Cooperative and Noncooperative R&D in Vertically Related Markets

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This paper compares R&D cooperation and R&D competition in a vertical market structure with an intermediate good producer and a final good producer. It is shown that cooperative R&D generates more R&D efforts, higher output and lower consumer price than non-cooperative R&D in a vertically related market.

Keywords: Cooperative R&D, Noncooperative R&D, Vertical market

JEL Classification D43, L13

I. Introduction

Inspired by the seminal papers by Katz (1986) and d'Aspremont and Jacquemin (1988), a rich literature has been developed to study issues related to R&D cooperation and R&D competition in the presence of R&D spillovers in an oligopolistic market. De Bondt (1996) contains an extensive review of this literature. Despite the fact pointing to the prevalence of vertical R&D relations, the literature has mostly focused on horizontal R&D where firms compete in the product market. The present paper attempts to study the issue of R&D cooperation in a vertical market structure.

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The fundamental question addressed in this paper is similar to that in d'Aspremont and Jacquemin (1988). That is, how does R&D cooperation affect R&D efforts, output levels and social welfare? While d'Aspremont and Jacquemin consider a horizontal market structure with two firms producing homogeneous goods, we intend to examine and compare R&D cooperation and R&D competition in a vertical market structure with an intermediate good producer and a final good producer in the presence of vertical R&D spillovers. To the best of our knowledge, Banerjee and Lin (2001) are among the first to have published work that studies vertical R&D cooperation. The focus of their paper is on the incentive to form a vertical research joint venture (RJV) to share R&D costs between upstream and downstream firms. There is no R&D spillover. In contrast, the present paper is focused on the effect of cooperative R&D between upstream and downstream firms in the presence of R&D spillover.2

The model developed in this paper involves a two-stage game. In the first stage, firms engage in cost-reducing R&D efforts either non-cooperatively or jointly. The R&D levels determine their unit costs of production. In the second stage, the upstream firm chooses its price to charge the downstream firm and the downstream firm then sets its price to charge final good consumers. The solution is obtained by using backward induction, starting from the second stage of the game.

1Veugelers (1993) and Vonortas (1997), among others, have reported empirical evidence supporting the importance of vertical R&D relationships in a number of industries. An important example of vertical relationship is the Windows-Intel relationship. The word Wintel coins the nature of the relationship between hardware and software in the IT industry. These vertically related industries as well as their R&D efforts develop hand in hand.

2Banerjee and Lin (2001) study a model with one intermediate good producer and an oligopolistic downstream industry of final good producers. Only the intermediate good producer innovates. A vertical RJV is an agreement between the intermediate good producer and some final good producers to share R&D costs and to internalize an externality created by any innovation by the intermediate good producer. Their main result is that there is a fundamental conflict between the incentives of the intermediate good producer and the downstream RJV member firms in that the intermediate good producer always desires for a larger RJV than that desired by each downstream member firm.
Our main result is that cooperative R&D generates larger R&D efforts and higher output than noncooperative R&D. However, R&D efforts and output are higher in a vertically integrated monopoly and social optimum demands still higher levels. The reverse holds for consumer price with the social optimum corresponding to the lowest level, the next closest being offered by a vertically integrated monopoly, followed by R&D cooperation, and with R&D competition yielding the highest level. These results hold for any levels of R&D spillover, and either symmetric or asymmetric R&D spillovers.

II. Setup

Consider a market with an intermediate good producer and a final good producer. The final good producer uses one unit of the intermediate good together with a fixed proportion of other inputs to produce a unit of the final good. The demand for the final good is \( p=a-bq \), where \( a>0 \) and \( b>0 \). Decisions are made in two stages. In the first stage, both firms choose their R&D levels simultaneously. In the second stage, the intermediate good producer chooses its price to charge the final good producer and then the final good producer chooses its price to charge consumers. The equilibrium concept adopted in the following is subgame perfect Nash equilibrium.

