저작자표시-비영리-변경금지 2.0 대한민국

이용자는 아래의 조건을 따르는 경우에 한하여 자유롭게

- 이 저작물을 복제, 배포, 전송, 전시, 공연 및 방송할 수 있습니다.

다음과 같은 조건을 따라야 합니다:

저작자표시. 귀하는 원저작자를 표시하여야 합니다.

비영리. 귀하는 이 저작물을 영리 목적으로 이용할 수 없습니다.

변경금지. 귀하는 이 저작물을 개작, 변형 또는 가공할 수 없습니다.

- 귀하는, 이 저작물의 재이용이나 배포의 경우, 이 저작물에 적용된 이용허락조건을 명확하게 나타내어야 합니다.
- 저작권자로부터 별도의 허가를 받으면 이러한 조건들은 적용되지 않습니다.

저작권법에 따른 이용자의 권리는 위의 내용에 의하여 영향을 받지 않습니다.

이것은 이용허락규약(Legal Code)을 이해하기 쉽게 요약한 것입니다.

Disclaimer
Behavior and Design of Eccentrically Loaded Group Bolted Connections Considering Bearing Boundary Conditions

경제조건을 고려한 편심하중을 받는 군볼트 접합부의 거동 및 설계

2014년 8월

서울대학교 대학원 건축학과 진승표
Behavior and design of Eccentrically Loaded Group Bolted Connections Considering Bearing Boundary Conditions

지도교수 이철호

이 논문을 공학석사 학위논문으로 제출함 2014 년 8 월

서울대학교 대학원 건축학과 진승표

진승표의 공학석사 학위논문을 인준함 2014 년 8 월

위원 장홍성 해 (고)
부위원장 이철호 (인)
위원 박홍수 (인)
Abstract

Behavior and Design of Eccentrically Loaded Group Bolted Connections Considering Bearing Boundary Conditions

Jin, Seung-Pyo
Department of Architecture and Architectural Engineering
College of Engineering
Seoul National University

Steel structures have many advantages of material and prefabrication characteristics than reinforced concrete structures. In steel structures, group bolted connections and welded connections are typical methods as these prefabrications. Of these, well-designed bolted connections can exhibit excellent ductile behavior through bearing mechanism until the occurrence of bolt shear rupture.

Two approaches exist for design group bolted connections under eccentric load: a traditional elastic analysis and the more accurate (but more complex) ultimate strength analysis. The elastic analysis is widely used for decades because it is very easy to apply. But the elastic analysis brings about more conservative results than the ultimate strength analysis. In the elastic analysis, the behavior between force and deformation of single bolted connections is assumed to be proportional although it shows a nonlinear relationship. To calculate a more accurate capacity of group bolted connection (ultimate strength design), the nonlinear relationship between force and deformation of single bolted connections has to be defined preferentially. However, the
current bolt load-deformation relationship, which the basis of the current ultimate strength method, was derived long ago by Crawford and Kulak (1971), is based on one-type of 3/4in-A325 bolt and A36 steel combination. Considering that combination of different bolt sizes and steel specifications are now widely used in designing group bolted connections, the applicability and reliability of the current approach needs to be evaluated.

Each fastener in group bolted connection under eccentric load has different boundary conditions depending on displacement of that. For example, in the case of the fasteners of which displacement is pointing to edge of the connection is governed by end tearing of base steel. Although the fasteners can have that boundary condition, which is ‘open boundary condition’, the current design practice only consider ‘closed boundary condition’ which is governed by bolt shear rupture and excessive bolt hole deformation. Therefore, in this study, single bolt tests with variables such as steel specification, thickness of a base steel, bolt diameter and edge distance were carried out to propose the force-deformation relationship depending on the boundary condition of fasteners.

The test results with the open boundary conditions revealed that the bolt force-deformation relationship is strongly affected by the edge distance, the strength and the thickness of the steel plates. On the other hand, in the tests with the closed boundary condition, the projected area of the bolt hole, the strength and the thickness of the plates were the main factor affecting the force-deformation relationship. From these approach, to propose generalized force-deformation equation of both types single bolted connection, normalization was conducted by current bearing strength, which are $3.0F_u dt$ and $1.2L_AtF_u$, of each specimen. From the normalized data based on the test results in this study, the force-deformation equation is proposed. Together with the normalized force-deformation relationship, the allowable
deformation limit of bolted connections was also proposed considering failure modes of the both boundary conditions. The available deformation capacity should be used as the limit deformation for redistributing the bolt forces in the ultimate strength design of eccentric group bolted connections. It was that the relationship proposed in this study can be used for a more accurate analysis and economical design of a variety of group bolted connections subjected to eccentric shear.

**Keywords:** Group bolted connection, bearing strength, force deformation relationship, boundary condition, ICRM

**Student Number:** 2012-23130
Table of Contents

Abstract .............................................................................................................................................. i
List of Figures .................................................................................................................................. vi
List of Tables ................................................................................................................................... viii
1. Introduction ................................................................................................................................. 1
   1.1 Background ............................................................................................................................. 1
   1.2 Objectives and scope .............................................................................................................. 4
2. Previous researches ..................................................................................................................... 8
   2.1 Elastic analysis ....................................................................................................................... 9
   2.2 Ultimate strength analysis ..................................................................................................... 10
      2.2.1 Relationship between force and deformation of single bolt ........................................ 11
      2.2.2 Design method of group bolted connection using ICRM (Crawford and Kulak, 1971) .... 18
3. Single bolt test and modeling : closed boundary condition ..................................................... 20
   3.1 Introduction ............................................................................................................................ 20
   3.2 Test set-up ............................................................................................................................ 21
   3.3 Test result and contemplation ............................................................................................... 22
   3.4 Force-deformation relationship ............................................................................................ 30
   3.5 Allowable deformation limit .................................................................................................. 32
4. Single bolt test and modeling : open boundary condition ........................................................ 35
   4.1 Introduction ............................................................................................................................ 35
   4.2 Test set-up ............................................................................................................................ 36
   4.3 Test result and contemplation ............................................................................................... 38
   4.4 Force-deformation relationship ............................................................................................ 52
   4.5 Allowable deformation limit .................................................................................................. 53
5. Application to design of group bolted connections ......................... 55
  5.1 Numerical method proposed to find ICR of connection ................. 55
  5.2 Design application to group bolted connection under eccentric load ........................................................................................................ 59
    5.2.1 Procedure of design application .............................................. 59
    5.2.2 Behavior of group bolted connection ...................................... 61
  5.3 Evaluation examples ....................................................................... 65
    5.3.1 Example 1 ................................................................................ 66
    5.3.2 Example 2 ................................................................................ 72
    5.3.2 Summary of results ................................................................ 76
6. Summary and conclusions .................................................................. 79
References ................................................................................................. 81
Korean Abstract .......................................................................................... 84
List of Figures

Figure 1 Ductile behavior of bolted connection (lap joint) ......................... 1
Figure 2 Catenary action of a moment frame .............................................. 3
Figure 3 Type of steel special bolted moment frame (Uang and Sato 2010) ............................................................................................................ 4
Figure 4 Boundary condition of fasteners in bolted connection................. 5
Figure 5 Behavior of single bolted connection under loading .................. 5
Figure 6 Failure mode of specimen having open boundary condition ...... 6
Figure 7 Failure mode of specimen having closed boundary condition..... 6
Figure 8 Scope of this research................................................................. 7
Figure 9 Group bolted connection under eccentric load ......................... 8
Figure 10 Concept of elastic analysis ....................................................... 9
Figure 11 Behavior of fastener using elastic analysis .............................. 10
Figure 12 Sawed section of a single bolt (Fisher, 1965) ......................... 11
Figure 13 analytical model using exponential function ......................... 12
Figure 14 Test set-up for compression test (Crawford and Kulak, 1971) . 15
Figure 15 Uang and Sato’s hysteresis rule (Uang and Sato, 2010) ......... 17
Figure 16 Cyclic testing of cold-formed SBMF (Uang and Sato, 2010)... 17
Figure 17 Instantaneous center of rotation method for design of eccentrically loaded fasteners ................................................................. 18
Figure 18 Test set-up for compression tests ......................................... 22
Figure 19 Deformed shape of jig after test ............................................. 22
Figure 20 Deformed shape of specimens after test ............................... 24
Figure 21 Measured load-displacement relationship of compression tests ............................................................................................................ 29
Figure 22 Comparison test results with Crawford and Kulak’s Equation . 30
Figure 23 Normalized force-displacement relationship of compression test results and proposed model......................................................... 31
Figure 24 Maximum allowable limit curve in accordance with plate
bearing strength vs. bolt shear strength ratio ............................................. 34
Figure 25 Test set-up for tension tests ....................................................... 36
Figure 26 Deformed shape of jig after test .............................................. 38
Figure 27 Lap plate prying mechanism (Wallaert and Fisher, 1964)........ 39
Figure 28 Deformed shape of specimens after tests .................................. 44
Figure 29 Measured load-displacement relationship of tension tests ....... 51
Figure 30 Normalized force-displacement relationship of tension test results and proposed model ................................................................. 53
Figure 31 Maximum allowable limit curve governed by bolt shear rupture in accordance with end tearing strength vs. bolt shear strength ratio ............................................................................................................ 54
Figure 32 Instantaneous center of rotation method for design .............. 56
Figure 33 Numerical algorithm or Flowchart for Instantaneous center .... 58
Figure 34 ICRM from the group-bolts which have different boundary conditions ......................................................................................... 59
Figure 35 Example of beam to column group bolted connection......... 61
Figure 36 Rotation of group bolted connection ........................................ 62
Figure 37 Evaluation example 1 (symmetry) ............................................. 65
Figure 38 Evaluation example 2 (asymmetry) ........................................... 66
Figure 39 Equivalent concentric load and moment for elastic analysis .... 67
Figure 40 Location of I.C. ........................................................................ 69
Figure 41 Strength reduction by nominal bolt shear strength in example. 70
Figure 42 Location of I.C. by proposed method ...................................... 71
Figure 43 Location of IC ......................................................................... 73
Figure 44 Location of ICR by proposed method ..................................... 75
Figure 45 Force distribution of group bolted connection under eccentric load (Evaluation example 1) ................................................................. 78
List of Tables

Table 1 Test matrix for compression tests ................................................. 21
Table 2 Results of compression tests .......................................................... 25
Table 3 Test variable and result of Yoon (2012) ........................................ 33
Table 4 Test matrix for tension tests ............................................................ 37
Table 5 Results of tension tests ................................................................. 40
Table 6 Result of elastic analysis ................................................................. 67
Table 7 The result of ICRM based on Crawford and Kulak’s approach ... 69
Table 8 Result of proposed method ............................................................. 72
Table 9 The result of elastic analysis ........................................................... 73
Table 10 The result of ICRM based on Crawford and Kulak’s approach . 74
Table 11 Result of proposed method ........................................................... 75
Table 12 Comparison among three design method ................................. 77
1. Introduction

1.1 Background

Steel structures have many more advantages of material and prefabrication characteristics than reinforced concrete structures. In steel structures, bolted connections and welded connections are typical methods as these prefabrications. Of these, the bolted connections are not mainly affected by a working environment because the construction process is very simple and clear. In addition, the bolted connections are massively used in every-day steel construction since this method is effective in shortening the construction period. Well designed bolted connections can exhibit excellent ductile behavior through bearing mechanism. But ductility of bolted connection is not well recognized in usual design practice. The ultimate strength analysis of the bolted connections is an economical and mechanistic approach which can take advantage of the excellent ductility of bolted connection.

