The Term Structure of Interest Rates and The Real Activity in a Sticky Price Model

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This paper sets up a sticky price model in which money is used to reduce the transaction costs. It shows that the contemporaneous correlations between interest rates and output of the sticky price model match well the data. It also shows that a flexible price model fails to generate interest rates as inverted leading predictors of real economic activity, while a sticky price model partly has a limited success. This paper also shows that the term spread of a sticky price model partly matches the data when there is a modest nominal rigidity.

Keywords: Leading predictors, Monetary policy, Spread, Sticky price

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I. Introduction

There is an extensive literature which examines the predictive and informational content of the term structure of interest rates or yield curve, with regard to the real activity. The flattening of the yield has been interpreted by many macroeconomists and financial economists as evidence that a recession is imminent. This interpretation presumes that a flattening of the yield curve predicts a

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drop of future interest rates which are associated with a lower level of real GNP. With the enormous academic literature on the term structure, the term structure moved to the center of debate on economic policy. When there is an extra information in the term structure that is not readily available in other published statistics, the policy makers must consider this in the conduct of policy.

Among others, Fama (1990), Mankiw and Miron (1986), and Hardouvelis (1988) provide empirical evidences that the term structure has the predictive power. In particular, Estrella and Hardouvelis (1991) provide a strong evidence about the predictive power of the term structure on the real activity using OLS and Probit model. They present evidence that the slope of the yield curve can predict cumulative changes in real output up to 4 years into the future and it has an additional predictive power over other economic variables. Estrella and Mishkin (1995) step further and examine the relationship between the term structure and monetary policy in France, Germany, Italy, the UK and the US. According to them, monetary policy is an important determinant of the term structure spread in these countries.

Notwithstanding this enormous empirical research on the term structure, there has been little theoretical research on this. Among them, Harvey (1988) claims that the consumption capital asset pricing model (CCAPM) is consistent with the observed predictability of consumption growth. Although CCAPM provides an elegant explanation of the relationship between the slope of the yield curve and future consumption growth, it cannot provide a full explanation of the empirical evidence such as the relationship between the term structure and GNP, investment, and so on. Kydland and Prescott (1988) construct a real business cycle (RBC) model that generates a positive correlation between the real interest rate and real output. But it is not clear how this model relates to the slope of the term structure. Since it does not consider monetary policy, one cannot say anything about the relationship between nominal interest rates and output, and the response of the yield curve to monetary policy shock. In recent, King and Watson (1996) as well as Cooley and Hansen (1995) document other interesting empirical facts about the serial correlation of monetary aggregates and real economic activity over the business cycles. They note that nominal interest rates move procyclically while real interest rates move countercyclically.

Moreover, both real and nominal interest rates are negatively correlated with future real output which means that they are inverted leading indicators for output.

In order to address these issues more fully, I will consider a sticky price model as well as an RBC model with a flexible price in this paper. I follow Calvo (1983), Woodford (1996), and Yun (1996) to set up a general model with sluggish price adjustment and discuss the relationship between the role of monetary policy and the term structure. The main results from this paper can be summarized as follows. First, the contemporaneous correlations between interest rates and output in a flexible price model as well as in a sticky price model match well with the data. Second, a flexible price model fails to generate interest rates as inverted leading predictors of real economic activity as in Cooley and Hansen (1995), while a sticky price model succeeds in generating interest rates as inverted leading predictors of real activity when there is modest nominal rigidity. Finally, this paper also shows that the term spread of the flexible price model does not match with data, while the sticky price model partly succeeds in generating the cross correlation.

This paper is composed as follows. In section II, I specify a sticky price model with a transaction cost, in which the transaction cost is reduced when a representative household use money in purchasing goods. In section III, I derive an equilibrium and discuss the implications of a capital-asset pricing formula as well as interest rates, and in section IV I discuss the quantitative implications of the model. Finally I give a concluding remarks in section V.

II. The Model

The model in this paper is based on monopolistically competitive markets in which there exist a continuum of differentiated goods indexed by [0, 1]. Assume that utilities of consumers and additions to the aggregate capital stock depend only on the amount of a single composite good. This is associated with introducing an aggregator of the differentiated goods. Consider the aggregator[1]

[1]See Rotemberg and Woodford (1992) for a detailed discussion of the specification of the aggregator in monopolistically competitive goods market.

suggested by Dixit and Stiglitz (1977) such that

$$J_t = \left[\int_0^1 d_t(j)^{1-\phi}\, dj \right]^{\frac{1}{1-\phi}}, \quad 0 < \phi < 1, \tag{1}$$

where J_t denotes the number of units of the composite good at time t, and $d_t(j)$ is the measure of the purchases of the jth good at time t. Then the purchases of differentiated goods over [0, 1] are the solutions to minimize the total cost of obtaining J_t such that

$$\min. \int_0^1 P_t(j) d_t(j) dj \quad s.t. \quad J_t \leq \left[\int_0^1 d_t(j)^{1-\phi} dj\right]^{\frac{1}{1-\phi}}, \tag{2}$$

where $P_t(j)$ is the price of the jth good at time t. The cost minimizing demand for each differentiated good and the minimized cost are

$$d_t^*(j) = \left[\frac{P_t(j)}{P_t} \right]^{\frac{-1}{\phi}} J_t, \quad P_t = \left[\int_0^1 P_t(j)^{\frac{\phi-1}{\phi}} dj \right]^{\frac{\phi}{\phi-1}}, \quad \int_0^1 P_t(j) d_t^*(j) dj = P_t J_t,$$

The demand for each differentiated goods is determined by the above cost minimization when the demand for the composite good is given. This allows the following optimization problem of the household to be described in terms of the composite good only.

