

## **A Real Currency Futures Hedging Instrument for the Very Short-Term Exposure Risk**

Myoung Shik Choi\*

Currency futures contracts hedger can reduce or remove the risk posed by the fluctuations in the cash exchange rate. Moreover, this hedger can minimize or eliminate the risk even if he/she is exposed to it for a very short hedging duration. Therefore, this paper derives an optimal hedging ratio, and presents a hedging tool against short-run risk.

According to hedging effective test, we find that the hedged risk is much smaller than the unhedged risk when the risk is measured for a very short period (e.g. 1, 2, 3, 5, 10 and 15 business days). In particular, we suggest that our hedging instrument may be able to be more effective in reducing short-term risk than other hedges such as naive hedge.

### Introduction

Official exchange-traded currency futures contracts are frequently used to hedge the risk posed by the fluctuations in the spot exchange rate over time. A futures hedger, by taking a specific position on these contracts, hopes to optimally minimize the risk or uncertainty posed by his/her position on the underlying cash asset. In fact, the hedger may be able to reduce the risk even if he/she is only exposed to it for a very short period of

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\* E-mail: address is msc50355@daum.net

time.

Generally, risk size is measured by the variance amount of the minimum-variance approach, and is associated with the functional relations of the cash and futures prices. This approach, in fact, produces the same results as maximization of the expected utility function. As such, this paper adopts the minimization of risk in the minimum-variance framework to determine optimal hedging

For optimal currency futures hedging, it should be noted that a crucial first step is the construction of appropriate hedge ratios. In many existing empirical studies, the authors have attempted to show that their hedge ratios reflect a close first-order correlation between the futures and spot prices only. However, the ratios used in these papers can be inappropriate and cannot work in creating a fully optimal hedge. Hence, this paper starts by forming a new hedge ratio that better captures the information needed by the hedger.

To see why these other measures of the hedge ratio are inappropriate, it is important to remember that the hedge ratio estimated by simple Ordinary Least Square equation with the futures price regressed on the cash price only reflects the degree of the correlation between the spot and futures prices over the sample period. This ratio, however, does not directly reflect other factors, such as the carry cost, that will make the futures price vary from the spot price. Moreover, the carry cost also causes the futures basis to fluctuate since it is also a random variable. Therefore, in this paper, we derive a hedge ratio that reflects the carry cost, which will be the source for the basis-posed risk. Our hedge ratio will simultaneously reflect the cash position-posed risk and the basis-posed risk, under the assumption that there is a short-run stable relationship between the foreign exchange markets and the financial markets.

In addition, two views on the basis risk to which a hedger is exposed are discussed. First, Castelino (2000) argues that, if interest rates unexpectedly rises, then both the cash and futures prices will decrease, and the basis will subsequently increase due to the increased carry costs. Hence, he asserts that the hedge ratio in the minimum-variance approach should have a maximum value of one since the futures price and basis are negatively correlated. This would imply that the change in the cash price caused by interest rate

variation is larger than that of the futures price.

In another important study, Pennings & Leuthold (2000) review the well-known motivations for exchange-traded futures contracts hedging. These include: the insurance against cash price-fluctuated risk, the liquidity of exchange-traded futures contracts, the role of the futures exchange as a loan market, the utility maximization that arises from simultaneous positions in both cash and futures, and the pursuit of futures basis-changed gains. In particular, this futures basis can be formed as a function of the carry cost, which has a significant effect on both the cash and futures price series.

Finally, we describe lengthy hedging-posed risk. At any time, a futures hedger will hold several co-existing futures contracts as well as a most liquid contract that is nearest to expiration in the exchanges. A hedger usually chooses the most liquid contract to have an expiration date in the regular maturity months, such as March, June, September, and December. Then, he will be subject to rolling over of futures contracts-posed risk whenever he rolls over the nearby contract, which will occur about every three months for a long hedging period. Neuberger (1999) argues that this rolling over-posed risk causes a significant amount of error over lengthy hedging periods. Hence, he presents a way to correct for it. However, we can avoid this issue since our hedge ratio is established only for short-run hedges, where nearby contracts of less than one month can be used.

Therefore, this study will present a specific hedging rule to optimally hedge against short-term foreign exchange rate-fluctuation risk. We find that this hedging tool is effective in reducing the very short-run risk. Moreover, our hedging ratio can be estimated using the simple OLS method, depending on the attributes of the sample data.