![Figure 1 Ductile behavior of bolted connection (lap joint)](image)
In the case of typical design practice, the shear tab which connects a beam and a column is considered as the pinned connection. In fact, the shear tab can transfer to some degree as a semi-jigid connection due to a plastic deformation by bearing mechanism. Therefore, if this approach is considered, bolted connections can be designed economically. But the bearing mechanism of bolted connections is not well reflected to current design practice because the behavior of bolted connection and modeling of that is not established. Also, the domestic researches of that are much scarce. Lee and Kim (2005) carried out the study focused on the effect by a slip of bolted connection experimentally and analytically. Shin at al (2011) conducted the study about finite element analysis of beam-column connection. In their paper, the behavior of the moment connection with bolted web and high strength steel was studied by using the finite-element analysis computer program of ABAQUS. Yang (2012) conducted the tests to evaluated strength of bolted connection having standard and over bolt holes and compared the results of the tests with design criterion strength. Kim and Kim (2013) carried out the experimental study on structural behaviors of double shear bolted connections fabricated with ferritic stainless steel and Kim et al (2013) investigated the ultimate strength and curling influence using finite element analysis based on the test results of ferritic stainless steel bolted connections.

When the column in the moment frame is collapsed, it is necessary that the structure should reach the catenary action through large deformation to prevent the building from progressive collapse. For that, the rotation capacity of the connection is required more than 10% radian. While, In the case of welded moment connection, there is high probability of brittle failure, the connection using bearing mechanism can reach more easily the catenary action without brittle failure than welded connection.
Consisdering ductile behavior of bearing type bolted connection, Crawford and Kulak (1971) suggested pioneering method which is ICRM(Instantaneous Center of Rotation Method) using force redistribution. However, the force-deformation relationship of bolted connections, which forms the basis of the current practice, was based on the test without variables and boundary conditions. Therefore, the fore-deformation relationship proposed by Crawford and Kulak is not applicable to current design practice. Uang and Sato (2010) also gave attention to the bearing mechanism of bolted connections. They proposed a numerical method which can predict the hysteresis curve of the SBMF (steel special bolted moment frame). To establish predictable method, they modified the force-deformation equation proposed by Crawford and Kulak. But this equation was came from the cyclic test data of nine beam-column specimens and this process limits the failure mode of bolted connectiton. In addition, only consider thin steel plate which is less than 6.4mm. It means that additional test is needed. Therefore, there is no way of knowing applicability of their method to the other typical structure.
1.2 Objectives and scope

Each fastener in group bolted connection under eccentric load has different boundary condition. It means that the fasteners with different boundary condition show the different behavior. For example, in the Figure 4, the fasteners in the upper box is governed by end tearing of base steel unlike the fasteners in the lower box. In this study, a boundary condition which is governed by edage tearing in base steel is named ‘Open boundary condition’. On the contrary to this, the case of the fasteners having fixed end (the lower box in Figure 4) is called ‘Closed boundary condition’.

As shown in Figure 5, the fasteners with different boundary condition show different behavior. In the case of closed boundary condition, the large deformation occure at vincinty of bolt hole in the loading direction and the bolt shear rupture take place finally (Figure 5). In the case of open boundary condition, the failure mode of bolted connection is determined according to bolt shear strength and tearing strength of base steel. But the bolted connection is sure to be governed by edge tearing in base steel. These are well shown in Figure 6 and Figure 7 which show the specimens after the test.
Figure 4 Boundary condition of fasteners in bolted connection

Figure 5 Behavior of single bolted connection under loading
Therefore, in this study, the single bolt test with variables such as steel specification, thickness of base steel, bolt diameter and edge distance was
carried out to propose the force-deformation relationship depending on the boundary condition of fasteners. In addition, the allowable deformation limit of fasteners was suggested.

Figure 8 Scope of this research
2. Previous researches

The column bracket connection shown in Figure 9 is an example of a bolted connection subjected to eccentric shear. Two approaches exist for the solution of this problem: the traditional elastic analysis and the more accurate (but more complex) ultimate strength analysis.

The elastic analysis is widely used for decades because it is very easy to apply. But elastic analysis brings about more conservative results than ultimate strength analysis. In elastic analysis, the behavior between force and deformation of single bolted connection is assumed to be proportional although it shows a nonlinear relationship. To calculate a more accurate strength of bolted connection, the relationship between force and deformation of single bolted connection has to be defined preferentially.

In this chapter, traditional design methods for group bolted connection under eccentric load were explained and studied the limitation of those.

Figure 9 Group bolted connection under eccentric load


2.1 Elastic analysis

The existing traditional elastic analysis is based on the assumptions below.

① The base plate is assumed to be rigid body and bolt is considered as elastic body.
② Bolt and plate show elastic behavior.
③ Each reaction in group bolt is proportionate to each distance from centroid of bolted connection.

From the assumption ① and ②, the fastener load-deformation relationship is linear and that the yield stress is not exceeded. Also, the eccentric load \( P \) can be replaced with the same load acting at the centroid plus the couple, \( M=Pe \), where \( e \) is the eccentricity. Then, the maximum strength of connection can be calculated by sum of force vector at each fastener (Figure 10). Figure 11 shows what linear behavior of the fastener means (assumption ③).

![Figure 10 Concept of elastic analysis]

As before mentioned, the relationship between force and deformation of fastener is proportional, but experimental evidence conducted by Yoon (2012) and Crawford and Kulak (1971) shows that this is not the case.
Consequentially, the maximum strength from the elastic analysis is inaccurate and conservative side although this procedure is very simple.

2.2 Ultimate strength analysis

The real relationship between force and deformation of the fastener shows nonlinear behavior contrary to the assumption in elastic analysis. Therefore, to calculate maximum strength of group bolted connection under eccentric load, the fastener load-deformation relationship and maximum allowable limitation have to be defined. If these are well defined, the maximum strength of group bolted connection can be calculated.

Figure 11 Behavior of fastener using elastic analysis
2.2.1 Relationship between force and deformation of single bolt

(a) Fisher (1965)

In the experiment results, the behaviors of fastener show inelastic shape unlike the assumption in elastic analysis. This nonlinear behavior of the fastener is influenced by tensile strength of plate, shear strength of bolt and geometrical configuration.

J.W. Fisher discussed the behavior of the individual components of bolted or reveted joints in his report (1965). He suggested analytical models for the behaviors that are applicable to low alloy, low carbon steels such as A7, A36, A242, A440, and A441.

Figure 12 Sawed section of a single bolt (Fisher, 1965)

The shape of analyticla models in this report is exponential function (Figure 13) and the shape of the curve was governed by the ultimate shear strength and two empirical parameters which are $\mu$ and $\lambda$. 
\[ R = R_{ult} (1 - e^{-\mu\lambda})^\lambda \]  

(1)

where,

- \( R_{ult} \) = bolt shear force at failure
- \( e \) = base of natural logarithms = 2.718
- \( \mu \) = a regression coefficient
- \( \lambda \) = a regression coefficient
- \( \Delta \) = deformation of fastener

For the analytical relationship developed (Equation 1), the following assumption are made.

The actual values of \( \lambda \), \( \mu \), and \( R_{ult} \) for several bolt and reivet lots were suggested. This report concluded that the exponent \( \lambda \) is affected only...
slightly by the variations in the connected material properties and the specimen geometry and the coefficient $\mu$ was mostly affected by the type of connected material. It is implied that the parameters $\lambda$ and $\mu$ can be related to the physical and geometrical properties of the plate and bolt.

For the analytical relationship developed (Equation 1), the following assumptions are made.

1. At zero loads the deformation is zero
2. For small values of deformation the relationship between load and deformation is approximately linear.
3. As $\Delta$ approaches ultimate $\Delta$, the bolt force increases at a decreasing rate.
4. The deformation $\Delta$ contains the components due to shear, bending, and bearing of the fastener as well as the shearing deformation of the plates.

Equation 1 satisfies the boundary condition that requires the load to be zero at a zero deformation. It means that assumption ① is satisfied. If the Equation 2 is expanded in Maclaurin’s series there is obtained if $\lambda$ is unity

$$f(\Delta) = \mu R_{ult} \Delta - \mu^2 R_{ult} \frac{\Delta^2}{2!} + \cdots + \frac{(-1)^{n+1} \mu^n R_{ult} \Delta^n}{n!}$$ (2)

This series is convergent as long as $\lambda \cdot \Delta < 1$, For small values of $\Delta$ this condition is satisfied and an approximate solution is obtained by considering only the first term, it is means that $\lambda$ is initial stiffness of fastener.

$$R = \mu R_{ult} \Delta$$ (3)

It also shows that Equation 1 satisfies assumption ②. These parameters
were found to vary for different fasteners and different types of connected material. As a result, he could not develop a general expression of the behaviors.

In this study, this analytical model (Equation 1) was used to propose a generalized expression of fastener’s behavior.

