18.65	18.6222	0.1491
5	10.26	10.2418	0.1774

Table 3. Split of Q-factor according to variation of radius (thickness 40μm)

Mode	radius(mm)	Single mass		Double mass	
		1%, 0deg	1%, 30deg	0.5% / 0.5% ,0deg / 30deg	1.5% / -0.5% 0deg / 30deg
H-mode	1.0	0.122253	0.396992	0.259679	-0.015290
	1.4	0.176214	0.571587	0.374092	-0.022050
	1.8	0.190832	0.618884	0.405085	-0.023880
	2.2	0.195731	0.634734	0.415472	-0.024490
	2.6	0.197704	0.641118	0.419655	-0.024740
	3.0	0.198615	0.644065	0.421587	-0.024850
	3.4	0.199081	0.645573	0.422575	-0.024910
	3.8	0.199339	0.646408	0.423122	-0.024950
	4.2	0.199491	0.646900	0.423445	-0.024960
	4.6	0.199585	0.647205	0.423645	-0.024980
	5.0	0.199646	0.647403	0.423774	-0.024980
L-mode	1.0	0.488470	0.213883	0.351234	0.625592
	1.4	0.703038	0.308175	0.505797	0.899901
	1.8	0.761164	0.333718	0.547668	0.974210
	2.2	0.780643	0.342278	0.561700	0.999113
	2.6	0.788489	0.345726	0.567351	1.009142
	3.0	0.792111	0.347318	0.569961	1.013773
	3.4	0.793965	0.348132	0.571296	1.016143
	3.8	0.794991	0.348583	0.572035	1.017455
	4.2	0.795596	0.348849	0.572470	1.018228
	4.6	0.795970	0.349014	0.572740	1.018707
	5.0	0.796213	0.349120	0.572915	1.019017

Table 4. Q-factors with respect to temperatures

Temperature			**240**	**258**	**298**	**320**	**348**
α $[10^{-6}K^{-1}]$			1.99	2.24	2.60	2.85	3.06
C_v $[10^6 J/(m^3 K)]$			1.51	1.52	1.64	1.68	1.73
χ $[10^5 m^2/s]$			14.30	11.70	8.60	7.92	6.97
Q-factor	Inext	Non-rotating	19632.31	14444.21	10730.65	8804.08	7690.09
		Rotating	20372.52	14988.80	11135.21	9136.00	7980.01
	Ext	Non-rotating	537101.80	482420.70	451630.80	388659.70	361798.90
		Rotating	537388.60	482678.30	451871.90	388867.20	361992.00

