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## 공학석사학위논문

# Three Dimensional Path Planning and Guidance for Aerial Refueling of Unmanned Aerial Vehicle 

## 무인기 공중급유를 위한 3 차원 경로계획 및 유도기법

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## Abstract

# Three Dimensional Path Planning and Guidance for Aerial Refueling of Unmanned Aerial Vehicle 

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In this thesis, a rendezvous path planning is considered for a UAV (Unmanned Aerial Vehicle). The objective of the path planning is to make the tanker and the UAV perform rendezvous at a preplanned point, and then the UAV should maintain the same speed with the tanker to accomplish aerial refueling mission. Since the aerial refueling mission is generally performed between one tanker and multiple aircraft, setting the pre-planned point for the rendezvous will be efficient in the operation management level. In this study, rendezvous path planning and guidance law are proposed such that the UAV can be prevented from turning sharply while minimizing an energy consumption. Numerical simulations are performed for rendezvous and aerial refueling mission to demonstrate the performance of the proposed path planning and guidance law.

Keywords : Rendezvous, Pursuit guidance law, Aerial refueling, Unmanned Aerial Vehicle

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## Chapter 1

## Introduction

### 1.1 Background

In 2015, Republic of Korea Air Force (ROKAF) determined to adopt 'A-330 MRTT (Multi Role Tanker Transport)' as a multirole tanker. The operating type of aerial refueling is the flying boom method as shown in Fig. 1.1 [1]. For aircraft receiving fuel in the air, the receiver pilot has to fly within the 'aerial refueling envelope', which means the safe area to perform the aerial refueling mission. By the adoption, mission area of various aircraft including unmanned aerial vehicles (UAVs) will expand over the limitation of fuel-capacity.


Figure 1.1: A-330 MRTT in the aerial refueling mission

It is expected that UAVs will be utilized in various applications, because UAV can perform dangerous and difficult missions in complex area such as rugged mountain areas or haz-
ardous zones. However, the limitation of UAVs is an important issue for performing various missions, and therefore many studies have dealt with various rendezvous problems and aerial refueling mission of the UAV. Especially, test-flight of the automated aerial refueling mission between a tanker and a UAV has been performed successfully by the US Navy; X-47B succeeded the automated aerial refueling mission and received over $4,000 \mathrm{lbs}$ of fuel from Omega K-707 tanker in the air in 2015 [2].

On the other hand, many studies on path planning problems have been performed for rendezvous regardless of the point where rendezvous should succeed. This kind of scheme could be useful to perform rendezvous quickly. However, it might have several limitations for the case that multiple UAVs perform rendezvous sequentially. Generally, aerial refueling mission is performed between one tanker and multiple aircraft, and therefore the rendezvous point should be set beforehand to efficiently and effectively conduct its mission.

### 1.2 Related Research

Guidance laws have been widely studied in the field of missile systems [3] [4]. The proportional navigation (PN) guidance has been widely used to deal with intercept problems of missile systems. The PN guidance constructs guidance command whose value is proportional to the line-of-sight (LOS) rate to a target. An interceptor or a missile using PN guidance can hit the target efficiently and effectively. For this reason, several researchers have studied rendezvous guidance using the PN guidance [5] [6]. However, the PN guidance can not guarantee the tail-chased rendezvous in general.

Pure pursuit (PP) guidance uses an intuitive concept of the interception. The PP guidance generates a guidance command that the velocity vector of an interceptor is toward a target. Since the UAV must approach to the tail-side of the tanker for aerial refueling mission, the PP guidance is more suitable than the PN guidance; in detail, PP guidance lets the UAV head towards the tanker at any time. Ratnoo proposed a rendezvous guidance law based on the PP guidance considering field of view (FOV) limitation and autopilot lag, and the performances of PP guidance and PN guidance were compared [7].

When the UAV comes close to the tanker, the PP guidance might fail a sensitive chase to the tanker. To solve this problem, Yamazaki et al. applied a sliding mode control for UAV rendezvous [8]. Then, the guidance laws were expanded to the three dimensional environment [9] [10]. Luo et al. used a guidance law based on the iterative computation method controlling a velocity of the UAV for rendezvous and formation flight. However, using only this method has some limitations. One of which is the possibility of generating miss-distance [9]. Kung et al. designed a rendezvous guidance law based on pursuit guidance with the feedback linearization method [10]. The guidance law makes the heading direction of the UAV follow the LOS; thereby, the guidance law not only reduces a miss-distance but also lets the heading angle of the UAV correspond with the tanker.

It can be expected that the PP guidance guarantees the chasing of the UAV to the tanker. However, the PP guidance needs more flight length for a rendezvous than the PN guidance. Park et al. introduced a nonlinear path-following guidance method, which
uses reference points on the desired path [11]. Nonlinear pathfollowing guidance can make the UAV chase to the tail-side of the tanker. Also, the nonlinear path-following guidance can reduce the turning radius in comparison to the PP guidance. Kim and Kim adopted this concept and designed the pseudo pursuit guidance law for target observation considering incidence angle constraint [12].

On the other hand, optimal desired path for minimum-time was also considered for the rendezvous problem [13] [14] [15]. Since the minimum-time path is related to the minimum-length of the UAV, 'Dubins path' has been widely used for the determination of optimal path [8] [13]. Weiss et al. considered the minimum effort pursuit guidance [16].

### 1.3 Contributions

For the autonomous aerial refueling mission, the UAV has to not only make rendezvous at the tail-side of the tanker but also maintain the same velocity with the tanker after the rendezvous. In this study, the path planning and guidance law based on pseudo pursuit guidance are proposed to perform rendezvous at the preplanned point. The velocity control scheme is also proposed to minimize energy consumption.

The contribution of this study is to propose a three dimensional path planning and guidance law, which can manage the aerial refueling operations. Scheduling for aerial refueling missions of multiple UAVs is also considered. Using the proposed method, the rendezvous at the pre-planned point can be performed to accomplish the aerial refueling missions between one tanker and mul-
tiple UAVs efficiently. Thereby, the UAVs do not need holding maneuver in the air, and can save the fuel and operation time.

### 1.4 Thesis Organization

This thesis is organized as follows. Chapter 2 presents two classical guidance laws. Chapter 3 describes the autonomous aerial refueling mission, and proposes path planning and guidance law for a rendezvous in two dimensions. Chapter 4 focuses on the expansion of the proposed rendezvous path planning to the three dimensional space. Chapter 5 provides the performance evaluation results of the proposed path planning and guidance law through numerical simulations for the autonomous aerial refueling mission between a tanker and multiple UAVs. Finally, concluding remarks and further research works are addressed in Chapter 6.

## Chapter 2

## Guidance Laws for UAV

## Rendezvous

### 2.1 Pure pursuit guidance law

PP guidance law is one of the widely used guidance laws. Figure 2.1 shows an engagement geometry of the rendezvous between a tanker and a UAV.