(b) Crawford and Kulak (1971)

Observing ductile behavior of bearing type bolted connection, Crawford and Kulak (1971) suggested pioneering method which is ICRM (Instantaneous Center of Rotation Method) using force redistribution. They suggested maximum deformation and analytical model from the single bolt tests conducted in person. The analytical model by the Crawford and Kulak was exponential curve in common with that of Fisher. But, the empirical parameters and maximum deformation (0.34 inch) were only from one-type of 3/4 inch-A325 ASTM bolt and A36 steel plate combination.

\[
R = R_{ult}(1 - e^{-\mu \lambda})^\lambda
\]

(4)

where

- \( R_{ult} \) = bolt shear force at failure = 74 kips
- \( e \) = base of natural logarithms = 2.718
- \( \mu \) = a regression coefficient = 10
- \( \lambda \) = a regression coefficient = 0.55
- \( \Delta \) = deformation of fastener \( \leq \) 0.34 inch
This study has has the limitations like below,

(1) The bolt load-deformation relationship was derived long ago by Crawford and Kulak based on one-type of 3/4in-A325 ASTM bolt and A36 steel plate combination. Considering that combination of different bolt sizes and steel specifications such as F10T bolt and SM490 are now widely used in designing bolted connections, the applicability and reliability of this relationship has to be modified.

(2) The relationship suggested by Crawford and Kulak was only came from results of compression type single bolt test. It means that they did not consider boundary condition of bolted connection. Using this method, the maximum strength of connection will be inaccurate.

(3) Finally, they defined the maximum deformation as 0.34inch, 8.64mm, equally. But if geometrical condition (diameter of bolt and thickness of steel plate) and tensile strength of bolt and steel plate are changed, this maximum deformation (8.64mm) is not applicable.
C.M. Uang and A. Sato (2010)

Uang and Sato carried out the modeling of single-bolt behavior to evaluate the seismic performance of cold-formed steel special bolted moment frame connections (SBMF). In Uang and Sato’s study, the plates used in Uang and Sato’s test are much more smaller (2.7mm~6.4mm) than that of Crawford and Kulak (19.05mm). It means that the deformation is contributed mainly by the connected webs, not the bolt, therefore, the $R_{ult}$ is determined as bearing strength of plate, not a Crawford and Kulak’s coefficients. In Crawford and Kulak’s case, the bearing deformation of the connections was contributed by both the connected plates and the bolts.

$$R = R_{ult}(1 - e^{-\mu \Delta})^\lambda$$

(5)

where

- $R_{ult} = 2.1F_u dt$
- $F_u$ = tensile strength of steel plate
- $t$ = thickness of steel plate
- $d$ = diameter of bolt
- $e$ = base of natural logarithms = 2.718
- $\mu$ = a regression coefficient = 10
- $\lambda$ = a regression coefficient = 0.55
- $\Delta$ = deformation of fastener

$R_{ult}$, $\lambda$, and $\mu$ determined in Uang’s study were came from the test data of nine beam-to-column specimens (Figure 16). So the coefficients are only applicable for SBMF and this process limits the failure mode of bolted connection. In other words, in SBMF, as previously mentioned, deformation are mainly affected by bearing of plate, which implies that bolt shear rupture is not considered.
Figure 15 Uang and Sato’s hysteresis rule (Uang and Sato, 2010)

Figure 16 Cyclic testing of cold-formed SBMF (Uang and Sato, 2010)
2.2.2 Design method of group bolted connection using ICRM (Crawford and Kulak, 1971)

As previously mentioned, considering ductile behavior of bearing type bolted connection, Crawford and Kulak (1971) suggested pioneering method which is ICRM (Instantaneous Center of Rotation Method) using force redistribution. To solve the problem, I.C (Instantaneous center) of group bolt connection preferentially is assumed. the bearing strength of each fastener (Figure 17) is calculated using the relationship between force and deformation of fastener by Crawford and Kulak. If the equilibrium condition at first I.C is not satisfied, the I.C of connection is reselected and equilibrium condition of that has to be reconfirmed (Equation 1). Through this trial-and-error procedure, determine accurate I.C of connection and calculate the bearing force and deformation of each fastener (Equation 2).

\[
\sum_{n=1}^{m} F_n = \sum_{n=1}^{m} (R_x)n - P_x = 0, \quad R_x = R(y/r)
\]  

(6.a)

Figure 17 Instantaneous center of rotation method for design of eccentrically loaded fasteners
\[
\sum_{n=1}^{m} F_y = \sum_{n=1}^{m} (R_y) n - P_y = 0, R_y = R(x / r) \tag{6.b}
\]

\[
M_{IC} = P(e + r_0) - \sum_{n=1}^{m} (r_n \times R_n) = 0 \tag{6.c}
\]

\[
\Delta_n = (r_n / r_{\text{max}}) \times \Delta_{\text{max}} \tag{7}
\]

where

- \( r_n \) = distance from the ICR to the fastener
- \( r_{\text{max}} \) = distance to the farthest fastener
- \( \Delta_n \) = deformation of fastener
- \( \Delta_{\text{max}} \) = deformation of the farthest fastener at ultimate = 0.34in, 8.64mm

(determined experimentally)

The specific procedure is like below (Figure 17).

1. Assume a value for \( r_0 \)
2. Solve for \( P \) from Equation 1.c
3. Substitute \( r_0 \) and \( P \) into Equation 1.a and 1.b
4. If these Equations are satisfied within an acceptable tolerance, the analysis is complete. Otherwise, a new trial value of \( r_0 \) must be selected and the process repeated.

This approach has been widely introduced in many textbooks. Especially, in LRFD STEEL DESIGN 3rd edition (W. T. Segui), if the group bolted connection is under only vertical eccentric load, the equilibrium condition in x-axis is automatically satisfied. But if bolted connection is not symmetric, the x-axis force equilibrium must be considered.
3. Single bolt test and modeling : closed boundary condition

3.1 Introduction

To supplement Crawford and Kulak’s limitations, the single bolt tests of 12 specimens with variables such as different bolt size, steel specifications and thickness of steel plate were conducted.

Yoon(2013) also carried out single bolt tests in common with this study considering different geometric configurations and steel specifications and suggested single bolt force-deformation relationship. Because the steel used in that test is thick plate (15mm and 20mm), applicability of this Equation is uncertain whether thickness of steel plate is less than 15mm. So in this chapter, the results of compression type single bolt test with variable including relative thin plate (9mm, 12mm) and Yoon’s test were integrated. This test considers the closed boundary condition in bolted connection.

In this chapter, generalized force-deformation Equation based on normalization by tensile strength of base metal was suggested. Together with the normalized force-deformation relationship, the available deformation capacity of bolted connections was also proposed based on the ratio of the bearing strength and the shear strength of the bolt.
3.2 Test set-up

As previously mentioned, the compression type single bolt tests with different bolt size and steel specifications was conducted unlike the Crawford and Kulak’ tests based on on-type bolt and steel combination. The tests was carried out by UTM(Universal Testing Machine), the force and deformation from UTM was used. Figure 4 and Table.1 show test setup and specification of bolt and steel plate. To control that the shear fracture of bolt do not occurred, design that threads excluded in shear plane. Jig was made of 30mm steel with 800 MPa nominal tensile strength to maintain elastic stage after test is over. It helps to minimize the effect of deformation of jig during the tests.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Steel plate</th>
<th>t (mm)</th>
<th>Diameter of bolt (mm)</th>
<th>Diameter of hole (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C400-9-20</td>
<td>SS400</td>
<td>9</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>C400-9-22</td>
<td>SS400</td>
<td>9</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>C400-9-24</td>
<td>SS400</td>
<td>9</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>C400-12-20</td>
<td>SS400</td>
<td>12</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>C400-12-22</td>
<td>SS400</td>
<td>12</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>C400-12-24</td>
<td>SS400</td>
<td>12</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>C490-9-20</td>
<td>SM490</td>
<td>9</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>C490-9-22</td>
<td>SM490</td>
<td>9</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>C490-9-24</td>
<td>SM490</td>
<td>9</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>C490-12-20</td>
<td>SM490</td>
<td>12</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>C490-12-22</td>
<td>SM490</td>
<td>12</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>C490-12-24</td>
<td>SM490</td>
<td>12</td>
<td>24</td>
<td>27</td>
</tr>
</tbody>
</table>
3.3 Test result and contemplation

Table 2 shows the results of the compression type single bolt test. The maximum strength of each specimen is proportional to thickness of plate and diameter of bolt. All of the tests stopped because UTM head was contact to top of the jig. After the tests, deformation of most specimens occurred in bolt hole and the deformation in the jig was almost never shown.
Figure 20 Deformed shape of specimens after test
### Table 2 Results of compression tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Failure Mode</th>
<th>Load at $\Delta = 15\text{mm}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C400-9-20</td>
<td>Stroke limit (excessive deformation)</td>
<td>245.9</td>
</tr>
<tr>
<td>C400-9-22</td>
<td>Stroke limit (excessive deformation)</td>
<td>287.9</td>
</tr>
<tr>
<td>C400-9-24</td>
<td>Stroke limit (excessive deformation)</td>
<td>292.8</td>
</tr>
<tr>
<td>C400-12-20</td>
<td>Stroke limit (excessive deformation)</td>
<td>263.1</td>
</tr>
<tr>
<td>C400-12-22</td>
<td>Stroke limit (excessive deformation)</td>
<td>279.1</td>
</tr>
<tr>
<td>C400-12-24</td>
<td>Stroke limit (excessive deformation)</td>
<td>298.4</td>
</tr>
<tr>
<td>C490-9-20</td>
<td>Stroke limit (excessive deformation)</td>
<td>299.4</td>
</tr>
<tr>
<td>C490-9-22</td>
<td>Stroke limit (excessive deformation)</td>
<td>324.7</td>
</tr>
<tr>
<td>C490-9-24</td>
<td>Stroke limit (excessive deformation)</td>
<td>330.3</td>
</tr>
<tr>
<td>C490-12-20</td>
<td>Stroke limit (excessive deformation)</td>
<td>392.7</td>
</tr>
<tr>
<td>C490-12-22</td>
<td>Stroke limit (excessive deformation)</td>
<td>428.3</td>
</tr>
<tr>
<td>C490-12-24</td>
<td>Stroke limit (excessive deformation)</td>
<td>427.5</td>
</tr>
</tbody>
</table>
(a) C400-9-20

(b) C400-9-20

(c) C400-9-24
(d) C400-12-20

(e) C400-12-22

(f) C400-12-24
(g) C490-9-20

(h) C490-9-22

(i) C490-9-24
Figure 21 Measured load-displacement relationship of compression tests
Figure 22 Comparison test results with Crawford and Kulak’s Equation

Figure 22 shows that the results of the tests is compared to single bolt force-deformation Equation by Crawford and Kulak (Equation 4). As previously mentioned in chapter 2, because the geometric configuration of these specimens is different with those of Crawford and Kulak’s test, their equation is not applicable.

