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이 논문을 경제학석사 학위논문으로 제출함.

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Abstract

Majority Voting Rules with Quorums and Status-quo Bias

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In this article, we investigate majority voting rules and majority voting rules with quorums which take no social abstention decision and characterize these voting rules in the framework of May(Econometrica 20:680-684,1952). Standard voting schemes allow society, confronting two alternatives, to choose abstention (namely social indifference between the two). However, in actual politics, a resolute decision between the two has to be made. In this regard, we define alternative majority voting rules which takes no abstention decision. -we call them "pseudo majority voting rules". Also, in actual politics, to obviate the case where an alternative is elected with very poor support, quorums are sometimes required of standard voting schemes. Regarding the problem of social abstention and quorums, we define "majority voting rules with quorums and status-quo bias" which which are similar to majority voting rules with quorums by Houy(Theory and decision 67(3):295-301, 2009) but which are biased in favor of the status-quo.

Keywords: Majority voting, Social choice theory, Quorum, Status-quo bias, Abstention

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Contents

1 Introduction

Standard voting schemes(such as majority voting by May(1952)) allow society, confronting two alternatives, to choose abstention (namely social indifference between the two). However, in actual politics especially in referenda where society has to choose between the status-quo and an amendment, no abstention is allowed; a resolute decision between the two has to be made. While individuals may abstain, society should not. There have been some attempts of providing realistic meaning of social abstention. Some articles interpret social abstention as the choice depending on anything else but the vote(agenda for instance).[1] Other articles interpret social abstention as postponement of the decision. All these interpretations make it clear that an alternative in the end has to be chosen; social abstention cannot be understood as being similar to individual abstention. In this regard, the way we define, for example, majority voting rule might not fully explain what is understood as the majority rule in actual politics. We define alternative voting rules, which are similar to majority voting rule but which takes no abstention decision. - we call them "pseudo majority voting rules". One of our main results is the characterization of these rules in the framework of May(1952).

To obviate the case where an alternative is elected with very poor support, quorums are sometimes required of standard voting schemes. Houy(2007,2009) provides two different ways to characterize majority voting rules with quorums when there are only two alternatives. Houy(2007) introduced a family of voting rules based on differences of votes with quorums and characterized such voting rules; an alternative is chosen if the number of abstainers is less than a quorum and the number of voters for the alternative is so large that the difference of

[1] Houy (2009)

votes obtained by two alternatives is (strictly) greater than some constant. The majority voting rules with quorums are special cases when the constant equals zero. Since this family of voting rules does not generally satisfy Monotonicity, he introduced Decisiveness cancellation. Houy(2009) also provides the characterization of majority voting rules with quorums in the framework of May(1952). Although these voting rules are widely implemented in the cases of referenda, relatively little work has been done on social choice theory. Moreover, considering the problem of social abstention discussed above, his characterization and axioms are not satisfied anymore. Instead, we define alternative majority voting rules with quorums which are similar to majority voting rules with quorums but which are biased in favor of the status-quo -we call them "majority voting rules with quorums and status-quo bias" and provide the characterization of these rules. When the required quorum is one, the majority voting rule with the quorum and status-quo bias is identical to the majority voting rule with status-quo bias which is a special case of pseudo majority voting rules and biased in favor of the status-quo. As a broad concept of voting rules including pseudo majority voting rules and majority voting rules with quorums and status-quo bias, we also define "pseudo majority voting rules with quorums" which are similar to majority voting rules with quorums and status-quo bias but which allow society to choose between two alternatives arbitrarily when the numbers of votes for two alternatives are the same and characterize these rules.

Since Neutrality which imposes equal treatments on alternatives is not satisfied anymore under voting rules taking no abstention decision, we introduce three weaker versions of neutrality. As a weaker version of Monotonicity, Weak monotonicity which considers the number of abstainers is also introduced.

Section 2 presents definitions of pseudo majority voting rules, majority voting rules with quorums, and pseudo majority voting rules with quorums. Section 3 states axioms used in the sequel and the characterizations of theorems.

