Optimal Reporting Frequency in Agencies

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We consider a multi-period principal-agent model, in which an agent's actions influence the mean and variance of the firm's future income stream. We characterize the optimal compensation contract under an information system, which reports periodic incomes. We show that the contractual efficiency increases as the reporting frequency increases, and derive the principal's optimal choice of the reporting frequency. Our comparative static analysis predicts that when the income stream is more variable, it is optimal to increase the reporting frequency, whereas when the variation of the income stream is more sensitive to the agent's action, the optimal reporting frequency decreases. We also provide comparative static implications associated with the incentive power placed on the estimated variance of the reported income.

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I. Introduction

The stewardship role of accounting reporting systems has been a central issue in the accounting literature. In particular, how to evaluate an accounting reporting system and how to use the signals generated by a reporting system for managerial contracts have been considered as important research issues.[1] An important aspect that characterizes a reporting system is its reporting frequency.

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Regulatory policies on interim reporting differ across countries. For example, listed firms are required to report their earnings quarterly on Form 10-Q and annually on Form 10-K in the U.S., whereas the firms are required to file their earnings on a half-year basis in the U.K.. On a non-mandatory basis, it also appears that there is a systematic difference in the reporting frequency not only across countries but also across industries and across divisions in a multi-divisional firm.[2] Recently, Frost and Pownall (1994) document that both mandatory and voluntary disclosures are more frequent in the U.S. than in the U.K., which they attribute to the U.S. investors' stronger information demand. However, the linkage between the stronger information demand and more frequent reporting remains unclear.[3] For example, if a top manager is paid solely according to the firm's annual earnings, there is no need for shareholders to request more frequent interim reporting since the sum of interim earnings is the same as the annual earnings. This leads to questions such as "What are the potential benefits, if any, that lead the shareholders in the U.S. to demand more frequent reporting?" and "What are the economic forces driving a variation in reporting frequency?"

The main objective of this paper is to offer an explanation to the above questions from an agency theory perspective. We show that there is a potential benefit from more frequent reporting and explain why such a benefit arises. We also identify the factors that determine a firm's reporting frequency, and characterize the optimal reporting frequency. We then provide some comparative static predictions about the optimal reporting frequency.

We present a multi-period dual agency model in which a risk-neutral principal hires a risk-averse agent who takes two actions at the beginning of the entire period. The agent's first action affects the mean value of each period's income, and the

[1] Earlier research on the evaluation of information systems includes Marschak and Miyasawa (1968), Feltham (1968), Demski and Feltham (1978), Holmstrom (1979, 1982), Gjesdal (1981), Grossman and Hart (1983), among others.

[2] For example, see Barr (1994) for the banking industry and Klein (1995) for the insurance industry.

[3] Crokett (1992) argues that a key element in determining the reporting frequency is organizational demand for performance measure/evaluation, although it is not quite clear why and how they are linked together.

second action affects its variance, where the income in each period is normally distributed. We define a reporting system by its reporting frequency. For example, an extreme reporting system reports only the aggregate income at the end of the periods. Another extreme reporting system reports the periodic income in every period, which is assumed to be a more costly system than the former reporting system.

It is not necessarily true that a more frequent reporting system is Pareto-superior to a less frequent system. In fact, a more frequent reporting system is strictly Pareto-inferior if the principal can directly observe and control the agent's second action that influences the variation of the income stream. This is because, when the agent's second action is observable, the agency problem arises only from the agent's unobservable first action. The optimal compensation contract is solely based on the firm's aggregate income, which is a sufficient statistic about the agent's first action. This implies that, to the extent that the aggregate income can be computed by adding periodic incomes regardless of reporting frequency, all reporting systems are equivalent in their informativeness about the agent's first action. Therefore, a more frequent reporting system achieves no informational improvement over a less frequent reporting system, while the former system is more costly.

However, the reporting frequency matters if the agent's second action that affects the variations of income stream is not observable. In this case, there is an additional layer of the moral hazard problem associated with the agent's second action, and signals generated by a more frequent reporting system indeed allow the principal to control the additional incentive problem more efficiently. To be more specific, as our analysis shows, the optimal compensation contract in this case is based not only on the aggregate income but also on the estimated variance of the income stream, both of which are sufficient statistics about the agent's two actions. We show that signals generated by a more frequent reporting system are of a higher quality in their informativeness about the agent's second action than those generated by a less frequent reporting system. Such an improvement on informational quality allows the principal to place a higher-powered incentive on the estimated variance. This implies that the principal can control the incentive problem associated with the agent's second action in a more forceful manner. This property then translates into an

improvement in the contractual efficiency in the sense that the agency cost decreases as the reporting frequency increases.

Given that a more frequent reporting system is more costly, the principal must balance the improvement in the contractual efficiency and the reporting cost when she chooses the optimal reporting frequency (hereafter, 'she' represents the principal and 'he' represents the agent). Our comparative static analysis predicts that when the income stream is more variable, the optimal reporting frequency increases. This follows from the fact that an increase in the variance of the income stream generates coarser information about the agent's second action, which in turn increases the principal's needs for finer information. We also find that as the variation of income stream becomes more sensitive to the agent's second action, the optimal reporting frequency decreases. This is because an increase in the sensitivity results in an increase in the informativeness of the estimated variance about the agent's second action, *ceteris paribus*. This reduces the principal's demand for a higher quality signal through frequent reporting.

We also examine the power of an incentive placed on the estimated variance in the optimal contract, *i.e.*, how sensitively the managerial compensation depends on the estimated variance. We find that the incentive power is decreasing in the variance of income stream when the reporting frequency is given. However, it is independent of the variance of income stream when the reporting frequency is endogenized. This is because an increase in the variance of income stream induces the principal to increase the reporting frequency, which results in an improvement on the informational quality of the estimated variance. Such an improvement exactly offsets the negative effect that an increase in the variance has on the quality of the estimated variance. Thus, the power of the incentive placed on the estimated variance in the optimal compensation contract remains unchanged.

