Welfare of Incomplete Markets Economy with Permanent as well as Transitory Shocks

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Under the assumption that shocks are transitory, the welfare properties of the incomplete markets economy with only bonds are well known. As agents become more patient, its behavior get closer to that of the complete markets economy. As shocks become more persistent, the incomplete markets economy behave much differently from the complete markets economy. This paper shows that, when shocks have permanent as well as transitory components, neither of these two properties is true. More patience does not always move the incomplete markets economy closer to the complete markets economy, and more persistence in growth rate shocks induces similarity between the incomplete markets economy and the complete markets economy.

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I. Introduction

In recent years, many papers have investigated the welfare properties of an incomplete markets economy in which risk-free bonds are the only available asset. Levine and Zame (2002) proved that the welfare loss of the incomplete markets (or incomplete risk-sharing) economy relative to the complete markets (or complete risk-sharing) economy disappears as agent become very patient. Kubler and Schmedders (2001) showed numerically that more persistence in shocks implies divergence between the two economies. Kim, Kim and Levin (2002) presented analytic results for these two properties and showed that the incomplete markets endowment economy behave like the autarky when shocks become very persistent.

These papers have considered shocks that are transitory. However, it is a common assumption in economics that shocks have permanent components as well as transitory components.[1] This paper studies the welfare properties of the incomplete markets economy when shocks have both components.[2] In particular, we investigate the comparative statics with respect to patience and persistence. When shocks occur to the growth rates of endowments (rather than their levels), more patience does not always move the incomplete markets economy towards the complete markets economy and more persistence induces similarity (rather than divergence) between the incomplete markets economy and the complete markets economy.

Our analysis is related to many wields in economics which deals with incomplete markets. Kim, Kim and Levin (2002) laid out the model following the tradition of international finance, and this paper is related to welfare costs of missing international asset markets. Recent literature on welfare gains from international risk sharing incorporates the model in which shocks occur to the growth rates instead of the levels; see van Wincoop (1999). In

[1]The terminology of transitory *vs.* permanent shock may be confusing, since some part of transitory shocks contributes to the 'permanent' income instead of the 'transitory' income. Incomplete markets insure against 'transitory' income only, but complete markets insure against both types of income. Transitory shocks can be called stationary shocks, and shocks with both components, nonstationary shocks.

[2]Willen (1999) also considers permanent as well as transitory shocks.

macroeconomics, Krusell and Smith (1998) investigated the implications of incomplete markets on income and wealth heterogeneity. The theoretical proof by Levine and Zame (2002) and the numerical analysis by Kubler and Schmedders (2001) adopted the general-equilibrium microeconomics literature. Our analysis is also closely related to the finance literature, since the incomplete markets economy would imply different pricing kernel from the complete markets economy; see Constantinides and Duffie (1996), and Heaton and Lucas (1996).

The remainder of the paper proceeds as follows. Section II presents the specification and analytic solution for our benchmark model of incomplete markets. Section III presents analytic results to elucidate the welfare properties when shocks are transitory. Section IV investigates how these properties change when shocks become permanent. Section V concludes.

II. The Benchmark Model

We first display the benchmark model of the incomplete markets economy, without reference to whether shocks are transitory or not.

A. Specification

We analyze a two-country endowment economy in which risk-free discount bonds are the only internationally traded asset. We assume that the two countries are completely symmetric, except that their endowment levels are different. The representative infinitely-lived household in each country chooses its consumption and bond holdings in order to maximize its present discounted utility, subject to the risk-free interest rate and to its exogenously given endowment sequence. Each country's initial endowment is subject to a random disturbance, which may have a persistent impact on its subsequent endowments.

The representative household in the home country solves the following problem:

$$\max \sum_{t=0}^{J-1} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma}, \tag{1}$$

subject to

$$A_{t+1}=R_t(A_t+Y_t-C_t) \quad \text{for every} \quad t=0,1,2,\ldots \tag{2}$$

where C_t indicates home-country consumption; Y_t indicates the exogenous home-country endowment level; A_t denotes the asset held at the beginning of period t; and R_t indicates the gross risk-free interest rate. The conditional expectations operator is omitted from equation (1) because we assume perfect foresight. There are J periods, which can be infinity. The representative agent in the foreign country solves the same maximization problem and its variables are denoted with the asterisk.

For a given sequence of interest rates, the household's optimal plan is described by equation (2) together with the following sequence of first-order conditions:

$$C_t^{-\gamma}=\beta R_t C_{t+1}^{-\gamma} \quad \text{for every} \quad t=0,1,2,\ldots \tag{3}$$

The optimal plan for the foreign household is given by expressions parallel to equations (2) and (3), involving C_t^* and Y_t^* instead of C_t and Y_t. Imposing the usual transversality condition on bond holdings, the household faces the following intertemporal budget constraint in period $t=0$:

$$C_0+\sum_{t=1}^{J-1}\left(\prod_{s=0}^{t-1}R_s\right)^{-1}C_t=Y_0+A_0+\sum_{t=0}^{J-1}\left(\prod_{s=0}^{t-1}R_s\right)^{-1}Y_t. \tag{4}$$