2 Formal framework

Let $N = \{1, \dots n\}$ be the set of individuals in the society. Each individual chooses between amendment(a) and status-quo(s). If for any individual $i \in N$ $V_i = 1$(resp. $V_i = -1$), individual i votes for a(resp. s). If $V_i = 0$, individual i abstains. Each individual can abstain while the society has to choose between a and s. The vector of all individuals' decisions is a voting configuration, $V = (V_1, \dots V_n) \in \{-1, 0, 1\}^n = \mathcal{V}$. The voting configuration $-V$ is defined by $-V = (-V_1, \dots - V_n)$. Let σ be a permutation of N, A voting configuration V_σ is defined by $V_\sigma = (V_{\sigma(1),} \dots V_{\sigma(n)})$. For $V \in \mathcal{V}$, we define $N^+(V) = \{i \in N | V_i = 1\}$, $N^-(V) = \{i \in N | V_i = -1\}$, and $N^0(V) = \{i \in N | V_i = 0\}$. The cardinality of each set $N^+(V)$, $N^-(V)$, and $N^0(V)$ are $n^+(V)$, $n^-(V)$, and $n^0(V)$ respectively.

A voting rule C is a mapping from $\mathcal{V}$ into $\{-1, 1\}$ so that the society can not abstain. $C(V)$ is the decision of the society when V is the voting configuration.

In this article, we concentrate on voting rules where the society as a whole is not allowed to abstain.

Definition. (Pseudo majority voting rule, M_p) The voting rule is a pseudo majority voting rule if

$$M_p(V) = \begin{cases} 1 & \text{if } n^+(V) > n^-(V), \\ -1 & \text{if } n^+(V) < n^-(V), \\ 1 \ \ or \ -1 & \text{if } n^+(V) = n^-(V). \end{cases}$$

M_p states that a is socially chosen if the number of votes in favor of a(resp. s) is strictly greater than the number of votes in favor of s(resp. a). If the number of votes for a equals the number of votes for s, either a or s is chosen by social conventional rules, agenda, etc. M_p is analogous to May's(1952) majority voting rule except that May's majority voting rule allows the society to abstain when a and s obtain the same number of votes. We note that M_p can be changed depending on the social decision when two alternatives obtain the same numbers of votes. We can generate a family of pseudo majority voting rules by imposing a different social decision on each situation where two alternatives obtain the same number of votes but the number of abstention is varied. The following voting rule called "Majority voting rules with status-quo bias, M_{sb}," is a special case of M_p in that the society chooses s when the numbers of votes for a and s are the same.

Definition. (Majority voting rule with status-quo bias, M_{sb}**)** The voting rule M_{sb} is defined by $\forall V \in \mathcal{V}$,

$$M_{sb}(V) = \begin{cases} 1 & \text{if } n^+(V) > n^-(V), \\ -1 & \text{otherwise.} \end{cases}$$

M_{sb} states that a is socially chosen if and only if the number of votes in favor of a is strictly greater than the number of votes in favor of s. If the number of votes for s is strictly greater than or equals the number of votes for

a, s is socially chosen.

Definition. (Majority voting rules with quorums and status-quo bias, M_q) Let $q \in \{0, \dots, n\}$. The voting rule M_q is defined by $\forall V \in \mathcal{V}$

$$M_q(V) = \begin{cases} 1 & \text{if } n^+(V) + n^-(V) \geq q \text{ and } n^+(V) > n^-(V), \\ -1 & \text{otherwise.} \end{cases}$$

If $q \in \{1, \dots, n\}$ is the quorum, the majority voting rule with quorum q and status-quo bias, M_q states that a is socially chosen if and only if (1) the number of votes in favor of a is strictly greater than the number of votes in favor of s and (2) the number of individuals abstaining is less than q. If either the quorum is not achieved or the number of votes for a is less than or equals the number of votes for s, s is socially chosen. The voting rule $M_{sb}$is also considered a family of M_q in that M_q is identical to M_{sb} when the quorum is 1.

As comprehensive voting rules including all voting rules state above, we can define pseudo majority voting rules with quorums.