As discussed earlier, our results are crucially based on the dual agency situation in which the agent determines not only the firm's average income stream but also its variance. There are several other papers that also study the dual agency setting. Hirshleifer and Suh (1992) show that the agent's indirect risk preferences are mainly determined by the curvature of his incentive contract. For example, if his incentive contract is sufficiently convex, then the risk-averse agent can be induced to be a risk-lover. Kim and

Titman (1997) show, in a single period setting, that the agent is paid according to both the absolute output level and the deviation from the anticipated output level. In particular, they show that the principal rewards the agent's deviation if she wants a higher risk level than the agent, but she penalizes the deviation otherwise. Sung (1995) extends Holmstrom and Milgrom's (1987) continuous-time agency model to a dual agency setting. He shows that a linear contract is optimal when the agent with constant absolute risk aversion chooses not only the drift ratio of the outcome process but also its variance. However, none of the above papers study the issue of a firm's optimal reporting frequency, which is the main focus of our paper.

The rest of the paper is organized as follows. In Section II, we formulate our dual agency model, and derive an optimal compensation contract in Section III. In Section IV, we show that there is a benefit from having a more frequent reporting system if there is no cost associated with reporting incomes. We then derive the optimal reporting frequency in a setting in which a more frequent reporting system is more costly. We also provide some comparative static analyses. Concluding remarks follow in Section V. All proofs are provided in the Appendix.

II. The Basic Model

We consider a dual principal-agent setting, in which a risk-and effort-averse agent works for a risk-neutral principal for T periods. At the beginning of the entire period, the agent takes two unobservable actions, and, $a_1 \in [0,\infty)$ and $a_2 \in [0,\bar{a}_2]$.[4] The agent's two actions

[4] It has to be noted that in our model the agent chooses his multiple actions only once at the beginning of the entire period. In other multi-period agency models (*e.g.*, Lambert (1983), Radner (1985), and Holmstrom and Milgrom (1987)) the agent selects his single action in every period. Assuming that the agent's actions are chosen at the beginning of the first period and then fixed for the rest of T periods is somewhat restrictive. However, a multi-period agency model with a general utility function for the agent is not tractable when the agent's two actions vary from period to period. Furthermore, our main focus is not on how the principal controls the agent's incentive to change actions period by period but on how the optimal reporting frequency is determined when the agent also affects the firm's risk level.

generate a stream of incomes, which is denoted by a vector $x=(x_1, x_2, \cdots, x_T)$, where x_t is the income realized in period t. The income in each period depends not only on the agent's two actions, (a_1, a_2), but also on the state of nature in that period, which is denoted by θ_t. For simplicity, we assume that the income generating process is described by the following stochastic function:

$$x_t = \phi(a_1) + \theta_t,$$

where $\theta_t \sim$ i.i.d.$N(0, \sigma^2(a_2))$ for all $t = 1,2,\cdots,T$. After T periods end, the principal pays the compensation, w, to the agent. The net interest rate is assumed to be zero. We assume that the income stream is verifiable without cost to both the principal and agent. Thus, the compensation contract for the agent must be based on x, *i.e.*, $w(x)$. In addition, we make the following assumptions.

Assumption 1: The agent's preferences on wealth and both activities are additively separable, *i.e.*,

$$U(w, a_1, a_2) = u(w) - v_1(a_1) - v_2(a_2),$$

and $u'>0$, $u''<0$, $v_i'>0$, $v_i''>0$, where $v_i(\cdot)$ denotes the agent's disutility function of taking action i for $i=1,2$.

Assumption 2: $\phi_1 > 0$ and $\phi_{11} < 0$.[5]

Assumption 3: $\sigma'<0$, $\lim_{a_2 \to 0} \sigma(a_2) < \infty$, and $\lim_{a_2 \to \bar{a}_2} \sigma(a_2) > 0$.

Assumption 1 represents that the agent is risk- and effort-averse, and the agent's disutility functions associated with both actions are convex. Assumption 2 indicates that a_1 is a productive activity in the sense that that a higher effort generates more income in terms of the first-order stochastic dominance. Assumption 3 states that an increase in a_2 reduces the variability of the income stream. In other words, a_2 is a risk reduction activity. It also states that the firm's risk level cannot be eliminated or increased without limit by the agent's choice of the second action.[6] In general, reducing risk

[5] The subscripts denote the derivatives.

[6] This assumption is needed to guarantee the existence of an optimal

is costly because of the usual trade-off relation between return and risk. In our model, such a trade-off relation is given by $v_2'>0$.[7]

III. The Analysis

In this section, we derive the optimal compensation contract that maximizes the combined utilities of the principal and the agent, which is referred to as joint surplus *SW*, given that the agent chooses both of his actions to maximize his own expected utility.[8] Assuming the first-order approach is valid, and there exists an interior solution for (a_1,a_2), we have the principal's optimization problem as follows:[9]

$$\max_{a_1,a_2,w(\mathbf{x})} SW = \int [\sum x_t - w(x)]h(x|a_1,a_2)dx + \lambda [\int u(w(x)h(x|a_1,a_2)dx - v_1(a_1) - v_2(a_2)]$$

$$\text{s.t.} \quad \text{(i)} \quad \int u(w(x))h_1(x|a_1,a_2)dx - v_1'(a_1) = 0$$

$$\text{(ii)} \quad \int u(w(x))h_2(x|a_1,a_2)dx - v_2'(a_2) = 0$$

$$\text{(iii)} \quad w(x) \geq \underline{w} \text{ for all } x,$$

where $h(x|a_1,a_2)$ is a joint density function of the income stream x given the agent's action combination (a_1,a_2), and λ is a weight placed on the agent's expected utility in the joint surplus. The first

contract, especially when the agent also affects the firm's risk level.