Let \( c_i \) denote the unit cost of production for the intermediate good producer, \( c_f \) the unit cost of production from other inputs for the final good producer.\(^3\) In the R&D stage, each firm chooses a level of R&D, denoted by \( x_i \) and \( x_f \) respectively. The firms’ R&D costs are assumed to take the quadratic form and given by \( \gamma_i(x_i)^2/2 \) and \( \gamma_f(x_f)^2/2 \) respectively, where \( \gamma_i \) and \( \gamma_f \) are R&D efficiency parameters. With R&D, the firms’ unit costs of production are given by

\[
\begin{align*}
c_i &= A_i - x_i - \beta_i x_f, \\
c_f &= A_f - x_f - \beta_f x_i.
\end{align*}
\]  

In (1), \( \beta_f \in [0, 1] \) denotes the level of R&D spillover from the final good producer to the intermediate good producer. \( \beta_i \in [0, 1] \) denotes

\(^3\)Throughout the paper, subscript \( I \) denotes for the intermediate good producer and subscript \( F \) denotes for the final good producer.
the level of R&D spillover from the intermediate good producer to 
the final good producer; \( A_l \) and \( A_r \) denote respectively the firms’ 
unit costs when there is no R&D by either firm. R&D spillover is 
symmetric if \( \beta_r \) and \( \beta_l \) are equal and asymmetric when they are 
not equal. Here, the R&D efficiency parameters as well as the 
spillover parameters are allowed to be different to account for the 
difference in the nature of the firms’ products.

With R&D, the intermediate good producer’s profit is

\[
\Pi_l = (p_l - c_l)q - \frac{\gamma_l}{2} (x_l)^2.
\]

and the final good producer’s profit is

\[
\Pi_r = (p_r - c_r - \alpha)q - \frac{\gamma_r}{2} (x_r)^2.
\]

In (2) and (3), \( c_l \) and \( c_r \) are as given in (1). \( p_l \) and \( p_r \) denote 
respectively the price of the intermediate good and the price of the 
final good, and \( q \) denotes the common output level of both firms.

III. Models and Results

In the following we consider four different models. The first two 
center noncooperative and cooperative R&D in the first stage of 
the game, assuming that there is no cooperation in the setting of 
prices. The third considers vertical integration in which all 
decisions are made by one integrated firm. The fourth deals with 
the social optimum. R&D and price levels in these solutions will be 
compared.

A. Noncooperative R&D

In the second stage of the game, the intermediate good producer 
first chooses its price to charge the final good producer, and the 
final good producer then chooses its price to sell to consumers. It 
is straightforward to obtain that the prices chosen by the firms are
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\[ \pi_i = \frac{\alpha + c_t - c_F}{2}, \quad \pi_F = \frac{3\alpha + c_t + c_F}{4}. \]  

(4)

and the common output level is

\[ q = \frac{\alpha - c_t - c_F}{4b}. \]  

(5)

By (1)-(5), the firms’ reduced form profit functions in the first stage of the game are given by

\[ \pi_i = \frac{1}{8b} (\alpha - c)^2 - \frac{\gamma_i}{2} (\omega)^2, \]  

(6)

\[ \pi_F = \frac{1}{16b} (\alpha - c)^2 - \frac{\gamma_F}{2} (\omega)^2, \]  

(7)

where \( c = c_t + c_F \) denotes the total cost of producing one unit of the final good. By (1),

\[ c = A_t + A_F - (1 + \beta)\lambda q_t - (1 + \beta_i)\lambda q_F. \]  

(8)

Note that since the total cost \( c \) plays the same role in both firms’ reduced form profit functions as shown by (6) and (7), (8) indicates that the terms \( (1 + \beta)\lambda q_t \) and \( (1 + \beta_i)\lambda q_F \) play the same role in the firms’ reduced form profit functions.

Under noncooperative R&D, the firms independently and simultaneously choose their R&D levels in the first stage of the game. The noncooperative choices in R&D are obtained by solving the equations \( \frac{\partial}{\partial x} \pi_i = 0 \) for \( i = I, F \). Simple algebra gives (superscripts \( N \) denote noncooperative solution)\footnote{Positive R&D in equilibrium requires that \( 8\gamma_1 \gamma_F - 2\gamma_4(1 + \beta)^2 - \gamma_4(1 + \beta)^2 \circ \circ > 0 \). This condition also guarantees that the equilibrium \( \{c_t, x_t^N\} \) is stable (see Henriques (1990)).}

\[ x_t^N = \frac{\gamma_4(1 + \beta)(A_t - A_F)}{4b \gamma_1 \gamma_F - \gamma_4(1 + \beta)^2 - 0.5 \gamma_4(1 + \beta)^2}. \]  