Definition. (Pseudo majority voting rules with quorums, M_{pq}) Let $q \in \{0, \dots, n\}$. The voting rule M_{pq} is defined by $\forall V \in \mathcal{V}$

$$M_{pq}(V) = \begin{cases} 1 & \text{if } n^+(V) + n^-(V) \geq q \text{ and } n^+(V) > n^-(V) \\ 1 \text{ } or \text{ } -1 & \text{if } n^+(V) + n^-(V) \geq q \text{ and } n^+(V) = n^-(V) \\ -1 & \text{otherwise.} \end{cases}$$

If $q \in \{0, \dots, n\}$ is the quorum, the pseudo majority voting rules with quorum q states that a is socially chosen if (1) the number of votes in favor of a is (strictly) greater than that of s and (2) the number of individuals abstaining is less than q. If either the quorum is not achieved or the number of votes for a is less than that of s, s is socially chosen. Contrary to M_{sb}, when the numbers of votes for a and s are the same, a may be chosen as long as the number of individuals abstaining is less than q. When the quorum is 0, M_{pq} are considered as some special cases of M_p.

3 Axioms and result

We consider the following axioms to characterize the voting rules

Anonymity For any permutation σ of N, $\forall V \in \mathcal{V}$, $C(V_\sigma) = C(V)$.

Anonymity is very usual since May(1952). C satisfies Anonymity if relabeling the names of individuals does not change the social choice.

Weak monotonicity. $\forall V, V' \in \mathcal{V}$ such that $N^0(V') \subseteq N^0(V)$, $N^+(V') \supseteq N^+(V)$ and $N^-(V') \subseteq N^-(V)$, $C(V') \geq C(V)$.

Weak monotonicity states that if a (sub)group consisting of voters for s and abstainers changes its votes into a, the decision of society does not respond to the change of the group's votes in a "negative" way. On the other hand, it also states that if a (sub)group of voters for a changes its votes into either s or abstention, the decision of society does not respond in a "positive" way. Weak monotonicity is a weaker axiom than usual Monotonicity,[2] in the sense that it depends on the change in the set of abstainers whereas Monotonicity is satisfied regardless of the change in the set of abstainers.

[2]The Monotonicity axiom can be stated as follows.
$\forall V, V' \in \mathcal{V}$, $N^+(V') \supseteq N^+(V)$ and $N^-(V') \subseteq N^-(V)$, $C(V') \geq C(V)$.

Nondegeneracy. $\exists V$ and $V' \in \mathcal{V}$ such that $C(V) = 1$, $C(V') = -1$.

Nondegeneracy excludes trivial voting rules that choose only one alternatives regardless of voting configurations.

Weak neutrality1. $\forall V \in \mathcal{V}$ such that $n^+(V) \neq n^-(V)$, $C(-V) = -C(V)$.

Weak neutrality2. $\forall V \in \mathcal{V}$, $C(V) = 1$ implies $C(-V) = -C(V)$ and $\forall V' \in \mathcal{V}$ such that $n^+(V') \neq n^-(V')$ and $N^0(V') \subseteq N^0(V)$, $C(-V') = -C(V')$.

Weak neutrality3. $\forall V \in \mathcal{V}$, $C(V) = 1$ implies $\forall V' \in \mathcal{V}$ such that $n^+(V') \neq n^-(V')$ and $N^0(V') \subseteq N^0(V)$, $C(-V') = -C(V')$.

Weak neutrality1 states that if when the numbers of votes obtained by a and s are different and every member of society reverses their voting between two alternatives, so does the decision of society. Weak neutrality1 is weaker than Neutrality since Weak neutrality1 considers only voting configurations which have the different numbers of vote for a and s while Neutrality is satisfied for all voting configurations.[3] Weak neutrality2 states that if society chooses a, then Neutrality is satisfied for this voting configuration and also when a (sub)group of abstainers changes its votes and the numbers of votes for a and s are different, the voting rule treats each alternative equally. Weak neutrality3 is analog to Weak neutrality2 but does not impose Neutrality on a voting configuration for which social decision is a, when the numbers of votes for a and s are the same. When the numbers of votes for a and s are different, Weak neutrality2 and Weak neutrality3 also state that once a voting configuration satisfies Neutrality, Neutrality is satisfied as the group of

[3] The Neutrality axiom can be state as follows.
$\forall V \in \mathcal{V}$, $C(-V) = -C(V)$.

abstainers decreases. Weak neutrality1 implies Weak neutrality3 and Weak neutrality2 implies Weak neutrality3, whereas the opposites are not true.