[7] That is, we can equivalently assume that the mean value of the income stream, $\phi(a_1,a_2)$, has a separable form such as $\phi(a_1,a_2) = \phi(a_1) - v_2(a_2)$.

[8] This is qualitatively equivalent to a model in which the principal maximizes her utility subject to the individual rationality constraint that ensures the optimizing agent to receive his reservation utility level.

[9] Grossman and Hart (1983) and Rogerson (1985) showed that MLRP (Monotone Likelihood Ratio Property) and CDFC (Convexity of the Distribution Function Condition) are sufficient for the validity of the first-order approach when the signal space is of one dimension. Jewitt (1988) found less restrictive conditions for the validity of the first-order approach, which are based on both the agent's risk preferences and the distribution function of the signal, and showed that many well-known families of distribution functions satisfy his conditions. Recently, Sinclair-Desgagne (1994) established more generalized versions of MLRP and CDFC in a multi-dimensional signal space, which are sufficient for the validity of the first-order approach.

constraint is the agent's incentive compatibility constraint of a_1, while the second constraint is that of a_2. The third constraint implies that the agent has limited liability.[10]

Let $(w^*(x), a_1^*, a_2^*, \mu_1, \mu_2)$ be a solution to the above optimization program. By solving the Euler equation for the above program, we have the optimal compensation contract, $w^*(x)$, satisfying

$$\frac{1}{u'(w^*(x))} = \lambda + \mu_1 \frac{h_1}{h}(x|a_1^*, a_2^*) + \mu_2 \frac{h_2}{h}(x|a_1^*, a_2^*), \tag{1}$$

for almost every x for which equation (1) has a solution $w^*(x) \geq \underline{w}$. Otherwise, $w^*(x) = \underline{w}$. In equation (1), μ_1 and μ_2 denote the Lagrangian multipliers of the agent's incentive constraints associated with a_1 and a_2, respectively. Since x_t's are identically, independently, and normally distributed, we have

$$\begin{aligned} \frac{h_1}{h}(x|a_1^*, a_2^*) &= \frac{f_1}{f}(x_1|a_1^*, a_2^*) + \cdots + \frac{f_1}{f}(x_T|a_1^*, a_2^*) \\ &= \frac{\sum_{t=1}^{T} x_t - T\phi(a_1^*)}{\sigma^2(a_2)} \phi_1(a_1), \end{aligned} \tag{2}$$

where $f(x_t|a_1^*, a_2^*)$ is a normal density function with mean $\phi(a_1)$ and variance $\sigma^2(a_2)$. Also, $f_1 \equiv \partial f / \partial a_1$, and $\sum x_t$ is an aggregate income for T periods. Similarly, it can be shown that

$$\begin{aligned} \frac{h_2}{h}(x|a_1^*, a_2^*) &= \frac{f_2}{f}(x_1|a_1^*, a_2^*) + \cdots + \frac{f_2}{f}(x_T|a_1^*, a_2^*) \\ &= \frac{-T\sigma'(a_2^*)}{\sigma(a_2^*)} + \frac{Ts^2}{\sigma^3(a_2^*)} \sigma'(a_2^*), \end{aligned} \tag{3}$$

where $f_2 \equiv \partial f / \partial a_2$, and $s^2 = \sum_{t=1}^{T} [x_t - \phi(a_1^*)]^2 / T$ denotes an estimated variance of the income stream x.[11] Substituting h_1/h given by (2) and h_2/h given by (3) into (1), we obtain the following proposition.

[10] This limited liability constraint is introduced to guarantee the existence of an optimal contract $w(x)$. See Mirrlees (1974) for details about the existence problem when the outcomes are normally distributed.

[11] The estimated variance here is different from the sample variance, which is commonly defined as $[\sum_{t=1}^{T} (x_t - \bar{x})^2]/(T-1)$, where $\bar{x} \equiv [\sum_{t=1}^{T} x_t]/T$.

Proposition 1

Let (a_1^*, a_2^*) be an optimal action combination that the principal wants to implement. Then, given Assumptions 1-3, the optimal contract, $w^*(x)$, satisfies

$$\frac{1}{u'(w^*(x))} = \lambda + \mu_1 \phi_1 \left[\frac{\sum x_t - T\phi(a_1^*)}{\sigma^2(a_2^*)} \right] + \mu_2 T \frac{\sigma'(a_2^*)}{\sigma(a_2^*)} \left[\frac{s^2}{\sigma^2(a_2^*)} - 1 \right] \quad (4)$$

Thus, the agent's compensation depends on x only through the aggregate income, $\sum x_t$, and the estimated variance, s^2.

Note that, in our dual agency model, the principal should provide the agent with appropriate incentives for both the productive effort a_1 and the risk choice a_2. As shown in equations (2) and (3), the aggregate income, $\sum x_t$, is a sufficient statistic for x about a_1, and the estimated variance, s^2, is a sufficient statistic for x about a_2. Therefore, the optimal compensation is characterized by those two sufficient statistics only.

One can show that the principal always rewards a higher aggregate income ($\mu_1 > 0$). But a question arises: "Can μ_2 in (4) be always zero?" In other words, "Will the agent voluntarily choose the firm's risk level that the principal wishes?" In a single period dual agency model, Kim and Titman (1997) show that the principal and the agent in general do not agree upon the firm's risk level in a continuous risk choice setting.[12] The principal wishes the agent to reduce the firm's risk level because it generates more precise information about the agent's hidden effort. But, the principal wishes the agent not to reduce the firm's risk too much because reducing risk is costly to the joint utilities via $\lambda v_2(a_2)$. On the other hand, the agent's own preferences on the firm's risk level mainly depend on his indirect risk preferences induced by the curvature of the compensation contract[13] and his personal cost of reducing risk via $v_2(a_2)$. Kim and Titman (1997) show that if the principal wants more risk reduction than the agent, then the principal penalizes the agent for having a large estimated variance, *i.e.*, $\mu_2 > 0$ in (4), but rewards a large estimated variance, *i.e.*, $\mu_2 < 0$, otherwise.

