The Social Benefit and Cost of Entry Contest in Oligopoly: Cournot-Nash Oligopoly with Linear Demand

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This paper examines the social benefit and cost of entry contest when government tries to remove entry regulation in a Cournot-Nash oligopoly. The deregulation process may be plagued with rent seeking by incumbents and potential entrants. When there is rent seeking in the deregulation process, collective contest between potential entrants and incumbents takes place. If the incumbents win, no entry occurs. However, when the potential entrants win, the government distributes licenses to the potential entrants. Given these conditions, this paper obtains the following results. The more potential entrants there are, the more likely it is that the deregulation increases expected social welfare. Moreover, the more licenses the government tries to issue, the higher the probability that the deregulation increases expected social welfare, if there is no fixed cost.

Keywords: Entry barrier, Entry contest, Deregulation, Oligopoly

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I. Introduction

This paper examines the social benefit and cost of entry contest when government tries to deregulate entry barriers in a regulated oligopoly. The deregulation process may be plagued with rent seeking. When there is rent seeking in the deregulation process, collective contest between incumbents and potential entrants takes place over the entry of new firms. Quite naturally, the incumbents are opposed to the issuance of more licenses, while the potential entrants try to induce the government to issue more licenses.

Entry deregulation is one of most important issues in the literature of regulation, as the entry barrier is the most preferred regulation of the incumbents (Rasmusen and Zupan 1991). Utilizing a rent-seeking contest model, Poitras and Sutter (1997) examine the efficiency gains from deregulation of monopolies when reformers expend resources to secure deregulation and the monopolist expends resources to defend his position. They find that the potential welfare gains from deregulation exceed the cost of deregulation. However, their analysis is confined to complete deregulation of entry barriers.

Governments might allow for a limited number of additional firms instead of complete abolition of the entry barriers. While governments are unlikely to revoke licenses already issued to firms in regulated industries, they can issue additional licenses to some of potential entrants. For example, many governments have recently issued licenses to potential entrants in industries traditionally viewed as natural monopolies, such as telecommunication services. This has raised conflict of interests between the incumbent monopolists and the potential entrants.¹

When the government tries to deregulate entry barriers, the deregulation process may be associated with rent seeking. While

¹The Korean government has recently issued licenses to new entrants in telecommunication service and broadcasting industries. In Korea, the state-run Korea Telecom was the only supplier of telephone services until the late 1980s. Competition was first introduced in 1990 with the entry of DACOM in the international telephone service market. The Ministry of Information and Communication of Korea introduced competition in the cellular phone service market in 1994 and in the long distance telephone service market in 1995, respectively. For a detailed explanation, see White Paper (1998) of Ministry of Information and Communication of Korea.
the process is always plagued with rent seeking in the model of Kang and Lee (2001), this paper allows for the possibility that it is free from rent seeking. This paper models the deregulation procedure as a collective contest involving three parties: incumbents, potential entrants and government. In the first stage the government announces its plan to issue more licenses in a regulated industry. When the process is free from rent seeking, as assumed in most literature on regulation, the deregulation proceeds as planned by the government. However, if the process is plagued with rent seeking, there follows a collective contest between the group of incumbents and the group of potential entrants. If the incumbents win, the government does not issue licenses and the market structure remains the same. If the potential entrants win, the government distributes licenses among the potential entrants.

The remainder of this paper is organized as follows. Section II sets out the basic Cournot-Nash oligopoly model with a linear demand. The effect of entry is examined when there is no rent seeking by incumbents and potential entrants. Section III analyzes the contest between the incumbents and the potential entrants. The case when firms adopt a non-cooperative Nash strategy in rent seeking is considered. Section IV examines the effect of entry contest on expected social welfare. The optimal competition policy is also analyzed. Section V offers concluding remarks.