Strong pareto. $\forall V \in \mathcal{V}$, $C(V) = 1$ whenever $N^+(V) \neq \emptyset$ and $N^-(V) = \emptyset$ and $C(V) = -1$ whenever $N^+(V) = \emptyset$ and $N^-(V) \neq \emptyset$.

Strong pareto states that if no individual votes for s(resp. a) and there is at least one who votes for a(resp. s), then society respects the decisions of individuals who cast valid votes, a(resp. s).

Theorem 1. *The voting rule C satisfies Anonymity, Weak monotonicity, and Weak neutrality1 if and only if C is a pseudo majority voting rule.*

Proof. Checking that any pseudo majority voting rule C satisfies Anonymity and Weak neutrality1 is straightforward.

We now show that C satisfies Weak monotonicity as well.

Suppose $C(V') < C(V)$,$\exists V', V \in \mathcal{V}$, such that $N^+(V') \supseteq N^+(V)$ and $N^-(V') \subseteq N^-(V)$.

If $C(V') = -1$ when $n^+(V') < n^-(V')$, $C(V) = -1$ since $n^+(V') \geq n^+(V)$ and $n^-(V') \leq n^-(V)$. If $C(V) = 1$ when $n^+(V) > n^-(V)$, $C(V') = 1$ since $n^+(V') \geq n^+(V)$ and $n^-(V') \leq n^-(V)$. Hence, the only possible case is $C(V') = -1$ and $C(V) = 1$ when $n^+(V') = n^-(V')$ and $n^+(V) = n^-(V)$, which implies $N^+(V) = N^+(V')$ and $N^-(V) = N^-(V')$ since $N^+(V) \subseteq N^+(V')$ and $N^-(V') \subseteq N^-(V)$. For $V' = V$, $C(V') = C(V)$, since C is a mapping from $\mathcal{V}$ into $\{-1, 1\}$. It is not possible to make a decision rule which generates different social decisions for the identical voting configuration. Thus, it is contradiction.

For the proof of the sufficiency part of Theorem 1, assume that the voting rule$C$ satisfies Anonymity, Weak monotonicity, and Weak neutrality1.

Since C satisfies Anonymity, C depends only on $n^+(V)$, $n^0(V)$, and $n^-(V)$. We can define $C(V) = c(n^-(V), n^0(V), n^+(V))$.

With Anonymity, Weak monotonicity can be represented as follows; The anonymous voting rule c satisfies Weak Monotonicity if and only if

$\forall V, V' \in \mathcal{V}$, such that $n^0(V') \leq n^0(V)$, $n^+(V') \geq n^+(V)$ and $n^-(V') \leq n^-(V)$, $c(n^-(V'), n^0(V'), n^+(V')) \geq c(n^-(V), n^0(V), n^+(V))$.

Suppose $n^+(V) < n^-(V)$ and $c(n^-(V), n^0(V), n^+(V)) = 1$. Note that $n^+(V) = n^-(-V)$, and $n^-(V) = n^+(-V)$.

By Weak neutrality1,

$c(n^-(-V), n^0(-V), n^+(-V)) = -1 = -c(n^+(V), n^0(V), n^-(V))$ and

$$n^+(V) + n^-(V) = n^+(-V) + n^-(-V)$$
$$n^-(-V) = n^+(V) < n^-(V) \text{ and } n^+(-V) = n^-(V) > n^+(V).$$

Therefore, by Weak monotonicity,

$1 = c(n^-(V), n^0(V), n^+(V))) \leq c(n^-(-V), n^0(-V), n^+(-V)) = -1$ which is contradiction.

Hence, $c(n^-(V), n^0(V), n^+(V)) = -1$ when $n^+(V) < n^-(V)$. Also, when $n^+(V) > n^-(V)$, $c(n^-(V), n^0(V), n^+(V)) = 1$ by Weak neutrality1. □

The following proposition shows that the characterization of M_p is minimal in the sense that all the axioms used for Theorem 1 are independent.

Proposition 2. *Anonymity, Weak monotonicity, and Weak neutrality1 are independent.*

Proof. Weak monotonicity, Weak neutrality1 $\nrightarrow$Anonymity.