[12] For a discrete risk choice setting, see Hirshleifer and Suh (1992).
[13] This has been well discussed in Hishleifer and Suh (1992).

IV. The Reporting Frequency

We now turn to our main focus of the paper, *i.e.*, a firm's optimal reporting frequency. By a reporting frequency, we mean how often the periodic incomes are reported. As discussed in the introduction, there is a systematic difference in a mandatory interim reporting policy across countries, and it appears that managers report their performance on a different periodic basis across industries as well as across divisions in a multi-divisional firm. In this section, we offer an explanation of why reporting frequency matters. In particular, we show that there is an efficiency gain from more frequent reporting. We also identify factors that determine the firm's optimal choice of reporting frequency, and provide some comparative static predictions.

For a formal analysis, we first define an accounting reporting system by its reporting frequency.

Definition 1

A reporting system $\eta^N=\{y_1^N,\cdots,y_N^N\}$ generates N income reports such that $y_i^N=x_{(i-1)k+1}+\cdots+x_{ik}$ for $i=1,\cdots,N$ where $k=T/N$ is an integer. That is, y_i^N is the aggregate income for k periods from $(i-1)k+1$ to ik.

Reporting system η^N equally divides the entire T periods into N sub-periods, each of which consists of k periods, and generates N signals $(y_1^N,\cdots,y_N^N)$. For example, a reporting system that reports the income in every period is η^T. On the other hand, a reporting system that reports only the aggregate income over T periods is η^1.

For analytical simplicity, we fix an action combination that the principal wants to implement, say (a_1^*,a_2^*). It can be shown from (4) that, given reporting system η^N, the optimal compensation contract that induces (a_1^*,a_2^*) is characterized by

$$\frac{1}{u'(w(\mathbf{y}^N))}=\lambda+\mu_1^N\phi_1\left[\frac{\sum x_t-T\phi(a_1^*)}{\sigma^2(a_2^*)}\right]+\mu_2^N N\frac{\sigma'(a_2^*)}{\sigma(a_2^*)}\left[\frac{s_N^2}{k\sigma^2(a_2^*)}-1\right], \quad (5)$$

where $y^N=(y_1^N,\cdots,y_N^N)$ and $s_N^2\equiv\sum_{i=1}^{T}[y_i^N-k\phi(a_1^*)]^2/N$. One can actually see that (4) is a special case of (5) in that (4) characterizes the

optimal contract under reporting system η^T. Observe that the optimal incentive contract characterized by (5) is a function of a non-linear aggregation of signals $(y_1^N,\cdots,y_N^N)$, which sharply contrasts with Banker and Datar (1989). This is because the agent's actions in our setting affect not only the average income but also the income variance.[14]

Having characterized the optimal compensation contract given a reporting system, we use backward induction to solve the principal's problem of choosing an optimal reporting system, *i.e.*, an optimal reporting frequency. Let $f^N(y^N|a_1^*,a_2^*)$ be a joint density function of y^N, and $C(N)=r\cdot N$ be the cost of reporting system η^N, where r is a unit cost of reporting income. Also let $SW(N)$ be the indirect joint surplus given reporting system η^N. That is,

$$SW(N)\equiv T\phi(a_1^*)-L(N)-\lambda\,[v_1(a_1^*)+v_2(a_2^*)]-C(N), \tag{6}$$

where

$$L(N)\equiv \int[w(y^N)-\lambda u(w(y^N)]f^N(y^N|a_1^*,a_2^*)dy^N.^{15} \tag{7}$$

In words, $SW(N)$ is the optimized joint surplus less the cost of reporting system η^N, when the optimal contract $w(y^N)$ is designed to motivate the agent to take (a_1^*,a_2^*) under reporting system η^N. Note that $L(N)$ represents the efficiency loss under reporting system η^N compared with the full information case in which the agent's action choice is observable. In other words, $L(N)$ measures the agency cost associated with inducing the agent to take (a_1^*,a_2^*) under reporting system η^N. We first establish the following result.

Proposition 2

Given Assumptions 1-3, the efficiency loss, $L(N)$, is decreasing in the reporting frequency, N.

[14] Technically speaking, in the class of density functions characterized by the proposition 1 in Banker and Datar (1989), the parameters affected by the agent's actions are additively separable in the exponent. On the other hand, the mean and variance of income in our normal density model are multiplicatively separable in the exponent.

[15] To focus on the firm's choice of the optimal reporting frequency, we express $SW(A)$ and $L(A)$ as functions of N, suppressing the action combination.

Proposition 2 shows that a more frequent reporting system increases the firm value if there is no cost of reporting incomes. Such a benefit arises from the fact that a more frequent reporting system provides more precise information about the agent's risk -reducing action a_2, and thus allows the principal to control the agent's a_2 more efficiently. To be specific, note that the information content associated with a_1 under η^N is captured by the likelihood ratio, f_1^N/f^N, and the amount of information is captured by its variance.[16] Since $f_1^N/f^N=[\sum x_t-T\phi(a_1^*)]/\sigma^2(a_2^*)$, the likelihood ratio is independent of N, and so is its variance. In fact, a reporting system with any frequency equally provides the aggregate income that the principal needs for controlling the agent's incentive problem associated with a_1. Thus, any reporting system is equivalent in its informativeness about the agent's productive effort a_1. This implies that if the agent chooses only a_1, or if a_2 is observable, the reporting frequency issue is irrelevant. Of course, this result crucially hinges upon our assumption that the agent does not change a_1 over the entire T periods. However, even in a multiperiod principal-agent model in which the agent with constant absolute risk aversion chooses the effort level period by period, the principal must base the agent's compensation on the aggregate income only.[17] Therefore, the reporting frequency issue is also irrelevant in such a setting.