Figure 2.1: Engagement geometry using PP guidance

In Fig. 2.1, $R$ denotes a distance between the tanker and the UAV, $\psi_{T}$ and $\psi_{u}$ denote heading angles of the tanker and the UAV, respectively, and $\lambda$ denotes LOS angle. The dynamics of the tanker
and the UAV can be represented as

$$
\begin{align*}
& \dot{R}=-v_{u} \cos \left(\psi_{u}-\lambda\right)+v_{T} \cos \left(\psi_{T}-\lambda\right)  \tag{2.1}\\
& \dot{\lambda}=-\left(\frac{v_{u}}{R}\right) \sin \left(\psi_{u}-\lambda\right)+\left(\frac{v_{T}}{R}\right) \sin \left(\psi_{T}-\lambda\right)  \tag{2.2}\\
& \dot{\psi_{u}}=\frac{a_{u}}{v_{u}}=a_{c u}  \tag{2.3}\\
& \dot{\psi_{T}}=\frac{a_{T}}{v_{T}}=a_{c T} \tag{2.4}
\end{align*}
$$

where $v_{T}$ and $v_{u}$ denote velocities of the tanker and the UAV, respectively, and $a_{T}$ and $a_{u}$ denote lateral acceleration commands of the tanker and the UAV, respectively.

The PP guidance law uses a lateral acceleration command to keep a UAV velocity vector heading to a tanker, and therefore the velocity vector of the UAV will match up the LOS. Using PP guidance law, the UAV can perform a tail-chase maneuver to the tanker regardless of initial heading angle, and the UAV can perform an aerial refueling mission behind the tanker. The lateral acceleration command normalized by the UAV velocity for the PP guidance can be written as

$$
\begin{equation*}
a_{c u}=-k\left(\psi_{u}-\lambda\right)+\dot{\lambda} \tag{2.5}
\end{equation*}
$$

where $k$ is a positive constant gain, and better performance can be achieved by using a term $\dot{\lambda}$. If the heading angle of the UAV corresponds to the LOS angle and the UAV has zero variation of the LOS, then the lateral acceleration command will become zero. After that, the UAV will fly straightly to the tanker.

However, the PP guidance law does not guarantee the accurate and sensitive tail-chase maneuver. For example, if a UAV has to perform a roll maneuver over the limitation of the bank angle or a tanker changes the flight direction abruptly when the UAV


Figure 2.2: Failed rendezvous situation
comes closer to the tanker, then the UAV fails to conduct the tailchase to the tanker. Figure 2.2 shows the situation when the UAV cannot perform the rendezvous due to the limitation of the bank angle. Although the UAV does not have the limitation of the bank angle as shown in Fig. 2.2, the UAV has to turn sharply for the completion of the rendezvous.

### 2.2 Proportional navigation guidance law

PN guidance law is the most widely used guidance law among the classical guidance laws. The PN guidance law uses a lateral acceleration command to make the LOS rate to zero. The lateral acceleration command normalized by the UAV velocity for the PN guidance can be represented as

$$
\begin{equation*}
a_{c u}=N \dot{\lambda} \tag{2.6}
\end{equation*}
$$

where $N$ denotes a navigation constant. The PN guidance is widely used in the missile system, and it makes missile hit the target on
the collision line, which means that the LOS rate maintains zero.
The PN guidance generates more direct flight path to the target than the PP guidance, but does not guarantee the tail-chase of a follower UAV.

### 2.3 PP guidance law and PN guidance law

Figure 2.3 shows the trajectories of PP guidance law and PN guidance law to a moving target. A tanker flies on a circular path, and two UAVs chase the tanker using PP guidance and PN guidance, respectively, while maintaining the same velocity with the tanker. Especially, when the tanker and UAVs have crossing heading angles in the initial condition as shown in Fig. 2.3, then the UAV using PN guidance finally collides with the tanker. In contrast, the UAV using PP guidance approaches to the tail-side of the tanker, but the UAV needs a longer flight path than the UAV using PN guidance.


Figure 2.3: A comparison of trajectories

## Chapter 3

## Rendezvous Path Planning in Two Dimensions

### 3.1 Path planning

A path planning algorithm for a rendezvous at a pre-planned point and time in two dimensions is proposed in this chapter. For this purpose, the 'CTA (Center of transition area)' between the preplanned 'RP (rendezvous point)' and an initial point of the tanker is defined. The tanker flies straightly to the RP while maintaining the constant velocity. Since the tanker maintains the same velocity during the flight, the rendezvous time can be calculated beforehand. In contrast, UAV flies to the RP, controlling its velocity until the completion of the rendezvous.

In this study, proposed path planning scheme consists of two phases: i) the approach phase, and ii) the rendezvous phase. Note that the final distance between a UAV and a tanker does not converge to zero in real aerial refueling considering the flying boom or probe-and-drogue systems. In this thesis, therefore, the position of a tanker means the combining point for the aerial refueling. The combining point is located at a certain distance behind the real cg (center of gravity) position of the tanker.

### 3.2 Approach phase

The approach phase is the phase from the initial position to the transition near the CTA. In this phase, a UAV uses a pseudo pursuit guidance law, which makes the UAV perform tail-chase to a tanker without a sharp turn. Also, the tanker and the UAV maintain a constant velocity in this phase.

### 3.2.1 Pseudo pursuit guidance law

The pseudo pursuit guidance law uses a virtual point, which locates between the tanker and the UAV as shown in Fig. 3.1. The virtual point becomes a reference point for generating acceleration commands of the UAV.


Figure 3.1: A relative position of a virtual point

In Fig. 3.1, $\eta$ denotes an angle between a heading direction of the UAV and the LOS to the virtual point, $\psi_{d}$ denotes a setting an-
gle for a trajectory of the virtual point, and $\overrightarrow{v_{i}}$ denotes a velocity vector of the virtual point.

The virtual point also moves to the CTA with $v_{i}$, which makes the virtual point arrive at the CTA simultaneously with the tanker. This guidance law uses a lateral acceleration command to make the UAV move toward the virtual point. The position of the virtual point is calculated using the following correlation formulae.

$$
\begin{equation*}
\overrightarrow{L_{d}} \times \overrightarrow{v_{i}}=0 \quad \vec{L} \cdot \overrightarrow{v_{i}}=0 \tag{3.1}
\end{equation*}
$$

where $\overrightarrow{L_{d}}$ denotes a vector from the virtual point to the CTA, and $\vec{L}$ denotes a vector from the UAV to the virtual point.

Using Eq. (3.1), the initial position of the virtual point can be obtained as follows

$$
\begin{align*}
& x_{i}=x_{C T A} \sin ^{2} \psi_{d}+\cos \psi_{d} \sin \psi_{d}\left(y-y_{C T A}\right)+x \cos ^{2} \psi_{d}  \tag{3.2}\\
& y_{i}=-\frac{\cos \psi_{d}}{\sin \psi_{d}}\left(x_{i}-x\right)+y \tag{3.3}
\end{align*}
$$

where $\left(x_{C T A}, y_{C T A}\right)$ denotes a position of the CTA. The desired angle to adjust the difference between the heading angle of the UAV and the LOS angle to the virtual point can be calculated as follows

$$
\begin{equation*}
\eta=\tan ^{-1}\left(\frac{y_{i}-y}{x_{i}-x}\right)-\psi_{u} \tag{3.4}
\end{equation*}
$$

The lateral acceleration command normalized by the UAV velocity can be rewritten as

$$
\begin{equation*}
a_{c u}=k_{a p p}\left(\frac{v_{u}}{L}\right) \sin \eta \tag{3.5}
\end{equation*}
$$

where $k_{\text {app }}$ denotes a positive constant guidance gain. The above acceleration command makes the UAV perform the tail-chase to the tanker for aerial refueling mission, and maintain the tail-side position of the tanker.