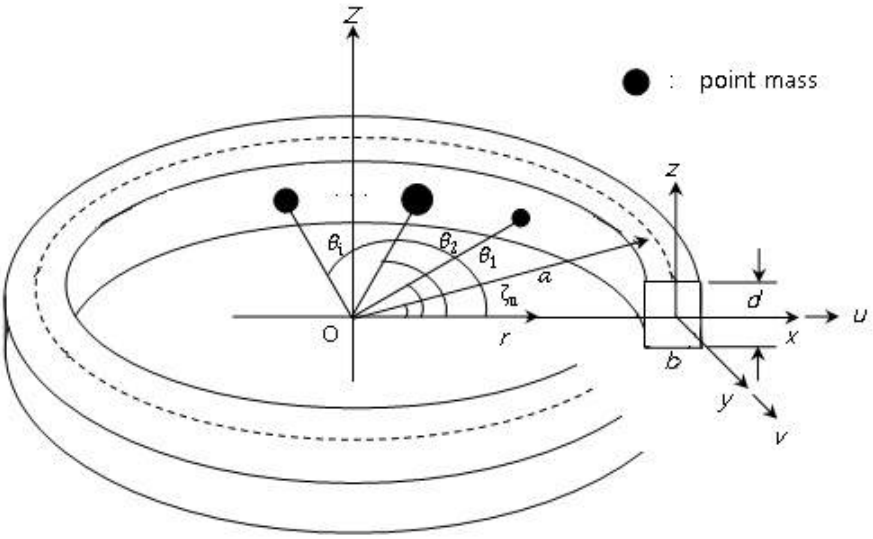


Figure 1. Global and local coordinate system of ring with point masses

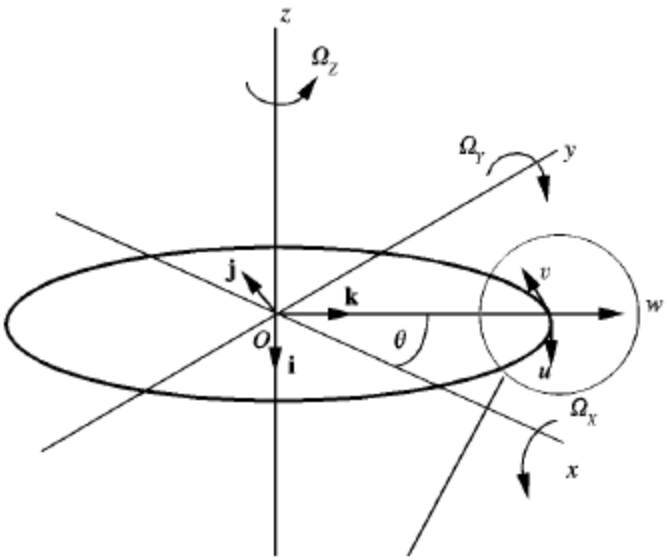


Figure 2. Coordinate system of rotating ring model

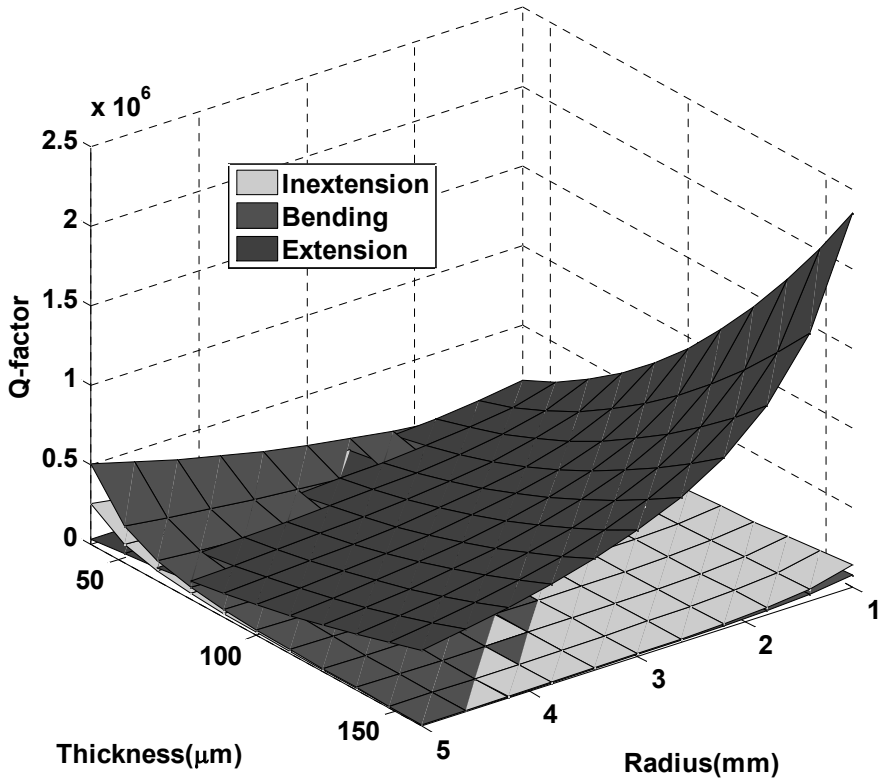
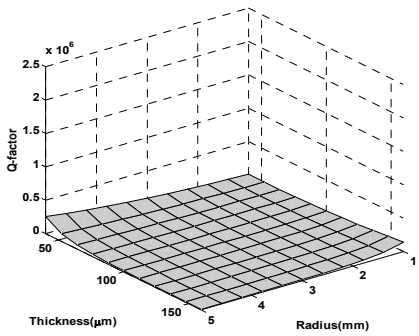
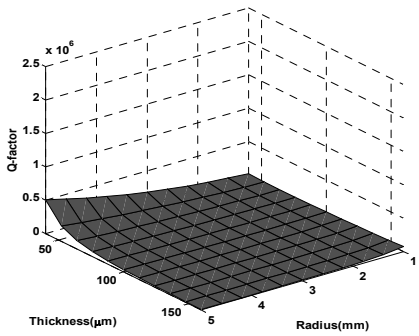