B. Equilibrium

In general equilibrium, the following bond market clearing conditions must be met in every period:

$$A_t+A_t^*=0 \quad \text{for every} \quad t=0,1,2,\ldots \tag{5}$$

It should be noted that these market-clearing conditions (combined with the home and foreign asset transition equations) are equivalent to the following sequence of global resource constraints:

$$C_t+C_t^*=Y_t+Y_t^* \quad \text{for every} \quad t=0,1,2,\ldots \tag{6}$$

The home and foreign consumption Euler equations can be expressed as

$$\begin{aligned} C_{t+1} &= \beta^{1/\gamma} R_t^{1/\gamma} C_t \\ C_{t+1}^* &= \beta^{1/\gamma} R_t^{1/\gamma} C_t^* \end{aligned} \quad \text{for all} \quad t=0,1,2,\ldots \tag{7}$$

By adding these two equations together, substituting the global resource constraint (6), we obtain the following expression for the equilibrium (*i.e.*, market clearing) path of the interest rate R_t:

$$R_t = \beta^{-1} \left(\frac{Y_{t+1}+Y_{t+1}^*}{Y_{t+1}+Y_t^*} \right)^{\gamma} \quad \text{for all} \quad t=0,1,2,\ldots \tag{8}$$

By substituting equation (8) into the home-country household's intertemporal budget constraint (4), we obtain the following solution for the entire path of its consumption as a function of the exogenous sequences of home and foreign endowments:

$$C_0 = \frac{Y_0 + \left(\frac{Y_0+Y_0^*}{2}\right)^{\gamma} \sum_{t=1}^{J-1} \beta^t \left(\frac{Y_t+Y_t^*}{2}\right)^{-\gamma} Y_t + A_0}{1 + \left(\frac{Y_0+Y_0^*}{2}\right)^{\gamma-1} \sum_{t=1}^{J-1} \beta^t \left(\frac{Y_t+Y_t^*}{2}\right)^{1-\gamma}}, \tag{9}$$

$$C_t = \left(\frac{Y_t+Y_t^*}{Y_{t-1}+Y_{t-1}^*} \right) C_{t-1} \quad \text{for all} \quad t=1,2,\ldots \tag{10}$$

The level of initial consumption is equal to the permanent income of the home country, where the permanent income is composed of the income from the endowment stream and the income from asset holdings. The foreign country's consumption path is symmetric to equations (9) and (10).

Finally, we note that under autarky, each country's representative household simply consumes its own endowment; that is, $C_t = Y_t$ for every period t. In the complete markets economy, there will be full risk sharing as implied by the second welfare theorem and the level of consumption is $C_t = C_t^* = (Y_t + Y_t^*)/2$.

III. Transitory Shocks

In this section, we illustrate the welfare properties of the incomplete markets economy. From a qualitative perspective, these results are not new. The contribution of this paper is to present these results with tractable analytic examples. We first use the endowment economy model with perfect foresight for these analytic results, and then extend these results into two settings. The first extension is stochastic shocks without assuming perfect foresight and the other is production economy with labor.

A. Endowment Economy with Perfect Foresight

We assume that the initial endowments Y_0 and Y_0^* are drawn from a bivariate lognormal distribution. In particular, log Y_0 and log Y_0^* each have mean $-\sigma^2/2$ and variance σ^2, and the two series are independent.[3] This specification enables us to consider mean-preserving spreads; that is, $E(Y_0)=E(Y_0^*)=\overline{Y}_0=1$ regardless of the specific value of σ^2.

In subsequent periods, the endowments Y_t and Y_t^* are determined by the following bivariate process:

$$\begin{bmatrix} \log Y_t \\ \log Y_t^* \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \log Y_{t-1} \\ \log Y_{t-1}^* \end{bmatrix} \quad \text{for} \quad t=1,2,\ldots, \tag{11}$$

where we refer to ρ as the persistence parameter.[4] Dynamic stability requires that $|\rho|<1$. We assume henceforth that no bonds are outstanding at the beginning of the initial period; that is, $A_0=0$.

B. Patience

To understand the intuition for the role patience in incomplete markets, it is useful to consider the special case in which the endowment shocks are purely transitory (*i.e.*, $\rho=0$) and the utility

[3]Correlation between the two shocks influences the absolute welfare levels of various financial market structures, but not the relative welfare weight of the incomplete markets economy; see Kim, Kim and Levin (2002).

[4]See Kim, Kim and Levin (2002) for a model with dynamic spillovers, i.e. nonzero off-diagonal coefficients in (11).

function is logarithmic (*i.e.*, $\gamma = 1$). In such a case, the level of consumption in incomplete markets would be

$$C_t = \omega_J \left(\frac{Y_t + Y_t^*}{2}\right) + (1-\omega_J)Y_t + (1-\omega_J)A_t,$$

where the weight on the complete markets is

$$\omega_J = \frac{\beta - \beta^{J+1}}{1-\beta^{J+1}} = 1 - \frac{1-\beta}{1-\beta^{J+1}}.$$

The last term in the above expression for consumption represents the permanent income from the asset holdings. Sum of the thirst two terms is the permanent income from the endowment stream. In other words, the initial income is decomposed into the permanent income and the transitory income:

$$Y_t \equiv \left[\omega_J \left(\frac{Y_t + Y_t^*}{2}\right) + (1-\omega_J)Y_t\right] + \omega_J \left(\frac{Y_t - Y_t^*}{2}\right).$$

Foreign income affects the permanent (and transitory) income of the home country, since it influences the world interest rate.