Let us have $i \in N$. Let us define the voting rule C_{-A} by $\forall V \in \mathcal{V}$,

$$C_{-A}(V) = \begin{cases} V_i & \text{if } V_i \neq 0 \\ 1 & \text{if } V_i = 0 \text{ and } n^+(V) > n^-(V) \\ -1 & \text{if } V_i = 0 \text{ and } n^+(V) \leq n^-(V). \end{cases}$$

Obviously, C_{-A} satisfies Weak neutrality1 while C_{-A} violates Anonymity.

We now show that C_{-A}satisfies Weak monotonicity. Suppose that $\forall V, V' \in \mathcal{V}$, such that $N^0(V') \subseteq N^0(V)$, $N^+(V') \supseteq N^+(V)$,and $N^-(V') \subseteq N^-(V)$,

$1 = C_{-A}(V) > C_{-A}(V') = -1$. Let V_i' and $V_i$are individual i's voting when the voting configuration are V' and V respectively. From the definition of C_{-A}, V has either $V_i = 1$or $V_i = 0$ when $n^+(V) > n^-(V)$ while V' has either $V_i' = -1$ or $V_i' = 0$ when $n^+(V) \leq n^-(V)$. If $V_i' = -1$, since $V_i \in N^-(V') \subseteq N^-(V)$, $V_i = -1$ which contradicts to the assumption that $C_{-A}(V) = 1$. If $V_i' = 0$ and $n^+(V') \leq n^-(V')$, since $V_i' \in N^0(V') \subseteq N^0(V)$, $V_i = 0$ and $n^+(V) > n^-(V)$ which contradicts to the assumption that $N^+(V') \supseteq N^+(V)$, and $N^-(V') \subseteq N^-(V)$. Hence, $C_{-A}(V) < C_{-A}(V')$.

Anonymity, Weak neutrality1$\nrightarrow$ Weak monotonicity.

Let us defined the voting rule C_{-WM} by $\forall V \in \mathcal{V}$,

$$C_{-WM}(V) = \begin{cases} 1 & \text{if } n^+(V) < n^-(V), \\ -1 & \text{if } n^+(V) > n^-(V), \\ 1 \ or \ -1 & \text{if } n^+(V) = n^-(V). \end{cases}$$

which is a family of anti-pseudo majority voting rules. Obviously, C_{-WM} satisfies Anonymity and Weak neutrality 1 while C_{-WM} violates Weak monotonicity.

Anonymity, Weak monotonicity $\nrightarrow$ Weak neutrality1.

Let us define the voting rule C_{-WN1} by $\forall V \in \mathcal{V}$,

$$C_{-WN1}(V) = \begin{cases} 1 & \text{if } qn^{+}(V) > n^{-}(V) \\ -1 & \text{otherwise.} \end{cases}$$

where $0 < q < 1$. It is straightforward to check that C_{-WN1} satisfies Anonymity, Weak monotonicity, but C_{-WN1}does not satisfies Weak neutrality1. □

We can provide the characterization of the majority voting rules with quorums and status-quo bias by replacing Weak neutrality1 with Weak neutrality2 and adding Nondegeneracy. The majority voting rules with quorums and status-quo bias is analog to the Houy's(2009) majority voting rules with quorums except that the society is in favor of status-quo when the numbers of votes for two alternatives are the same.

Theorem 3. *The voting rule C satisfies Anonymity, Weak monotonicity, Nondegeneracy, and Weak neutrality2 if and only if C is the majority voting rule with quorum and status-quo bias.*

To prove Theorem 3, we need the following lemma in the sequel.

Lemma 4. *If any voting rule C satisfies Weak monotonicity, Nondegeneracy, and Weak neutrality2, then C satisfies Weak Pareto.*[4]

Proof. Suppose $C(V) = -1$ when $N^{+}(V) = N$. By Weak monotonicity $C(V) = -1\ \forall V \in \mathcal{V}$ which contradicts to Nondegeneracy. Hence $C(V) = 1$.