### 3.2.2 Transition around the CTA

The approach phase performs to the rendezvous maneuver within the transition area. The transition area is the space in the certain distance from the CTA, like the circle of a dashed line in Fig. 3.2. There are two criteria of the transition on the basis of the CTA: i) the distance between the UAV and the CTA, and ii) the difference between a LOS angle of the UAV to the tanker and the heading angle of the UAV.
(1) First criterion: Distance $R_{c}$

The criterion of a distance can make nearly straight trajectory of the UAV after passing the CTA. At the time of transition, the UAV can depart to the RP near the trajectory of the tanker, because the UAV is located nearer to the CTA than the criterion of a distance.


Figure 3.2: The criterion of a distance

After the transition around the CTA, the trajectory of the

UAV is almost straight. Therefore, the distance from the UAV to the RP can be approximately calculated. Thereby, velocity control using the calculated distance will be precisely conducted.
(2) Second criterion: Angle $\theta_{c}$

At the transition, the reference point for the acceleration commands of the UAV changes from the virtual point to the tanker. In other words, the acceleration commands in the approach phase are generated by using the relation between the virtual point and the UAV, but the acceleration commands in the rendezvous phase are generated by using the relation between the tanker and the UAV. If the tanker is located ahead of the UAV at the transition, the UAV does not have to suddenly change the heading angle right after the transition as shown in Fig. 3.3. Therefore, the criterion of an angle between an angle of the relative position of the tanker from the UAV and the heading angle of the UAV is suggested in this study.


Figure 3.3: The criterion of an angle

### 3.2.3 Determination of UAV initial velocity range

Note that the criteria for the transition cannot always be satisfied. For example, if the UAV lies within the transition area, which has a radius of the criterion of the distance, then the tanker might not pass ahead of the UAV. Therefore, it is necessary to calculate the initial velocity range of the UAV for the satisfaction of the criteria.

In the approach phase, a lateral acceleration command of the UAV is generated by using the virtual point; therefore, Eqs. (2.1) and (2.2) can be reformulated as follows

$$
\begin{align*}
& \dot{R}_{i}=-v_{u} \cos \left(\psi_{u}-\lambda_{i}\right)+v_{i} \cos \left(\psi_{d}-\lambda_{i}\right)  \tag{3.6}\\
& \dot{\lambda_{i}}=-\left(\frac{v_{u}}{R_{i}}\right) \sin \left(\psi_{u}-\lambda_{i}\right)+\left(\frac{v_{i}}{R_{i}}\right) \sin \left(\psi_{d}-\lambda_{i}\right) \tag{3.7}
\end{align*}
$$

where $R_{i}$ denotes the distance between the UAV and the virtual point, and $\lambda_{i}$ denotes the LOS angle of the UAV to the virtual point. Using Eqs. (3.6) and (3.7), $v_{R i}$ and $v_{\lambda i}$ can be defined as follows

$$
\begin{align*}
& v_{R i}=\dot{R}_{i}=-v_{u} \cos \left(\psi_{u}-\lambda_{i}\right)+v_{i} \cos \left(\psi_{d}-\lambda_{i}\right)  \tag{3.8}\\
& v_{\lambda i}=R_{i} \dot{\lambda_{i}}=-v_{u} \sin \left(\psi_{u}-\lambda_{i}\right)+v_{i} \sin \left(\psi_{d}-\lambda_{i}\right) \tag{3.9}
\end{align*}
$$

or

$$
\begin{align*}
& v_{R i}+v_{u} \cos \left(\psi_{u}-\lambda_{i}\right)=v_{i} \cos \left(\psi_{d}-\lambda_{i}\right)  \tag{3.10}\\
& v_{\lambda i}+v_{u} \sin \left(\psi_{u}-\lambda_{i}\right)=v_{i} \sin \left(\psi_{d}-\lambda_{i}\right) \tag{3.11}
\end{align*}
$$

Using Eqs. (3.10) and (3.11), the following equation of a circle can be obtained.

$$
\begin{equation*}
\left\{v_{R i}+v_{u} \cos \left(\psi_{u}-\lambda_{i}\right)\right\}^{2}+\left\{v_{\lambda i}+v_{u} \sin \left(\psi_{u}-\lambda_{i}\right)\right\}^{2}=v_{i}^{2} \tag{3.12}
\end{equation*}
$$

Equation (3.12) means that the radius is $v_{i}$, and the center of the circle is $\left(-v_{u} \cos \left(\psi_{u}-\lambda_{i}\right),-v_{u} \sin \left(\psi_{u}-\lambda_{i}\right)\right)$. Figure 3.4 shows the velocity circle.


Figure 3.4: A velocity circle of the approach phase

Figure 3.4 shows that the center of the velocity circle moves along the circle of the dashed line which has a radius of $v_{u}$. When the UAV approaches to the CTA, $\psi_{u}$ becomes closer to $\lambda_{i}$. It means that the center of the circle gradually moves to $\left(-v_{u}, 0\right)$.

Depending on the velocities of the virtual point and the UAV, there exist two cases of the velocity circle. Figure 3.5 shows these two cases: i) $v_{i}$ is bigger than $v_{u}$, and ii) $v_{u}$ is bigger than $v_{i}$.

When the UAV approaches to the CTA, the virtual point also approaches to the CTA, which means that $\left(\psi_{u}-\lambda_{i}\right)$ becomes zero. Then, the following equations can be obtained by differentiating Eqs. (3.8) and (3.9).

$$
\begin{align*}
& v_{R i}=-v_{i} \sin \left(\psi_{d}-\lambda_{i}\right)\left(-\dot{\lambda_{i}}\right)=\dot{\lambda_{i}} v_{i}  \tag{3.13}\\
& v_{\dot{\lambda} i}=v_{i} \cos \left(\psi_{d}-\lambda_{i}\right)\left(-\dot{\lambda_{i}}\right)=-\dot{\lambda_{i}}\left(v_{R i}+v_{u}\right) \tag{3.14}
\end{align*}
$$



Figure 3.5: The two cases of the velocity circle near the CTA

Let us multiply $R_{i}$ to the both sides of Eqs. (3.13) and (3.14).

$$
\begin{align*}
& R_{i} v_{R i}=R_{i} \dot{\lambda_{i}} v_{\lambda i}=v_{\lambda i}^{2}  \tag{3.15}\\
& R_{i} v_{\dot{\lambda i}}=-R_{i} \dot{\lambda_{i}}\left(v_{R i}+v_{u}\right)=-v_{\lambda i}\left(v_{R i}+v_{u}\right) \tag{3.16}
\end{align*}
$$

When the UAV approaches near the CTA, the following conditions can be derived using Eqs. (3.15) and (3.16).
$v_{\lambda i}>0$ if $\left\{v_{\lambda i}>0 \& v_{R i}<-v_{u}\right\}$ or $\left\{v_{\lambda i}<0 \& v_{R i}>-v_{u}\right\}$
$\dot{v}_{\lambda i}<0$ if $\left\{v_{\lambda i}>0 \& v_{R i}>-v_{u}\right\}$ or $\left\{v_{\lambda i}<0 \& v_{R i}<-v_{u}\right\}$

Note that $v_{R i}$ is always bigger than zero due to Eq. (3.15). Therefore, the direction of movement of the $\left(v_{c R i}, v_{c \lambda i}\right)$ point in Fig. 3.5 is upward.