Note that ω_J is increasing in both β and J. That is, the consumption behavior of the incomplete markets becomes similar to that of the complete markets as people get more patient or time horizon gets longer.[5] The intuition for the length of time horizon is transparent if we consider the behavior as β approaches unity:

$$\lim_{\beta \to 1} \omega_J = \frac{J}{J+1}.$$

Even though our derivation for the consumption behavior for the incomplete markets are exact, we should rely on approximations to derive the welfare implications of market incompleteness. A word of

[5]In a partial-equilibrium model, an increase in the discount factor would imply a lower level of consumption since future becomes more important. However, in our general-equilibrium setup, there is an offsetting effects due to endogenous interest rates.

warning is needed at this point: We should not adopt the conventional linear approximation method (*e.g.*, King, Plosser and Rebelo 1988). As shown in Kim and Kim (2002), this method can severely distort the welfare implications; for example, this method can yield the nonsensical result that the level of welfare is lower under complete markets than under autarky.

To evaluate the welfare implications of alternative financial market structures, it is natural to consider the expected lifetime utility of the representative household at the beginning of period $t=0$, prior to observing the specific values of the endowment shocks.[6] Given the symmetry between the home and foreign countries, expected lifetime utility is identical for households in both countries. Since an exact analytic solution is not tractable, we derive the second-order Taylor approximation of lifetime utility with respect to the deviations of the endowments from their steady state, and then take the mathematical expectation with respect to the initial endowment shocks. Using V to denote this expected lifetime utility, we express the welfare of incomplete markets as a convex combination of the welfare of complete markets and that of autarky:

$$V^{inc} = \Omega_J V^{com} + (1-\Omega_J) V^{aut},$$

where

$$\Omega_J = \omega_J = \frac{\beta - \beta^{J+1}}{1-\beta^{J+1}}.$$

In this paper, we call chi weight 'the relative welfare index' of the incomplete markets economy and Ω with an appropriate subscript will represent this index.[7] While it is also interesting to investigate the determinants of all three levels of welfare as in Kim, Kim and Levin (2002), we focus only on the relative welfare index in this paper.[8]

[6]That is, we analyze the household's expected lifetime utility conditional on its information set in period $t=-1$.

[7]That is, $\Omega = (V^{inc} - V^{aut})/(V^{com} - V^{aut})$.

[8]In contrast, van Wincoop (1999) and Athanasoulis and van Wincoop (2000) focused on absolute gains under the complete markets economy.

In the case without shock persistence, the welfare weight is the same as the consumption weight. Therefore, our results regarding the consumption behavior carries over the welfare implications. In the extreme case in which people are very patient ($\beta \to 1$) *and* the time horizon is arbitrarily large ($J \to \infty$), the behavior of incomplete markets converge to that of the complete markets.[9] From now on, we assume that the time horizon is infinity.

In the infinite horizon case, the intuition for the role of patience can be easily obtained if we observe the behavior of asserts. An increase in β has two conflicting effects on asset holdings. First, since future becomes more important, people will accumulate more assets. Second, however, an endogenous decrease in the interest rates will lead to lower asset holdings. These two effects are cancelled out and the equilibrium asset holding equation is independent of the discount factor:

$$A_{t+1} = \frac{2A_t + Y_t - Y_t^*}{Y_t + Y_t^*}.$$

Therefore, as people become more patient, consumption path converges to that of the complete markets economy and the annuity return from the asset holdings converges to zero.

The preceding results regarding patience are also true even if we do not restrict the utility function into a logarithmic one. In this case, the behavior of consumption is

$$C_t = \omega_\gamma \left(\frac{Y_t + Y_t^*}{2}\right) + (1-\omega_\gamma)Y_t + (1-\omega_\gamma)A_t,$$

where the weight on the complete markets is

$$\omega_\gamma = \frac{\beta}{\beta + (1-\beta)\left(\frac{Y_t + Y_t^*}{2}\right)^{1-\gamma}}.$$

[9]This part can be viewed as a simple example of Levine and Zame (2002). However, our results regarding extreme patience are true whether there is non-traded aggregate uncertainty or not. In our perfect foresight setting, what agents do is intertemporal smoothing rather than risk sharing. It would be both in the stochastic setting.

The degree of relative risk aversion affects the permanent endowment income and the permanent asset income when the degree is not equal to unity, due to its potential influence on the interest rate.