II. Cournot-Nash Oligopoly

Consider a Cournot-Nash oligopoly consisting of $n$ identical firms licensed by the government. The firms produce a homogeneous product. Production takes place only once. For computational tractability this paper assumes that (inverse) demand is given by a linear function

$$P = a - Q(n),$$

(1)

$^2$Focusing on a single-period model, this paper assumes away difficult problems associated with multi-period models, one of which is whether oligopoly is contestable or not. Several papers deal with issues related to deregulation in multi-period models. See, for example, Crew and Rowley (1986, 1988).
where \( a > 0 \) and \( Q(n) = \sum q_i \) denotes the industry output when there are \( n \) firms. Marginal costs of the incumbent firms and the potential entrants are assumed to be identical and constant. Incumbent firm's profit when there are \( n \) firms is denoted \( \Pi(n) \), and is given by

\[
\Pi(n) = (a - Q(n) - c)q_i - F,
\]

where \( q_i \) represents firm \( i \)'s output level, \( c \) the constant marginal cost and \( F \) the fixed cost. For the incumbents, without loss of generality, \( F \) is assumed to be zero. Alternatively, \( F \) can be viewed as sunk cost. The first-order conditions for profit maximization are

\[
\frac{\partial \Pi}{\partial q_i} = a - Q(n) - c = 0, \quad \text{for} \quad i = 1, \ldots, n.
\]

The second-order conditions are satisfied. Solution of eq. (3) for \( Q(n) \) gives

\[
Q(n) = n(a - c)/(n + 1).
\]

Substitution of \( Q(n) \) into \( \Pi(n) \) yields

\[
\Pi(n) = (a - c)^2/(n + 1)^2.
\]

Since all the incumbents are identical, the subscript \( i \) denoting the firm is dropped in eq. (5).

Consumer surplus with \( n \) firms is given by

\[
CS(n) = \frac{(a - c)^2}{2} - \frac{n^2(a - c)^2}{2(n + 1)^2}.
\]

Social welfare is defined to be the sum of aggregate profits and consumer surplus, which is given by

\[
W(n) = CS(n) + n\Pi(n) = \frac{n^2 + 2n}{2} \frac{(a - c)^2}{2(n + 1)^2}.
\]

Now suppose that the government has issued \( k \) more licenses, where \( k \geq 1 \). Industry output, each incumbent's profit, and consumer surplus with \( k \) entrant(s), are, respectively, given by:
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\[ Q(n,k) = (n+k)(a-c)/(n+k+1) \] (4-1)

\[ I(n,k) = (a-c)^2/(n+k+1)^2 \] (5-1)

and

\[ CS(n,k) = (Q(n,k))^2/2 = (n+k)(a-c)^2/2(n+k+1)^2 \] (6-1)

If the government has issued \( k \) more licenses, each incumbent's profit decreases by

\[ \Delta I_k = I(n) - I(n,k) = k(2n+k+2)(a-c)^2/(n+1)^2(n+k+1)^2. \]

In other words, \( \Delta I_k \) denotes the benefit each incumbent obtains by blocking entry of \( k \) more firms.

Each entrant's profit is denoted \( I_d(n,k) \), which is given by

\[ I_d(n,k) = (a-c)^2/(n+k+1)^2 - F, \]

where \( F \) can be positive. From the non-negativity constraint \( I_d(n,k) \geq 0 \), we find the upper limit to the fixed cost as \( F \leq F^0 = (a-c)^2/(n+k+1)^2 \). This weak inequality is assumed to be satisfied. The fixed cost \( F \) can also be viewed as entry cost. Note that there is no difference in variable cost functions between the incumbents and new entrants. Social welfare with \( k \) entrants is again the sum of aggregate profits and consumer surplus:

\[ W(n,k) = CS(n,k) + nI(n,k) + kI_d(n,k) \]

\[ = n^2 + 2kn + 2n + k^2 + 2k(a-c)^2/(n+k+1)^2 - kF \] (7-1)