A. Model with Transaction Cost

a) Transaction Cost

Among the assets available to agents are money, equities, and bonds. The total supply of equity is normalized to unity. Assume that there exist n kinds of bonds with a risk-free bond in this economy, and that money reduces the costs of consumption transactions, as in Feenstra (1986). That is, when the household has real balance holdings equal to m_t, it must expend additional $\varphi(C_t, m_t)$ units of goods to consume C_t units as transaction costs. Here the real balance is defined as $m_t = M_t/P_t$. As in Feenstra (1986), we assume that each transaction cost function is homogenous of degree one in both arguments with $\varphi_1 \geq 0$, $\varphi_2 < 0$, $\varphi_{11} \geq 0$, $\varphi_{22} \geq 0$, and $\varphi_{12} < 0$.

b) Preferences and Budget Constraints

Suppose that a representative household enjoys consumption streams $\{C_t\}_{t=0}^{\infty}$

$$U_t(C_t,\ L_t,\ C_{t+1},\ L_{t+1},\cdots)\equiv E_t\left[\sum_{j=0}^{\infty}\beta^j u(C_{t+1},\ L_{t+1})\right],\ 0<\beta<1, \tag{3}$$

where β is the household's discount factor, and E_t denotes the conditional expectations operator on the information available in period t. L_{t+j} represents the leisure of the household at time $t+j$.

The household also faces a time constraint such that

$$L_t+H_t\leq\overline{H}, \tag{4}$$

where H_t and $\overline{H}$ denote the hours worked and time endowment of the household respectively.

The household starts at period t with nominal wealth Θ_t carried over from period $t-1$ and receives a lump-sum transfer of money, T_t before the goods market opens. That is, the household observes the current disturbance to real activity and the current (gross) rates of monetary expansion, ω_t at the beginning of each period. At the goods market the household buys goods using currencies, subject to a shopping transaction cost, φ. At the asset market all interest and dividend payments are made, and assets are traded. Assuming that there is a risk-free bond, the household chooses currency, M_{t+1} as well as bonds of various maturities, $B_{k,t+1}$ ($k=1,\cdots,n$) and securities, Z_{t+1} at the asset markets.

Moreover, each household owns only its own capital stock to rent to a firm and there is no firm specific capital stock. Since we do not empirically observe large discrete capital stock adjustments, it is reasonable to introduce an adjustment cost in capital stock installments. If there are costs of installing capital, the capital stock will move more sluggishly. I assume that there are dead-weight costs of installing capital stock. To preserve the simple model structure as far as possible, I will adopt the Uzawa-Lucas-Prescott form of investment adjustment costs.

$$K_{t+1}=\psi\left(\frac{I_t}{K_t}\right)K_t+(1-\delta_K)K_t, \tag{5}$$

where $\psi(I_t/K_t)$ is a positive, concave function, I_t is the composite investment of the household at period t, and K_t is the composite capital stock of the household at period t. At the end of each period, the household receives wages, rents for capital, and dividends from each firm. Thus its wealth at the beginning of the period $t+1$ is given by,

$$\int_0^1 P_t(s)[c_t(s)+I_t(s)+\varphi\{c_t(s),m_t\}]ds+M_{t+1}+Q_tZ_{t+1}+P_t\sum_{k=1}^{n}B_{k,t+1}\gamma_{k,t}\leq\Theta_t+T_t, \quad (6)$$

The household's wealth at the beginning of the period t is given by,

$$\Theta_t=M_t+P_tB_{0t}+\sum_{k=1}^{n}P_tB_{k,t}\gamma_{k,t+1}+(Q_t+P_td_t)Z_t+W_tH_t+R_tK_t+\Pi_t \quad (7)$$

Here $\gamma_{k,t+1}$ is the real prices of an indexed bond maturing in k periods from time $t+1$. Π_t, W_t, and R_t denote the firm's nominal profits, nominal wages and nominal rental rate for capital stock given to the household, respectively.

c) Firms

In my model differentiated goods and monopolistic competition are introduced along the lines of Dixit and Stiglitz (1977). Suppose that there are a continuum of firms producing differentiated goods, and each firm indexed by j, $0\leq j\leq 1$, produces its product with constant returns to scale, concave production technology. Each firm j takes P_t and the aggregate demand as given, and chooses its own product price $P_j(t)$. Since the input markets are perfectly competitive, the demands for labor and capital are determined by its cost minimization as follows,

$$C\{W_t,\ R_t,\ Y_t(j),\ H_0,\ z_t\}\equiv\min_{H_t(j),K_t(j)}\{R_tK_t(j)+W_tH_t(j)\} \quad (8)$$

$$s.t.\ \ Y_t(j)\leq A_tF[K_t(j),\ z_t\{H_t(j)-H_0\}].$$

Here H_0, z_t and A_t are the household's fixed overhead cost in units of labor hours, labor augmenting permanent technology progress, and transitory technology process at period t. $Y_t(j)$[2] is the output of

[2] $A_tF[K_t(j),\ z_t\{H_t(j)-H_0(j)\}]$ is strictly concave, twice continuously differentiable, and CRS in $K_t(j)$ and $H_t(j)-H_0(j)$ but it is IRS in $K_t(j)$ and $H_t(j)$.

the jth firm in the economy. I assume that the technology shock follows an $AR(1)$ process, the permanent changes in the total factor productivity, z_t are taken as growing deterministically, *i.e.* $\gamma = z_t/z_{t-1}$ for all t as in King, Plosser, and Rebelo (1988).

$$\log A_t = \rho \log A_{t-1} + \xi_{At}, \quad -1 < \rho < 1, \tag{9}$$

where $E(\xi_{At}) = 0$, and ξ_{At} is i.i.d. over time.[3]

From the firm's first-order condition,

$$\begin{aligned} R_t &= MC_t(j) A_t F_1[K_t(j),\ z_t\{H_t(j) - H_0\}] \\ W_t &= MC_t(j) A_t F_2[K_t(j),\ z_t\{H_t(j) - H_0\}], \end{aligned} \tag{10}$$

The marginal cost of each firm is equal, *i.e.* $MC_t(j) = MC_t$ for each j as the production function is *CRS*. This also implies that $\{H_t(j) - H_0\}/K_t(j) = (H_t - H_0)/K_t$ for all j, and thus the wage and rental rate can also be represented by (10).