The relative welfare index under rick aversion is

$$\Omega_\gamma = \beta.$$

We have shown that the welfare implications of the incomplete markets is independent of the risk aversion parameter.[10]

C. Persistence

Now we investigate the role of persistence when the shocks are temporary. To allow for an exact expression for the consumption dynamics, we go back to the model of logarithmic utility. The consumption behavior would be[11]

$$C_t = \omega_\rho \left(\frac{Y_t + Y_t^*}{2}\right) + (1-\omega_\rho)Y_t + (1-\beta)A_t, \tag{12}$$

where

$$\omega_\rho = \frac{\beta(1-\rho)}{1-\beta\rho} = 1 - \frac{1-\beta}{1-\beta\rho}, \tag{13}$$

and the relative welfare index is

$$\Omega_\rho = \frac{\beta(1-\rho)^2}{(1-\beta\rho)^2} = 1 - \frac{(1-\beta)(1-\beta\rho^2)}{(1-\beta\rho)^2}. \tag{14}$$

[10]This result needs be modified when there is deterministic growth. In such a model, the welfare weight is the growth-adjusted discount factor as used in Barro and Sala-i-Martin (1995) and Romer (1996). Risk aversion affects the welfare implications as far as it changes the growth-adjusted discount factor. See the Appendix for a model with risk aversion, deterministic growth, and persistent shocks.

[11]If the endowment process follows (11), then $[(Y_t - Y_t^*)/(Y_t + Y_t^*)]$ is autocorrelated with coefficient ρ up to the second order. Under the latter process, the consumption behavior of (12) is exact.

The intuition for the role of persistence is simple. If shocks become more persistent, a change in the initial endowment means a bigger change in the permanent endowment income and the country with a good (bad) shock has less necessity (ability) to smooth the level of consumption and there will be less risk-sharing.[12] In the extreme case of difference stationary shocks ($\rho = 1$), there is no risk sharing by the bond market.

With our analytic results, it is easy to compute the convergence speed of the incomplete markets as discussed in Kubler and Schmedders (2001). If the discount factor converges to unity at the rate of h_β (*i.e.*, $\beta = 1 - h_\beta$) and the persistence parameter converges to unity at the rate of h_ρ (*i.e.*, $\rho = 1 - h_\rho$), the relative welfare index is approximately $[h_\rho / (h_\beta + h_\rho)]^2$ which is smaller than the consumption weight.

Since the welfare implication depends critically on the parameter values for patience and persistence, Kim, Kim and Levin (2002) presented empirical analysis on the these two parameter values. Using the quarterly data for the United States and the European Union, the empirical estimates for the persistence parameter is 0.939. It is conventional to set the quarterly discount factor at 0.99, and the implied value for the relative welfare index is 0.62. That is, the incomplete markets economy is closer to the complete markets economy than to autarky in terms of welfare.

D. Stochastic Shocks: No Perfect Foresight

The foregoing analysis has focused on an economy in which each country is only subject to a single endowment chock; now we extend the model to analyze the case in which endowment shocks occur in every period. To generate a stationary equilibrium in the case of incomplete markets, we assume that internationally-traded bonds are subject to quadratic holding costs.[13] In particular, instead of equation (2), the household's budget constraint is given as follows:

[12]The results on persistence are robust to the non-logarithmic utility function; see the Appendix.

[13]Schmitt-Grohe and Uribe (2002) analysed various assumptions (including bond holding costs) for ensuring stationarity in a small open economy, and found that each of the alternatives yields very similar dynamic properties. Holding costs are popular in the finance literature, *e.g.*, Heaton and Lucas (1996).

$$\frac{A_{t+1}}{R_t}+\frac{\varsigma}{2}\left(\frac{A_{t+1}}{R_t}\right)^2=A_t+Y_t-C_t+\tau_t, \tag{15}$$

where the parameter ς determines the costs associated with a particular asset position. Note that bond holding costs do not affect the aggregate resource constraint, because the bond holding costs are rebated to the representative domestic household via a lump-sum transfer τ_t. The representative foreign household is subject to a parallel budget constraint involving the corresponding foreign variables. The detrended endowments Y_t and Y_t^* are determined by the bivariate Markov process

$$\begin{bmatrix} \log Y_t \\ \log Y_t^* \end{bmatrix}=\begin{bmatrix} \rho & 0 \\ 0 & \rho \end{bmatrix}\begin{bmatrix} \log Y_{t-1} \\ \log Y_{t-1}^* \end{bmatrix}+\begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix}. \tag{16}$$

The innovations ε_t and ε_t^* follow an *i.i.d.* bivariate normal distribution with identical means and variances, with no contemporaneous correlation.[14]

In contrast to the previous analysis, it is no longer feasible to obtain an exact solution for the model's dynamic equilibrium. Thus, we proceed by log-linearizing the model around its deterministic steady state, and then obtain a second-order approximation for welfare using the method of Kim and Kim (2002).[15] To enable comparison with the complete-markets and autarkic economies, we assume that the incomplete-market economy has zero initial bond holdings and that the initial endowments log Y_t and log Y_t are drawn from their stationary distribution.

Table 1 presents the value of Ω, the relative welfare index of the incomplete-market economy for a range of values of several key parameters. When bond holding costs are very small ($\delta=10^{-8}$), the relative welfare results in all cases (except for the case of $\rho=1$ and $\beta=0.99$) are identical (at least up to the fifth decimal place) to the value given by equation (14) for the economy with one-time

[14]Suppose that the spillover parameter is set at θ, rather than at zero. To preserve the unconditional mean of the detrended endowments, we assume that each innovation has mean $-\sigma_\varepsilon^2(1-\rho^2-\theta^2)/[2(1+\rho+\theta)\{1-(\rho-\theta)^2\}]$ and variance σ_ε^2.