Bolt shear rupture did not take place because thickness of steel plate used in the tests was thin. Therefore, maximum allowable limit proposed by Crawford and Kulak (0.34in, 8.64mm) cannot apply to the case that has the different combination of bolt and steel specification. The force-deformation relationship by Crawford and Kulak is likely to overestimate capacity of bolted connection as shown in Figure 22.

3.4 Force-deformation relationship

To propose generalized force-deformation equation of compression type
single bolted connection, normalization was conducted by bearing strength of each specimen. From the normalized data based on the test results in this study and Yoon’s test results, the force-deformation equation is proposed. The mathematical expression for the load-deformation relationship low fits the data when values of $\mu = 0.1$ and $\lambda = 0.55$ are used as the two empirical coefficients.

$$R = R_{ult} (1 - e^{-\mu \Delta})^\lambda$$  \hfill (8)

where

- $R_{ult} = 3.0 F_u dt$
- $F_u$ = tensile strength of steel plate
- $t$ = thickness of steel plate
- $d$ = diameter of bolt
- $e$ = base of natural logarithms = 2.718
\[ \mu = \text{a regression coefficient} = 0.1 \]
\[ \lambda = \text{a regression coefficient} = 0.55 \]
\[ \Delta = \text{deformation of fastener} \]

### 3.5 Allowable deformation limit

As previously mentioned, because the maximum deformation of single bolted connection (0.34 inches) suggested by Crawford and Kulak was from result of the single bolt shear test with one-type of 3/4 inches-A325 ASTM bolt and A36 steel combination applicability of this value is uncertain in case of bolted connection with different condition. This limit of Crawford and Kulak’s maximum deformation is clearly shown in Yoon’s test result. Furthermore, as shown in Table 2, since the bolt shear rupture did not occur due to thin plate, the maximum deformation cannot be unific in this case.

On the other hand, in the case of the specimens having bolt shear rupture, as the diameter of bolt is bigger, the deformation capacity improve. Also, in the same diameter of bolt, the specimen with thick steel plate shows low deformation capacity. It means that the deformation at the maximum load is affected by geometric configuration of connection such as strength of bolt, thickness of steel plate and edge distance. From this approach, the test results was normalized by bolt shear strength and bearing strength. As a result, the maximum deformation of specimens takes into account bolt shear strength and bearing strength (Equation 9).

The proposed Equation came from low fitted experimental data because the maximum deformation is inversely proportional to ratio of bearing strength of steel plate to bolt shear strength. But the upper bound of maximum deformation is limited to 20mm since the bigger deformation than 20mm is not meaningful. This upper bound prevents the connection from excessive deformation.
Table 3 Test variable and result of Yoon (2012)

<table>
<thead>
<tr>
<th>Steel grade</th>
<th>Specimen</th>
<th>Steel thickness t (mm)</th>
<th>Diameter of bolt d (mm)</th>
<th>Maximum load kN</th>
<th>Maximum displacement mm</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS400</td>
<td>1</td>
<td>15</td>
<td>20</td>
<td>457.2</td>
<td>18.08</td>
<td>Bolt shear rupture</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>15</td>
<td>22</td>
<td>521.6</td>
<td>20.16</td>
<td>Stroke limit</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>15</td>
<td>24</td>
<td>542.8</td>
<td>20.12</td>
<td>Stroke limit</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>20</td>
<td>20</td>
<td>456.5</td>
<td>7.48</td>
<td>Bolt shear rupture</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>20</td>
<td>22</td>
<td>565.1</td>
<td>11.58</td>
<td>Bolt shear rupture</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>20</td>
<td>24</td>
<td>728.6</td>
<td>21.63</td>
<td>Stroke limit</td>
</tr>
<tr>
<td>SM490</td>
<td>7</td>
<td>15</td>
<td>20</td>
<td>427.6</td>
<td>9.56</td>
<td>Bolt shear rupture</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>15</td>
<td>22</td>
<td>519.6</td>
<td>13.54</td>
<td>Bolt shear rupture</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>15</td>
<td>24</td>
<td>668.2</td>
<td>21.10</td>
<td>Stroke limit</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>20</td>
<td>20</td>
<td>554.3</td>
<td>10.00</td>
<td>Bolt shear rupture</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>20</td>
<td>22</td>
<td>673.92</td>
<td>12.98</td>
<td>Bolt shear rupture</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>20</td>
<td>24</td>
<td>823.4</td>
<td>19.26</td>
<td>Bolt shear rupture</td>
</tr>
</tbody>
</table>

\[
\alpha = 3F_{u,n} \cdot dt / (2F_{v,n} \cdot A_b) \tag{9.a}
\]

\[
\Delta_{allow,comp} = 10 \alpha^{-3} + 5 \quad (5mm \leq \Delta_{allow,comp} \leq 20mm) \tag{9.b}
\]

where,

\( F_{u,n} \) = nominal tensile strength of steel plate

\( F_{v,n} \) = nominal shear strength of bolt
Figure 24 Maximum allowable limit curve in accordance with plate bearing strength vs. bolt shear strength ratio
4. Single bolt test and modeling: open boundary condition

4.1 Introduction

The boundary condition of fastner in group bolted connection under eccentric load is divided according to direction of fastner’s displacement. It means that each fastner having different boundary condition shows different force-deformation behavior.

On the other hand, Crawford and Kulak proposed force-deformation relationship as the basis of design the eccentrically loaded bolted connection. But this relationship is based on experimental data which only came from compression type single bolt test. It implies that the capacity of connection cannot accurately be evaluated by this relationship.

In this chapter, to evaluate bolted connection under eccentric load accurately, the tension type single bolt test was conducted. This test considers the open boundary condition in bolted connection.

Yoon(2013) also carried out single bolt tests in common with this study considering different geometric configurations and steel specifications and suggested single bolt force-deformation relationship. Because the steel used in that test is thick plate (15mm and 20mm), applicability of this Equation is uncertain whether thickness of steel plate is less than 15mm. In addition, that test did not consider edge distance of steel plate.

In common with chapter 3, the results of tension type single bolt test with variable including relative thin plate(9mm, 12mm) and Yoon’s test were integrated. The force-deformation relationship for open boundary condition was proposed through normalization by bearing strength of each specimen and the maximum deformation Equation was also proposed.
4.2 Test set-up

As previously mentioned, tension type specimen has different boundary condition with compression type specimen. It means that the force-deformation behavior is different with behavior of closed boundary condition. Not merely bolt shear rupture but also tearing fracture of steel plate can occur according to geometric configuration of specimen.

The variables in this test are steel specification, thickness of steel plate, edge distance and diameter of bolt. To control that bearing failure only occurs in specimen, edge distance and thickness of steel plate was determined. Jig was made of 30mm steel with 800 MPa nominal tensile strength to maintain elastic stage after test is over. It helps to minimize the effect of deformation of jig during the tests.

Figure 25 Test set-up for tension tests
<table>
<thead>
<tr>
<th>Specimen</th>
<th>Steel plate</th>
<th>Diameter of bolt (mm)</th>
<th>Diameter of hole (mm)</th>
<th>Edge distance (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T400-9-20</td>
<td>SS400</td>
<td>9</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T400-9-22</td>
<td>SS400</td>
<td>9</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>T400-9-24</td>
<td>SS400</td>
<td>9</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>T400-12-20a</td>
<td>SS400</td>
<td>12</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T400-12-20b</td>
<td>SS400</td>
<td>12</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T400-12-20c</td>
<td>SS400</td>
<td>12</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T400-12-22</td>
<td>SS400</td>
<td>12</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>T400-12-24</td>
<td>SS400</td>
<td>12</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>T400-15-20a</td>
<td>SS400</td>
<td>15</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T400-15-20b</td>
<td>SS400</td>
<td>15</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T400-15-20c</td>
<td>SS400</td>
<td>15</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T400-15-22b</td>
<td>SS400</td>
<td>15</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>T400-15-24b</td>
<td>SS400</td>
<td>15</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>T490-9-20</td>
<td>SM490</td>
<td>9</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T490-9-22</td>
<td>SM490</td>
<td>9</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>T490-9-24</td>
<td>SM490</td>
<td>9</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>T490-12-20a</td>
<td>SM490</td>
<td>12</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T490-12-20b</td>
<td>SM490</td>
<td>12</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T490-12-20c</td>
<td>SM490</td>
<td>12</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T490-12-22</td>
<td>SM490</td>
<td>12</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>T490-12-24</td>
<td>SM490</td>
<td>12</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>T490-15-20a</td>
<td>SM490</td>
<td>15</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T490-15-20b</td>
<td>SM490</td>
<td>15</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T490-15-20c</td>
<td>SM490</td>
<td>15</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>T490-15-22b</td>
<td>SM490</td>
<td>15</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>T490-15-24b</td>
<td>SM490</td>
<td>15</td>
<td>24</td>
<td>27</td>
</tr>
</tbody>
</table>
4.3 Test result and contemplation

The failure mode of tension type specimens was bolt shear rupture or tearing fracture of steel plate and deformation around bolt hole in jig made of steel with 800MPa nominal strength was not shown (Figure 26). As shown in Table 4, in the case of bolt shear rupture, the specimen’s capacity of maximum deforation is less than that of specimen with tearing fracture of steel plate. It means that the maximum deforation is affected by the edge distance of steel plate.