d) Staggered Price Setting

In this subsection, I use a discrete time variant of a model introduced by Calvo (1983), which has become a standard baseline model of nominal rigidity in quantitative general equilibrium model. The monopolistic competition firms in the product markets set their own prices in advance by maximizing the present discounted value of profits. Suppose that only a fraction $1-\alpha$ of the firms sets the new price, $P_{t,t}$ while the other fraction of firms, α sets its price by multiplying the average inflation rate or average monetary growth rate (ω) by their previous price level. Those firms that get to set new prices are chosen randomly each period, with each having an equal probability of being selected. The probability of an optimal price change is independent of both the time that has elapsed since the last optimal price change, and the degree to which costs and other market conditions have changed since then. Let $D_{t,t+k}$ denote the demands at period $t+k$ facing firms that set their prices at time t, and $P_{t,t+k}$ the prices at period at $t+k$ that are predetermined at time t. As the price applies in period t with certainty, in period $t+1$ with a probability α, in period $t+2$ with α^2,

[3] $\rho = 0.95$ and $\sigma_A = 0.007$ are used in the artificial economy.

and so on, the firm's maximization problem can be written as follows:

$$\max.\ E_t \sum_{k=0}^{\infty} (\alpha)^k \Lambda_{t+k}\{R_{t,t+k} P_{t,t+k} D_{t,t+k}(P_{t,t+k})\} - \beta^k MC_{t+k} D_{t,t+k}(P_{t,t+k}), \quad (11)$$

where Λ_{t+k} is the marginal utility for the household of additional money income at $t+k$.

As total demand for a good j is given by

$$D_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{\frac{-1}{\psi}} Y_t, \quad (12)$$

where

$$Y_t = C_t + I_t, \quad (13)$$

the first order condition of this newly determined price at time t implies the following recursive price level

$$P_t^{-\varphi} = (1-\alpha) P_{t,t}^{-\varphi} + \alpha\,\omega^{-\varphi} P_{t-1}^{-\varphi}. \quad (14)$$

When $\alpha = 0$ the optimization conditions are reduced to that of a flexible price level such that

$$P_t = \frac{MC_t}{1-\psi}. \quad (15)$$

If the price level is flexible, then the markup — the ratio of price to marginal cost — is constant at each period, while it responds to monetary and real shocks when prices are predetermined.

e) Monetary Authority

Suppose that the central bank of prints currency and distributes it to the residents in lump-sum transfer fashion, T_t, before the opening of the asset markets at every period. That is, the money stock at time t is given by

$$M_t^s = M_{t-1}^s + \overline{T}_t, \qquad \overline{T}_t = (\omega_t - 1) M_{t-1}^s. \quad (16)$$

Moreover, I assume that the monetary authority sets its own money supply growth rate according to the following rule as in Eichenbaum and Evans (1995), and Yun (1996).

$$\log(\omega_t)=Ç(\Omega_t)+\xi_t. \tag{17}$$

Here $Ç$ is a linear function and Ω_t is the information set available to the monetary authority when ω_t is set and ξ_t is a serially uncorrelated shock that is orthogonal to the elements of Ω_t. That is, I regard (17) as the monetary authority's decision rule for setting $\log(\omega_t)$, which means that ξ_t is a policy disturbance.

III. Equilibrium

A. First-Order Conditions

The first-order necessary conditions for a solution to the household problem posed by (1) to (3) are the budget constraints with the following Euler equations:

$$U_1=\Lambda_t\{1+\varphi_1(C_t,\ m_t)\}, \tag{18}$$

$$\Lambda_t=\beta E_t\left[\Lambda_{t+1}(1-\varphi_2)\frac{P_t}{P_{t+1}}\right], \tag{19}$$

$$U_2=\Lambda_t\omega_t, \tag{20}$$

$$\Lambda_t=\beta E_t\left[\frac{P_{t\,t+1}}{P_{t+1}}\Lambda_{t+1}\right], \tag{21}$$

$$\frac{\Lambda_t}{\gamma_{k,t}}=\beta E_t\left[\frac{\Lambda_{t+1}}{\gamma_{k-1,t+1}}\right]\ \text{for}\ k=1,2,\cdots,n, \tag{22}$$

$$\Lambda_t q_t=\beta E_t\{\Lambda_{t+1}(q_{t+1}+d_{t+1})\}, \tag{23}$$

$$\Lambda_t\psi'^{-1}(X_t)=\beta E_t\{\Lambda_{t+1}\psi'^{-1}(X_{t+1})x(X_{t+1})+\gamma_{t+1}\Lambda_{t+1}\}, \tag{24}$$

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{\frac{-1}{\phi}} C_t, \quad I_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{\frac{-1}{\phi}} I_t, \tag{25}$$

where $q_t = Q_t/P_t$ is the real price of shares and Λ_s, Δ_s are Lagrange multipliers associated with the household's budget constraint and capital stock adjustment respectively, and $X_t = I_t/K_t$. Here $x(X_{t+1}) = \phi(X_{t+1}) - \phi'(X_{t+1})X_{t+1} + 1 - \delta_k$, ϕ'^{-1} equals Tobin's q, $r_t = R_t/P_t$, and $w_t = W_t/P_t$. Equations (19) and (20) say that the household equates the marginal rate of substitution between current and next period's consumption to the marginal rate of transformation. Equation (26) says that the jth consumption goods and investment goods are determined by the cost minimization demands when the composite demands are given.