A simple comparison of (7) and (7-1) shows that

\[ W(n,k) > W(n), \]

if \( F < (2n+k+2)(a-c)^2/2(n+1)^2(n+k+1)^2 \). Let us define the critical value of the fixed cost

\[ F^* = (2n+k+2)(a-c)^2/2(n+1)^2(n+k+1)^2. \]
For a given number of entrants $k$, if the fixed cost $F$ is smaller than the critical value $F^*$, then entry of $k$ firms increases social welfare. However, if the fixed cost is larger than the critical value $F^*$, entry of $k$ firms decreases social welfare. The critical value of the fixed cost decreases in the number of entrants, i.e., $\frac{\partial F^*}{\partial k} < 0$. Moreover, if $F$ belongs to the range $[\frac{(\alpha - \delta^2)/(\eta n + k + 1)^2}{F^*}]$, then $\partial W(n,k)/\partial k < 0$. Thus, the optimal number of the new entrants is one in this range. We further assume that $k < 2n(n+1)$. This condition will be satisfied provided that the number of new entrants is not too large compared to the number of incumbents. Then, simple comparison shows that $F^* < F_0$. If the fixed cost lies in the range $[F^*, F_0]$, entry of $k$ firms reduces social welfare, i.e., $W(n,k) < W(n)$, even though entry is profitable from each entrant's point of view. If the fixed cost is $F^*$, social welfare remains the same with entry of $k$ firms. When the fixed cost belongs to the range $[0, F^*)$, entry increases social welfare. From now on, we focus on this case.

### III. Entry Contest

The government plans to issue $k$ more licenses to $m$ potential entrants. If the deregulation process is free from rent seeking, the deregulation proceeds as planned by the government. In this case the government simply distributes $k$ licenses to $m$ potential entrants. However, if the process is plagued with rent seeking, incumbents compete with potential entrants over the entry of $k$ firms. The incumbents are, of course, against issuing additional licenses. If the incumbents win, the regulation regime remains the same. If the potential entrants win, $k$ potential entrants out of $m$ potential entrants receive entry licenses. Consumers have no influence on the government decision-making regarding entry.

Kang and Lee (2001) assume that the deregulation process is always plagued with rent seeking. In the present paper, however,

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3We thank the referee for pointing out this condition.

4This case has been extensively examined in the literature on "excessive entry," See, for example, Mankiw and Whinston (1986).

5While this paper focuses on interaction between groups of firms, there can be a variety of scenarios on deregulating process. For example, Poitras and Sutter (1997) consider interaction between the reformer and the incumbent.
the process may or may not be plagued with rent seeking. The probability that the process is plagued with rent seeking is exogenously given and is denoted \( r \), where \( 0 \leq r \leq 1 \) (The model of Kang and Lee (2001) corresponds to the case when \( r = 1 \)). An implicit assumption in most papers on excessive entry is that \( r = 0 \), i.e., there is no rent seeking. If the process is free from rent seeking, the deregulation always increases social welfare in the present model as, by assumption, \( F \in [0,F^*] \). However, if the process is plagued with rent seeking with probability of \( r \), the deregulation decreases social welfare when \( k = 1 \) (Kang and Lee 2001).

The contest that takes place when the deregulation process is plagued with rent seeking is now examined. The prize of the contest for each incumbent is the incremental profit secured by blocking entry of \( k \) firms, \( \Delta P_i \). As each potential entrant receives the license with probability \( \frac{k}{m} \) when the government issues the licenses, the expected prize of the contest to each potential entrant is simply \( \frac{k}{m} \ln(\ln k) \).

The incumbent firm \( i \) contributes \( x_i \) to entry-preventing activity, \( i = 1, \ldots, n \). The potential entrant \( j \) contributes \( y_j \) to entry-promoting activity, \( j = 1, \ldots, m \). Following Tullock (1980), the probability that the government does not issue \( k \) more licenses, \( P_k \), is given by a logit-form function of aggregate outlays of the two groups as

\[
P_k = \frac{X}{X+Y},
\]  

(8)

where \( X (= \sum x_i) \) and \( Y (= \sum y_j) \) denote, respectively, the aggregate outlay of the incumbents and the potential entrants. The probability of deregulation is denoted \( P_Y = 1 - P_k \).

If the government decides not to issue additional licenses, social welfare decreases exactly by the amount of first-stage expenditures \( X + Y \), with no change in market structure. When the government decides to issue more licenses, consumer surplus increases while rent-seeking expenditures are incurred.