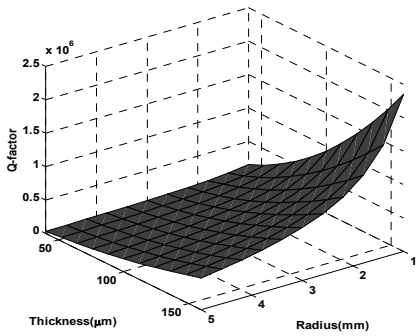
Figure 3. Q-factors for perfect ring



(a) In-extensional

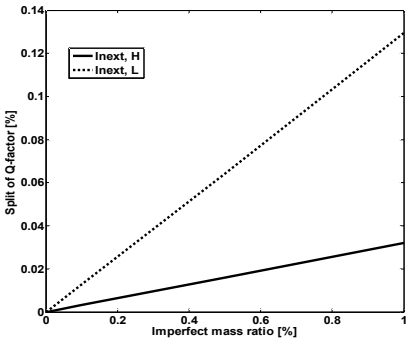


(b) Bending (no in-plane term for extensional)

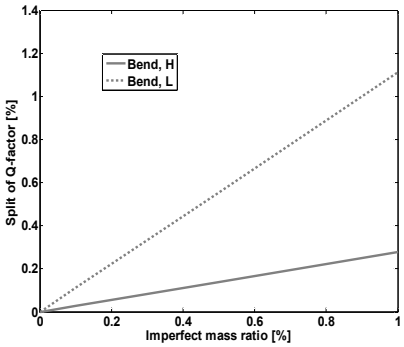


(c) Extensional

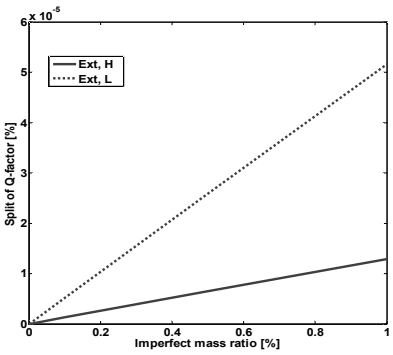
Figure 4. Q-factors for imperfect ring (1% mass)



(a) In-extensional



(b) Bending



(c) Extensional

Figure 5. Q-factor split according to imperfect mass ratio

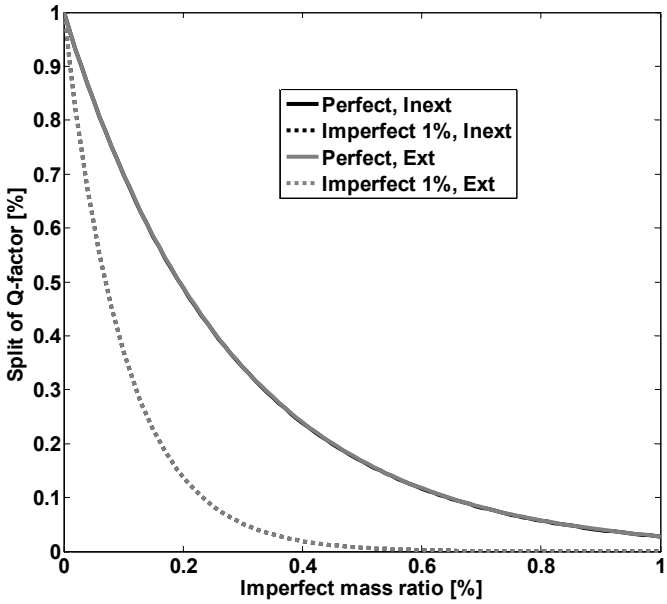
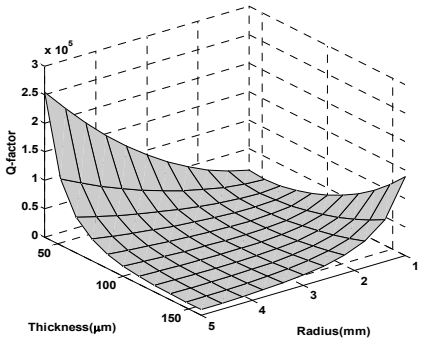
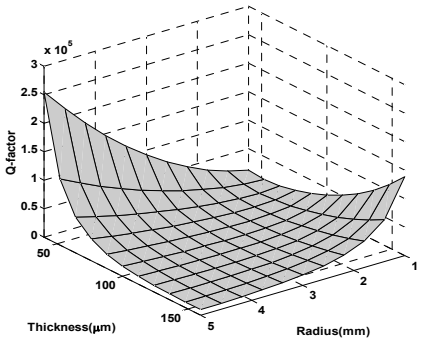