[15]Details are available in the technical appendix for Kim, Kim and Levin (2002) available on the author's homepage.

TABLE 1

RELATIVE WELFARE INDEX (Ω): NO PERFECT FORESIGHT

	$\beta=0.95$		$\beta=0.99$	
ρ	$\varsigma=10^{-4}$	$\varsigma=10^{-8}$	$\varsigma=10^{-4}$	$\varsigma=10^{-8}$
0	0.95	0.95	0.99	0.99
0.90	0.45	0.45	0.82	0.83
0.95	0.25	0.25	0.68	0.70
0.99	0.03	0.03	0.22	0.25
1	0.00	0.00	0.04	0.04

endowment shocks. Specifying a higher cost of bond holding ($\varsigma=10^{-4}$) has negligible effects on relative welfare when the discount factor $\beta=0.95$, and only reduces the relative welfare slightly when $\beta=0.99$. In all cases, the qualitative properties of welfare for the stationary economy match those obtained in the previous section: the welfare under incomplete markets is identical to that of autarky when shocks are permanent, and approaches that of complete markets when the degree of persistence is substantially smaller than the discount factor.

E. Production Economy with Labor

This part extends the endowment economy model to the production economy with endogenous labor supply. An extension to a production economy is particularly interesting since there is no microeconomic studies for incomplete markets with production. A technical difficulty is that it would be hard to obtain the case of the constant (or deterministic) interest rates as done easily in the endowment economy. In the production case, the analytical solution is not available and we solve the nonlinear model by using the stacked Newton algorithm and perform numerical analysis to evaluate the welfare implications of incomplete markets.[16]

Each country faces the following maximization problem with a separable utility function:

[16]See Kim, Kim and Levin (2001) for details on this production economy with labor.

$$\max\sum_{t=0}^{\infty}\beta^t U(C_t,L_t), \text{ where } U(C,L)=\frac{C^{1-\gamma}-1}{1-\gamma}+\nu_0\,\nu\,(1-L^{1/\nu}), \tag{17}$$

subject to

$$C_t+B_t=R_{t-1}B_{t-1}+Y_t, \tag{18}$$

$$Y_t=X_tL_t^{1-\alpha}, \quad \text{for all } t. \tag{19}$$

where L_t is the amount of labor and X_t is productivity.[17] The labor production technology can be nonlinear with a nonzero α. The parameter ν, taking a value between 0 and 1, represents how elastic the labor supply is. The elasticity of labor supply is $\nu/(1-\nu)$, which is increasing in ν and takes a value between 0 and ∞.

To analyze the welfare implications of alternative financial market structures, we consider a single shock to the home country's total factor productivity, leaving the foreign country's total factor productivity unchanged. For simplicity, we abstract from growth, so that steady-state productivity $\overline{X}_t=1$ for every period t. The initial productivity disturbance only takes one of two values with equal probability, that is, $\log(X_0)=\pm 0.03-0.00036$. Thus, initial total factor productivity shifts by roughly 3 percent, with a small adjustment to achieve a mean-preserving spread. The productivity disturbance decays at the rate ρ; that is, $\log(X_t)=\rho\log(X_{t-1})$ for every period $t>0$. For the baseline calibration, we set $\alpha=0.33$, $\beta=0.986$, $\gamma=2$, and $\nu=0.5$. Using the numerical methods outlined above, we compute the expected lifetime utility of the representative household in each country at the beginning of period $t=0$, prior to observing the specific value of the total factor productivity shock. Finally, we express world welfare (that is, the average of expected lifetime utility across the two countries) in terms of the certainty-equivalent shift in consumption in the initial period, holding leisure at its steady-state value.

For alternative values of γ, ν, and ρ, Table 2 presents the value of Ω, the relative welfare loss of the incomplete-market economy. In particular, we consider $\gamma\in\{1,2\}$; $\nu\in\{0,0.5,0.9\}$; and $\rho\in\{0,0.9,0.95,1.0\}$. In all cases, these results confirm the qualitative properties

[17]We assume that the technology is multiplicative. It would also be interesting when the shocks are additive, as in the consumption literature.

TABLE 2

RELATIVE WELFARE INDEX (Ω): LABOR PRODUCTION ECONOMY

	$\gamma=1$				$\gamma=2$			
ρ	0	0.9	0.95	1	0	0.9	0.95	1
$\nu=0.0$	0.98600	0.77763	0.61508	0	0.98599	0.77745	0.61468	0
$\nu=0.5$	0.98600	0.77762	0.61503	0	0.98599	0.77742	0.61457	0
$\nu=0.9$	0.98600	0.77759	0.61493	0	0.98600	0.77734	0.61432	0

obtained previously; that is, the welfare under incomplete markets is identical to that of autarky when $\rho=1$, and approaches that of complete markets when ρ is substantially smaller than β. It is also interesting to note that an increase in the degree of relative risk aversion, γ (or equivalently, a reduction in the intertemporal elasticity of consumption) has negligible impact on the relative welfare lose of the incomplete-market economy. The welfare results are also quite insensitive to the specific vague of ν, that is, the intertemporal elasticity of labor supply.