![Figure 26 Deformed shape of jig after test](image)

Figure 28 shows force-deformation behavior of specimens. In the case of the specimens with bolt shear rupture, the maximum strength of a single bolted tested in a tension jig was lower than compression type single bolt test. This phenomenon is due to prying action which tends to bend the lap plates of the tension jig outward. The lap plate prying mechanism is shown in Figure 27.
Due to the bearing deformation of the test bolt, the resistance force $P/2$ does not act at the centerline of the lap plate, but acts at a distance “$e$” to the left of it. Besides, another reason is the bending deflection which occurred in bolt because of separation distance between steel plate and lap plate of tension jig.
<table>
<thead>
<tr>
<th>Specimen</th>
<th>Failure Mode</th>
<th>Maximum Load (kN)</th>
<th>Maximum displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T400-9-20</td>
<td>Tearing fracture</td>
<td>221.1</td>
<td>24.06</td>
</tr>
<tr>
<td>T400-9-22</td>
<td>Tearing fracture</td>
<td>222.4</td>
<td>20.36</td>
</tr>
<tr>
<td>T400-9-24</td>
<td>Tearing fracture</td>
<td>255.2</td>
<td>24.18</td>
</tr>
<tr>
<td>T400-12-20a</td>
<td>Tearing fracture</td>
<td>201.8</td>
<td>23.58</td>
</tr>
<tr>
<td>T400-12-20b</td>
<td>Tearing fracture</td>
<td>237.7</td>
<td>31.52</td>
</tr>
<tr>
<td>T400-12-20c</td>
<td>Tearing fracture</td>
<td>278.9</td>
<td>43.54</td>
</tr>
<tr>
<td>T400-12-22</td>
<td>Tearing fracture</td>
<td>280.6</td>
<td>43.8</td>
</tr>
<tr>
<td>T400-12-24</td>
<td>Suspended</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>T400-15-20a</td>
<td>Tearing fracture</td>
<td>342.1</td>
<td>27.6</td>
</tr>
<tr>
<td>T400-15-20b</td>
<td>Suspended</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>T400-15-20c</td>
<td>Bolt shear rupture</td>
<td>285.8</td>
<td>13.74</td>
</tr>
<tr>
<td>T400-15-22b</td>
<td>Bolt shear rupture</td>
<td>369.9</td>
<td>21.02</td>
</tr>
<tr>
<td>T400-15-24b</td>
<td>Suspended</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>T490-9-20</td>
<td>Bolt shear rupture</td>
<td>256.6</td>
<td>23.56</td>
</tr>
<tr>
<td>T490-9-22</td>
<td>Tearing fracture</td>
<td>315.8</td>
<td>29.88</td>
</tr>
<tr>
<td>T490-9-24</td>
<td>Tearing fracture</td>
<td>293.95</td>
<td>22.2</td>
</tr>
<tr>
<td>T490-12-20a</td>
<td>Tearing fracture</td>
<td>311.1</td>
<td>25.8</td>
</tr>
<tr>
<td>T490-12-20b</td>
<td>Bolt shear rupture</td>
<td>356.45</td>
<td>29.08</td>
</tr>
<tr>
<td>T490-12-20c</td>
<td>Bolt shear rupture</td>
<td>306.7</td>
<td>17.8</td>
</tr>
<tr>
<td>T490-12-22</td>
<td>Tearing fracture</td>
<td>364.4</td>
<td>31.2</td>
</tr>
<tr>
<td>T490-12-24</td>
<td>Tearing fracture</td>
<td>414.5</td>
<td>39.4</td>
</tr>
<tr>
<td>T490-15-20a</td>
<td>Bolt shear rupture</td>
<td>303</td>
<td>-</td>
</tr>
<tr>
<td>T490-15-20b</td>
<td>Bolt shear rupture</td>
<td>318.6</td>
<td>-</td>
</tr>
<tr>
<td>T490-15-20c</td>
<td>Suspended</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>T490-15-22b</td>
<td>Bolt shear rupture</td>
<td>426.6</td>
<td>19.38</td>
</tr>
<tr>
<td>T490-15-24b</td>
<td>Bolt shear rupture</td>
<td>488.6</td>
<td>25.28</td>
</tr>
</tbody>
</table>
(g) T400-15-20c
(h) T400-15-22
(i) T490-9-20
(j) T490-9-22
(k) T490-9-24
(l) T490-12-20a
Figure 28 Deformed shape of specimens after tests
(a) T400-9-20

(b) T400-9-22

(c) T400-9-24
(d) T400-12-20a

(e) T400-12-20b

(f) T400-12-20c
(g) T400-12-22

(h) T400-15-20a

(i) T400-15-20c
(j) T400-15-22

(k) T490-9-20

(l) T490-9-22
(m) T490-9-24

(n) T490-12-20a

(o) T490-12-20b
(p) T490-12-20c

(q) T490-12-22

(r) T490-12-24
Figure 29 Measured load-displacement relationship of tension tests

(s) T490-15-22

(t) T490-15-24
4.4 Force-deformation relationship

In common with the compression type single bolt tests considering closed boundary condition, to the propose generalized force-deformation equation of tension type sing bolted connection, normalization was conducted by bearing strength of each specimen. As shown in Figure 30, most of specimens show similar pattern of force-deformation behavior. In the case of the specimens with bolt shear rupture, the force-deformation behavior is similar to that of others, but the maximum deformation is relatively low. The maximum strength of the specimens with tearing fracture of steel plate comes close to the current steel design codes which are KBC 2009 and AISC 2010 \( R = 1.2L_t tF_u \). From this approach, the force-deformation relationship of tension type single bolt is suggested below (Equation 10).

\[
R = R_{ult} (1 - e^{-\mu \Delta})^\lambda
\]  

(10)

where

\[ R_{ult} = 1.2L_t tF_u \]

\[ L_t \] = edge distance

\[ F_u \] = tensile strength of steel plate

\[ t \] = thickness of steel plate

\[ d \] = diameter of bolt

\[ e \] = base of natural logarithms = 2.718

\[ \mu \] = a regression coefficient = 0.08

\[ \lambda \] = a regression coefficient = 0.8

\[ \Delta \] = deformation of fastener
4.5 Allowable deformation limit

In common with compression single bolt test, the test results was normalized by bolt shear strength and end tearing strength of steel plate. As a result, the maximum deformation takes into account bolt shear strength and end tearing strength (Equation 11). The upper bound of maximum deformation is limited to 20mm since the bigger deformation than 20mm is not meaningful. This upper bound prevents the connection from excessive deformation.

\[
\beta = 1.2 \frac{F_{u,n} \cdot L_t}{(2F_{v,n} \cdot A_p)}
\]

\[
\Delta_{\text{max, tens}} = 10 \beta^{-2} + 5 \quad (5\text{mm} \leq \Delta_{\text{max, tens}} \leq 20\text{mm})
\]

where,

\[
F_{u,n} = \text{nominal tensile strength of steel plate}
\]
$F_{v,n} =$ nominal shear strength of bolt

The proposed force-deformation relationship and maximum allowable limit of both compression and tension type single bolt with ICRM (Instantaneous center of rotation method) can be reasonable method to design bolted connection under eccentric load.

**Figure 31** Maximum allowable limit curve governed by bolt shear rupture in accordance with end tearing strength vs. bolt shear strength ratio
5. Application to design of group bolted connections

As previously mentioned, to evaluate ultimate strength of bolted connection under eccentric load, Crawford and Kulak suggested pioneering method which is the ICRM(Instantaneous Center of Rotation Method) using the force-deformation relationship proposed by them.

But it is very difficult to use their method by hand calculation because trial-error method is needed to find instantaneous center of group bolted connection. And when the connection has many fastener, it is very inefficient and wastes time.

Therefore, in this chapter, the method using the force-deformation relationship suggested in this study is introduced after the numerical algorithm which can find instantaneous center of bolted connection is suggested.

After that, solving two evaluation examples can help understanding how to use this method.

5.1 Numerical method proposed to find ICR of connection

The ICR (instantaneous center of rotation) of connection has to be defined to evaluate maximum strength of group bolted connection using ICRM. To calculate the maximum strength of connection, the ICR of the connection has to be assumed priorly and force equilibrium of connection must be satisfied at that point. If the force equilibrium is not satisfied within an acceptable tolerance, a new trial ICR must be selected and process repeated by the time force equilibrium is satisfied.
Figure 32 Instantaneous center of rotation method for design of eccentrically loaded fasteners

\[
\sum_{n=1}^{m} F_x = \sum_{n=1}^{m} (R_x)n - P_x = 0, R_x = R(y / r) 
\]

\[
\sum_{n=1}^{m} F_y = \sum_{n=1}^{m} (R_y)n - P_y = 0, R_y = R(x / r) 
\]

\[
\sum_{n=1}^{m} M_{IC} = P(e + r_0) - \sum_{n=1}^{m} (r_n \times R_n) = 0 
\]

where

\( r_n \) = distance from the ICR to the fastener

\( r_{max} \) = distance to the farthest fastener

\( \Delta_n \) = deformation of fastener
\[ \Delta_{\text{max}} = \text{deformation of the farthest fastener at ultimate} \]

Firstly, obtain the force-deformation relationship and maximum deformation of group bolted connection limits from the geometrical configuration of connection and steel specification. After that, the general procedure is to assume the location of the IC, then determine if the corresponding value of \( P \) satisfies the equilibrium Equation 12. If so, this location is correct and \( P \) is the capacity of the connection. As a result, the unconstrained multivariable vector (residual of equilibrium) was derived (Equation 13).

\[
\text{Res} = \begin{bmatrix} \sum F_x(r_i) \\ \sum M_{IC}(r_i) \end{bmatrix} = \begin{bmatrix} \text{Res}_1 \\ \text{Res}_2 \end{bmatrix}
\]

(13)

Equation 14 was came from Equation 13.

\[
r_i = (x_i, y_i) \quad \text{(14.a)}
\]

\[
f(r_i) = \text{Res}^T \cdot \text{Res} \quad \text{(14.b)}
\]

\( r_i \) is an initial location of ICR of the connection (Figure 32). The residual of equilibrium \( f(r_i) \) is function of \( r_i \). To minimize \( f(r_i) \), the Quasi-Newton method was used. From this approach, the numerical algorithm which can find instantaneous center of bolted connection is suggested (Figure 33).
Figure 33 Numerical algorithm or Flowchart for Instantaneous center

\[ R = 3.0 F_u dt (1 - e^{-0.14})^{0.55} \]
\[ \Delta_{allow, comp} = 10 \alpha^{-3} + 5 \]
\[ \Delta_{allow, tens} = 10 \beta^{-2} + 5 \]

Initial guess
\[ r_0 = [x_0, y_0]^T \]

\[ i = 0 \]

Subroutine based on Quasi-Newton Method

\[ i = i + 1 \]

No

Yes

Determine IC

End
5.2 Design application to group bolted connection under eccentric load

5.2.1 Procedure of design application

In the case of using hand calculation, the ICRM procedure is like below.

(1) Assume the ICR of group bolted connection. When the group bolted connection is x-axis and y-axis symmetry, the ICR can move along not only the x-axis but also y–axis. This is because both open boundary condition and closed boundary condition are considered to calculate ICR of connection unlike Crawford and Kulak’s method in this study.