B. Term Structure of Interest Rates

The real gross rate of return of the sure bond is given by

$$\gamma_{t+1} = \frac{\Lambda_t}{\beta E_t \Lambda_{t+1}}. \tag{26}$$

The real gross interest rate defined is the present marginal utility of wealth over the discounted conditional expected marginal utility of next period's wealth. It is related to the expected intertemporal marginal rate of substitution between wealth in the current and next period, not between consumption in current and next period. The real gross interest rate from time t into the k periods ahead is the present marginal utility of wealth over the discounted conditional expected marginal utility of the k periods ahead wealth.

$$\gamma_{k,t+1} = \frac{\Lambda_t}{\beta^k E_t \Lambda_{t+k}}. \tag{27}$$

If one denotes the nominal interest rate by R_{t+1}, then the real interest rate is related to the nominal interest rate by $R_{t+1} = \gamma_t(P_{t+1}/P_t)$. If one substitutes this relation into (27), the nominal interest rate is given by the conditional expected utility of the liquidity services of money over the conditional expected utility of the wealth of money. That is,

$$R_{t+1}=\frac{\Lambda_t}{P_t}\cdot\beta E_t\left[\frac{\Lambda_{t+1}}{P_{t+1}}\right]=\frac{E_t\left[\frac{\Lambda_{t+1}\{1-\varphi_2(c_{t+1},\ m_{t+1})\}}{P_{t+1}}\right]}{E_t\frac{\Lambda_{t+1}}{P_{t+1}}}. \tag{28}$$

It is evident from the above equation that the gross nominal interest rate cannot be less than one, *i.e.* $R_{t+1}\geq 1$, and it is strictly greater than one when the expected marginal utility of liquidity service is positive, *i.e.* $-\varphi_2(c_{t+1},\ m_{t+1})>0$.

In the above equation the effects of all future variables, *i.e.* their expectations are reflected in the marginal utility of wealth, Λ_t. That is, the household's demand for consumption goods and labor supply depend on her own wealth, current wage rate, and interest rate. This is comparable to that of the traditional international IS-LM model. The price of money can be interpreted in terms of capital-asset-pricing equation as follows:

$$P_t=\sum_{j=1}^{\infty}\beta^j\frac{E_t[\Lambda_{t+j}\{1-\varphi_2(c_{t+j},\ m_{t+j})\}]}{\Lambda_t} \tag{29}$$

Here money is priced in complete symmetry with other assets. That is, the real price of a unit of money is the discounted sum of all future period's conditional expected marginal utility of money, divided by the current marginal utility of wealth. The direct return of money is the value of the liquidity services provided by the value of reducing transaction costs, $-\varphi_2(c_t,\ m_t)$.

(23) says that when the household buys one unit of security at q_t at time t and sells it next period, the increase of a utility of $\Lambda_t q_t$ at time t which it gets equals the decrease of a utility of $\beta E_t\{\Lambda_{t+1}(q_{t+1}+d_{t+1})\}$ at time $t+1$ at equilibrium. That is, the price of the claim equals the discounted expected next period's marginal utility of real wealth times the gross return on the claim, divided by the current marginal utility of wealth. Using recursion on (23) and the law of iterated expectations, one gets a standard capital asset pricing model (CAPM)

$$q_t=E_t\sum_{j=1}^{\infty}\left[\beta^j\frac{\Lambda_{t+j}}{\Lambda_t}d_{t+j}\right], \tag{30}$$

in which the share price is an expected discounted stream of dividends with time varying and stochastic discount rates. If $q_t > E\sum_{j=1}^{\infty}\{\beta^j(\Lambda_{t+j}/\Lambda_t)d_{t+j}\}$, the household can make the present value of consumption as large as it wants by selling the security price at q_t. However, in this economy, it is not feasible for the household to achieve any such desired present value of consumption. Therefore, in equilibrium one cannot have $q_t > E\sum_{j=1}^{\infty}\{\beta^j(\Lambda_{t+j}/\Lambda_t)d_{t+j}\}$. If $q_t < E\sum_{j=1}^{\infty}\{\beta^j(\Lambda_{t+j}/\Lambda_t)d_{t+j}\}$, the household can make the utility as large as it wants by buying the security at q_t which is impossible in this economy. Thus (23) holds in equilibrium. These capital-asset-pricing equations are standard except that the marginal utility of consumption is not always equal to the marginal utility of wealth. As the household needs money to reduce the transaction cost, there is a wedge between the marginal utility of wealth and that of consumption, and the marginal utility of wealth is less than the marginal utility of consumption.

IV. State Space Analysis

First, I will represent the economy system in a state space to explore the dynamics of the economy. Next, I will analyze the response of the economy to shocks of technology and monetary policy using essentially the method of King, Plosser, and Rebelo (1988). That is, I restrict my attention to the case of small fluctuations of the endogenous variables around a steady state growth path.

Since most of the following analysis will be done in stationary terms, it is more convenient to define a symmetric rational equilibrium in terms of a stationary one. Finally, I will explore the implications of asset pricing in this dynamic sticky price model.

Though I need not specify the functional form for adjustment cost function, ϕ, I should specify three parameters which describe the behavior around the steady state. First, I must specify the steady state value of Tobin's q and the share of investment in national product. Since the steady state value of Tobin's q is 1.0, I also set the value of this variable to 1.0 in steady state. And I will take the same investment share in the steady state as in a model without adjustment cost. Next, I have to specify the parameter which determines the elasticity of marginal adjustment cost function. Though there is little study about this adjustment cost parameter value, I will present results when there is a moderate adjustment cost as in King and Wolman (1996) and Baxter and Crucini (1993) in next section.

In this system, the state vector at period t, s_t consists of a technology shock (A_t), a monetary shock (ω_t), a predetermined capital stock (k_t), and a previous price level (p_{t-1}) (All in log forms). Since each firm in each group sets the same price in symmetric equilibrium, it is desirable to divide consumption and investment goods into groups on the basis of the staggered prices setting decisions times. Let $\{c_t(j),\ i_t(j)\}_{t=0}^{s}$ and $s=\infty$ denote the groups of firms that sets their prices of time t at time $t-j$.