If $v_{i}$ is bigger than $v_{u}$, then the $\left(v_{c R i}, v_{c \lambda i}\right)$ point crosses the $v_{\lambda i}$-axis, and then enters to a positive area of the $v_{R i}$. It means the 'miss-distance', the closest distance between the virtual point and the UAV, is generated. Crossing the $v_{\lambda i}$-axis means that $R_{i}$ gradually decreases to a certain distance, and then increases. Therefore, there is no collision between the virtual point and the UAV.

If $v_{u}$ is bigger than $v_{i}$, then the $\left(v_{c R i}, v_{c \lambda i}\right)$ point moves to a top of the circle (A) which is on the $v_{R i}$-axis. Since $v_{\lambda i}$ is $R_{i} \dot{\lambda_{i}}$, positioning on the $v_{R i}$-axis means that the UAV flies along the LOS to the virtual point; because $\dot{\lambda_{i}}$ is zero. At the ' A ' point, since $\dot{R}_{i}$ is negative and the UAV is on the LOS to the virtual point, a collision between the virtual point and the UAV occurs. If the collision occurs in the approach phase, the UAV may pass the virtual point and then arrive at the CTA before the tanker. This is because the velocity of the virtual point is set to make the virtual point arrive at the CTA simultaneously with the tanker. To accomplish an aerial refueling mission, the tanker passes the CTA earlier than the UAV does, like the tail-chase maneuver of the UAV to the tanker. Therefore, in this study, the case that $v_{i}$ is bigger than $v_{u}$ is considered. In other words, if the UAV has a lower velocity than the virtual point, the UAV will arrive at the CTA later than the tanker. Therefore, the first condition of the initial velocity range of the UAV can be set as follows

$$
\begin{equation*}
v_{u}<v_{i} \tag{3.19}
\end{equation*}
$$

To obtain the minimum value of the possible initial velocity range of the UAV, the following situations are considered: i) when the UAV has the minimum velocity, and ii) when the UAV has the maximum velocity in the possible velocity range.

Let us consider the first case that the UAV flies with its minimum velocity. In this case, at the transition, the UAV will be on the velocity circle and the tanker will fly ahead of the UAV until the difference angle between the LOS angle of the UAV to the tanker and the heading angle of the UAV becomes to $\theta_{c}$. Figures 3.6 and 3.7 show the geometry of the UAV and the tanker at


Figure 3.6:
The transition when the UAV has the minimum velocity


Figure 3.7: Relative angle and distance
the transition when the UAV flies with its minimum velocity.
From Fig. 3.7, the transition time when the UAV has the minimum velocity, $t_{\text {tran }_{\text {min }}}$, can be calculated using a flight distance of the tanker as follows

$$
\begin{equation*}
t_{\text {tran }_{\text {min }}}=t_{C T A}+\frac{R_{c} \sin \left(\theta_{c}\right)}{\sin \left(\psi_{T}-\frac{\pi}{2}-\theta_{c}\right) v_{T}} \tag{3.20}
\end{equation*}
$$

where $t_{C T A}$ denotes the time when the tanker passes the CTA.
In contrast, if the UAV has the maximum velocity in the possible range, the tanker and the UAV will pass the CTA almost at the same time, because the UAV cannot pass ahead of the tanker in the approach phase. Therefore, in this case, the transition time when the UAV has the maximum velocity, $t_{\text {tran }_{\text {max }}}$, can be obtained as follows

$$
\begin{equation*}
t_{\operatorname{tran}_{\max }} \approx t_{C T A} \tag{3.21}
\end{equation*}
$$

Now, let us consider a real flight distance of the UAV. In this case, the trajectory is approximated to a straight line as shown in Fig. 3.8.

The relation for the angle $\alpha$ can be obtained using the geometry shown in Fig. 3.9.

$$
\begin{align*}
S & =s_{1}+s_{2}=s_{1}+l_{u}-s_{1} \sin \alpha=s_{1}(1-\sin \alpha)+l_{u} \\
& =\frac{l_{1}}{\cos \alpha}(1-\sin \alpha)+l_{u} \tag{3.22}
\end{align*}
$$

where $S$ denotes the real flight distance before the transition. Note that $\alpha$ is almost same even though a velocity of the UAV changes.

The flight distance, when the UAV has the minimum velocity or the maximum velocity, can be obtained using $\alpha$. Figure 3.10 shows each situation. The flight distances can be calculated as follow


Figure 3.8: Approximation of a real trajectory of the UAV


Figure 3.9: Approximated trajectory of the UAV


Figure 3.10:
Two cases of $v_{u}$ ( L : the minimum / R: the maximum)


Figure 3.11: A relation for selecting a random velocity
a) Distance when the UAV has the minimum velocity, $S_{\text {min }}$

$$
\begin{equation*}
S_{\min }=\frac{l_{1}}{\cos \alpha}(1-\sin \alpha)+l_{2}-R_{c} \tag{3.23}
\end{equation*}
$$

b) Distance when the UAV has the maximum velocity, $S_{\max }$

$$
\begin{equation*}
S_{\max }=\frac{l_{1}}{\cos \alpha}(1-\sin \alpha)+l_{2} \tag{3.24}
\end{equation*}
$$

Using Eqs. (3.20) and (3.21) and Eqs. (3.23) and (3.24), the possible initial velocity range of the UAV for the transition can be obtained as follow
a) The possible minimum velocity of the UAV, $v_{u_{\text {min }}}$

$$
\begin{equation*}
v_{u_{\min }}=\frac{S_{\min }}{t_{t r a n_{m i n}}} \tag{3.25}
\end{equation*}
$$

b) The possible maximum velocity of the UAV, $v_{u_{\max }}$

$$
\begin{equation*}
v_{u_{\max }}=\frac{S_{\max }}{t_{\operatorname{tran}_{\max }}} \tag{3.26}
\end{equation*}
$$

The initial guess of the flight distance is needed to calculate the possible velocity range, which can be obtained through numerical calculation. For this calculation, the random velocity is selected through the following procedure.

From Fig. 3.11, the random velocity ( $v_{\text {random }}$ ) can be set as

$$
\begin{equation*}
v_{\text {random }}=\operatorname{mean}\left(\frac{l_{2}-R_{c}}{\sin \beta \cdot t_{\text {tran }_{\min }}}, \frac{l_{2}}{\sin \beta \cdot t_{\text {tran }_{\max }}}\right) \tag{3.27}
\end{equation*}
$$

Combining Eq. (3.19), the possible velocity range of the UAV can be obtained as

$$
\begin{equation*}
v_{u_{\min }}<v_{u}<\min \left(v_{u_{\max }}, v_{i}\right) \tag{3.28}
\end{equation*}
$$

### 3.3 Rendezvous phase

The rendezvous phase is the phase from the transition to a rendezvous. In this phase, the UAV uses a pure pursuit guidance law
and a velocity controller. Using the pure pursuit guidance law, a lateral acceleration command normalized by the UAV velocity can be obtained as follow

$$
a_{c u}=-k_{r e n d}\left(\psi_{u}-\lambda\right)+\dot{\lambda}
$$

where $k_{\text {rend }}$ denotes a positive constant guidance gain. By controlling the velocity, the UAV can arrive at the RP simultaneously with the tanker, and then maintain the same velocity with the tanker. Thereby, the UAV can perform an aerial refueling mission after passing the RP.