There can be a variety of scenarios on possible interaction between the group of incumbents and the group of potential entrants, depending upon institutional environment. This paper focuses on the case in which the firms of the two groups employ a non-cooperative Nash strategy in the rent-seeking contest. Since the firms are Cournot-Nash competitors in the output market, it is reasonable to assume that they employ the non-cooperative Nash
strategy in rent seeking as well. This assumption is also adopted in many papers in the literature on collective contest (See, e.g., Katz and Tokatlidou (1996), Lee (1995), and Nitzan (1991)).

The objective of the risk-neutral incumbent firm \( i \) is:

\[
\max_{\lambda_i} V_i = P_i / (k \pi n) + (1 - P_i) / (\pi n k) - x_i, \text{ for } i = 1, \ldots, n. \tag{9}
\]

Similarly the objective of the risk-neutral potential entrant \( j \) is

\[
\max_{\lambda_j} W_j = P_j / m \pi n k - y_j, \text{ for } j = 1, \ldots, m. \tag{10}
\]

The first-order conditions for (9) and (10) are, respectively, given by

\[
\frac{\partial V_i}{\partial \lambda_i} = \frac{Y(1 - \pi n k)}{\pi n k} / (X + Y)^2 - 1 \\
= Yk(2n + k + 2) / (n + 1)^2(n + k + 1)^2(X + Y)^2 - 1 = 0, \tag{11}
\]

for \( i = 1, \ldots, n \) and

\[
\frac{\partial W_j}{\partial \lambda_j} = Xk / m \pi n k / (X + Y)^2 - 1 \\
= Xk[(\alpha - c)^2 - F(n + k + 1)^2] / (n + k + 1)^2(X + Y)^2 - 1 = 0, \tag{12}
\]

for \( j = 1, \ldots, m \).

Simultaneously solving eqs. (11) and (12), we obtain

\[
X^* = \frac{mk(2n + k + 2) / (n + 1)^2(\alpha - c)^2A}{[m(\alpha - c)^2(2n + k + 2) + (n + 1)^2A]^2(n + k + 1)^2}, \tag{13}
\]

\[
Y^* = \frac{k(2n + k + 2) / (n + 1)^2(\alpha - c)^2A^2}{(n + k + 1)^2[m(2n + k + 2)(\alpha - c)^2 + (n + 1)^2A]^2}, \tag{14}
\]

and

\[
X^* + Y^* = \frac{k(2n + k + 2) / (n + 1)^2(\alpha - c)^2A}{(n + k + 1)^2[m(2n + k + 2)(\alpha - c)^2 + (n + 1)^2A]}, \tag{15}
\]

where \( A = [(\alpha - c)^2 - F(n + k + 1)^2]. \)
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Substituting eqs. (13), (14) and (15) into \( P_x \) and \( P_y \), we obtain the equilibrium probability of winning of each group.

**Lemma 1.** When all the firms in the two groups employ a non-cooperative Nash strategy, the equilibrium probability of winning of each group is, respectively, given by

\[
P_x^* = \frac{m(2n+k+2)(a-c)^2}{m(2n+k+2)(a-c)^2 + (n+1)^2[(a-c)^2-F(n+k+1)^2]}, \text{ and}
\]

\[
P_y^* = \frac{(n+1)^2[(a-c)^2-F(n+k+1)^2]}{m(2n+k+2)(a-c)^2 + (n+1)^2[(a-c)^2-F(n+k+1)^2]}.
\]

From Lemma 1 comparative static results can be derived. It is easy to find that

\[
\frac{\partial P_x^*}{\partial n} < 0 \text{ if } F < (a-c)^2/(3n^2 + k^2 + 3kn + 6n + 3k + 3), \quad (16-1)
\]

\[
\frac{\partial P_x^*}{\partial k} > 0, \text{ and} \quad (16-2)
\]

\[
\frac{\partial P_x^*}{\partial m} > 0. \quad (16-3)
\]

Simple calculation shows that \( F < (a-c)^2/(3n^2 + k^2 + 3kn + 6n + 3k + 3) \). Thus, in the relevant range, inequality (16-1) is always satisfied. This result can be easily explained as follows. The free-rider problem the incumbents suffer becomes severer with more incumbents. As a result, the probability of entry blocking decreases in the number of incumbents. The inequalities (16-2) and (16-3) can be easily explained as well. The potential entrants also face the free-rider problem, (16-3). Moreover, the expected payoff to each potential entrant decreases with more entrants. This induces the potential entrants to reduce expenditures on rent seeking. Then the probability of winning of the incumbents increases, (16-2).