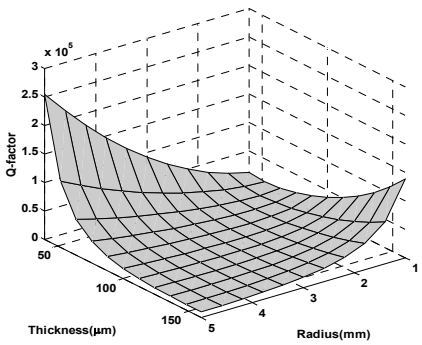
Figure 6. Decreasing of amplitude according to time



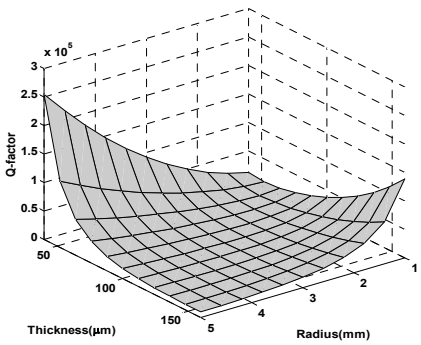
(a) Q-factor of single point mass (1%, 0 degree)



(b) Q-factor of single point mass (1%, 30 degree)

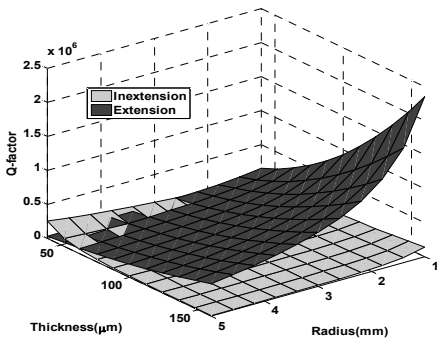


(c) Q-factor of double point masses (0.5% / 0.5%, 0 degree / 30 degree)

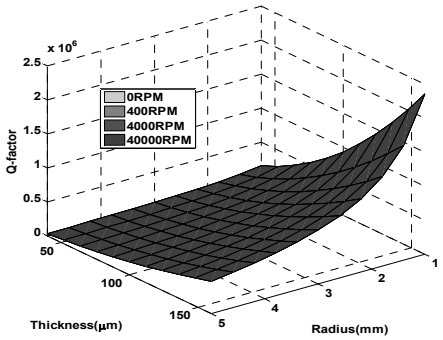


(d) Q-factor of double point masses (1.5% / -0.5%, 0 degree / 30 degree)

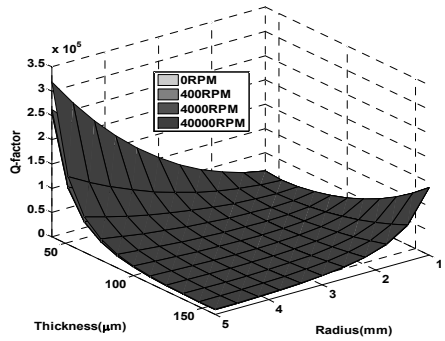
Figure 7. Q-factor according to the variation of mass number and location