### 3.3.1 Velocity control for the rendezvous



Figure 3.12: A desired velocity of the UAV

Let us define $v_{d}$ as a desired velocity. Then, an acceleration command in the direction of a velocity of the UAV, $a_{u 1}$, can be given as follows

$$
\begin{equation*}
a_{u 1}=k_{1} \frac{v_{d}-v_{u}}{t_{R}}=k_{1}\left(v_{d}-v_{u}\right) \frac{v_{T}+v_{d}}{R_{T}+R_{u}} \tag{3.29}
\end{equation*}
$$

where $k_{1}$ denotes a positive constant, $t_{R}$ denotes a required time for a rendezvous, and $R_{T}, R_{u}$ denote a remaining distance to the

RP of the tanker and the UAV, respectively. And $t_{R}$ is obtained by calculation of $\frac{R_{T}}{v_{T}}$. Note that the trajectory of the UAV after the transition is nearly straight.

Figure 3.12 shows a history of desired velocity $v_{d}$. From Fig. 3.12, the $v_{d}$ can be obtained as follows

$$
\begin{equation*}
\left\{k_{2} v_{d}+\left(1-k_{2}\right) v_{T}\right\} t_{R}=R_{u} \tag{3.30}
\end{equation*}
$$

or

$$
\begin{equation*}
v_{d}=\frac{1}{k_{2}}\left\{\frac{R_{u}}{t_{R}}-\left(1-k_{2}\right) v_{T}\right\} \tag{3.31}
\end{equation*}
$$

where $k_{2}$ denotes a positive constant gain, which range is $0<k_{2}<1$. Substituting Eq. (3.31) into Eq. (3.29), $a_{u 1}$ can be obtained as $a_{u 1}=\frac{k_{1}}{R_{T}+R_{u}}\left[\frac{1}{k_{2}}\left\{\frac{R_{u}}{t_{R}}-\left(1-k_{2}\right) v_{T}\right\}-v_{u}\right]\left[v_{T}+\frac{1}{k_{2}}\left\{\frac{R_{u}}{t_{R}}-\left(1-k_{2}\right) v_{T}\right\}\right]$

After a rendezvous at the RP, the UAV has to maintain the same velocity with the tanker for the aerial refueling mission. Therefore, the acceleration command has to be changed, and the following acceleration command, $a_{u 2}$, is proposed.

$$
\begin{align*}
a_{u 2} & =k_{3} \frac{v_{T}-v_{u}}{t_{R}} \\
& =k_{3}\left(v_{T}-v_{u}\right) \frac{v_{T}+v_{u}}{R_{T}+R_{u}} \\
& =k_{3} \frac{v_{T}^{2}-v_{u}^{2}}{R_{T}+R_{u}} \tag{3.33}
\end{align*}
$$

where $k_{3}$ denotes a positive constant gain.
When the desired velocity becomes almost same as the velocity of the tanker, the first acceleration command $a_{u 1}$ is switched to the second acceleration command $a_{u 2}$. Then, $a_{u 2}$ is used continuously after the rendezvous. For making the transition of these acceleration commands smoothly, $k_{3}$ is obtained as follows

$$
\begin{equation*}
k_{3}=\frac{a_{u 1_{t r}}\left(R_{T}+R_{u}\right)}{v_{T}^{2}-v_{u}^{2}} \tag{3.34}
\end{equation*}
$$

where $a_{u 1_{t r}}$ denotes $a_{u 1}$ at the transition of the acceleration commands, and the calculation of $k_{3}$ is performed using the values at the transition.

## Chapter 4

## Rendezvous Path Planning in Three Dimensions

### 4.1 Approach phase



Figure 4.1: A trajectory of a virtual point in three dimensions

Expanding the concept of rendezvous path planning in two dimensions, a trajectory of the virtual point is considered as shown in Fig. 4.1. Using the similar method in two dimensions, the position of the virtual point is needed, but $z_{i}$ position of the virtual point has to be set as follows
a) Before passing the CTA

$$
\begin{equation*}
z_{i}=z_{C T A}-\left(d-\frac{d}{t_{C T A}} t\right) \sin \left(\gamma-\frac{\gamma}{t_{C T A}}\right) \tag{4.1}
\end{equation*}
$$

b) After passing the CTA

$$
\begin{equation*}
z_{i}=z_{C T A} \tag{4.2}
\end{equation*}
$$

where $z_{C T A}$ denotes the $z$-position of the CTA, $d$ denotes a distance between the CTA and the initial point of the virtual point, and $\gamma$ denotes the angle between a trajectory of the tanker and the straight line from the initial point of the virtual point to the CTA as shown in Fig. 4.1. By setting the trajectory of the virtual point using Eqs. (4.1) and (4.2), it is expected that the UAV will level off near the CTA. And $v_{i}$ is updated to make the virtual point arrive at the CTA simultaneously with the tanker as follows

$$
\begin{equation*}
v_{i}=\frac{\left\|\left(x_{C T A}, y_{C T A}, z_{C T A}\right)-\left(x_{i}, y_{i}, z_{i}\right)\right\|}{t_{C T A}} \tag{4.3}
\end{equation*}
$$

where $\left(x_{C T A}, y_{C T A}, z_{C T A}\right)$ denotes a position of the CTA.
Since the virtual point is set to move at a same altitude with the tanker after the CTA, the UAV can level off near the CTA. The UAV maintains a constant velocity according to the x-y plane during the approach phase, similar to the two dimensional case. Therefore, a possible initial velocity range of the UAV, obtained in the section 3.2.3, can be also applied to the three dimensional environment.

In Fig. 4.2, $\zeta$ denotes a desired angle to adjust the difference between a UAV pitch angle $\theta_{u}$ and a vertical angle to the virtual point. Therefore, the vertical acceleration command can be derived as

$$
\begin{equation*}
a_{v e r t}=k_{a p p}\left(\frac{v_{u}}{R_{i}}\right) \sin \zeta \tag{4.4}
\end{equation*}
$$



Figure 4.2: Vertical acceleration in three dimensions
where $R_{i}$ denotes the distance between a virtual point and the UAV, and $k_{\text {app }}$ denotes a positive constant gain.

Similarly, the lateral acceleration command is given as follows

$$
\begin{equation*}
a_{l a t}=k_{a p p}\left(\frac{v_{u_{x y}}}{R_{i_{x y}}}\right) \sin \eta \tag{4.5}
\end{equation*}
$$

where $v_{u_{x y}}$ denotes a velocity of the UAV according to the $\mathrm{x}-\mathrm{y}$ plane, and $R_{i_{x y}}$ denotes the distance between a virtual point and the UAV according to the $\mathrm{x}-\mathrm{y}$ plane.

Criteria for the transition are similar to those in two dimensional case. In detail, the criterion of a distance uses the distance in three dimensions, and the criterion of an angle uses the angle according to the $\mathrm{x}-\mathrm{y}$ plane.

### 4.2 Velocity optimization to minimize energy consumption

In this study, it is assumed that the UAV maintains a constant velocity in the $x-y$ plane during the approach phase, and therefore the velocity of the UAV should increase when the UAV climbs to the altitude of the tanker. During the level-off, the velocity of the UAV decreases as the pitch angle decreases. Moreover, the UAV has to increase its velocity to meet the tanker in the rendezvous phase. Therefore, 'increasing and then decreasing the velocity in the approach phase' is not efficient to manage the energy. In this study, the following performance index $\left(J_{v e l}\right)$ is considered to optimize the velocity of the UAV.

$$
\begin{equation*}
J_{v e l}=\int_{0}^{t_{f 1}} a^{2}{ }_{v e l}(t) d t \tag{4.6}
\end{equation*}
$$

where $t_{f 1}$ denotes the time at the transition, and $a_{v e l}$ denotes the acceleration in the direction of velocity of the UAV.