(9)
From (9), it is obvious that the intermediate good producer's R&D level \( x^{I}\) rises as the spillover rate \((\beta)\) from firm \(F\) to firm \(I\) increases. The reason for this effect is that as \(\beta\) increases, innovation becomes more rewarding for firm \(I\) and this induces it to expend a higher level of R&D effort. Similar relationship holds between firm \(F\)'s R&D effort \(x^{F}\) and the spillover rate \((\beta)\) from firm \(I\) to firm \(F\), as indicated by (10). The effect of the spillover rate \((\beta)\) from firm \(I\) to firm \(F\) on firm \(I\)'s R&D effort \(x^{I}\) is indirect. As \(\beta\) rises firm \(F\) will increases its R&D effort, leading to a larger spillover from firm \(F\) to firm \(I\), as a result firm \(I\) also has an incentive to increase its R&D effort. This result can be checked from (9) as (9) implies \(\frac{3a}{\beta} > 0\). Similar relationship holds between the spillover rate \((\beta)\) and firm \(F\)'s R&D effort \(x^{F}\).

The resulting price for the final good under noncooperative R&D is

\[
p^{F} = \frac{3a + A_{I} + A_{F} - (1 + \beta) x^{I} - (1 + \beta) x^{F}}{4}. \tag{11}
\]

Note that in (10) the terms \((1 + \beta) x^{I}\) and \((1 + \beta) x^{F}\) play the same role. This is for the same reason as discussed above following equations (6) and (7) regarding profits.

**B. Cooperative R&D**

Under cooperative R&D, the firms jointly determine their R&D levels in the first stage of the game. The cooperative R&D solution is obtained by maximizing \(\pi^{I} + \pi^{F}\) to get (superscripts \(C\) denote cooperative solution)\(^5\)

\(^5\)Positive R&D requires that \(8d_{I} \gamma_{F} - 3 \gamma_{F} (1 + \beta)^{2} - 3 \gamma_{F} (1 + \beta)^{2} > 0\). Note that this condition implies the stability condition in the previous footnote.
The resulting final good price is

$$p_F^V = \frac{3\alpha + A_t - A_F - (1 + \beta)X_I^V - (1 + \beta)X_F^V}{4}. \quad (14)$$

\section*{C. Vertical Integration}

The integrated firm chooses R&D levels and the final good price to maximize its total profit $\Pi + \Pi_F$. Direct calculations yield (superscripts V denote solution for the vertically integrated firm)

$$x_I^V = \frac{\gamma_t(1 + \beta)(\alpha - A_t - A_F)}{2b \gamma_t \gamma_F - \gamma_t(1 + \beta)^2 - \gamma_t(1 + \beta)^2}. \quad (15)$$

$$x_F^V = \frac{\gamma_t(1 + \beta)(\alpha - A_t - A_F)}{2b \gamma_t \gamma_F - \gamma_t(1 + \beta)^2 - \gamma_t(1 + \beta)^2}. \quad (16)$$

$$p_F^V = \frac{\alpha + A_t - A_F - (1 + \beta)x_I^V - (1 + \beta)x_F^V}{2}. \quad (17)$$

The main difference between the integrated solution given by (15)-(17) and the cooperative R&D solution given by (12)-(14) is that in finding the integrated solution the sum of original profit functions $\Pi$ and $\Pi_F$ given respectively by (2) and (3) is maximized, while the sum of reduced form profit functions $\pi_I$ and $\pi_F$ given respectively by (6) and (7) is maximized in finding the cooperative
R&D solution. The integrated solution obtained by choosing \((x_i, x_F, p_F)\) to maximize \(\Pi_i + \Pi_F\) (note that the price of the intermediate good \(p_i\) is cancelled out in the sum of \(\Pi_i\) and \(\Pi_F\)) is the same as the solution found by the following two stage procedure. In the first stage, \(x_i\) and \(x_F\) are chosen to maximize \(\Pi_i + \Pi_F\). Obviously, the solutions for \(x_i\) and \(x_F\) from the first stage are functions of \(p_F\).