By Weak neutrality2, $C(-V) = -C(V) = -1$when $N^{-}(-V) = N$. Thus, $C(V) = -1$ when $N^{-}(V) = N$. □

[4]In this framework, C satisfies Weak Pareto if $C(V) = 1$ when $N^{+}(V) = N$ and $C(V) = -1$ when $N^{-}(V) = N$.

Proof of Theorem 3

Proof. Checking that any C satisfies Anonymity, Nondegeneracy, and Weak neutrality2 is straightforward.

We now show that C satisfies Weak monotonicity as well.

Suppose that $\exists V', V \in \mathcal{V}$, such that $N^+(V') \supseteq N^+(V), N^-(V') \subseteq N^-(V)$, and $N(V') \subseteq N^0(V)$ $C(V') < C(V)$.

$C(V) = 1$ only when $n^+(V) + n^-(V) \geq q$ and $n^+(V) > n^-(V)$. Since $N^+(V') \supseteq N^+(V)$ and $N^-(V') \subseteq N^-(V)$, $n^+(V') \geq n^+(V) > n^-(V) \geq n^-(V')$. Hence, $C(V') = 1$ which contradicts the assumption that $C(V') < C(V)$. Thus, $C(V') > C(V)$ $\forall V', V \in \mathcal{V}$, such that $N^+(V') \supseteq N^+(V)$, $N^-(V') \subseteq N^-(V)$, and $N^0(V') \subseteq N^0(V)$.

For the proof of the sufficiency part of Theorem 3, assume that C satisfies Anonymity, Weak monotonicity, Nondegeneracy, and Weak neutrality2.

Since C satisfies Anonymity, $C(V)$ depends only on $n^+(V)$, $n^0(V)$, and $n^-(V)$. Let $C(V) = c(n^-(V), n^0(V), n^+(V))$ $\forall V \in \mathcal{V}$.

As we can see in Theorem 1, with Anonymity, Weak monotonicity can be represented as follows;

The anonymous voting rule c satisfies Weak Monotonicity if and only if

$\forall V, V' \in \mathcal{V}$, such that $n^0(V') \leq n^0(V)$, $n^+(V') \geq n^+(V)$ and $n^-(V') \leq n^-(V)$, $c(n^-(V'), n^0(V'), n^+(V')) \geq c(n^-(V), n^0(V), n^+(V))$.

With Anonymity, Weak neutrality2 can be represented as follows;

The anonymous voting rule C satisfies Weak neutrality2 if and only if $\forall V \in \mathcal{V}$, $c(n^-(V), n^0(V), n^+(V)) = 1$ implies $c(n^-(-V), n^0(-V), n^+(-V)) = -c(n^-(V)$ $, n^0(V), n^+(V))$ and $\forall V' \in \mathcal{V}$ such that $n^+(V') \neq n^-(V')$ and $n^0(V') \leq n^0(V)$, $c(n^-(-V'), n^0(-V'), n^+(-V')) = -c(n^-(V'), n^0(V'), n^+(V'))$.

Suppose that $c(0, n^0(V), 0) = 1$. By Weak neutrality2, $c(0, n^0(-V), 0) = -c(0, n^0(V), 0) = -1$ which is contradiction because $n^0(V) = n^0(-V)$ and $c(0, n^0(-V), 0) = c(0, n^0(V), 0) = 1$ by Anonymity. Also, as the same way, we can prove that $\forall V \in \mathcal{V}$ such that $n^-(V) = n^+(V)$, $c(n^-(V), n^0(V), n^+(V)) = -1$. That is, when the numbers of vote for two alternatives are the same, the decision of society is -1(maintaining the status quo).

By the assumption and lemma 4, we know that C satisfies Weak pareto.

Since $c(0, n, 0) = -1$ and $c(0, 0, n) = 1$, there exists $p \in \{1, \ldots, n\}$ such that

$$\begin{aligned} c(0,0,n) \geq c(0,1,n-1) \geq \ldots \geq c(0,p-1,n-p+1) = 1 \\ > c(0,p,n-p) \geq c(0,p+1,n-p-1) \geq \ldots \geq c(0,n,0) = -1 \end{aligned} \tag{3.1}$$

Because of the second row of (3.1), $C(V) = -1$ $\forall V \in \mathcal{V}$ such that $p \leq n^0(V)$ by Weak monotonicity.