(a) Q-factor

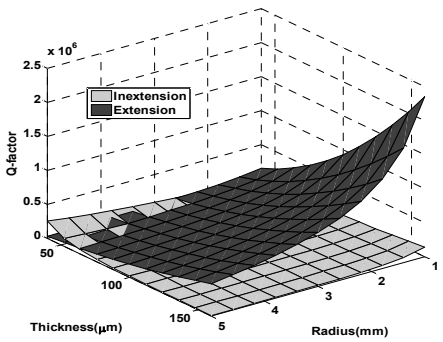


(b) Q-factor of extensional mode according to rotating speed

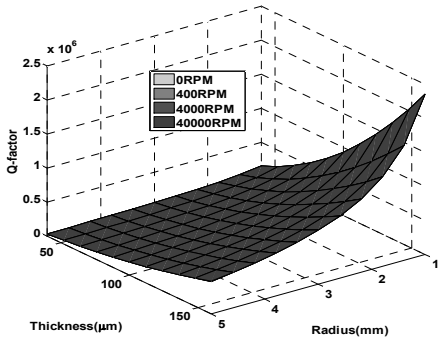


(c) Q-factor of inextensional mode according to rotating speed

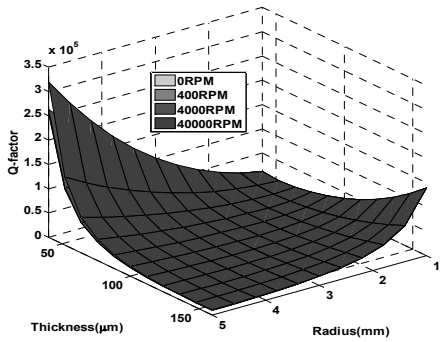
Figure 8. Q-factors for rotating perfect ring



(a) Q-factor



(b) Q-factor of extensional mode according to rotating speed



(c) Q-factor of inextensional mode according to rotating speed

Figure 9. Q-factors for rotating imperfect ring (1% mass)

국 문 초 록

본 논문에서, 불균질한 질량 분포를 갖는 링 구조물의 열-탄성 댐핑(thermo-elastic damping)에 관하여 연구하였다. 연구에서 균일한 링의 in-plane vibration 을 가정한 열-탄성 링 모델을 적용하였다. 모델은 사각형의 단면을 가진 것으로 가정하였다. 그리고, 미소질량은 불균형적인 질량분포와 같은 불완전성으로 표현하였다. 고유진동수에 대한 해석적인 표현을 도출하기 위한 분석이 이루어졌다. 원형 링에 대한 해석적 모델은 진동의 모드쉐입을 이용하여 소개되었다. 링 모델에 대한 extensional, bending 및 inextensional 가정을 고려하고 논의하였다. 링의 고유진동수를 결정하기 위하여, 링에 대한 에너지 방정식을 활용하였다. 에너지 방정식으로서, Rayleigh-Ritz 기법을 적용하였다. 또한, 회전하는 링에 대한 운동방정식을 도출하였다. 회전하는 모델에 대하여, extensional 및 inextensional 가정을 적용하고 논의하였다. 미소질량을 갖는 마이크로 링은, 공식에서 링에 대해 임의적으로 위치한 미소질량으로 표현되었다. 또한, 열 전도 방정식을 이용하여, 열-탄성 댐핑 효과를 연구하였다. 댐핑 효과에 대한 향상된 이해를 얻기 위하여, Quality factor(Q-factor)라는 개념이 활용되었다. 덧붙여, 미소질량의 영향은 모델의 고유진동수 분리로서 나타내어졌다. Q-factor 및 고유진동수는 링에 대한 모드넘버의 명백한 함수로서 표현되어질 수 있었다. 더불어, 모델에 대한 더 나은 물리적 해석을 위하여 다양한 사례 연구를 수행하였다.

주요어 : 링, 열-탄성 댐핑, 미소질량, Quality factor(Q-factor), 진동 분리

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