On the other hand, observe that the likelihood ratio with respect to the agent's second action is given by $f_2^N/f^N=N(\sigma'(a_2^*)/\sigma(a_2^*))[s_N^2/k\sigma^2(a_2^*)-1]$, which crucially depends on the reporting frequency N. The proof of Proposition 2 in the Appendix shows that f_2^N/f^N is a mean-preserving spread of f_2^M/f^M if $N>M$, implying that reporting system η^N provides more precise information about a_2 than η^M. Consequently, a more frequent reporting system improves on the contractual efficiency (*i.e.*, reduces the agency cost) if there is no cost of reporting incomes. However, as one might expect, a more frequent reporting system is more costly. Hence, the principal has to balance the efficiency gain and the cost associated with more frequent reporting in determining optimal reporting frequency.

From (6), we know that the efficiency loss and the reporting cost depends upon the reporting frequency N. Therefore, the optimal

[16] For details, see Kim (1995) and Kim and Suh (1991).
[17] See Holmstrom and Milgrom (1987).

reporting frequency, N^*, is the solution of the the principal's problem:

$$\min_N L(N)+C(N).$$

To make our analysis tractable, we assume that the agent has a square-root utility function.

Proposition 3

Assume $u(w)=2\sqrt{w}$. Then,

(i) the optimal compensation contract that implements (a_1^*,a_2^*) under reporting system 0^N is characterized by

$$w(y^N)=[\beta_0^N+\beta_1^N(\sum x_t)+\beta_2^N\cdot s_N^2]^2, \tag{8}$$

where $\beta_1^N=(v_1'/2)/T\phi_1>0$, and $\beta_2^N=N(v_2'/2)/2T\sigma\sigma'<0$, and

(ii) the efficiency loss associated with reporting system η^N is given by

$$\begin{aligned} L(N) &= -\lambda^2+\frac{(v_1'/2)^2}{\mathrm{Var}[f_1^N/f^N]}+\frac{(v_2'/2)^2}{\mathrm{Var}[f_2^N/f^N]} \\ &= -\lambda^2+\frac{(v_1'/2)^2\sigma^2}{T\phi_1^2}+\frac{(v_2'/2)^2\sigma^2}{2N(\sigma')^2} \end{aligned} \tag{9}$$

where $v_i'\equiv v_i'(a_i^*)$ and $\sigma'\equiv\sigma'(a_2^*)$.

The first part of proposition 3 shows that the principal rewards the agent for a higher aggregate income ($\beta_1^N>0$), but penalizes the agent for having a larger estimated variance ($\beta_2^N>0$). To see why it is optimal for the principal to penalize the agent when the estimated variance is large, it is useful to consider a situation, in which the agent's second action is observable. In such a case, the optimal contract is given by $\hat{w}^N(y^N)=[\beta_0^N+\beta_1^N(\sum x_t)]^2$, which is convex in the aggregate income.[18] Observe that the contract $\hat{w}^N$ induces the agent to be risk-neutral with respect to the aggregate

[18] This is because the incentive compatibility constraint for the agent's second action is not binding, *i.e.*, $\mu_2^N=0$, which results in $\beta_2^N=0$. See the proof of Proposition 3 in the Appendix.

income, *i.e.*, $u(\hat{w}^N(\cdot))$ is linear in $\sum x_t$. This implies that the agent is inclined to increase the variance of the aggregate income by reducing a_2 because of his disutility associated with a_2. Thus, if the principal cannot observe but still offers $\hat{w}^N(y^N)$, then the agent will cut down the level of a_2 as low as possible. This reduces the informational quality of the aggregate income about the agent's productive action a_1. Therefore, the principal must provide an appropriate incentive to discipline the agent's incentive to increase the variance of income stream by penalizing a larger estimated variance, *i.e.*, $\beta_2^N<0$.

Observe that β_1^N is, in fact, independent of N, which implies that the reporting frequency does not affect the incentive power placed on the aggregate income. This is because any reporting system is equivalent in its informativemess about the agent's action a_1. On the other hand, the incentive power placed on the estimated variance, which is measured by the absolute value of β_2^N, is increasing in the reporting frequency N. This follows from the fact that the estimated variance generated by a more frequent reporting system provides more precise information about a_2 (see Proposition 2), which allows the principal to use that signal to control a_2 in a more forceful manner. On the other hand, $|\beta_2^N|$ is decreasing in σ^2 and $|\sigma'|$. The former result can be explained by the fact that the estimated variance becomes less informative about a_2 as σ^2 increases. This limits the principal's use of the estimated variance as an incentive device for the agent's risk influencing action a_2. The latter result can be understood by the fact that when the variance of income is very sensitive to a_2, it is relatively easy for the principal to implement a_2^*. This is because in such a case a small deviation from a_2^* will have a large impact on the agent's expected payoff given β_2^N, which allows the principal to implement a_2^* with relatively less risk imposed on the agent (*i.e.*, saving on the risk premium that she has to pay).

The second part of proposition 3 states that the efficiency loss is inversely related to the variances of likelihood ratios for both a_1 and a_2. Kim and Suh (1991) show that, when the agent has a square-root utility function, the variance of likelihood ratio is a measure of information amount contained in signal y^N about the agent's productive action,[19] and an information system is more

[19] In the statistics literature, it is referred to as Fisher information. See

valuable if its likelihood ratio is more variable. In our dual agency model, however, the variance of the likelihood ratio associated with a_1, var $[f_1^N/f^N]$, is the same across the reporting systems (*i.e.*, independent of the reporting frequency N). As discussed earlier, this is driven by the fact that all reporting systems in our model are equivalent in their informativeness about a_1. Therefore, the ranking of reporting systems is solely determined by var $[f_2^N/f^N]$, which is increasing in the reporting frequency N. In sum, the efficiency loss is decreasing in the reporting frequency.