Figure 34 ICRM from the group-bolts which have different boundary conditions
(2) Calculate each distance ($r_n$) from ICR to each fastener.

(3) Calculate maximum deformation of each bolt using proposed Equation in this study. The deformation of an individual fastener is proportional to distance from ICR to each fastener. Because the boundary condition of the fasteners in upper box (Figure 34) is similar to that of tension type single bolted connection in chapter 4, that is open boundary condition, the tension type maximum allowable limit has to be used (Equation 11). On the other hand the fasteners in the lower box have boundary condition like closed boundary condition, the compression type maximum allowable limit (Equation 9) in chapter 3 should be used.

(4) Calculate bearing force of each fastener from the each force-deformation relationship. The bearing force of the fasteners having open boundary condition is obtained from Equation 10 and other fastener’s bearing force is calculated from Equation 8.

(5) If the force equilibrium of connection is not satisfied, a new trial ICR must be selected and the Process repeated.
5.2.2 Behavior of group bolted connection

In this study, to define behavior of bolted connection, the analytical model which can predict behavior of group bolted connection was proposed. The proposed force-deformation relationships shown in Chapter 3 and 4 were used to establish analytical model. Figure 35 shows the example of beam to column group bolted connection. If the connection is under eccentric load, the fasteners will rotate on ICR (Figure 36).
Before bolts bear against the connected web of the beam, the amount of rotation is affected by size of the bolt hole (the bolt oversize), the distance between outermost bolt. Therefore, the initial slip range \( \theta_{slip} \) has to be calculated. This amount of rotation in slip range can be obtained from the bolt oversize \( h_{OS} \) and the distance from the ICR to the farthest bolt \( d_{max} \) (Equation 15).

\[
\theta_{slip} = \frac{\sum h_{OS}}{d_{max}} = \frac{2h_{OS}}{d_{max}}
\]

where,

\( \theta_{slip} \) = initial slip range of the connection

\( h_{OS} \) = bolt hole oversize
The bearing force of the connection is like below.

\[
R_{\text{total}} = R_{\text{open}} + R_{\text{closed}} = \sum R_i
\]  \hspace{1cm} (16.a)

\[
R_{\text{open}} = \sum 1.2LcF_ut(1-e^{-0.08\Delta_i})^{0.8}
\]  \hspace{1cm} (16.b)

\[
R_{\text{closed}} = \sum 3.0F_utd(1-e^{-0.1\Delta_i})^{0.55}
\]  \hspace{1cm} (16.c)

where,

\begin{align*}
R_{\text{total}} & = \text{total bearing force of the connection} \\
R_{\text{open}} & = \text{bearing force of the connection in open boundary condition} \\
R_{\text{closed}} & = \text{bearing force of the connection in closed boundary condition} \\
F_u & = \text{tensile strength of steel plate} \\
Lc & = \text{edge distance} \\
t & = \text{thickness of steel plate} \\
d & = \text{diameter of bolt} \\
e & = \text{base of natural logarithms} = 2.718 \\
\mu & = \text{a regression coefficient} = 0.08 \\
\lambda & = \text{a regression coefficient} = 0.8 \\
\Delta_i & = \text{deformation of fastener}
\end{align*}

The reaction moment of the connection is like below.
\[ M_{\text{total}} = M_{\text{open}} + M_{\text{closed}} = \sum R_i d_i \]  
(17.a)

\[ M_{\text{open}} = \sum d_i \cdot [1.2 L_f F_i t (1 - e^{-0.08 \Delta_i})^{0.8}] \]  
(17.b)

\[ M_{\text{closed}} = \sum d_i \cdot [3.0 F_i d t (1 - e^{-0.1 \Delta_i})^{0.55}] \]  
(17.c)

where,

- \( M_{\text{total}} \) = total reaction moment of the connection
- \( M_{\text{open}} \) = reaction moment of the connection in open boundary condition
- \( M_{\text{closed}} \) = reaction moment of the connection in closed boundary condition
- \( d_i \) = distance from the ICR to the fastener

In bearing stage, the rotation of the connection can be defined by deformation of fastener \( \Delta_i \) and distance from the ICR to the fastener \( d_i \).

\[ \theta_{\text{bearing}} = \frac{\Delta_i}{d_i} \]  
(18)

where,

- \( \theta_{\text{bearing}} \) = rotation due to bearing

Consequently, the sum of the initial rotation \( \theta_{\text{slip}} \) and the bearing rotation \( \theta_{\text{bearing}} \) is equal to the total rotation \( \theta_{\text{total}} \) of the beam to column connection (Equation 19).

\[ \theta_{\text{total}} = \theta_{\text{slip}} + \theta_{\text{bearing}} \]  
(19)
5.3 Evaluation examples

In this chapter, bracket example helps understanding how to apply this method in this study to design and shows the validity of this approach. The braket connection shown in Figure 37 is under the eccentric load. The connection was designed to have vertical rows of four bolts which are 20mm diameter F10T bearing type and 15mm SS400 steel plate. The problem is what the maximum load which the connection can resist is. Assume that the flange of column is rigid body. To solve this example, the method of ultimate strength design in LRFD STEEL DESIGN 3rd edition (W. T. Segui) based on Crawford and Kulak’s Equation was used. Three design methods which are elastic analysis, Crawford & Kulak’s method and proposed method were compared to show validity of proposed method.

![Figure 37 Evaluation example 1 (symmetry)](image)
5.3.1 Example 1

(a) Elastic analysis

As previously mentioned in chapter 2, in the existing elastic analysis, the basic assumptions like below.

① Bolt and plate show elastic behavior.
② Each reaction in group bolt is proportionate to each distance from centroid of bolted connection.
③ The ICR of connection is centroid of connection.

From these assumptions, the maximum strength of connection can be calculated by sum of force vector at each fastener (Figure 39).
Figure 39 Equivalent concentric load and moment for elastic analysis

Table 6 Result of elastic analysis

<table>
<thead>
<tr>
<th>Bolt No.</th>
<th>Origin at centroid</th>
<th>Concentric load term</th>
<th>Moment term</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x (mm)</td>
<td>y (mm)</td>
<td>p cx/P x</td>
</tr>
<tr>
<td>1</td>
<td>-37.5</td>
<td>-112.5</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>37.5</td>
<td>-112.5</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-37.5</td>
<td>-37.5</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>37.5</td>
<td>-37.5</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>-37.5</td>
<td>37.5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>37.5</td>
<td>37.5</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>-37.5</td>
<td>112.5</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>37.5</td>
<td>112.5</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \phi R_n = \phi F_v A_n = 0.75 \cdot 0.4 \cdot 1000 \cdot 314 = 94.25kN \quad (15) \]

\[ P_{max} = \frac{\phi R_n}{(p/P_y)_{max}} = \frac{94.25}{0.387} = 243.21kN \quad (16) \]

Solving the this example in elastic analysis, the maximum load \( P_{max} \) is 243.21kN, when the bearing force of the 2 and 8 fastener reaches the nominal bolt shear strength.
(b) **ICRM using the force-deformation relationship of Crawford and Kulak (LRFD STEEL DESIGN 3rd)**

When the value of \( r_0 \) is equal to 42.117mm, the force equilibrium of the connection is satisfied. The specific procedure is as follows to calculated accurate IC.

Firstly, Assume a value of \( r_0 \). After that, determine the accurate ICR by the time the equilibrium condition is satisfied. For example, when a value of \( r_0 \) is equal to 40.75mm, the bearing force of fasteners in connection is shown in Table 7. Outermost fastener’s maximum strength is 8.64mm.

The force equilibrium in direction of x-axis is automatically satisfied because the group bolted connection is symmetry structure, both x-direction and y-direction, and there is no external force vector in the direction of x-axis. And the equation of equilibrium in direction of y-axis was applied to the fastener group like below.

\[
P_v = \sum (R_y)_n = 1019.76kN \quad \text{(20.a)}
\]

\[
P = \frac{\sum (r_n \cdot R_n)}{e + r_0} = \frac{232515kN \cdot mm}{(187.5 + 40.5)mm} = 981.28kN \quad \text{(20.b)}
\]

Because \( P \) is not equal to \( P_v \), the force equilibrium was not satisfied. Therefore, a new trial value of \( r_0 \) must be selected and the process repeated. When the accurate ICR is obtained, the capacity of the connection will be 1019.76kN and the bearing force of outermost fastener(no.2 and no.8) reach the ultimate strength simultaneously.

\[
P = \frac{\sum (r_n \cdot R_n)}{e + r_0} = \frac{234155kN \cdot mm}{(187.5 + 42.117)mm} = 1019.76kN \quad \text{(21)}
\]
Figure 40 Location of I.C.

Table 7 The result of ICRM based on Crawford and Kulak’s approach

<table>
<thead>
<tr>
<th>Bolt No</th>
<th>Boundary condition</th>
<th>Origin at centroid</th>
<th>Origin at I.C.</th>
<th>Α (mm)</th>
<th>R (kN)</th>
<th>rR (kN.mm)</th>
<th>R₁ (kN)</th>
<th>R₂ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x (mm)</td>
<td>y (mm)</td>
<td>x’ (mm)</td>
<td>y’ (mm)</td>
<td>r (mm)</td>
<td>Δ (mm)</td>
<td>R (kN)</td>
<td>rR (kN.mm)</td>
</tr>
<tr>
<td>1</td>
<td>-375</td>
<td>-1125</td>
<td>4.6</td>
<td>-112.5</td>
<td>112.6</td>
<td>7.1</td>
<td>318.3</td>
<td>35838.0</td>
</tr>
<tr>
<td>2</td>
<td>375</td>
<td>-1125</td>
<td>79.6</td>
<td>-112.5</td>
<td>137.8</td>
<td>8.8</td>
<td>323.5</td>
<td>44582.8</td>
</tr>
<tr>
<td>3</td>
<td>-375</td>
<td>-375</td>
<td>4.6</td>
<td>-37.5</td>
<td>37.8</td>
<td>2.4</td>
<td>251.2</td>
<td>9489.5</td>
</tr>
<tr>
<td>4</td>
<td>-375</td>
<td>-375</td>
<td>79.6</td>
<td>-37.5</td>
<td>88.0</td>
<td>5.6</td>
<td>308.7</td>
<td>27167.4</td>
</tr>
<tr>
<td>5</td>
<td>-375</td>
<td>375</td>
<td>4.6</td>
<td>37.5</td>
<td>37.8</td>
<td>2.4</td>
<td>251.2</td>
<td>9489.5</td>
</tr>
<tr>
<td>6</td>
<td>375</td>
<td>375</td>
<td>79.6</td>
<td>37.5</td>
<td>88.0</td>
<td>5.6</td>
<td>308.7</td>
<td>27167.4</td>
</tr>
<tr>
<td>7</td>
<td>-375</td>
<td>1125</td>
<td>4.6</td>
<td>112.5</td>
<td>112.6</td>
<td>7.1</td>
<td>318.3</td>
<td>35838.0</td>
</tr>
<tr>
<td>8</td>
<td>375</td>
<td>1125</td>
<td>79.6</td>
<td>112.5</td>
<td>137.8</td>
<td>8.8</td>
<td>323.5</td>
<td>44582.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
<td>234155.4</td>
<td></td>
</tr>
</tbody>
</table>

\[(R_n)_{\text{max}} = 74\text{kips} = 329.39\text{kN} \tag{22}\]
From Equation 8 (the nominal bolt shear strength), the design strength of one bolt (based on shear) is 291.86kN (Equation 23).