IV. Permanent Shocks

It is well known in the time series literature that the shocks in macroeconomic data contain not only temporary but also permanent components. Therefore, international finance literature measuring welfare gains of rick sharing have investigated the process for output (or consumption) that has permanent as well as temporary components; see van Wincoop (1999) and Athanagoulis and van Wincoop (2000). However, such literature has emphasized the absolute gains of risk sharing rather than the relative welfare index of the incomplete markets economy. This paper fills the gap and compute this relative welfare index under permanent shocks.

Since the relative welfare index does not depend on the contemporaneous correlation of shocks between the two countries, we assume that the two shocks are negatively correlated. To be more precise, the ensuing section on growth rate regressions assumes that the two growth rates (or the logs of endowments) are perfectly negatively correlated, and the levels of endowments are

negatively correlated when we adopts the Beveridge-Nelson decomposition.

A. Growth Rate Regressions

In investigating how big the welfare gains from international risk sharing are, van Wincoop (1999) considered many stochastic processes for endowments including the temporary shocks we analyzed in Section III. Another process he also considered implies that the growth rates of endowments follow an autoregressive process, Autoregressive growth rates imply that shocks to endowments have permanent as well as temporary components.

This paper considers the following stochastic process for home endowments:

$$\log Y_t - \log Y_{t-1} = \eta(\log Y_{t-1} - \log Y_{t-2}),$$

where $\eta(>0)$ represents the autoregressive parameter for growth rates. We assume that the initial endowments Y_0 and Y_0^* are drawn from a bivariate log-normal distribution with mean zero and correlation coefficient of -1.

By plugging the endowment process into (9), we express the level of consumption in the initial period up to the second order as follows:

$$\log C_0 \approx \omega_\eta \log\left(\frac{Y_0 + Y_0^*}{2}\right) + (1-\omega_\eta)\log Y_0 - \frac{\beta\eta}{(1-\beta\eta)^2}\,\frac{(\log Y_0)^2}{2},$$

where the weight on the complete markets outcome is

$$\omega_\eta = \frac{-\beta\eta}{1-\beta\eta}.$$

It is interesting that this weight is negative. That is, a country faced with a good shock will borrow (rather that lend) from the foreign country.[18] This is in contrast to the economy with only temporary shocks: a country faced with a good shock will lend

[18]The permanent income from the endowment stream is bigger than the change in the initial endowment.

some of its extra endowment to the foreign country in order to receive interests in the future.

When the growth raters follow the autoregressive process, the relative welfare index of the incomplete markets economy is

$$\Omega_\eta = \frac{\beta(1-\beta)\,\eta^2}{1-\beta^2\,\eta^2}. \tag{20}$$

In comparison to the results on the relative welfare index in the models with only temporary shocks. Two observations are noticeable. First, an increase in the patience parameter (β) does not lead to an increase in the relative welfare index. It is still true that a very low patience implies similarity between the incomplete markets economy and the autarky, but high patience does not lead to similarity between the incomplete markets economy and the complete markets economy. As people become very patient ($\beta \to 1$), the incomplete markets economy has welfare implications similar to the autarky. Second, more persistence now implies a higher relative welfare index. This is in contrast with the transitory case, since more persistence implied a lower relative welfare index with transitory shocks. Using the same data set as in Kim, Kim and Levin (2002), the estimate for the persistence of growth rates is 0.205, and the relative welfare index is less than 0.001.

To reconcile these two observations with the intuitions we obtained from the model with only transitory shocks, we consider a more general characterization of the stochastic process with both transitory and permanent shooks.

B. Beveridge-Nelson Decomposition

Beveridge and Nelson (1981) observed that stochastic processes can be decomposed into the permanent components and the temporary components.[19] We will present intuition on how to understand the welfare properties under the autoregressive growth rates using the

[19]Considering a potential confusion between the permanent/transtory shock and the permanent/transitory income, we may use the forecastable and the unforecastable component for the Beveridge-Nelson decomposition. However, in our perfect foresight setting, both components are 'forecastable' in some sense. No such problems exist with stochastic shocks as in Section III.D.

idea of Beveridge-Nelson decomposition. Specifically, we consider the following process for the level of endowment for the home country:[20]

$$\{Y_t\}_{t=0}^{\infty}=\{1+y_0, 1+\pi y_0+\rho(y_0-\pi y_0), 1+\pi y_0+\rho^2(y_0-\pi y_0), \cdots\}, \quad (21)$$

where the foreign endowments are perfectly negatively correlated,

$$Y_t+Y_t^*=2. \quad (22)$$

The latter condition implies a constant interest rate in our perfect foresight setting.[21] The initial shock, y_0, is assumed to have a zero mean and a small valiance. The home endowment converges to $(1+\pi y_0)$, and $\pi(>0)$ captures the ratio between the permanent shock and the initial shock.[22] The initial shock (y_0) is decomposed into a permanent component (πy_0) and a transitory component $(y_0-\pi y_0)$. In the case of positive initial shock, the transitory component is positive when $\pi<1$, and negative when $\pi>1$. We call ρ the persistence parameter, since it captures the persistence of transitory components. That is, it indicates how slowly the endowment converges from the initial shock to the permanent shock. In the special case of no permanent shocks $(\pi=0)$, this endowment process is analogous to (11) with only temporary shocks.