Now, the optimization problem can be defined as

$$
\begin{equation*}
\text { Minimize } \int_{0}^{t_{f 1}} a^{2}{ }_{v e l}(t) d t \tag{4.7}
\end{equation*}
$$

subject to

$$
\begin{equation*}
S_{1}=\int_{0}^{t_{f 1}} v_{u}(t) d t \tag{4.8}
\end{equation*}
$$

Let us define $z(t) \equiv \int_{0}^{t} v_{u}(t) d t$, then we have

$$
\begin{equation*}
\dot{z}(t)=v_{u}(t), \quad z(0)=0, \quad z\left(t_{f 1}\right)=S_{1} \tag{4.9}
\end{equation*}
$$

The Lagrangian can be defined as

$$
\begin{equation*}
L_{a}=a^{2}{ }_{v e l}(t)+\chi(t)\left\{\dot{z}(t)-v_{u}(t)\right\}=\dot{v}_{u}(t)^{2}+\chi(t)\left\{\dot{z}(t)-v_{u}(t)\right\} \tag{4.10}
\end{equation*}
$$

where $S_{1}$ denotes a flight length of the UAV in the approach phase, and $\chi(t)$ denotes a Lagrange multiplier.

Necessary conditions for the optimization can be written as follows

$$
\begin{align*}
& \frac{\delta L_{a}}{\delta v_{u}}-\frac{d}{d t}\left(\frac{\delta L_{a}}{\delta \dot{v}_{u}}\right)=-\chi(t)-\frac{d}{d t}\left(2 \dot{v}_{u}(t)\right)=0  \tag{4.11}\\
& \frac{\delta L_{a}}{\delta z}-\frac{d}{d t}\left(\frac{\delta L_{a}}{\delta \dot{z}}\right)=-\frac{d}{d t}(\chi(t))=0  \tag{4.12}\\
& \frac{\delta L_{a}}{\delta \chi}-\frac{d}{d t}\left(\frac{\delta L_{a}}{\delta \dot{\chi}}\right)=\dot{z}(t)-v_{u}(t)=0 \tag{4.13}
\end{align*}
$$

Using Eqs. (4.11) - (4.13), the acceleration command can be obtained as

$$
\begin{align*}
& \frac{d}{d t}\left(2 \dot{v}_{u}(t)\right)=2 \dot{a}_{\text {vel }}(t)=C  \tag{4.14}\\
& a_{v e l}=c_{1} t+c_{2}=c_{1} t  \tag{4.15}\\
& v_{u}(t)=\frac{c_{1}}{2} t^{2}+c_{2} t+c_{3}=\frac{c_{1}}{2} t^{2}+v_{u_{0}} \tag{4.16}
\end{align*}
$$

Note that $\chi(t)$ is constant, $a_{v e l}(0)=0$, and $v_{u}(0)=v_{u_{0}}$.
Substituting Eq. (4.16) into Eqs. (4.8) and (4.9) yields

$$
\begin{align*}
& S_{1}=\frac{c_{1}}{6} t_{f 1}^{3}+v_{u_{0}} t_{f 1}  \tag{4.17}\\
& z(t)=\frac{c_{1}}{6} t^{3}+v_{u_{0}} t \tag{4.18}
\end{align*}
$$

and

$$
\begin{equation*}
c_{1}=\frac{6}{t_{1}^{3}}\left(S_{1}-v_{u_{0}} t_{1}\right) \tag{4.19}
\end{equation*}
$$

where $C, c_{1}, c_{2}, c_{3}$ are constants. Using the Eq. (4.19), the optimized acceleration command and velocity can be obtained. The optimized acceleration command makes the velocity of the UAV continue to increase in the approach phase.

### 4.3 Rendezvous phase

After the transition of phases, the UAV may not level off to the same altitude with the tanker as shown in Fig. 4.3. Therefore, the following vertical acceleration command has to be used continuously.

$$
\begin{equation*}
a_{v e r t}=k_{r e n d}\left(\frac{v_{u}}{R}\right) \sin \xi \tag{4.20}
\end{equation*}
$$

where $\xi$ denotes a desired angle to adjust the difference between a UAV pitch angle and a vertical angle to the tanker.


Figure 4.3: A relation of angle in the rendezvous phase

The lateral acceleration command, $a_{l a t}$, and the acceleration command in the direction of velocity, $a_{v e l}\left(a_{u 1} \& a_{u 2}\right)$, are obtained using by the same formula in the two-dimensional case as

$$
\begin{gather*}
a_{l a t}=-k_{r e n d}\left(\psi_{u_{x y}}-\lambda_{x y}\right)+\dot{\lambda}_{x y}  \tag{4.21}\\
a_{u 1}=\frac{k_{1}}{R_{T}+R_{u}}\left[\frac{1}{k_{2}}\left\{\frac{R_{u}}{t_{R}}-\left(1-k_{2}\right) v_{T}\right\}-v_{u}\right]\left[v_{T}+\frac{1}{k_{2}}\left\{\frac{R_{u}}{t_{R}}-\left(1-k_{2}\right) v_{T}\right\}\right]
\end{gather*}
$$

$$
a_{u 2}=k_{3} \frac{v_{T}^{2}-v_{u}^{2}}{R_{T}+R_{u}}
$$

where $\psi_{u_{x y}}$ denotes a heading angle of the UAV according to the $\mathrm{x}-\mathrm{y}$ plane, and $\lambda_{x y}$ denotes the LOS angle according to the $\mathrm{x}-\mathrm{y}$ plane.

### 4.4 Pseudo pursuit guidance vs. straight climb in the approach phase

In this section, two cases, the case of straight climb and the case of climb using the pseudo pursuit guidance in the approach phase, are compared.


Figure 4.4: A failure of the transition (straight climb)

In the case of straight climb, there are some cases which fail the transition of phases. For example, a UAV may climb to near the altitude of the RP after passing the trajectory of the tanker as shown in Fig. 4.4. Since the UAV has the maximum limit of pitch angle, $\theta_{\max }$, the UAV may pass the trajectory of the tanker
before the transition. Therefore, it can be stated that failure occurs due to the maximum limit of the pitch angle of the UAV and a difference in initial heading angles.

Moreover, if the UAV climbs straightly in the approach phase, the UAV can scarcely catch up with the tanker during the climb. Therefore, the UAV needs much more acceleration for the rendezvous with the tanker than the case of climb using the pseudo pursuit guidance.

As a result, using the pseudo pursuit guidance in the approach phase is more efficient than the straight climb.

## Chapter 5

## Numerical Simulations

### 5.1 Simulation Environment

Numerical simulations are conducted to demonstrate the performance of the proposed path planning and guidance method. In the simulation, the following assumptions are considered. First, RP is located between the positions of the tanker and the UAV, second, RP is located far enough away from the initial positions of the tanker and the UAV. It is also assumed that no wind exists, and the dynamics of the tanker and the UAV are not considered.

It is assumed that the UAV knows the information of the tanker and a rendezvous point by receiving the data from the tanker, which are summarized in Table 5.1.