\[ P_{\text{max}} = 1019.76 \cdot \frac{94.25}{329.30} = 291.86kN \]

(23)

Figure 41 Strength reduction by nominal bolt shear strength in example 1

The major flow in this analysis is the implied assumption that all fastener’s force-deformation behavior agree with the result of Crawford and Kulak’s test based on one-type of 3/4inches A325 bolt and A36 steel combination. In other words, in \( R = R_{ult} (1 - e^{-\Delta})^\mu \), the regression coefficients are unchanged and \( R_{ult} \) is only changed. If the geometric configuration is extremely changed, from weak bolt-strong plate to strong bolt-weak plate, the maximum deformation of fastener is not changed.
(c) Proposed method

In this study, the maximum strength limit and the force-deformation relationship is depending on geometric configuration, boundary condition of fastener. In this example, the force-deformation behavior of no.1~no.6 fastener is determined by the Equations 8 from the results of compression type single bolt test (closed boundary condition) and that of no.7 and no.8 fasteners is determined by the Equations 10 from the results of tension type single bolt test (open boundary condition).

From the Equation 9 and Equation 11, the maximum deformation of no.1~no.6 fastener is 6.44mm and that of no.7 and no.8 fastener is 9.51mm. The ICR of connection and maximum strength of the connection was calculated from the maximum deformation of fasteners. Table. 8 shows the results of calculation.

From Equation 12.b

\[
P = \sum \left( \frac{r_n \cdot R_n}{e + r_0}\right) = \frac{142564.16kN \cdot mm}{(187.5 + 39.8)mm} = 627.13kN
\]

(24)

To satisfy force equilibrium, ICR is determined like below (Figure 40).
The maximum external load \( P \) (Equation. 25) is reduced by the nominal bolt shear strength.

\[
\sum_{n=1}^{8} (R_{y,n})(\phi F_v A_n / (R_n)_{max}) = 247.37kN
\]  

(25)

<table>
<thead>
<tr>
<th>Bolt No.</th>
<th>Boundary condition</th>
<th>Origin at centroid</th>
<th>Origin at I.C.</th>
<th>( x ) (mm)</th>
<th>( y ) (mm)</th>
<th>( x' ) (mm)</th>
<th>( y' ) (mm)</th>
<th>( r ) (mm)</th>
<th>( \Delta ) (mm)</th>
<th>( R ) (kN)</th>
<th>( rR ) (kNmm)</th>
<th>( R_x ) (kN)</th>
<th>( R_y ) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Compression</td>
<td>-375 -1125</td>
<td></td>
<td>2.3</td>
<td>-99.3</td>
<td>99.3</td>
<td>5.1</td>
<td>216.9</td>
<td>21538.7</td>
<td>-216.9</td>
<td>5.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Compression</td>
<td>375 -1125</td>
<td></td>
<td>77.3</td>
<td>-99.3</td>
<td>125.8</td>
<td>6.4</td>
<td>238.9</td>
<td>30062.6</td>
<td>-188.5</td>
<td>146.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Compression</td>
<td>-375 -375</td>
<td></td>
<td>2.3</td>
<td>-24.3</td>
<td>24.4</td>
<td>1.2</td>
<td>110.7</td>
<td>2697.4</td>
<td>-110.2</td>
<td>10.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Compression</td>
<td>375 -375</td>
<td></td>
<td>77.3</td>
<td>-24.3</td>
<td>81.0</td>
<td>4.1</td>
<td>198.7</td>
<td>16101.5</td>
<td>-59.5</td>
<td>189.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Compression</td>
<td>-375 375</td>
<td></td>
<td>2.3</td>
<td>50.7</td>
<td>50.8</td>
<td>2.6</td>
<td>160.0</td>
<td>8125.8</td>
<td>159.8</td>
<td>7.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Compression</td>
<td>375 375</td>
<td></td>
<td>77.3</td>
<td>50.7</td>
<td>92.5</td>
<td>4.7</td>
<td>210.5</td>
<td>19468.9</td>
<td>115.5</td>
<td>176.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Tension</td>
<td>-375 1125</td>
<td></td>
<td>2.3</td>
<td>125.7</td>
<td>125.8</td>
<td>8.1</td>
<td>155.3</td>
<td>19529.0</td>
<td>155.3</td>
<td>2.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Tension</td>
<td>375 1125</td>
<td></td>
<td>77.3</td>
<td>125.7</td>
<td>147.6</td>
<td>9.5</td>
<td>169.6</td>
<td>25040.3</td>
<td>144.5</td>
<td>88.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8 Result of proposed method

5.3.2 Example 2

(a) Elastic analysis

Solving the this example in elastic analysis, the maximum load \( P_{max} \) is 195.27kN, when the bearing force of the 2 and 8 fastener reaches the nominal bolt shear strength.
Table 9 The result of elastic analysis

<table>
<thead>
<tr>
<th>Bolt No.</th>
<th>Origin at centroid</th>
<th>Concentric load term</th>
<th>Moment term</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x (mm)</td>
<td>y (mm)</td>
<td>pₓₓ/ₚ₀</td>
</tr>
<tr>
<td>1</td>
<td>-32.1</td>
<td>-96.4</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>42.9</td>
<td>-96.4</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-32.1</td>
<td>-21.4</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>42.9</td>
<td>-21.4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>-32.1</td>
<td>53.6</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>42.9</td>
<td>53.6</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>-32.1</td>
<td>128.6</td>
<td>0</td>
</tr>
</tbody>
</table>

(b) ICRM using the force-deformation relationship of Crawford and Kulak (LRFD STEEL DESIGN 3rd)

Figure 43 Location of IC
Table 10 The result of ICRM based on Crawford and Kulak’s approach

<table>
<thead>
<tr>
<th>Bolt No.</th>
<th>Boundary condition</th>
<th>Origin at centroid</th>
<th>Origin at I.C.</th>
<th>( x ) (mm)</th>
<th>( y ) (mm)</th>
<th>( x' ) (mm)</th>
<th>( y' ) (mm)</th>
<th>( r ) (mm)</th>
<th>( \Delta ) (mm)</th>
<th>( R ) (kN)</th>
<th>( rR ) (kN mm)</th>
<th>( R_x ) (kN)</th>
<th>( R_y ) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-32.1</td>
<td>-96.4</td>
<td>2.8</td>
<td>-96.4</td>
<td>96.5</td>
<td>6.6</td>
<td>315.4</td>
<td>-315.3</td>
<td>30426.1</td>
<td>-315.3</td>
<td>9.3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>42.9</td>
<td>-96.4</td>
<td>77.8</td>
<td>-96.4</td>
<td>123.9</td>
<td>8.4</td>
<td>322.7</td>
<td>-251.1</td>
<td>39992.0</td>
<td>-251.1</td>
<td>202.7</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-32.1</td>
<td>-21.4</td>
<td>2.8</td>
<td>-21.4</td>
<td>21.6</td>
<td>1.5</td>
<td>209.5</td>
<td>-207.7</td>
<td>4529.6</td>
<td>-207.7</td>
<td>27.6</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>42.9</td>
<td>-21.4</td>
<td>77.8</td>
<td>-21.4</td>
<td>80.7</td>
<td>5.5</td>
<td>307.9</td>
<td>-81.7</td>
<td>24861.2</td>
<td>-81.7</td>
<td>296.9</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-32.1</td>
<td>53.6</td>
<td>2.8</td>
<td>53.6</td>
<td>53.6</td>
<td>3.7</td>
<td>283.7</td>
<td>283.3</td>
<td>15217.0</td>
<td>15217.0</td>
<td>15.1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>42.9</td>
<td>53.6</td>
<td>77.8</td>
<td>53.6</td>
<td>94.5</td>
<td>6.4</td>
<td>314.6</td>
<td>29732.1</td>
<td>178.4</td>
<td>259.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>-32.1</td>
<td>128.6</td>
<td>2.8</td>
<td>128.6</td>
<td>128.6</td>
<td>8.8</td>
<td>323.5</td>
<td>41600.3</td>
<td>323.4</td>
<td>7.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum</td>
<td>186358</td>
<td>-70.8</td>
<td>817.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Equation 8 (the nominal bolt shear strength), the design strength of one bolt (based on shear) is 234.09kN (Equation 26).

\[
P_{\text{max}} = 817.90 \times \frac{94.25}{329.30} = 234.09kN
\] (26)

Table 9 shows that a force equilibrium in x-axis is not satisfied. In LRFD STEEL DESIGN 3rd, the process of solving problem was introduced: if the external force vector has only vertical component (y-axis), a force equilibrium in horizontal component (x-axis) will be automatically satisfied. But, in the case of asymmetric bolted connection, a force equilibrium in both direction should be considered. Resultingly, if a force equilibrium is satisfied, the ICR of the connection will get out of centroid axis of the connection.

(c) Proposed method

From Equation 12.b

\[
P = \sum \frac{(r_x \cdot R_x)}{e + r_0} = 530.40kN
\] (27)
To satisfy force equilibrium, ICR is determined like below (Figure 44).

\[
\sum_{n=1}^{8} (R_{y,n})(\phi F_{v}A_{b} / (R_{n})_{\text{max}}) = 209.22\text{kN}
\]  

(28)

Figure 44 Location of ICR by proposed method

The maximum external load \( P \) is reduced by the nominal bolt shear strength (Equation 28).