Using Proposition 3, we also obtain:

Proposition 4

The optimal reporting frequency, N^*, is given by

$$N^* = \frac{(v_2'/2)\,\sigma}{\sqrt{2r}\,|\,\sigma'|}\,.^{20} \tag{10}$$

Thus, the optimal reporting frequency increases when

(i) the income stream is more variable (*i.e.*, σ^2 is large);

(ii) the variability of the income stream is less sensitive to the agent's action (*i.e.*, $|\,\sigma'|$ is small);

(iii) the reporting cost is small (*i.e.*, r is small).

Intuitive explanations for the comparative static predictions are in order. When the income stream is highly variable, the estimated variance provides little information about the agent's second action. Hence, in such a case, the principal has stronger needs for improving the quality of the signal. One way of improving the quality is to request more frequent reports from the agent. On the other hand, the second result in the above proposition can be explained by the fact that if the variation of income stream is highly sensitive to the agent's action, it is relatively easy for the principal to control the agent's incentive problem. Hence, *ceteris paribus*, it is optimal to reduce the reporting frequency. Lastly, the third result states that an increase in the reporting cost induces the principal to reduce the reporting frequency, which is rather obvious.

Lehmann (1983).

[20] We ignore the fact that N^* may not be an integer, but focus on the comparative statics.

We conclude our analysis with the following observation. As discussed earlier, the incentive power placed on the estimated variance, $|\beta_2^N|$, decreases in the income variance when the reporting frequency is fixed. However, substituting (10) into β_2^N in Proposition 3, we have

$$\beta_2^{N*} = \frac{N^*(v_2'/2)}{2T\sigma\sigma'} = \frac{(v_2'/2)^2}{2\sqrt{2r}T\sigma'|\sigma'|}, \tag{11}$$

which is independent of σ^2. In other words, the incentive power placed on the estimated variance under the optimal reporting frequency is independent of the income variance, which is in sharp contrast with what we have seen in proposition 3 when the frequency of reporting is fixed.

V. Conclusion

In this study we considered a multi-period principal-agent model, in which the agent's actions influence the mean and variance of the firm's future income stream. We derived the optimal compensation contract under an information system that reports income periodically. We then examined the principal's optimal choice of reporting frequency and identified the factors that determine the optimal reporting frequency. Our comparative static analysis predicts that when the income stream is more variable, it is optimal to increase the reporting frequency. However, the reporting frequency decreases when the variation of the income stream is more sensitive to the agent's action.

Our study contributes to the accounting literature by identifying the relation between the reporting frequency and the contractual efficiency. It will be interesting to empirically test our comparative static predictions. For example, investors are expected to react positively to a voluntary increase in a firm's reporting frequency when the risk associated with the firm's future income has been increased. On the other hand, the market response to an increase in the reporting frequency is expected to be negative when the variability of future income becomes more sensitive to the manager's action. Testing the incentive power placed on the variation of the income stream is also an interesting research topic.

Finally, note that while we focus on the benefit of more frequent reporting that arises from the principal's informational needs for controlling the agent's risk influencing action, our consideration on the reporting cost is rather mechanical. We observe that there may be other costs associated with more frequent reporting. For example, if one extends our model to a market setting in which a firm's interim earnings are observed by its competitors, more frequent reporting is likely to provide them with more refined information about the firm's risk level. Such information is often proprietary, which implies that there might be an additional cost associated with more frequent reporting. In such a case, the firm' owner, *i.e.*, the principal, has a strategic consideration for the choice of reporting frequency because it affects other firms' production decisions. We leave a formal analysis on this issue as an interesting research topic.

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Appendix

Proof of Proposition 2: To prove this proposition, we need the following lemma.

Lemma A1: Let $\varphi_1(z_1,z_2)$ and $\varphi_2(z_1,z_2)$ be two joint density functions defined on $Z_1 \times Z_2$. Suppose that $\varphi_1(z_1) = \varphi_2(z_1)$ for almost all z_1, where $\varphi_j(z_1) \equiv \int \varphi_j(z_1,z_2)dz_2$, $j=1,2$, and $\varphi_1(z_2|z_1)$ is a mean preserving spread of $\varphi_2(z_2|z_1)$ for almost all z_1. Then $E_{\varphi_2}[\Psi(z_1,z_2)] > E_{\varphi_1}[\Psi(z_1,z_2)]$ for any function Ψ that is concave in z_2.

Proof of Lemma A1: Observe that

$E_{\varphi_j}[\Psi(z_1,z_2)] = \iint \Psi(z_1,z_2)\,\varphi_j(z_1,z_2)dz_1dz_2 = \iint \Psi(z_1,z_2)\,\varphi_j(z_2|z_1)dz_2\,\varphi_j(z_1)dz_2$, $j=1,2$. Since $\Psi(z_1,z_2)$ is concave in z_2 and $\varphi_1(z_2|z_1)$ is a mean preserving spread of $\varphi_2(z_1|z_2)$ for almost all z_1, we have

$$\int \Psi(z_1,z_2)\,\varphi_2(z_2|z_1)dz_2 > \int \Psi(z_1,z_2)\,\varphi_1(z_2|z_1)dz_2 \quad \text{for all } z_1.$$

Thus, since $\varphi_1(z_1) = \varphi_2(z_2)$ for almost all z_1, we must have $E_{\varphi_2}[\Psi(z_1,z_2)] > E_{\varphi_1}[\Psi(z_1,z_2)]$.

Q.E.D.