Table 5.1: Information provided to UAV

| Arrival time at the RP of the tanker |
| :---: |
| Velocity of the tanker |
| Heading angle of the tanker after the rendezvous |
| Flight altitude of the tanker |
| Position of combining point from the tanker |

Until the RP, the tanker flies straightly and the UAV performs maneuver using the proposed guidance laws. Success of the rendezvous is judged when the distance, $R$, between the tanker and the UAV, is nearer than 0.1 m and the relative velocity, $\dot{R}$, is less
than $0.016 \mathrm{~m} / \mathrm{s}[7]$.

Table 5.2: Data of A330 MRTT

| Airbus A330 MRTT |  |
| :---: | :---: |
| Maximum speed | $880 \mathrm{~km} / \mathrm{h}(475 \mathrm{knots}, 244 \mathrm{~m} / \mathrm{s})$ |
| Cruise speed | $860 \mathrm{~km} / \mathrm{h}(464 \mathrm{knots}, 239 \mathrm{~m} / \mathrm{s})$ |
| Take-off speed | $356 \mathrm{~km} / \mathrm{h}(192 \mathrm{knots}, 99 \mathrm{~m} / \mathrm{s})$ |
| Landing speed | $333 \sim 352 \mathrm{~km} / \mathrm{h}(180 \sim 190 \mathrm{knots}, 93 \sim 98 \mathrm{~m} / \mathrm{s})$ |
| Holding speed | $445 \mathrm{~km} / \mathrm{h}(240 \mathrm{knots}, 123 \mathrm{~m} / \mathrm{s})$ |

Table 5.3: Performance data of UAVs

| MQ-9 Reaper |  |
| :---: | :---: |
| Maximum speed | $482 \mathrm{~km} / \mathrm{h}(260 \mathrm{knots}, 134 \mathrm{~m} / \mathrm{s})$ |
| Cruise speed | $313 \mathrm{~km} / \mathrm{h}(169 \mathrm{knots}, 87 \mathrm{~m} / \mathrm{s})$ |
| HFT-60A |  |
| Maximum speed | $600 \mathrm{~km} / \mathrm{h}(324 \mathrm{knots}, 167 \mathrm{~m} / \mathrm{s})$ |

Tables 5.2 and 5.3 summarize the data of A330 MRTT and several UAVs [17] [18] [19]. Considering the data, the initial conditions are considered in the simulation. The maximum limitation speed is set as $160 \mathrm{~m} / \mathrm{s}$.

The CTA is located between the RP and the initial position of the tanker. If the CTA is closer to the initial position of the tanker, the period for the velocity control shown in Fig. 5.1 will become longer. Then, the maximum velocity of the UAV in the rendezvous phase will be smaller. However, the longer the velocity control period becomes, the smaller the period of the approach phase becomes. The position of the CTA is determined by the following equation in this study.

$$
\mathrm{CTA}=\text { Tanker0 }+k_{C T A}(\mathrm{RP}-\text { Tanker0 })
$$

where $k_{C T A}$ denotes a positive constant, which range is $0<k_{C T A}<1$.


Figure 5.1: Position of the CTA

### 5.2 Simulation Results

### 5.2.1 Single UAV case

Considering the data in the Tables 5.2 and 5.3 , the initial conditions considered in the simulation are summarized in Table 5.4. In Table 5.4, 'Tanker0' and 'UAV0' denote the initial positions of the tanker and the UAV, respectively. The constant guidance command gains of the acceleration commands used in this study are positive, which are summarized in Table 5.5.

Table 5.4: Initial values for single UAV case

| RP $(m)$ | $[5000,250000,6000]$ |
| :---: | :---: |
| Tanker0 $(m)$ | $[40000,0,6000]$ |
| UAV0 $(m)$ | $[0,0,0]$ |
| $v_{t 0}(m / s)$ | 120 |
| $v_{u 0}(m / s)$ | 100 |
| $\psi_{u 0}($ degree $)$ | 20 |

Table 5.5: Guidance command gains for single UAV case

| $k_{\text {app }}$ | $k_{\text {rend }}$ | $k_{C T A}$ | $k_{1}$ | $k_{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 5.0 | 0.07 | 0.2 | 5.5 | 0.56 |

Figures 5.2-5.8 show the simulation results. The rendezvous time is $2,103 \mathrm{~s}$, and the maximum speed of the UAV is $153.72 \mathrm{~m} / \mathrm{s}$. At the rendezvous time, the distance is about $0.016 m$, and the relative velocity of the UAV with respect to the tanker is about $-5.4 \cdot 10^{-4} \mathrm{~m} / \mathrm{s}$. Therefore, it can be stated that the rendezvous between the tanker and the UAV is successful conducted.

Figure 5.2 shows the trajectories of the tanker and the UAV in the approach phase, and Fig. 5.3 shows the trajectories of the tanker and the UAV in the rendezvous phase. Figures 5.4 - 5.6 show the history of the distance between the tanker and the UAV, relative velocity, velocity of the UAV, respectively. Figure 5.7 shows the history of acceleration commands of the UAV, and Fig. 5.8 shows the three dimensional trajectories of the tanker and the UAV.


Figure 5.2: Trajectories in the approach phase


Figure 5.3: Trajectories in the rendezvous phase


Figure 5.4: History of relative distance between tanker and UAV


Figure 5.5:
Relative velocity of the UAV with respect to the tanker


Figure 5.6: History of UAV velocity




Figure 5.7: History of UAV acceleration commands


Figure 5.8: Three dimensional trajectory

### 5.2.2 Multiple UAVs case

Now, an aerial refueling mission between one tanker and four UAVs is considered for the second numerical simulation. The initial values and guidance command gains of the second scenario are summarized in Table 5.6 and 5.7. Flight trajectories of the multiple UAVs scenario are shown in Fig. 5.9. As shown in Fig. 5.9, the tanker turns with $30^{\circ}$ bank angle.

In this simulation, an aerial refueling time is set as $90 s$, which is decided based on the half scaling of the combat aircraft refueling case. During the aerial refueling, the tanker and the UAV fly straightly. After the aerial refueling, the UAV will break out from the trajectory of the tanker. The histories of velocity and acceleration commands of the four UAVs are shown in Figs. 5.105.17. Simulation results are summarized in Table 5.8. In Table 5.8, $v_{u m a x}$ denotes the maximum velocity of the UAV.