This paper is organized into five sections as follows: Section 2 discusses the short period exchange-traded futures contract hedging model. Section 3 describes the data. The testing results are reported in section 4. Finally, concluding words are presented in section 5.

## Model

This section discusses the procedure used to create the hedging instrument presented in this paper. To hedge against the risk caused by a cash open position, a hedger will open an exchange-traded currency futures contract in the opposite position of the cash position. However, in addition, the hedger will face the risk caused by basis fluctuations, as Castelino (2000) gives a similar explanation. The reason for this basis-exposed risk is the two random variables that compose the basis, the cash and futures prices, vary irregularly with one another over time. Hence, the futures hedger must recognize that the basis is not constant when making his/her hedging decision.

To create the optimal futures hedge, we create a hedge ratio that reflects the basis-posed risk as well as the spot currency-posed risk. This hedge ratio will be modeled for the hedging duration using the relationship between the cash and futures prices. After this, the hedge ratio will be computed based upon the sample realizations. Finally, we will compute the optimal hedging quantity of the futures contract to the hedged cash quantity.

We start by forming the hedging ratio from the following definition of the futures basis:

$$(1) \quad F_t = S_t + B_t$$

$$(2) \quad F_{t-i} = S_{t-i} + B_{t-i}$$

Where

$F_t$  = currency futures contract at time  $t$

$S_t$  = cash foreign exchange rate at time  $t$

$B_t$  = currency futures basis in the form of  $(F_t - S_t)$  at time  $t$

$(t - i)$  = hedging duration till time  $t$

The futures basis term indicates a short-run deviation from the stable long-run relationship between the cash and futures prices. We take  $i = 1, 2, 3, 5, 10$  and  $15$  business

days, indicating a short hedging duration. From the equations (1) and (2), we have that the difference-formed equation for the hedging period is as follows:

$$(3) \quad \Delta F_t - \Delta S_t - \Delta B_t = 0$$

Where

$\Delta F_t$  = changed currency futures contract for the hedging period

$\Delta S_t$  = changed cash foreign exchange rate for the hedging period

$\Delta B_t$  = changed basis for the hedging period

The equation (3) indicates that, during the hedging duration, the change in the futures price will offset both the change in the cash price and change in the basis exactly. This would imply a perfect hedge.

To realize this perfect hedge, we form the fundamental deterministic function of the futures price as follows:

$$(4) \quad \Delta F_t = f(\Delta S_t, \Delta B_t) = a_0 + a_1 \Delta S_t + a_2 \Delta B_t + u_t$$

Where

$f(.)$  = functional notation

$a_0$  = constant term

$a_1$  and  $a_2$  = slope coefficients

$u_t$  = stochastic error term at time  $t$

The changed futures price in the equation (4) indicates a stable linear relation between the changed cash price and the changed basis over the duration of the hedge.

Also, the basis in the equation (4) indicates the linear relationship of the difference in domestic and foreign interest rates multiplied by the cash price at time  $t$   $\{(i_t - i_t^*) S_t\}$ , where  $i_t$  = the domestic interest rate and  $i_t^*$  = the foreign interest rate at time  $t$  suggested by

covered interest rate parity. This interest rate parity condition addresses the dynamic equilibrium relationships between the foreign exchange and financial markets under the assumption of no short-term arbitrage transaction.

Specifically, the exchange-traded futures contract is adopted here instead of the foreign exchange market-traded forward contract because, under this type of parity, both contracts take the same functional role for foreign exchange risk hedging. Additionally, opening a position for a very short period is much easier using a future contract than a forward contract since the forward contract is usually opened for 30, 60, 90 days, or more. It, also, can be argued that a futures contract is often less costly than a forward contract.

Therefore, the specific testable equation will be as follows:

$$(5) \quad \Delta F_t = a_0 + a_1 \Delta S_t + a_2 \{\Delta[(i_t - i_t^*) S_t]\} + u_t$$

Using the equations (4) and (5) to measure the changed futures price at either the changed cash price or the changed basis, we find that  $dE(\Delta F_t | \Delta S_t, \Delta[(i_t - i_t^*) S_t]) / d \Delta S_t$  equals  $a_1$  given the carry cost term fixed and  $dE(\Delta F_t | \Delta S_t, \Delta[(i_t - i_t^*) S_t]) / d \Delta B_t$  equals  $a_2$  given  $(S_t$  fixed, where  $E(\Delta F_t | \Delta S_t, \Delta[(i_t - i_t^*) S_t])$  indicates the average value of  $\Delta F_t$  conditional to  $(\Delta S_t, \Delta[(i_t - i_t^*) S_t])$ . In particular, as we take the natural logarithm of all of the variables in the equation (5),  $a_1$  and  $a_2$  represent the values of the average elasticities that the variables have with one another. Moreover, if  $(i_t - i_t^*) = 0$ , then our hedge ratio equals the OLS hedge ratio because the OLS hedge is calculated to be the slope value of the simple equation where  $F_t$  is regressed on  $S_t$ . If  $(i_t - i_t^*) > 0$ , then our hedge ratio is larger than the OLS ratio, and vice versa.