The stochastic symmetric stationary equilibrium consists of the bounded time invariant decision rules $\{c^j(s_t),\ i^j(s_t),\ k^j(s_t),\ H^j(s_t),\ c(s_t),\ i(s_t),\ \lambda(s_t)\}_{j=0}^{s}$ and prices $\{p_{t,\ t}(s_t),\ p(s_t),\ q(s_t),\ \iota(s_t),\ w(s_t),\ r^k(s_t),\ \lambda(s_t)\}_{k=1}^{n}$ with state of the economy s_t such that

1) The households decision rules$\{c^j(s_t),\ i^j(s_t),\ H^j(s_t),\ c(s_t),\ i(s_t),\ \lambda(s_t)\}_{j=0}^{s}$ solve their optimization problem given the states and the prices.
2) The demands for labor and capital $H^j(s_t)$, $k^j(s_t)$ solve each firm's cost minimization problem, and price setting rules $p_{t,t}$ solves its present value maximization problem given the states and the prices.
3) Each goods market, capital rental market, labor market, bond market, and money market is cleared at $\{p_{t-j,t}(s_{t-j}),\ q(s_t),\ r^k(s_t),\ w(s_t),\ \iota(s_t)\}$ respectively.

A. Dynamics around Steady State

Now I characterize a stationary equilibrium in the case of small fluctuations of the detrended state variables around their steady state values. In order to do that, I approximate a stationary equilibrium involving small fluctuations around it by the solution of the log-linear approximation to the equilibrium condition.

Following Blanchard and Kahn (1980) and King, Plosser, and Rebelo (1988), I regard consumption (c_t) and labor supply (H_t) as well as the investment-capital stock ratio (X_t) as control variables, and both countries output and wage rate and interest rate as flow variables. Thus the model can be simplified as follows.

$$B\begin{bmatrix} \hat{k}_{t+1} \\ \hat{p}_t \\ E_t\ \hat{p}_{t+1} \\ E_t\ \hat{\lambda}_{t+1} \\ E_t\ \hat{\mu}_{t+1} \\ E_t\ \hat{q}_{t+1} \end{bmatrix} = D\begin{bmatrix} \hat{k}_t \\ \hat{p}_{t-1} \\ \hat{p}_t \\ \hat{\lambda}_t \\ \hat{\mu}_t \\ \hat{q}_t \end{bmatrix} + F\begin{bmatrix} E_t\ \hat{A}_{t+1} \\ E_t\ \hat{w}_{t+1} \end{bmatrix} + G\begin{bmatrix} \hat{A}_t \\ \hat{w}_t \end{bmatrix} \tag{31}$$

$$\Phi\begin{bmatrix} \hat{c}_t \\ \hat{H}_t \\ \hat{x}_t \end{bmatrix} = \Gamma\begin{bmatrix} \hat{k}_t \\ \hat{p}_{t-1} \\ \hat{p}_t \\ \hat{\lambda}_t \\ \hat{\mu}_t \\ \hat{q}_t \end{bmatrix} + Y\begin{bmatrix} \hat{A}_t \\ \hat{w}_t \end{bmatrix}.$$

This state space representation corresponds to the economy system in a state space:

$$S_{t+1} = \Pi S_t + C\varepsilon_{t+1} \tag{32}$$

$$Z_t = \Xi S_t, \tag{33}$$

where S_t and Z_t are the vector of state variables and the vector of control variables at time t, *i.e.* $S_t = [\hat{k}_t\ \hat{A}_t\ \hat{p}_{t-1}]'$ and $Z_t = [\hat{c}_t\ \hat{H}_t\ \hat{x}_t\ \hat{p}_t\ \hat{\lambda}_t\ \hat{\mu}_t\ \hat{q}_t]'$. ε_{t+1} is the vector of innovations at time $t+1$.

V. Quantitative Evaluation of the Model

A. Parameter Values

Since money is defined as M_1, I have $M_1/PC = 0.34$. From the money market equilibrium, and $C_t = s_{ct}Y_t$ at steady state, it follows that $h(\iota_t) = M_t/s_{ct}P_tY_t \Rightarrow \log(M_t/P_t) = \log(s_{ct}) + \log h(\iota_t) + \log(Y_t)$. I will take Lucas (1988)'s estimates of the elasticity of money with respect to interest rate. Lucas obtained -0.07 for 1958-85 and -0.09 for 1900-85 for M_1. That is, $h_i = -7$ or -9. The monetary growth rate is estimated using US monetary base data for a monetary policy. Assuming that monetary base growth rate of US follows an $AR(1)$, its estimates over 1972:1-1991:4 is given by

$$\log \omega_t = \underset{(0.00146)}{0.00646} + \underset{(0.08674)}{0.60569} \log \omega_{t-1} + \underset{(0.00503)}{\varepsilon_{Mt}}, \tag{34}$$

where the numbers in the parenthesis represent the standard errors.

I will use the parameter values of the US economy for the model. All parameter values used in this paper are reported in Table 1. Most of them are taken from King, Plosser, and Rebelo (1988), Lucas (1988), and Rotemberg and Woodford (1992). In particular, one needs to note the intertemporal elasticities of consumption and labor supply because these parameter values are important in the quantitative implications of the model. Even though many RBC models assume that unit elasticity of intertemporal substitution ($\varepsilon_c = \sigma_c^{-1} = 1$) which is taken from Hansen and Singleton (1982), many empirical studies on consumption tell us to be more cautious and conservative in choosing the value. Thus the baseline model of this paper takes lower values of intertemporal elasticity of consumption, $\sigma_c = 2$, *i.e.* $\varepsilon_c = 1/2$. This intertemporal elasticity of consumption is much smaller than those in Rotemberg and Woodford (1992). The value of elasticity of i/k with respect to Tobin's q, η_q is the cost adjustment elasticity which reflects the volatility of investment. Since previous studies have not estimated this cost adjustment parameter, I will choose $10 as the value in the baseline model.[4] This value can be considered conservative for two reasons. On the one hand, higher values of this elasticity perform better since investment responds more to shocks. On the other hand, various second moments of the model match well with those of data even when there is only a small degree of price rigidity in the model. Finally, I will choose 1.1 as the benchmark average size of markup, μ. Though this value is much lower than the value that many sources of evidence suggest,[5] it is consistent with the average markup

[4]Baxter and Crucini (1993) used the elasticity of 15 as a benchmark parameter value.