We first denote

$$q(y^N) \equiv \lambda + \mu_1^N \frac{f_1^N}{f^N}(y^N) + \mu_2^N \frac{f_2^N}{f^N}(y^N).$$

Then the first-order condition for the optimal contract under the reporting system η^N can be written as

$$u'(w(y^N)) \cdot q(y^N) = 1, \tag{A1}$$

for all y^N. Also note that we have two binding incentive compatibility constraints

$$\int [u((w(y^N)) \frac{f_1^N}{f^N}(y^N)] f^N(y^N) dy^N = v_1', \tag{A2}$$

and

$$\int [u((w(y^N)) \frac{f_2^N}{f^N}(y^N)] f^N(y^N) dy^N = v_2', \tag{A3}$$

Using the definition of $q(y^N)$, (A2) and (A3), we have

$$\begin{aligned} &\int u(w(y^N)) q(y^N) f^N(y^N) dy^N \\ &= \int u(w(y^N)) \left[\lambda + \mu_1^N \frac{f_1^N}{f^N}(y^N) + \mu_2^N \frac{f_2^N}{f^N}(y^N) \right] f^N(y^N) dy^N \\ &= \lambda \int u(w(y^N)) f^N(y^N) dy^N + \mu_1^N v_1' + \mu_2^N v_2'. \end{aligned} \tag{A4}$$

Let M be an integer greater than N such that $T = Ml$. It follows from (11) in the text that

$$\begin{aligned} L(N) - L(M) = &\int [\, w(y^N) - \lambda\, u(w(y^N))\,] f^N(y^N) dy^N \\ &- \int [\, w(y^M) - \lambda\, u(w(y^M))\,] f^M(y^M) dy^M, \end{aligned} \tag{A5}$$

where $w(y^J)$ is the optimal contract under the reporting system η^J, $J = N, M$. Using (A4), we can rewrite (A5) as

$$\begin{aligned} L(N) - L(M) = &\int \Psi(q(y^N)) f^N(y^N) dy^N - \int \Psi(q(y^M)) f^M(y^M) dy^M \\ &+ v_1'(\mu_1^N - \mu_1^M) + v_2'(\mu_2^N - \mu_2^M), \end{aligned} \tag{A6}$$

where $\Psi(q(y^J)) \equiv w(y^J) - u(w(y^J))q(y^J)$ for $J=N,M$. Since $w(y^J)$ depends on y^J through $q(y^J)$, we define $w(y^J)=r(q(y^J))$. It is easy to see that $\Psi'(q)=-u(r(q))$ since $u'(r(q)) \cdot q=1$ by (A1). Also, note that $\Psi''(q)=-u'(r(q)) \cdot r'(q)<0$ since $r'(q)>0$. Therefore, Ψ is concave in q.

Now, define

$$q^O(y^N) \equiv \lambda + \mu_1^M \frac{f_1^N}{f^N}(y^N) + \mu_2^M \frac{f_2^N}{f^N}(y^N).$$

Observe that

$$\begin{aligned}\int [\Psi(q^O(y^N)) - \Psi(q(y^N))] f^N(y^N) dy^N &\leq \int \Psi'(q(y^N)[q^O(y^N) - q(y^N)] f^N(y^N) dy^N \\ &= -\int u(r(q(y^N)) \cdot [q^O(y^N) - q(y^N)] f^N(y^N) dy^N \\ &= v_1'(\mu_1^N - \mu_1^M) + v_2'(\mu_2^N - \mu_2^M) \qquad \text{(A7)}\end{aligned}$$

where the first inequality comes from the concavity of P, and the second and third equalities from the fact that $\Psi'(q)=-u(r(q))$ and (A4), respectively. Therefore, we have

$$\begin{aligned}L(N) - L(M) &\geq \int \Psi(q(y^N)] f^N(y^N) dy^N - \int \Psi(q(y^M)] f^M(y^M) dy^M \\ &\quad + \int [\Psi(q^O(y^N)) - \Psi(q(y^N))] f^N(y^N) dy^N \\ &= \int \Psi(q^O(y^N) f^N(y^N) dy^N - \int \Psi(q(y^M)) f^M(y^M) dy^M \qquad \text{(A8)} \\ &= E[\Psi(q^O(y^N))] - E[\Psi(q(y^M))],\end{aligned}$$

Note that y_i^N is normally distributed with the mean $k\phi$ and variance $k\sigma^2$ for all $i=1,\cdots,N$, and that y_j^M is normally distributed with the mean $l\phi$ and variance $l\sigma^2$ all $j=1,\cdots,M$. Moreover, for $J=N,M$, y_i^J is independent of y_k^J for all $i \neq k$. Hence, it follows that

$$z_1^N \equiv \frac{f_1^N}{f^N}(\mathbf{y}^N) = \frac{\sum_{t=1}^{N}(y_i^N - T\phi)}{\sigma^2} \phi_1 = \frac{f_1^M}{f^M}(\mathbf{y}^M) \equiv z_1^M; \qquad \text{(A9)}$$

$$z_2^N \equiv \frac{f_2^N}{f^N}(\mathbf{y}^N) = \frac{\sigma'}{\sigma}\left[\frac{\sum_{t=1}^{N}(y_i^N - k\phi)^2}{k\sigma^2} - N\right] = \frac{\sigma'}{\sigma}\left[\chi^2(N) - N\right]; \qquad \text{(A10)}$$

and

$$z_2^M \equiv \frac{f_2^M}{f^M}(y^M) = \frac{\sigma'}{\sigma}\left[\frac{\sum_{j=1}^{N}(y_j^M - \lambda\phi)^2}{\lambda\sigma^2} - M\right] = \frac{\sigma'}{\sigma}\left[\chi^2(M) - M\right], \quad \text{(A11)}$$

where $\chi^2(J)$, $J=N,M$, denotes the Chi-square random variable with J degrees of freedom.