If the coefficients,  $a_1$  and  $a_2$  are optimally determined for the short hedging duration within the believable statistical confidence level ranges, then the optimal hedge ratio,  $H_t^*$ , can be calculated as the absolute estimated value of  $a_1$  plus the absolute estimated value of  $a_2$ . Therefore, the optimal hedging position of the futures contract at time  $t$ , equals  $H_t^* Q_t$ , where  $Q_t$  is the spot currency position at time  $t$ .

## Data

In the following analysis, a series of 174 daily points of the Japanese Yen against the US dollar over the period September 17, 2001 through June 14, 2002 are used to conduct the empirical tests. In this series, the currency futures contracts are the settlement prices traded on the Chicago Mercantile Exchange, while the spot exchange rates are the noon-closing prices from New York. This one-hour difference between the closing times of these US currency markets is assumed to be trivial in a noise in the measurement.

For series of futures prices, Lien & Tse (2001) address that, near the maturity date of the futures contract, the futures prices tend to exhibit serial noisy. Hence, the lengthy data points of the futures prices are usually constructed by combining the nearby contracts at several days before the expiration date. In this study, we address this issue by switching the nearest contract to the next nearest contract two weeks before the maturity date.

When constructing the series of the interest rates used to measure the cost of the carry, one must consider the fact that short-term interest rates can be manipulated by a country's central bank and are also affected by arbitragers. Furthermore, the US based-investors and speculators are not considered here. So, we use the shortest interest rates available from the central bank of each nation, 30 days-constant maturity Treasury bills for the United States and the Repo rate for Japan. As a final note, the daily interest rate used in this paper is computed as the annual rate divided by 360.

## Results

This section discusses the measurement of the hedging risk and reports the effectiveness of the hedges. To examine the validity of our hedging tests, we first need to confirm that the variables used in the OLS estimation are stationary since the data generating process is not known.

**Table 1.** Dickey-Fuller testing results

Hedging duration	Futures price	Spot price	Carry cost
1 business days	-4.47	-6.61	-6.84
	-3.96	-6.62	-4.4
2 business days	-5.95	-5.97	-3.87
	-5.71	-5.86	-3.99
3 business days	-6.16	-5.53	-4.91
	-5.75	-5.38	-4.27
5 business days	-2.77	-5.88	-4.13
	-2.32	-5.58	-3.35
10 business days	-3.52	-2.98	-5.01
	-3.47	-2.76	-4.72
15 business days	-5.04	-1.97*	-6.63
	-5.28	-0.9*	-5.76

\* The tested variables are defined in the model description chapter.

\* The T-statistic values of the testing results of the models with drift only and with both drift and trend are presented at each section.

\* The critical level values of 1%, 5% and 10% in the model with drift only are -3.51, -2.89, and -2.58.

\* The critical values of 1%, 5% and 10% in the model with drift and trend are -4.04, -3.45, and -3.15.

\* The mark \* indicates that the null hypothesis is not rejected at standard significance levels.

Using the null hypothesis of the unit root-contained series, Dickey-Fuller tests (e.g. testing the models for drift only or for both drift and trend) are taken. The Augmented Dickey-Fuller tests are omitted because the series seem to be well represented by the standard Dickey-Fuller test results. These are presented in Table 1.

Table 1 indicates that all of the variables except one appear to be stationary at the given significance levels. Hence, we can test our hedging model without concern for certain issues, such as spurious regressions.

To obtain an optimal hedge ratio for the hedging duration, we test our model using OLS estimation. The results are reported in Table 2.

For the hedge ratio tests based on the sample series, first in the joint test of hypothesis,



**Table 2.** Hedge ratio testing results

Hedging duration	1 days	2 days	3 days	5 days	10 days	15 days
Hedge ratio (Ht*)	0.837	0.853*	0.988	0.911*	0.646*	0.984*
Computed P-values of F-statistic test	0.00095	0.00001	0.0	0.0	0.0	0.0

\* The hedging duration is constructed based upon the business traded days.