Table 5.6: Initial values of multiple UAVs case

| UAV1 |  | UAV2 |  |
| :---: | :---: | :---: | :---: |
| RP $\left(10^{3} \mathrm{~m}\right)$ | $[5,180,6]$ | $\operatorname{RP}\left(10^{3} \mathrm{~m}\right)$ | $[1,150,6]$ |
| Tanker0 $\left(10^{3} \mathrm{~m}\right)$ | $[32,0,6]$ | Tanker0 $\left(10^{3} \mathrm{~m}\right)$ | $[-65,236,6]$ |
| UAV0 $\left(10^{3} \mathrm{~m}\right)$ | $[0,0,0]$ | UAV0 $\left(10^{3} \mathrm{~m}\right)$ | $[-65000,261000,0]$ |
| $v_{t 0}(\mathrm{~m} / \mathrm{s})$ | 120 | $v_{t 0}(\mathrm{~m} / \mathrm{s})$ | 120 |
| $v_{u 0}(\mathrm{~m} / \mathrm{s})$ | 100 | $v_{u 0}(\mathrm{~m} / \mathrm{s})$ | 100 |
| $\psi_{u 0}($ degree $)$ | 20 | $\psi_{u 0}($ degree $)$ | 30 |
| UAV3 |  | UAV4 |  |
| RP $\left(10^{3} \mathrm{~m}\right)$ | $[3,160,6]$ | $\operatorname{RP}\left(10^{3} \mathrm{~m}\right)$ | $[2,170,6]$ |
| Tanker0 $\left(10^{3} \mathrm{~m}\right)$ | $[139,340,6]$ | Tanker0 $\left(10^{3} \mathrm{~m}\right)$ | $[255,128,6]$ |
| UAV0 $\left(10^{3} \mathrm{~m}\right)$ | $[168,340,0]$ | UAV0 $\left(10^{3} \mathrm{~m}\right)$ | $[2550,98,0]$ |
| $v_{t 0}(\mathrm{~m} / \mathrm{s})$ | 120 | $v_{t 0}(\mathrm{~m} / \mathrm{s})$ | 120 |
| $v_{u 0}(\mathrm{~m} / \mathrm{s})$ | 100 | $v_{u 0}(\mathrm{~m} / \mathrm{s})$ | 100 |
| $\psi_{u 0}($ degree $)$ | 30 | $\psi_{u 0}($ degree $)$ | 40 |

Table 5.7: Guidance command gains for multiple UAVs case

| UAV1 | $k_{\text {app }}$ | $k_{\text {rend }}$ | $k_{C T A}$ | $k_{1}$ | $k_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5.0 | 0.12 | 0.2 | 5.5 | 0.56 |
| UAV2 | $k_{\text {app }}$ | $k_{\text {rend }}$ | $k_{C T A}$ | $k_{1}$ | $k_{2}$ |
|  | 5.0 | 0.15 | 0.2 | 5.2 | 0.54 |
| UAV3 | $k_{\text {app }}$ | $k_{\text {rend }}$ | $k_{C T A}$ | $k_{1}$ | $k_{2}$ |
|  | 5.0 | 0.12 | 0.2 | 5.2 | 0.57 |
| UAV4 | $k_{\text {app }}$ | $k_{\text {rend }}$ | $k_{C T A}$ | $k_{1}$ | $k_{2}$ |
|  | 5.0 | 0.12 | 0.2 | 5.3 | 0.56 |



Figure 5.9: Rendezvous between one tanker and four UAVs

Table 5.8: Simulation results of multiple UAVs case

|  | UAV1 | UAV2 | UAV3 | UAV4 |
| :---: | :---: | :---: | :---: | :---: |
| $R(\mathrm{~m})$ | 0.0190 | 0.0309 | 0.0047 | 0.0105 |
| $\dot{R}(\mathrm{~m} / \mathrm{s})$ | $-9.53 \cdot 10^{-5}$ | $5.69 \cdot 10^{-5}$ | $-5.17 \cdot 10^{-4}$ | $-5.21 \cdot 10^{-5}$ |
| $t_{R}(s)$ | 1516.8 | 1265.9 | 1350.8 | 1435.8 |
| $v_{\text {umax }}(\mathrm{m} / \mathrm{s})$ | 155.7715 | 153.2394 | 155.1266 | 154.5505 |

The departure times of the UAVs are calculated for the sequential aerial refueling. By calculating the departure times, UAVs do not need a holing time until each order of the aerial refueling mission comes. The calculated departure times of the simulation are summarized in Table 5.9. UAVs leave the each initial positions at the each departure time. Then, the UAVs perform the sequential rendezvous without holding maneuver successfully.

Table 5.9: Departure times of the UAVs

|  | UAV1 | UAV2 | UAV3 | UAV4 |
| :---: | :---: | :---: | :---: | :---: |
| Departure time $(s)$ | 0 | 1552.99 | 2773.70 | 3994.09 |



Figure 5.10: History of UAV1 velocity


Figure 5.11: History of UAV1 acceleration commands


Figure 5.12: History of UAV2 velocity


Figure 5.13: History of UAV2 acceleration commands


Figure 5.14: History of UAV3 velocity


Figure 5.15: History of UAV3 acceleration commands


Figure 5.16: History of UAV4 velocity


Figure 5.17: History of UAV4 acceleration commands

The total time for performing aerial refueling missions between one tanker and four UAVs, which is from the beginning of the first UAV to the completeness of the total aerial refueling mission, is 5, 519.84s.

## Chapter 6

## Conclusions

### 6.1 Concluding Remarks

In this thesis, rendezvous path planning and guidance schemes of UAV for aerial refueling are proposed. Many existing studies have dealt with proportional navigation guidance law and pursuit guidance law for rendezvous, but most of them have shown rendezvous without specified rendezvous position. Since one tanker generally has to perform aerial refueling mission with multiple UAVs, UAVs engaged in the aerial refueling mission must hold in the air until each turn comes. In this study, a path planning that can make UAVs perform aerial refueling mission without holding maneuver is proposed. Therefore, the energy used in the holding maneuver can be saved. The rendezvous time can be also calculated using the distance between the RP and the tanker and the velocity of the tanker, thereby, the departure time of UAVs can be assigned beforehand. It is expected that the results of this study can make UAV operation management of the aerial refueling efficient.

### 6.2 Future Work

To apply the proposed method to the real operation, there exist several issues to be considered: i) dynamics of the tanker and the UAV, ii) the feasible area to perform rendezvous and aerial
refueling mission, iii) wind condition, etc.
Also, the position of the CTA affects the acceleration commands of the UAV. In other words, the determination of the position of the CTA influences on the energy consumption during the UAV maneuver. Therefore, the problem in regard to the position of the CTA is important to minimize the energy consumption.

By considering the above items in detail, the proposed path planning scheme will become more practical for the UAV aerial refueling mission.

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## 국문초록

체공 중인 항공기에 연료를 공급하는 공중급유는 항속거리와 작전 가능 시간을 늘려 항공기에게 다양한 임무를 효율적으로 수행할 수 있게 한다. 특히, 무인기의 경우 연료의 제한이 크기 때문에 임무수행 영역에 제한이 생긴다. 일반적으로 한 대의 급유기와 다수의 항공기 간 공중급유가 이루어지므로, 효율적인 공중급유 수행을 위해서는 사전 계획된 지점에서 계획된 시각에 랑데부가 이루어지도록 해야 한다.

본 연구에서는 무인기 공중급유를 위한 경로를 두 단계로 나누어 제안하고, 무인기의 속도 제어를 통해 사전 계획된 지점에서 랑데 부를 수행하는 유도기법을 제안하였다. 랑데부 이후에는 공중급유 수행을 위해 급유기와 무인기가 동일한 속도를 유지하도록 하여 일 정 간격을 유지시켰다. 제안한 2 차원에서의 랑데부 경로 계획 및 유도기법을 3 차원으로 확장시켜 적용하였으며, 무인기의 에너지 사 용을 최소화하기 위한 최적화를 수행하였다. 제안한 기법은 무인기의 급격한 선회를 방지하며, 공중급유를 위해 무인기가 급유기의 후방 에 위치하도록 한다. 수치 시뮬레이션을 통해 제안한 랑데부 기법의 성능을 검증하였으며, 최종적으로 한대의 급유기와 다수 무인기 간 공중급유 시나리오를 통해 제안한 기법의 효율성을 확인하였다.

주요어 : 랑데부, 추적유도기법, 공중급유, 무인기
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