[5]See Rotemberg and Woodford (1992) for more detailed discussion and

TABLE 1

THE CALIBRATED PARAMETERS

Parameter	Value	Description of Parameters
γ	1.004	Steady state quarterly growth rate of technology
s_H	0.58	Steady state labor share
δ	0.025	Rate of depreciation of capital stock
r	0.016	Steady state rate of return
$\varepsilon_c(\sigma_c^{-1})$	1/2	Intertemporal elasticity of consumption
h_i	-9	Semi-elasticity of demand for money (percent)
$h(i)$	0.34	Inverse of steady state consumption velocity
μ	1.1	Steady state markup
ε_{nk}	1	Elasticity of substitution between capital and labor
η_q	10	Elasticity of i/k to Tobin's q

estimates in Basu and Fernald (1993).

B. Implications of the Model

In this subsection I review the main goal of this paper and see whether the nominal price rigidity model with monopolistic competition can explain the relationship between the term structures and real economy.

a) Persistent Effects of Monetary Shock

The first issue that I address is if actual data impulses correspond to the dynamic responses of interest rates and real activities to monetary shocks implied by this "sticky-price" model. The main results of Leeper and Gordon (1992)'s VAR and Estrella and Hardouvelis (1991), and Estrella and Mishkin (1995)[6] can be summarized as follows. A positive shock to monetary policy is associated with persistent nominal and real interest rate declines, and increases output. Second, a positive slope of the yield curve is

references about markup.

[6]Eichenbaum and Evans (1995) use NBR (Non Borrowed Reserves), NBRX (ratio of NBR to Total Reserves), and Romer and Romer index to measure monetary shocks. Their empirical results are robust to the measure of a monetary policy shock.

associated with a future increase in real economic activity. Table 2 taken from Stock and Watson (1999) shows that the marginal R^2 of the candidate series such as 3 month Treasury Bill rate and term spread between 10 years Treasury Bond rate and 3 month Treasury Bill rate to forecast output growth one and four quarter ahead is substantial. In particular, the marginal R^2 of spot rate and that of term spreads exceed 0.25 in 4 quarter ahead output growth. Moreover, as Table 3 and Figure 1 show, the serial correlation between current nominal interest rates and 4-period ahead output are strongly positive. Third, an increase of monetary growth rate, *i.e.* a decrease of federal fund rate tends to make the yield curve steeper, and the fall of long term interest rate is smaller than that of short term interest rate (See Figure 2).

Here I discuss the model's implications. First, let's consider the response of the real activity to monetary shocks. In the flexible price monetary model with no sluggish adjustment in portfolio, when there is a positive monetary shock, investment slightly increases, while consumption decreases a little bit. This is because the consumption good is a cash good while the investment good is a credit good in the transaction cost model as in cash-in-advance models.

The impulse response of the endogenous variables to exogenous shocks varies depending on the degree of nominal rigidity in the model as well as some deep parameter values such as intertemporal elasticities of labor supply and consumption. As the degree of nominal rigidity increases, the endogenous variables which are the household's decision rules respond more to a monetary shock. This result is due to the fact that the sluggish price adjustment to a positive monetary shock causes the markup to move much more negatively as the degree of price rigidity increases, and this leads the demand for labor to move upward more as the markup adjusts more negatively. Thus both investment and output increase as marginal product of capital goes up with the increase in labor demand. This response of real variables to a positive monetary shock becomes stronger as more and more firms depend on rule of thumb markup adjustments rather than optimal price setting rules as shown in Figure 3 to Figure 5. With these responses of markup and labor to a monetary shock, the reactions of capital stock and output also become much more volatile as the degree of price rigidity increases. Thus as the real quantities respond more to

TABLE 2

RESULTS OF PREDICTIVE REGRESSION

Variable	$R^2_{y_{t+1}S_t \cdot Y_t}$	$R^2_{y_{t+4}S_t \cdot Y_t}$
R_{n1}	0.10	0.27
$R_{n10}-R_{n1}$	0.10	0.28

Note: R_{n1} and $R_{n10}-R_{n1}$ denote 3-month Treasury Bill rate, and term spread of 10-years Treasury Bond rate and 1-year Treasury Bill rate.
Source: Stock and Watson (1999).

TABLE 3

MOMENTS OF DATA

Variable	Std. Dev. (Relative)	Autocorr. $t-1$	Cross Corr. with GDP X_{t-4}	X_{t-1}	X_t	X_{t+1}	X_{t+4}
Output	1.65(1.00)	0.85	0.26	0.85	1.00	0.85	0.26
Consumption	0.84(0.76)	0.85	0.37	0.82	0.85	0.71	0.14
Investment	5.56(3.37)	0.89	0.27	0.80	0.89	0.79	0.16
R_{n1}	1.31(0.75)	0.72	-0.60	0.01	0.27	0.35	0.38
$R_{n4}-R_{n1}$	0.39(0.22)	0.47	0.28	-0.18	-0.23	-0.27	-0.28

Note: R_{n1} and $R_{n4}-R_{n1}$ denote term spread of 1-year Treasury Bill rate and 3-month Treasury Bill rate.
Source: CITIBASE TAPE (1960:1-1991:4).