Plugging this stochastic process with both permanent and transitory components into (9), we derive that the level of consumption is constant at

$$C_t=\omega_\pi\left(\frac{Y_t+Y_t^*}{2}\right)+(1-\omega_\pi)Y_0,$$

[20]An alternative process is $\log Y_t-\log Y_{t-1}=u_t-u_{t-1}+v_t$, which is more popular in the empirical consumption literature. See Willen (1999) for the comparison of incomplete and complete markets with this process.

[21]Note also that, if the endowment process follows (21) and (22), then $[(Y_t-Y_t^*)/(Y_t+Y_t^*)]$ is autocorrelated with coefficient ρ and the consumption processes are exact.

[22]The intial shock is bounded between $-\min\{1, \pi^{-1}\}$ and $\min\{1, \pi^{-1}\}$. In the special case of $\pi=0$, corresponds to the case of transitory shocks. Results can be easily extended when time horizon is finite.

where the weight on the complete markets economy is

$$\omega_\pi = \omega_\rho(1-\pi) < \omega_\rho.$$

Note that ω_ρ in (13) is the consumption weight on the complete markets economy when shocks are temporary and persistent. When the transitory component is negative, this weight is negative. In the previous case of autoregressive growth rates, high endowments imply even higher endowments in the future, which implies that the transitory components are negative.

There are two more intuitive expressions for the level of consumption one expression is in terms of a convex combination between the permanent shock and the initial shock:

$$C_t = \omega_\rho Y_\infty + (1-\omega_\rho)Y_0. \tag{23}$$

The other involves the current income rather than the initial income:

$$C_t = Y_\infty + (1-\omega_\rho)(Y_t - Y_\infty) + (1-\beta)A_t.$$

Consumption is equal to the permanent income, which is a convex combination between the permanent shock and the initial shock. This expression reduces to (12) when there are no permanent components (*i.e.*, $\pi = 0$). As in our analysis of models with only transitory shocks, we thirst analyze a model without persistence to understand the role of patience.

Above expression for consumption implies the following decomposition of the current income into the permanent and the transitory income:

$$Y_t \equiv [Y_\infty + (1-\omega_\rho)(Y_t - Y_\infty)] + \omega_\rho(Y_t - Y_\infty).$$

The permanent income consists of the permanent part of the endowment process and the uninsured portion of its transitory part. The rest (*i.e.*, the insured portion) of the transitory part is the transitory income.

a) Patience: Permanent but not Persistent Shocks

In the special case without persistence ($\rho=0$), the endowment process for the home is

$$\{Y_t\}_{t=0}^{\infty}=\{1+y_0, 1+\pi y_0, 1+\pi y_0, \cdots\},$$

and the three expressions for consumption are

$$\begin{aligned} C_t &= \beta(1-\pi)\left(\frac{Y_t+Y_t^*}{2}\right)+[1-\beta(1-\pi)]Y_0 \\ &= \beta Y_\infty+(1-\beta)Y_0 \\ &= Y_\infty+(1-\beta)(Y_t-Y_\infty)+(1-\beta)A_t \end{aligned}$$

As people become more patient, the level of consumption gets closer to the permanent shock rather than the temporary shock. Only when there are no permanent components ($\pi=0$), this zero permanent shock happens to be the same as the outcome in the complete markets economy.

Now we are ready to give an intuition on what happens when people become more patient. As far as there are permanent components ($\pi>0$), the behavior of consumption does not converge to the full risk sharing when people are very patient ($\beta\rightarrow 1$). Under high patience, the level of consumption is equal to the size of the permanent shock and such a path for consumption would imply the same level of welfare as autarky. If the variance of the initial endowment is unity, the level of welfare in the incomplete markets economy is $-0.5[\beta\pi+(1-\beta)]^2$, which converges to that of autarky ($-0.5[\beta\pi^2+(1-\beta)]$) as $\beta\rightarrow 1$. The identical level of welfare is strictly positive, while the level of welfare under the complete markets economy is zero.[23] Note that this logic doers not go through when

[23]The relative welfare index is $[1-(1-\beta+\beta\pi)^2/(1-\beta+\beta\pi^2)]$, which increase in β when it is less than $1/(1+\pi)$ and then decrease in β. With respect to π, the index decreases when $\pi<1$ and then it increases. An alternative relative welfare index would take the temporarily complete economy as a benchmark for the complete markets economy; see Willen (1999). The incomplete markets economy would converge to this temporarily complete market economy as β goes to unity. However, this alternative complete market economy would yield a lower level of welfare than the incomplete markets economy if $\pi>1$. This alternative index would be $\beta(1-$

there are no permanent components in the shock process (*i.e.*, $\pi = 0$).

b) Persistence: Permanent and Persistent Shocks

Recall that more persistence in growth rates implies that the behavior of the incomplete markets economy becomes similar to that of the complete markets economy. If we define persistence as in (*B*.a), more persistence implies movements away from the complete markets economy. The relative welfare index of the incomplete markets economy when shocks have both transitory and permanent components as in (*B*.a) is

$$\Omega_\pi = 1 - \frac{\dfrac{[\omega_\rho \pi + (1-\omega_\rho)^2]}{1-\beta}}{\dfrac{\pi^2}{1-\beta} + 2\dfrac{\pi + (1-\pi)}{1-\beta_\rho} + \dfrac{(1-\pi)^2}{1-\beta_\rho{}^2}},$$

which encompasses all the previous discussion on the relative welfare index in this paper.