In the second stage, these functions for \(x_i\) and \(x_F\) are substituted into \(\Pi_i + \Pi_F\) and the resulting function is maximized by choosing \(p_F\). The solution for the whole problem is obtained by substituting the optimal choice for \(p_F\) obtained in the second stage into the functions for \(x_i\) and \(x_F\) obtained in the first stage to obtain the optimal choice for \(x_i\) and \(x_F\).

D. Social Optimum

The social optimum is obtained by maximizing the sum of consumer’s surplus and both firms’ profits, \(W = CS + \Pi_i + \Pi_F\), where \(CS = (\alpha - p_F)^2 / (2b)\) denotes consumer’s surplus. Direct calculations yield (superscripts * denote socially optimal solution)

\[
x^*_i = \frac{\gamma (1 + \beta) (\alpha - A_i - A_F)}{b \gamma_i \gamma_F - \gamma (1 + \beta)^2 - \gamma (1 + \beta)^2},
\]

\[
x^*_F = \frac{\gamma (1 + \beta) (\alpha - A_i - A_F)}{b \gamma_i \gamma_F - \gamma (1 + \beta)^2 - \gamma (1 + \beta)^2},
\]

\[
p^*_F = A_i + A_F - (1 + \beta) x^*_i - (1 + \beta) x^*_F.
\]

The result in (20) indicates the expected result that the socially optimal final good price is equal to the total marginal cost of production.

\(^6\)Note that since the reduced form profit functions \(\pi_i\) and \(\pi_F\) are independent of the final good price \(p_F\) the solutions for \(x_i\) and \(x_F\) from the problem of maximizing the sum \(\pi_i + \pi_F\) are not functions of \(p_F\).

\(^7\)It is assumed that the denominator in (18) and (19) is positive, which also implies that the denominator in (15) and (16) is positive.
E. Comparison

Since \((8/3) \beta > \gamma (1 + \beta \delta) - \gamma (1) \beta \delta < 4 \beta \gamma (1 + \beta \delta) - (1/2) \gamma (1 + \beta \delta)^2\), by (9), (10), (12) and (13), \(x_i^e > x_i^e\) and \(x_i^e > x_i^e\). That is, both firms engage in more R&D under R&D cooperation than under R&D competition. It follows immediately from (11) and (14) that the final good price is lower under cooperative R&D than under noncooperative R&D.

Since \(2b \gamma (1 + \beta \delta) - \gamma (1 + \beta \delta)^2 < (8/3) \beta \gamma (1 + \beta \delta)^2 - \gamma (1 + \beta \delta)^2\), by (12), (13), (15) and (16), the integrated firm engages in more R&D at both the intermediate good level and the final good level than the separate firms do when they cooperate on R&D. By (14) and (17),

\[
p_{f}^e - p_{f}^e = \frac{1}{4} \left[ (c_i^e + c_i^e - c_i^e - c_i^e) + (c_i^e + c_i^e - c_i^e) \right]. \tag{21}
\]

where \(c_i^e\) and \(c_i^e\) denote respectively firm \(i\)'s unit cost under vertical integration and under cooperative R&D. The first bracketed term in (21) is negative since both firms have lower cost under vertical integration due to higher R&D, the second bracketed term in (21) has to be negative because the total unit cost cannot exceed the price intercept of the demand curve. Hence, \(p_{f}^e < p_{f}^e\), i.e., vertical integration entails lower price for consumers compared to either cooperative or noncooperative R&D. Here, two sources contribute to the lower consumer price under vertical integration. One is because of the standard double marginalization argument.\(^8\) The other is because of lower cost due to higher R&D levels.

From (15), (16), (18) and (19), it is easy to see that the social optimal R&D levels are higher than those chosen by an integrated firm. By (17) and (20),

\[
p_{f}^e - p_{f}^e = \frac{1}{2} \left[ (c_i^e + c_i^e - c_i^e) + (c_i^e + c_i^e - c_i^e) \right]. \tag{22}
\]

\(^8\)See, for example, Tirole (1989) for an exposition of double marginalization.
which is negative for similar reasons as advanced above for (21). Hence, $p_i^Y < p_i^V$, i.e., socially optimal final good price is lower than that under vertical integration.