<table>
<thead>
<tr>
<th>Bolt No.</th>
<th>Boundary Condition</th>
<th>Origin at centroid</th>
<th>Origin at I.C.</th>
<th>( \Delta ) (mm)</th>
<th>( R ) (kN)</th>
<th>( rR ) (kN·mm)</th>
<th>( R_{x} ) (kN)</th>
<th>( R_{y} ) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Compression</td>
<td>-32.1 -96.4</td>
<td>0.2 -86.3</td>
<td>86.3</td>
<td>4.9</td>
<td>212.8</td>
<td>18375.4</td>
<td>-212.8</td>
</tr>
<tr>
<td>2</td>
<td>Compression</td>
<td>42.9 -96.4</td>
<td>75.2 -86.3</td>
<td>114.5</td>
<td>6.4</td>
<td>238.9</td>
<td>27357.7</td>
<td>-180.2</td>
</tr>
<tr>
<td>3</td>
<td>Compression</td>
<td>-32.1 -21.4</td>
<td>0.2 -11.3</td>
<td>11.3</td>
<td>0.6</td>
<td>77.9</td>
<td>883.6</td>
<td>-77.8</td>
</tr>
<tr>
<td>4</td>
<td>Compression</td>
<td>42.9 -21.4</td>
<td>75.2 -11.3</td>
<td>76.0</td>
<td>4.3</td>
<td>201.4</td>
<td>15315.9</td>
<td>-30.0</td>
</tr>
<tr>
<td>5</td>
<td>Compression</td>
<td>-32.1 53.6</td>
<td>0.2 63.7</td>
<td>63.7</td>
<td>3.6</td>
<td>185.9</td>
<td>11834.5</td>
<td>185.9</td>
</tr>
<tr>
<td>6</td>
<td>Compression</td>
<td>42.9 53.6</td>
<td>75.2 63.7</td>
<td>98.5</td>
<td>5.5</td>
<td>224.9</td>
<td>22159.7</td>
<td>145.3</td>
</tr>
<tr>
<td>7</td>
<td>Tension</td>
<td>-32.1 128.6</td>
<td>0.2 138.7</td>
<td>138.7</td>
<td>9.5</td>
<td>169.6</td>
<td>23519.4</td>
<td>169.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Sum</td>
<td>119446</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 11 Result of proposed method
5.3.2 Summary of results

The force-deformation behavior of a fastener is depending on geometric configuration of connection, steel specification and boundary condition of a fastener. But the force-deformation relationship of Crawford and Kulak is only applicable to the case using A325 bolt (3/4inch diameter) and A36 steel since the results of Crawford and Kulak’s method did not consider these variables. Also, in their approach, not only a fastener’s boundary condition is not considered but also the maximum deformation of fasterner is 8.64mm uniformly, irrespective of geometric configuration.

As shown in Table 12, Crawford and Kulak’s method overestimate the capacity of connection. In AISC Specification and other text books, the ICRM using Crawford and Kulak’s approach has been introduced as ultimate strength design method of group bolted connection under eccentric load because there is no suitable behavior model of force-deformation of single bolt.

Figure 45 shows the distribution of force in group bolted connection (evaluation example 1). In the case of elastic analysis, the force is distributed according to the assumptions of elastic analysis. The reaction of moment occur from centroid of the connection and vertical force is uniformly distributed to each fastener. In the case of ultimate strength analysis using Crawford and Kulak’s method, the fasteners rotate on ICR of the connection and the force is distributed unlike the case of elastic analysis. However, the ICR of the connection is on the neutral axis of the connection because Crawford and Kulak’s method does not consider boundary condition of fasteners in group bolted connection. It means that the bearing force of the fasteners is symmetric with respect to the neutral axis of the connection. Lastly, when the proposed method considering fastener’s boundary condition are used, each fastener has different value of bearing force.

Therefore, the proposed force-deformation realtionship and allowable deformation limit in this study can be applicable to the connection having different variables because the results were came from the tests having variables such as bolt diameter, steel specification, steel plate thickness and
edge distance. This method can be a reasonable approach to design group bolted connection under eccentric load.

**Table 12 Comparison among three design methods**

<table>
<thead>
<tr>
<th>Result of Example 1</th>
<th>Elastic analysis</th>
<th>Ultimate strength analysis based on ICRM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Crawford and Kulak’s method (LRFD STEEL ESIGN)</td>
</tr>
<tr>
<td></td>
<td>242.81kN</td>
<td>291.86kN</td>
</tr>
<tr>
<td>Deformation was not considered.</td>
<td></td>
<td>$\Delta_{\text{allow}} = 8.64\text{mm}$ (0.0635 rad)</td>
</tr>
<tr>
<td>Result of Example 2</td>
<td>195.27kN</td>
<td>234.09kN</td>
</tr>
<tr>
<td>Deformation was not considered.</td>
<td></td>
<td>$\Delta_{\text{allow}} = 8.64\text{mm}$ (0.0680 rad)</td>
</tr>
</tbody>
</table>
Figure 45 Force distribution of group bolted connection under eccentric load
(Evaluation example 1)
6. Summary and conclusions

In this study, the force deformation relation of bolted connection was established experimentally and mechanically. The reasonable design method which supplement limitation of current design method also was suggested. The summary of this study is like below.

(1) The current elastic analysis does not consider redistribution of group bolted connection. It means that the result of elastic analysis is commonly conservative. The ultimate strength analysis of eccentric bolted connections is an economical and mechanistic approach which can take advantage of the excellent ductility of bolted connection. However, the force-deformation relationship of single bolt, which forms the basis of the current practice, was derived long ago by Crawford and Kulak (1971) based on one-type of 3/4in-A325 ASTM bolt and A36 steel combination. Therefore, an applicability of this model is uncertain.

(2) Two-sets of experiments was conducted to investigate the influence of the end boundary conditions and surmount the limitations of the current practice as well. The first program was conducted with the “closed boundary condition” which is governed by bolt shear rupture or excessive deformation of bolt hole. The second program was a tensile test with the “open boundary condition” which is mainly affected by end tearing failure.

(3) The force-deformation relationship of each boundary condition was suggested. In closed boundary condition, the force-deformation relationship is \( R = 3.0F_u dt(1 - e^{-0.1t})^{0.55} \) came from the normalized data of compression type single bolt test. In open boundary condition, the force-deformation relationship is \( R = 1.2LtF_u(1 - e^{-0.08t})^{0.8} \) came from the normalized data of tension type single bolt test.
(4) The allowable deformation limit of single bolt was also proposed based on ratio between the bearing strength and the bolt shear strength. This allowable deformation should be sued as the limit deformation for redistributing the bolt bearing forces in the ultimate strength design of eccentric bolted connection. This result of the test shows that the deformation capacity of single bolt is affected by gemetric configuration such as bolt diameter, steel specification and edge distance.

(5) The numerical algorithm based on the Quasi-Newton method to find ICR of the connection was suggested. This approach can be resonable method to design group bolted connection under eccecntric load.

(6) The proposed force-deformation relationship was came from the tests with variables such as geometric configuration and steel specification. Therefore, the proposed model in this study can be used for a more accurate analysis and economical design of a variety of group bolted connections subjected to eccentric shaer. Finally, the proposed force-deformation relationship can be suitable model as ultimate strength design method.
References


15. Hong, J.K., and Uang, C.M. (2004) “Cyclic Testing of A Type of Cold-Formed Steel Moment Connections for Pre-Fabricated Mezzanines”, Report No. TR-04/03, University of California.


국 문 초 록

강구조물을 접합하는 대표적인 두 가지 방법으로는 용접접합과 볼트접합이 있다. 이 중 볼트접합은 볼트를 접합제로 사용하는 방법으로써 볼트를 조이는 것만으로 접합이 가능하기 때문에 시공시 특별한 기술을 요구하지 않는다. 따라서 품질을 확보하는데 있어 작업 숙련도와 작업환경에 큰 영향을 받지 않을뿐만 아니라 건설공기의 절감에도 유리하여 강구조 현장에서 선호되고 있다. 또한 구조적인 측면에서 볼때 볼트접합은 파괴모드만 잘 제어하여 지압과괴로 유도한다면 지압 메커니즘에 의한 강도상승과 소성변형에 의한 연성거동을 기대할 수 있다.

편심하중을 받는 군볼트 접합부의 설계방법에는 탄성해석법과 극한강도 설계법이 존재한다. 탄성해석법은 계산방법이 간편하나 단일볼트의 소성변형 및 연성거동을 고려하지 않는 보수적인 결과를 보인다. 군볼트 접합부의 성능을 더욱 정확하게 평가하기 위해서는 극한강도 설계법을 활용해야 한다. 하지만 현행 극한강도 설계법의 근간이 되는 단일볼트의 힘-변형 관계는 Crawford와 Kulak이 제안한 것으로 이는 미국 소재 A325 단일볼트 (직경 3/4in, 19.05mm) 및 A36 단일강재만을 이용한 실험으로부터 도출되었기 때문에 국내의 다양한 강종 및 설계조건에는 적용가능성 여부가 불분명하다.

따라서 본 논문은 극한강도 설계법의 근간이 될 수 있는 합리적인 단일볼트의 힘-변형 관계를 제안하고 이를 바탕으로 지압형식 군볼트 접합부의 설계법 및 거동을 예측할 수 있는 모델을 제안하였다. 군볼트 접합부 내에서 각각의 볼트들의 기하학적인 조건에 따라 파괴모드가 달라진다는 것에 착안하여 ‘열린경계’와 ‘닫힌경계’로 경계조건을 구분하였다. 각각의 경계조건을 고려한 두 가지 종류의 단일볼트 실험을 수행하고 이를 토대로 단일볼트의 힘-
변형 관계식을 제안하고 조건에 따른 볼트 접합부의 최대 변형량 산정식을 제안하였으며 볼트접합부의 기하학적인 요소 (볼트의 직경, 플레이트의 두께) 및 강도가 힘-변형 관계에 주된 영향을 미치는 요소라는 것을 보였다.

본 연구의 결과를 활용한다면 편심하중을 받는 군볼트 접합부의 합리적인 설계가 가능할 뿐만 아니라 거동의 예측에도 활용될 수 있다.

주요어: 군볼트 접합부, 지압강도, 힘-변형 관계, 경계조건, 순간중심회전법

학번: 2012-23130