\* The hedge ratio values are computed as defined in the model description chapter.

\* The mark \* indicates that, in the t-statistic test, the carry cost terms are not statistically significant at the 30% critical level according to the computed P-values while all of the cash price terms are statistically significant with zero of the computed P-values.

we find that, for all six hedging duration, the joint null hypothesis that all the slope coefficients are simultaneously zero is rejected according to the computed P-values of F-statistic test. In the individual test of hypothesis, we find that all of the spot price terms for all six hedging duration are quite statistically significant. However, only two of the carry cost terms seem to be significant with P-values of 12% and 29.5%. If these terms are insignificant, this would imply that our hedge ratio would collapses to the simple OLS hedge ratio. Hence, the hedge ratios are computed for all the specific hedging duration as shown in table 2. In addition, we find that the futures price tend to be correlated negatively with the basis for only 1, 2 and 15 hedging duration as Castelino (2000) asserts.

To compare hedging effectiveness, Kroner and Sultan (1993) use the variance for measuring the degree of risk posed by the cash open position, as most approaches in the minimum variance framework. As a result, they define the formula  $\text{Var}(S_t - b_t F_t)$  as their hedging effectiveness measurement, where  $\text{Var}$  = variance,  $S_t$  = the spot price,  $b_t$  = the calculated hedge ratio, and  $F_t$  = the futures price.

We modify this formula to measure variance for specific hedging duration, and, then, use it to measure hedging effectiveness over the relevant hedging duration. These results for the comparison of the hedged and unhedged risks are presented in Table 3.

Three kinds of hedges are tested per hedging duration. Additionally, the naive hedge indicates that the hedge ratio is one.

**Table 3.** Hedging effectiveness testing results

Hedging Duration	Risk from zero in No hedge	Risk from zero in Naïve hedge	Risk from zero in Our hedge
1	8.08E-05	6.08E-05	6.07E-05
2	0.000103	6.16E-05	6.2E-05
3	0.00012	6.16E-05	6.15E-05
5	0.00017	6.13E-05	6.13E-05
10	0.00032	6.5E-05	9.48E-05
15	0.00044	3.59E-05	3.63E-05

According to the results of Table 3, all of the hedged risks are much smaller than the unhedged risk. Moreover, they exhibit very similar great effectiveness in reducing risk for all hedging duration. Thus, without considering any transaction costs (e.g. barriers to using the exchange-traded futures contract, trading margins etc.) futures hedging will realize benefits even if the hedging period is quite short.

Moreover, comparing the hedging effectiveness of the naive hedge to our hedge, we find that our hedge is slightly more effective for the one and three day hedging duration.

### Conclusion

The hedging method that this paper proposes has been shown to be an appropriate short-term hedging instrument for the Japanese Yen. Also, we find that our hedge can be more effective in reducing the risk from holding the Japanese currency than the naive hedge. In particular, the testing results suggest that futures hedging will realize benefit in the reduction of risk than no hedge even if hedging duration is quite short.

In addition, our hedge will be much more effective in reducing very short-term risk if the estimated values of the coefficient variables ( $a_1$  and  $a_2$ ) that compose the hedge ratio are captured within more statistically significant level in the individual significance test. As a final note, further study of this issue will include a test using intra-day data.

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## 초단기 환위험 헤지기법에 관한 연구

최명식\*

외환시장에서 현물환 변동으로 야기되는 환위험은 파생상품시장의 통화선물 헤저에 의해서도 제거 또는 감소될 수 있다. 또한 통화선물 헤저에게 노출된 환위험이 초단기간일지라도 헤저는 환위험을 제거 또는 최소화할 수 있다. 따라서 본 논문에서는 이러한 단기 환위험에 적합한 헤징기법 제공을 목적으로 한다.

헤지효과를 검증한 결과, 환위험에 노출되는 기간이 매우 짧더라도 헤지를 하면 환위험이 상당히 감소하는 것으로 나타났다. 특히 본 논문에서 제시하는 헤징기법은 다른 헤징기법과 비교해서 단기 헤지에 더 효과적일 수 있음을 암시하고 있다.

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\* 미국 University of Missouri-Columbia, 경제학박사, 단국대학교 무역학과 외래교수(E-mail: msc50355@daum.net).