Now, we rewrite (A8) as

$$E[\Psi(q^O(z_1^N, z_2^N))] - E[\Psi(q(z_1^M, z_2^M))]. \quad \text{(A12)}$$

Since $\Psi(q)$ is concave in q, and $q^O(z_1^N, z_2^N)$ and $q(z_1^M, z_2^M)$ are linear in z_2^N and z_2^M, respectively, $\Psi(q^O(z_1^N, z_2^N))$ and $\Psi(q(z_1^M, z_2^M))$ are concave in z_2^N and z_2^M, respectively. From (A9)-(A11), it follows that $z_2^J | z_1^J$, $J=N,M$, is a random variable with mean zero and variance $2(\sigma'/\sigma)^2(J-1)$, which is derived from the Chi-square random variable with $(J-1)$ degrees of freedom. Therefore,

$$z_2^M | z_1^M = z_2^N | z_1^N + \varepsilon, \text{ where } \varepsilon = \sigma'/\sigma\,[\chi^2(M-N) - (M-N)] \text{ and}$$

$$E[\varepsilon \mid (z_2^N | z_1^N)] = 0.$$

Thus, by Rothschild and Stigliz (1970), $z_2^M | z_1^M$ is a mean preserving spread of $z_2^N | z_1^N$. Consequently, from lemma A1, we have

$$L(N) - L(M) \geq E[\Psi(q^O(z_1^N, z_2^N))] - E[\Psi(q(z_1^M, z_2^M))] > 0,$$

which completes the proof of proposition 2.

Q.E.D.

Proof of Proposition 3: Given the agent's square-root utility function $u(w) = 2\sqrt{w}$, it follows from (1) that

$$w^N(y^N) = \left[\lambda + \mu_1^N \frac{f_1^N}{f^N}(y^N) + \mu_2^N \frac{f_2^N}{f^N}(y^N)\right]^2. \quad \text{(A13)}$$

Also two binding incentive compatibility constraints can be written as

$$\int \left[\lambda + \mu_1^N \frac{f_1^N}{f^N}(y^N) + \mu_2^N \frac{f_2^N}{f^N}(y^N)\right] f_1^N(y^N) dy^N = \frac{v_1'}{2},$$

and

$$\int \left[\lambda + \mu_1^N \frac{f_1^N}{f^N}(y^N) + \mu_2^N \frac{f_2^N}{f^N}(y^N) \right] f_2^N(y^N) dy^N = \frac{v_2'}{2}.$$

Using the fact that $E[f_i^N(y^N)]=0$, $i=1,2$, we rewrite the incentive compatibility constraints as two simultaneous equations with two unknowns, μ_1^N and μ_2^N :

$$\mu_1^N \mathrm{var}[z_1^N] + \mu_2^N E[z_1^N \cdot z_2^N] = \frac{v_1'}{2} \tag{A14}$$

$$\mu_1^N E[z_1^N \cdot z_2^N] + \mu_2^N E[z_2^N] = \frac{v_2'}{2}. \tag{A15}$$

where $z_i^N \equiv f_i^N / f^N(\mathbf{y}^N)$, $i=1,2$. From (A9)-(A11) in the proof of proposition 2, It follows that

$$E[z_1^N] = E[z_2^N] = 0, \text{ and} \tag{A16}$$

$$\mathrm{var}[z_1^N] = (T\phi_1^2)/\sigma^2 \text{ and } \mathrm{var}[z_2^N] = \frac{\sigma'}{\sigma}(2N). \tag{A17}$$

Moreover,

$$E[z_1^N \cdot z_2^N] = \frac{\sigma'}{\sigma} E_{z_1^N}(z_1^N \cdot E_{\chi^2}[\chi^2(N-1) \mid z_1^N]) - \frac{\sigma'}{\sigma} E[z_1^N] N = 0, \tag{A18}$$

where we used iterated expectation (note that the Chi-square random variable loses one degree of freedom upon conditioned on z_1^N) and $E[z_1^N]=0$. We then can solve (A14) and (A15) for the Lagrangian multipliers, and obtain

$$\mu_i^N = (v_i'/2)(\mathrm{var}[z_i^N])^{-1}, \ i=1,2. \tag{A19}$$

Since

$$\mu_1^N \frac{f_1^N}{f^N} + \mu_2^N \frac{f_2^N}{f^N} = \mu_1^N z_1^N + \mu_2^N z_2^N,$$

we obtain β_1^N and β_2^N by substituting (A17) into (A19) and using (A9) and (A10).

To calculate the efficiency loss under the information system η^N, note that

$$L(N) = \int [w^N(y^N) - \lambda u(w^N(y^N))] f^N(y^N) dy^N$$
$$= \int w^N(y^N) f^N(y^N) dy^N - \lambda \int u(w^N(y^N)) f^N(y^N) dy^N$$
$$= \int [\lambda + \mu_1^N z_1^N + \mu_2^N z_2^N]^2 g^N(z^N) dz^N - 2\lambda \int [\lambda + \mu_1^N z_1^N + \mu_2^N z_2^N] g^N(z^N) dz^N$$

where $g^N(z^N)$ is the joint density function of $z^N \equiv (z_1^N, z_2^N)$, which is transformed from $f^N(y^N)$. Since $E[z_i^N]=0$, $i=1,2$, and $E[z_1^N \cdot z_2^N]=0$, it follows that

$$L(N) = -\lambda^2 + (\mu_1^N)^2 (\mathrm{var}[z_1^N])^{-1} + (\mu_2^N)^2 (\mathrm{var}[z_2^N])^{-1}. \tag{A20}$$

Substituting (A17) and (A19) into (A20) establishes the result.

Q.E.D.

Proof of Proposition 4: Observe that the principal's problem is to solve

$$\min_N L(N) + r \cdot N,$$

where $L(N)$ is given by proposition 3. Since $L(N)$ is convex in N, the unique solution, N^*, is characterized by the first-order condition:

$$\frac{(\upsilon_2'/2)^2 \sigma^2}{2(\sigma')^2} \cdot \frac{1}{N^2} = r.$$

Thus, we obtain $N^* = (\upsilon_2'/2)\sigma / \sqrt{2r}|\sigma'|$. The comparative static results follow from differentiating N^* with respect to the parameters of interest.

Q.E.D.

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