This relative welfare index is a decreasing function in ρ and converges to zero as the temporary components become more persistent ($\rho \to 1$). The intuition on the persistence parameter given in the context of only temporary shocks still holds in the model with permanent shocks. In terms of (23), as the temporary shocks become more persistent, the level of consumption becomes closer to the initial endowment which would be outcome of the autarky.

Finally, we are ready to understand why more persistence in growth rates implies a higher relative welfare index as in (20). Changes in the persistence in the growth rates imply two simultaneous movements in our specification following the Beveridge-Nelson decomposition. First, the persistence of the temporary shocks is determined by the persistence in growth rates:

$$\rho = \eta .$$

Second, the ratio between the permanent shocks and the initial shock is an increasing function of the persistence in growth rates:

$\pi)/(1+\pi)$.

$$\pi = \frac{1}{1-\eta}.$$

By plugging these two equations into the properties derived in the context of the Beveridge-Nelson decomposition, we produce the results under the autoregressive growth rates.

V. Conclusion and Further Research

In this paper, we have investigated average world welfare in open economies with three alternative financial market structures: autarky; complete markets; and incomplete markets in which sovereign bonds are the only internationally-traded asset. Our simple examples in the transitory shock case confirms the results previously known in the literature. When agents are very patient and shocks are not very persistent, the welfare under incomplete markets is nearly as high as that under complete markets; in contrast, when agents are relatively impatient and shocks are highly persistent, the welfare level is similar to that of the autarkic economy. We extend these results into the setting where shocks have permanent components as well as transitory components and have two findings. First, more patience does not always imply that incomplete markets economy becomes closer to the complete markets economy, Second, more persistence in the growth rate shocks increases the relative welfare index. We have explained these two findings using the Beveridge-Nelson decomposition.

In future work, it will be interesting to extend this analysis in many directions. In terms of utility functions, non-expected utility preferences and habit persistence can be considered. We investigated the labor production economy, but a further extension to an economy with capital is also interesting from the business cycle perspective.[24] Another interesting extension is the case of multiple goods, as in Cole and Obstfeld (1991). It will also be useful to consider the implications of bond market frictions such as exogenous bounds on asset levels (*cf.* Huggett 1993; Levine and

[24]A recent contribution is Angeletos and Calvet (2002). Kim and Kim (2002) analyzed the capital production economy in the cases of autarky and the complete markets economy, but not for the incomplete markets economy.

Zame 2002; Kubler and Schmedders 2001), endogenous solvency constraints (*cf.* Alvarez and Jermann 2000), and enforcement constraints (*cf.* Kehoe and Perri 2002).

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Appendix

In a model with deterministic growth, the endowment fluctuates around the deterministic growth path. If the growth rate is μ, then the equilibrium level of consumption in the first period is

$$\begin{aligned}\log C_0 \approx\ & \omega_\mu \left(\frac{\log Y_0 + \log Y_0^*}{2}\right) + (1-\omega_\mu)\log Y_0 \\ & + \Theta(\log Y_0)^2 + \Lambda(\log Y_0)(\log Y_0^*) + \Phi(\log Y_0^*)^2,\end{aligned} \tag{A.1}$$

where

$$\omega_\mu = \frac{\beta\mu^{1-\gamma}(1-\rho)}{1-\beta\mu^{1-\gamma}\rho},$$

$$\Theta = \frac{\omega_\mu}{8}(4-3\omega_\mu-2\gamma+2\omega_\mu\gamma) - \frac{(1-\gamma)(1-\beta\mu^{1-\gamma})\rho}{4(1-\beta\mu^{1-\gamma}\rho^2)}\omega_\mu,$$

$$\Lambda = -\frac{1}{2}\omega_\mu + \frac{1}{4}\omega_\mu^2,$$

$$\Phi = \frac{\omega_\mu}{8}(\omega_\mu+2\gamma-2\omega_\mu\gamma) + \frac{(1-\gamma)(1-\beta\mu^{1-\gamma})\rho}{4(1-\beta\mu^{1-\gamma}\rho^2)}\omega_\mu,$$

The relative welfare index

$$\Omega_\mu = \frac{\beta\mu^{1-\gamma}(1-\rho)^2}{(1-\beta\mu^{1-\gamma}\rho)^2}.$$

It is clear that the deterministic growth and the rick aversion affects the behavior of incomplete markets economy only through the growth-adjusted discount factor: $\beta\mu^{1-\gamma}$.

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