In summary, we have established the following relationships

\[ x_i^Y < x_i^V < x_i^N \quad \text{for } i = I, F; \]  \hfill (23)

\[ p_i^Y > p_i^V > p_i^N. \]  \hfill (24)

It follows immediately from (24) the following relationships on final good output levels:

\[ q_i^Y < q_i^V < q_i^N. \]  \hfill (25)

Note that the results in (23)-(25) hold for any levels of $\beta$ and $\beta_F$, symmetric or asymmetric, and any levels of $\gamma_I$ and $\gamma_F$. Finally, it can be easily shown that the levels of total welfare (sum of total profits and consumer surplus) under the four regimes follow the same relationship as in (23).

In a seminal paper, d’Aspremont and Jacquemin (1988) characterized the main effects of R&D cooperation in a horizontal market structure. In a horizontal market, R&D cooperation generates two opposing effects on R&D efforts. The on one hand, R&D cooperation internalizes the positive externality resulting from spillovers. In this sense, R&D cooperation should increase the level of R&D. On the other hand, R&D cooperation also internalizes the negative externality of R&D: one firm’s increased R&D level leads to increased profits partly at the expense of its rival. In this sense, R&D cooperation should decrease the level of R&D. The positive externality overrides the negative externality when the spillover rate is large. In the present paper with a vertical market structure, the main difference with respect to d’Aspremont and Jacquemin (1988) is that the negative externality is absent.$^9$ As a result, only the positive externality matters and the effect of R&D cooperation is always an increase in R&D.

$^9$In the horizontal model, each firm’s profit is a decreasing function of its own unit cost but an increasing function of its rival’s unit cost. In contrast, in the vertical model, each firm’s profit is a decreasing function of its own unit cost as well as its rival’s unit cost, as shown by (6) and (7).
IV. Discussion

In the following we discuss briefly generalizations of several results obtained in the preceding section. In comparing the cooperative R&D solution with the noncooperative R&D solution, equal weight is given to the profit of the intermediate good producer and the profit of the final good producer. Suppose, instead, the weighted sum \( a \pi_I + \pi_F \) is used to find the cooperative R&D levels. Here, \( a > 0 \) denotes the relative weight of the intermediate good producer’s profit. For \( a < 1 \), the final good producer’s profit is weighted more heavily; for \( a > 1 \), the intermediate good producer’s profit is weighted more heavily. The corresponding cooperative R&D levels are given by

\[
x_I(a) = \frac{\gamma(1 + \beta)(a - \Lambda_I - \Lambda_F)}{2a + 1} - b \gamma(1 + \beta)^2 - a \gamma(1 + \beta)^2.
\]

(26)

\[
x_F(a) = \gamma(1 + \beta)(a - \Lambda_I - \Lambda_F)
\]

(27)

Using the fact that \( 8a/(2a+1) < 4 \) for any \( a > 0 \), it is easy to see that if \( a > 0.5 \) the denominator of (26) is less than that of (9) and the denominator of (27) is less than that of (10). Hence, by comparing (26) with (9) and (27) with (10), we have \( x_I(a) > x_I^* \) and \( x_F(a) > x_F^* \) for all \( a > 0.5 \). This implies that all conclusions obtained in the preceding section regarding comparison between the noncooperative R&D solution and the cooperative R&D solution continue to hold as long as the intermediate good producer’s profit is weighted at least half as heavily as the final good producer’s profit.

Next, we discuss the welfare measure used in deriving the socially optimal solution in the preceding section. Consider a generally weighted welfare measure given by \( W = \lambda CS + \Pi_I + \Pi_F \) for
\( \lambda > 0 \). Obviously, the welfare measure used in the preceding section corresponds to \( \lambda = 1 \). For \( \lambda < 1 \), total profit is weighted more heavily than consumer surplus; for \( \lambda > 1 \), consumer surplus is weighted more heavily than total profit. Using the general welfare measure, the corresponding socially optimal R&D levels are

\[
X^\lambda(\lambda) = \frac{\gamma (1+\beta)(a-A_L-A_D)}{(2-\lambda)b \gamma_L \gamma_U - \gamma (1+\beta)^2 - \gamma (1+\beta)^2}, \quad \text{(28)}
\]

\[
X^\lambda(\lambda) = \frac{\gamma (1+\beta)(a-A_L-A_D)}{(2-\lambda)b \gamma_L \gamma_U - \gamma (1+\beta)^2 - \gamma (1+\beta)^2} \quad \text{(29)}
\]