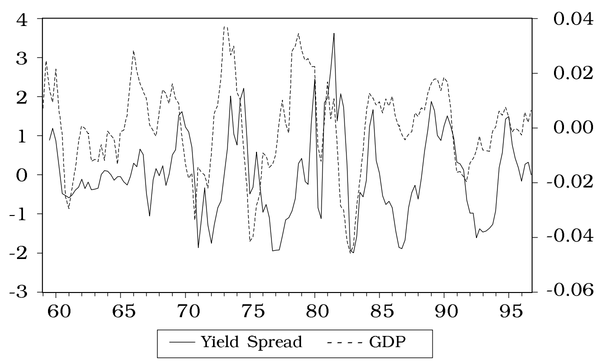

FIGURE 1

YIELD SPREAD AND GDP

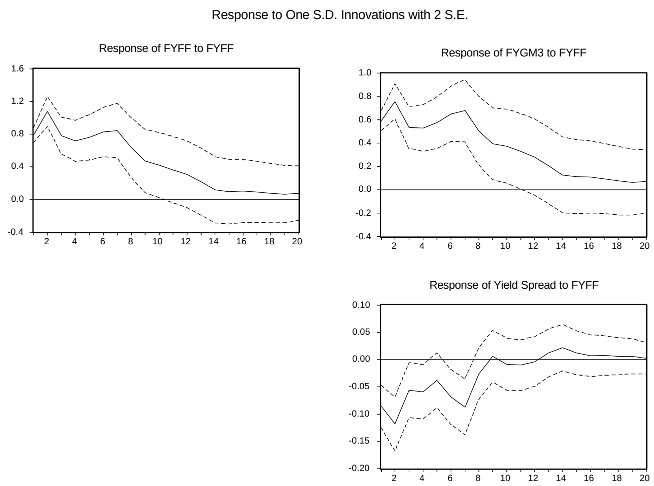

FIGURE 2

IMPULSE RESPONSE FUNCTION TO A SHOCK TO A FEDERAL FUND RATE

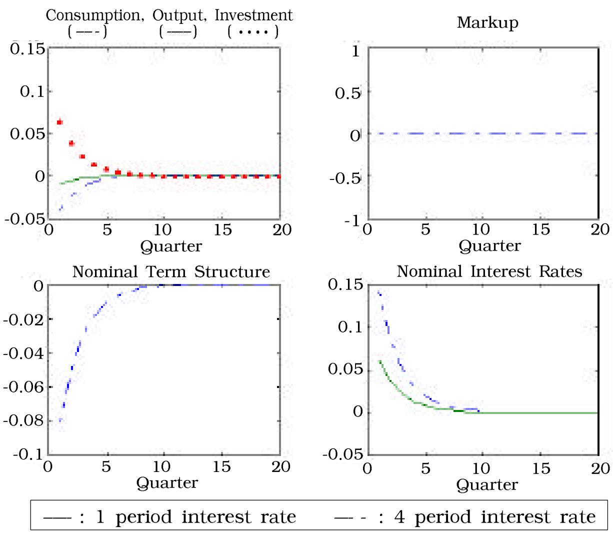

FIGURE 3

IMPULSE RESPONSE FUNCTION TO A POSITIVE MONETARY SHOCK IN A FLEXIBLE PRICE MODEL

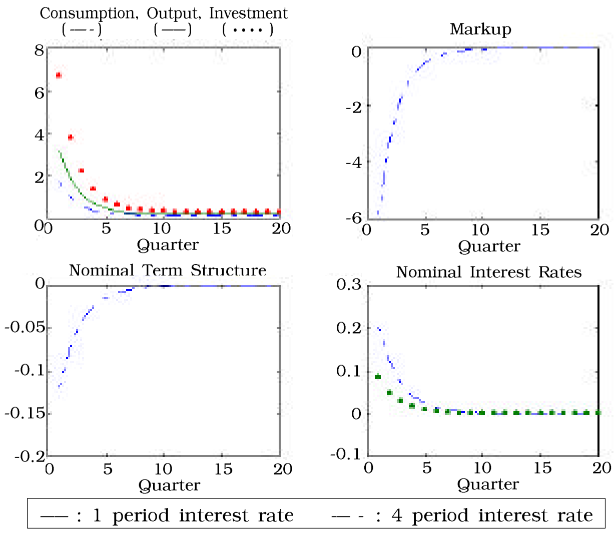

FIGURE 4

IMPULSE RESPONSE FUNCTION TO A POSITIVE MONETARY SHOCK IN A STICKY PRICE MODEL (α =0.75)

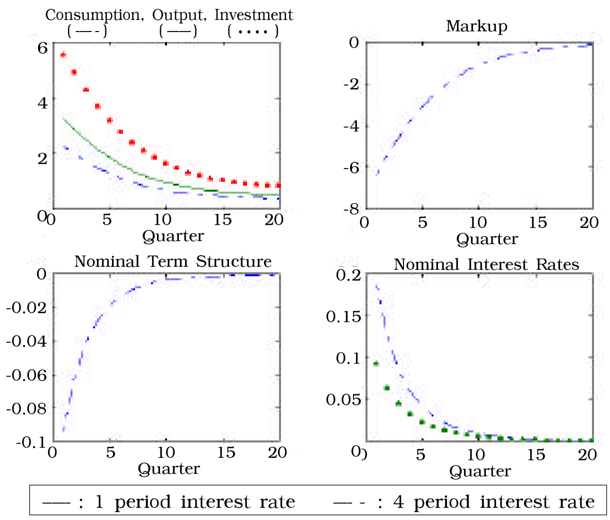

FIGURE 5

IMPULSE RESPONSE FUNCTION TO A POSITIVE MONETARY SHOCK IN A STICKY PRICE MODEL (α =0.9) monetary shocks, price responds less to monetary shocks with the increase of price rigidity α.

The impulse responses of nominal interest rates, however, are not satisfactory in that the nominal interest rates, whether they are long-term or short-term, increase instantaneously to a positive monetary shock. The inflation effect in this sticky price model or in the flexible price model seems to dominate the liquidity effect. In regard to the term structures of interest rates, things go the other way around. The short term nominal interest rate increases more than the long term real interest rate to a positive shock. This means that the term spread decreases to a positive monetary shock. In this way, the impulse response functions do not match with data because the term spread increases to a positive monetary shock in a 3 variable VAR model.