**IV. Social Benefit and Cost of Entry Contest**

This section examines the effect of entry contest on expected social welfare. When the incumbents win the contest, the market
structure remains the same. The expenditures on entry contest constitute social waste. When the potential entrants win the contest, the consumer surplus increases.

When the deregulation process is plagued with rent seeking, the expected social welfare $E(W)$ is given by

$$
E(W) = F_k[W(n) - (X + Y)] + (1 - P_0)W(n, k) - (X + Y)
$$

$$
= \frac{m(2n+k+2)(n^2+2n)(a-c)^4}{2(n+1)^2(m(2n+k+2)(a-c)^2 + (n+1)^2A)}
$$

$$
+ \frac{(n+1)^2A[n^2 + 2k(1 + 2n + k^2 + 2k)(a-c)^2 / 2(n+k+1)^2 - kF]}{m(2n+k+2)(a-c)^2 + (n+1)^2A}
$$

$$
- \frac{k(2n+k+2)(a-c)^2A}{(n+k+1)^2[m(2n+k+2)(a-c)^2 + (n+1)^2A]}.
$$

(17)

where $A = [a - c]^2 - F(n+k+1)^2$.

Straightforward calculation shows

**Lemma 2.**

$$
E(W) < W(n)
$$

(18)

Thus, the expected social welfare when the deregulation process is plagued with rent seeking is smaller than the social welfare in the status quo. This result generalizes the result in Kang and Lee (2001, p. 150: Proposition 1) to cases where the government issues more than a single license and where fixed costs are present. Even in such a case the attempt to deregulate entry barrier results in a decrease in expected social welfare, regardless of the number of new licenses.

Lemma 2 does not necessarily indicate that entry into regulated industries should not be allowed, however. Note that the deregulation process may or **may not** be plagued with rent seeking. As noted earlier, the probability of the presence of rent seeking is
exogenously given and is denoted \( r \), where \( 0 \leq r \leq 1 \). If the process is free from rent seeking, the deregulation can increase social welfare. The expected social welfare that takes into account the possibility of rent seeking is given by

\[
E(SW) = rE(W) + (1-r)W(n,k).
\]  
(19)

As \( E(W) < W(n) < W(n,k) \), \( E(SW) \) is larger than \( W(n) \) if \( r \) is close to 0. In this case, the deregulation increases expected social welfare, even though the process is plagued with rent seeking with probability \( r \). If \( r \) is close to 1, the deregulation always decreases social welfare.

The critical value of \( r \), \( r^* \), is defined as such that satisfies

\[
r^*E(W) + (1-r^*)W(n,k) = W(n).
\]  
(20)

Thus \( r^* = [W(n,k) - W(n)]/[W(n,k) - E(W)] \). As \( E(W) < W(n) < W(n,k) \), it follows that \( 0 < r^* < 1 \). Specifically, if \( F = 0 \), \( r^* \) is given by

\[
r^* = \frac{mn(2n+k+2)+(n+1)^2}{mn(2n+k+2)+2(n+1)^2}.
\]  
(21)

With the definition of \( r^* \), the following result is obtained.

**Proposition 1.**

1. If \( r < r^* \), the attempt by government to deregulate entry barrier increases expected social welfare.
2. If \( r = r^* \), the deregulation attempt by government does not change expected social welfare.
3. If \( r > r^* \), the deregulation attempt by government decreases expected social welfare.

Proposition 1 confirms our intuition. If the probability of rent seeking in the deregulation process is high, then the deregulation can decrease social welfare. Otherwise, it can increase expected social welfare.

The numbers of new licenses and potential entrants affect the critical value \( r^* \). From eq. (21), the following result is obtained.