Comparing (28) with (15) and (29) with (16), it is easy to see that \( X^\lambda(\lambda) > X^\lambda(\lambda) \) and \( X^\lambda(\lambda) > X^\lambda(\lambda) \) for all \( \lambda : 0 < \lambda < 2 \).\(^{11}\) Hence, as long as consumer surplus is not weighted no more than twice that of total profits, all conclusions obtained in the preceding section regarding comparison between the noncooperative R&D solution, the cooperative R&D solution, the vertically integrated solution, and the socially optimal solution continue to hold.

**V. Concluding Remarks**

This paper has shown that in a vertically related market structure with an intermediate good producer and a final good producer, R&D efforts and output levels follow a descending order from social optimum to vertical integration to cooperative R&D to noncooperative R&D. Consumer prices follow the reverse order.\(^{12}\) Though they are consistent with the outcomes obtained in the horizontal R&D model studied by d'Aspremont and Jacquemin

\(^{11}\)An additional requirement for this conclusion is that \( \lambda \) must be bounded from above to ensure that the denominator of (28) is positive. Inspection of the denominator of (28) indicates that this upper bound should be less than 2. Note also that the denominator of (28) becomes negative if \( \lambda \geq 2 \).

\(^{12}\)It is rather straightforward to show that all the results obtained in this paper for vertically related markets also hold for a complementary monopoly market where goods produced by two different firms are perfectly complementary in consumption.
(1988), the results from our vertical R&D cooperation analysis is even stronger than those from horizontal R&D models. For example, in d’Aspremont and Jacquemin the same result on R&D is obtained only for large spillovers, whereas the results in the vertical R&D model hold for any levels of R&D spillover, and either symmetric or asymmetric R&D spillovers. Also, for large spillovers, output in the horizontal model follows a similar order as in the vertical model except that the integrated monopoly produces the smallest quantity. Though the model developed in this paper is restrictive in many respects, the results nevertheless demonstrate the benefits that R&D cooperation can bring to both consumers and producers in a vertically related market.\footnote{The recent rift between Bridgestone/Firestone and Ford over the massive recall of Firestone tires serves as an example demonstrating that lack of cooperation between an upstream firm and a downstream firm can hurt both firms as well as consumers. (See BUSINESS WEEK, September 18, 2000 and June 25, 2001 for coverage on this story.)}

Further research is required to examine the validity of the conclusions reached in the simple model considered here in more general settings. One possible area is to build a model with more general demand and cost functions. Another is to relax the strong assumption that the downstream firm uses the intermediate good in a fixed combination with other inputs. Yet another issue is concerned with how the price of the intermediate good is determined. In the present model, this price is determined by the intermediate good producer to maximize its own profit. Given that both firms are monopolies, it may be more reasonable to assume that they negotiate the price of the intermediate good (e.g., Horn and Wolinsky, 1988). It is believed that the basic conclusion in this paper that vertical R&D cooperation raises the R&D efforts by both firms should continue to hold under all of these additional considerations.

Finally, we comment on the market structure assumed in this paper. A model with a more general market structure may assume that both the intermediate good market and the final good market are oligopolistic. With such a market, an analysis of R&D cooperation will have to consider R&D cooperation at several levels: between intermediate good producers, between final good producers, and between intermediate good and final good producers. Furthermore, the size of each coalition may involve only a subgroup
of firms. Obviously, the analysis will be much more involved than the one provided in this paper. We believe in such a hybrid model, both the issues and the properties of the d'Aspremont and Jacquemin (1988) paper on horizontal R&D cooperation and the present paper on vertical R&D cooperation are present. In particular, at the horizontal level (between intermediate good producers and between final good producers), effects of R&D cooperation will depend on the levels of R&D spillover; at the vertical level (between intermediate good and final good producers), R&D cooperation will be more likely to be beneficial.

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