The responses of other real variables to a monetary shock also last long enough and become excessively large compared to data as the nominal rigidity (α) increases above a half. These results follow from the fact that as the price rigidity becomes excessive, the response of real variables also becomes excessive because markup responds too excessively to monetary shock. Overall, the sticky price model with monopolistic competition generates successfully the empirical findings that Estrella and Hardouvelis (1991) and Estrella and Mishkin (1995) present.

b) Variabilities and Serial Correlations

In this subsection, I compare volatilities and serial correlations of the real variables of the baseline model with those of data to see the overall performance of the model. First, consider the standard deviation of the variables in model and data. A prominent feature about the investment is its (excessive) volatility relative to other real variables as can be seen in Table 2 where some selected moments of data are presented. When half of firms in the economy adjust their prices optimally, and the other half adjusts their prices by a simple markup with previous prices, *i.e.* when $\alpha = 0.5$ the standard deviations of consumption, investment and output are 1.64, 1.56, and 2.99 respectively. While the consumption volatility of the model is slightly larger than that of data, the investment volatility is relatively smaller than that of data. It is noteworthy that the volatilities of real variables increase as the degree of price stickiness increases. The volatility of investment increases to match

TABLE 4

MOMENTS OF BENCHMARK MODEL

Variable	Std. Dev. (Relative)	Autocorr. $t-1$	Cross Corr. with GDP X_{t-4}	X_{t-1}	X_t	X_{t+1}	X_{t+4}
$\varepsilon_c=1/2$	$\mu=1.0$	$\eta_q=\infty$					
$\alpha=0$							
Output	1.95(1.00)	0.80	0.32	0.80	1.00	0.80	0.32
Consumption	1.08(0.55)	0.83	0.55	0.75	0.97	0.83	0.45
Investment	4.15(2.30)	0.78	0.36	0.80	0.99	0.76	0.23
R_{n1}	0.12(0.05)	0.52	0.12	0.23	0.27	0.20	0.03
$R_{n4}-R_{n1}$	0.06(0.03)	0.50	-0.05	-0.04	-0.06	-0.05	-0.01
$\varepsilon_c=1/2$	$\mu=1.1$	$\eta_q=10$					
$\alpha=0.5$							
Output	1.64(1.00)	0.55	0.16	0.55	1.00	0.55	0.16
Consumption	1.56(0.095)	0.80	0.31	0.73	0.88	0.56	0.16
Investment	2.99(1.82)	0.17	-0.08	0.12	0.78	0.33	0.10
R_{n1}	0.13(0.08)	0.43	-0.09	0.04	0.64	0.45	-0.06
$R_{n4}-R_{n1}$	0.08(0.05)	0.41	0.09	-0.04	-0.64	-0.30	0.00
$\varepsilon_c=1/2$	$\mu=1.1$	$\eta_q=10$					
$\alpha=0.75$							
Output	2.78(1.00)	0.51	-0.01	0.51	1.00	0.51	-0.01
Consumption	1.81(0.65)	0.69	0.15	0.58	0.88	0.45	-0.02
Investment	6.02(2.61)	0.44	-0.13	0.38	0.94	0.47	0.00
R_{n1}	0.16(0.06)	0.48	-0.09	0.40	0.92	0.45	-0.06
$R_{n4}-R_{n1}$	0.09(0.03)	0.48	0.09	-0.40	-0.92	-0.45	0.06
$\varepsilon_c=1/2$	$\mu=1.1$	$\eta_q=10$					
$\alpha=0.9$							
Output	3.32(1.00)	0.72	0.13	0.72	1.00	0.72	0.13
Consumption	2.29(0.69)	0.74	0.20	0.72	0.95	0.67	0.09
Investment	6.25(1.88)	0.71	0.07	0.66	0.96	0.71	0.16
R_{n1}	0.16(0.05)	0.56	0.18	0.68	0.91	0.46	-0.10
$R_{n4}-R_{n1}$	0.08(0.02)	0.54	-0.19	-0.67	-0.89	-0.43	0.13

Note: α denotes the probability that a firm sets its price through rule of thumb.

nicely with data, while the standard deviation of output increases much compared to data. This is due to the fact that markups respond more to monetary shocks and to real shocks as the degree of nominal rigidity increases.

Next, note that the autocorrelations of the model have the same sign as those of data, although the values are a little bit lower compared to data. The first-order autocorrelation coefficients for real variables such as consumption, and investment drawn from the model are in the range of 0.6 and 0.8 whose values are comparable to those of data. These values do not change in the sensitivity analysis. In the contemporaneous correlation with output, consumption and investment comove with output as in data whether it is a flexible price or a sticky price model. In regard to contemporaneous correlation of output and interest rates, and nominal interest rates of a flexible price model move procyclically, which matches well with data. In the sticky price model, nominal interest rates move procyclically much compared to data. In regard to cross correlation of nominal interest rates with output, the nominal interest rates are negatively correlated with 4 quarter ahead output when there is a modest nominal rigidity as in data. As more and more firms, however, depend on rule of thumb markup adjustments rather than optimal price setting rules, interest rates tend to comove negatively with future output unlike data. These failures are expected in some sense, because liquidity effects cannot be generated from a sticky price model. Finally, the term spread from the sticky price model comoves as in data, even though the comovement tendency is much strong compared to data. Moreover, the current term spread and the future output comove positively as in data, which says that the flattening of the yield in the sticky price can be interpreted as a signal that a recession is imminent. However, the comovements of the current term spread with the future output tend to be at odds with data as the degree of price stickiness increases much.

VI. Concluding Remarks

This paper specifies a general equilibrium model such that firms that get to set new prices are chosen randomly at each period, with each having an equal probability of being selected, and the probability of an optimal price change is independent of both the time that has elapsed since the last optimal price change, and the degree to which costs and other market conditions have changed since then. It investigates whether a sticky price model can generate the relationship between term structures of interest rates and real economy as well as the relationship between nominal interest rates and real activity.

The sticky price model partly succeeds in generating a serial relationship between financial variables and output as in the data. It is a little bit satisfactory in that nominal interest rate moderately acts as a leading predictor for real economic activity in a sticky price model. In these respects, a sticky price model performs better than a flexible price model. But it fails to generate impulse response function of nominal interest rates to a positive monetary shock, which has been expected in the failure of liquidity effect in sticky price model.

In the future research agenda, it is desirable to consider a time varying risk premium and analyze the effect of this risk premium on the serial correlations of financial variables such as output, consumption, and term structure of interest rates as in Campbell and Cochrane (1995), Abel (1990), and Constantinides (1990) who have recently proposed a simple asset pricing model with habit formation.

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