**Lemma 3.**

1. \( \partial r^* / \partial m > 0 \), and
2. \( \partial r^* / \partial k > 0 \) if \( F = 0 \).
Lemma 3 has important implications on deregulation. The government may not have prior information about the probability that the deregulation process is plagued with rent seeking. That is, the exact value of \( r \) is not known. In such a situation, the more potential entrants there are, the more it is likely that the deregulation process increases expected social welfare. This result can be explained as follows. With more potential entrants, the potential entrants have less incentive to spend on rent seeking. This in turn reduces rent seeking by the incumbents. Moreover, the more licenses the government tries to issue, the more it is likely that the deregulation increases expected social welfare, if there is no fixed cost. An increase in \( k \) in such a case decreases success probability of deregulation. However, an increase in social welfare, in case of success in deregulation, is large enough to offset a decrease in the success probability. As a result, the expected social welfare increases.

A. Optimal Competition Policy

The optimal competition policy of government is now examined. Competition policy refers to the determination of the number of new entrants, \( k \), by the government. When there is no rent seeking, the optimal number of licenses is obtained by solving the equation \( \partial W(n,k)/\partial k=0 \). Let \( k^0 \) denote the solution to the equation \( \partial W(n,k)/\partial k=0 \). Then \( k^0 \) denotes the optimal competition policy when there is no rent seeking. If the fixed cost is zero, the optimal competition policy is to set \( k^0 \) as large as possible. That is, free entry is the optimal policy. When there are fixed costs, the value of \( k^0 \) is bounded from above.

The optimal competition policy in the presence of rent seeking is obtained by solving the following problem:

\[
\text{Max } E[SW] = rE(W) + (1 - r)W(n,k) \text{ with respect to } k. \tag{22}
\]

The first-order condition to (22) is given by

\[
\begin{align*}
\frac{\partial E[SW]}{\partial k} &= r(\frac{\partial P_x}{\partial k}W(n)) + (\frac{\partial W(n,k)}{\partial k})(1 - rP_X) \\
&\quad - r(\frac{\partial P_x}{\partial k}W(n,k) - r(\partial X + Y))/\partial k = 0. \tag{23}
\end{align*}
\]

Denote by \( k^e \) the solution to (23). Then \( k^e \) is the optimal number
of licenses to be distributed when rent seeking is present with probability $r$. At $k^0$, we find:

$$\frac{\partial E(SW)}{\partial k} = rt \frac{\partial P_X}{\partial k} [W(t) - W(t, k)] - r \frac{\partial (X+Y)}{\partial k}.$$  \hspace{1cm} (24)

Since $\frac{\partial W(t, k)}{\partial k} = 0$.

From (24) we obtain the relationship between $k^*$ and $k^0$ when $X+Y$ increases in $k$.

**Lemma 4.** If $\frac{\partial (X+Y)}{\partial k} \geq 0$, then $k^* < k^0$.

Lemma 4 can be explained as follows. Suppose that the government decreases the number of licenses from $k^0$ when $\frac{\partial (X+Y)}{\partial k} > 0$. A decrease in $k$ reduces $(X+Y)$. Moreover, the probability of deregulation increases, (16-2). As a result, $E(W)$ and $E(SW)$ increase. Thus, the optimal number of licenses is smaller than $k^0$. That is, the optimal number of new licenses in the presence of rent seeking is smaller than that in the conventional case without rent seeking. When an increase in the number of licenses decreases rent-seeking expenditures, the definite relationship between $k^0$ and $k^*$ cannot be obtained.

**V. Concluding Remarks**

This paper has examined the social benefit and cost of entry contest in a regulated Cournot-Nash oligopoly when the government deregulates entry barriers. The deregulation process may or may not be plagued with rent seeking by incumbents and potential entrants. This paper has shown that the deregulation reduces expected social welfare if the probability of rent seeking is high. Otherwise, the deregulation process turns out to increase expected social welfare.

This paper can be extended in several directions. Extension of the model to nonlinear demand seems to be an interesting one. Another possible extension would be to endogenize the probability that rent seeking occurs. Case studies of recently deregulated industries might generate valuable information about validity of this study.
References