Because of the first row of (3.1), $c(n, 0, 0) = c(n-1, 1, 0) = \ldots c(n-p+1, p-1, 0) = -1$ by Weak neutrality2.

Finally, we prove by contradiction that $C(V) = 1$ $\forall V \in \mathcal{V}$ such that $p - 1 \geq n^0(V)$ and $n^+(V) > n^-(V)$.

Suppose there exists $C(V^c) = -1$ such that $p - 1 \geq n^0(V^c)$ and $n^+(V^c) > n^-(V^c)$. Since $c(0, p-1, n-p+1) = 1$ and $p - 1 \geq n^0(V^c)$, $1 = -C(V^c) = C(-V^c)$ by Weak neutrality 2. Note that $n^+(V^c) = n^-(-V^c)$, and $n^-(V^c) = n^+(-V^c)$. By Weak monotonicity,

$$n^+(V^c) + n^-(V^c) = n^+(-V^c) + n^-(-V^c)$$

$$n^+(V^c) > n^-(V^c) = n^+(-V^c) \text{and } n^-(V^c) < n^+(V^c) = n^-(-V^c)$$

$$\therefore C(V^c) \geq C(-V^c)$$

which contradicts to the assumption that $C(V^c) = -1$and $C(-V^c) = 1$. Hence, $C(V) = 1\ \forall V \in \mathcal{V}$ such that $p-1 \geq n^0(V)$ and $n^+(V) > n^-(V)$. Since $C(V') = 1\ \forall V' \in \mathcal{V}$ such that $p-1 \geq n^0(V')$ and $n^+(V') > n^-(V')$,$\forall V \in \mathcal{V}$ such that $p-1 \geq n^0(V)$ and $n^+(V) < n^-(V)$, $C(V) = -1$ by Weak neutrality2. □

M_{sb} is a family of M_q in the sense that M_{sb} is the majority voting rule with the quorum 1. Lemma 4 states that any voting rules satisfying Weak monotonicity, Nondegeneracy, and Weak neutrality2, also satisfy Weak pareto. We can characterize M_{sb} by imposing stronger axiom, Strong pareto, instead of Weak pareto on M_q and eliminating a redundant axiom, Nondegeneracy which Strong pareto implies.

Corollary 5. *The voting rule C satisfies Anonymity, Weak monotonicity, Weak neutrality2 and Strong pareto if and only if C is the majority voting rule with status-quo bias.*

Proof. Checking that any C satisfies Anonymity, Weak neutrality2 and Strong pareto is straightforward.

We now show that C satisfies Weak monotonicity as well.

Suppose that $\exists V', V \in \mathcal{V}$, such that $N^+(V') \supseteq N^+(V), N^-(V') \subseteq N^-(V)$, and $N(V') \subseteq N^0(V)$, $C(V') < C(V)$.

$C(V) = 1$ only when $n^+(V) > n^-(V)$. Since $N^+(V') \supseteq N^+(V)$ and

$N^-(V') \subseteq N^-(V)$, $n^+(V') \geq n^+(V) > n^-(V) \geq n^-(V')$. Hence, $C(V') = 1$ which contradicts the assumption that $C(V') < C(V)$.

For the proof of the sufficiency part of Corollary 5, assume that C satisfies Anonymity, Weak monotonicity, Weak neutrality2 and Strong pareto.

As in Theorem 3, by Weak neutrality2 we know that $\forall V \in \mathcal{V}$ such that $n^+(V) = n^-(V)$, $C(V) = -1$. Also, $\forall V \in \mathcal{V}$, $C(V) = 1$ whenever $n^+(V) > 0$ and $n^-(V) = 0$ by Strong pareto. We prove by contradiction that $C(V) = 1$ $\forall V \in \mathcal{V}$ such that $n - 1 \geq n^0(V)$ and $n^+(V) > n^-(V)$.

Suppose there exists $C(V^c) = -1$ such that $n-1 \geq n^0(V^c)$ and $n^+(V^c) > n^-(V^c)$. Since $c(0, n-1, 1) = 1$ and $n-1 \geq n^0(V^c)$, $1 = -C(V^c) = C(-V^c)$ by Weak neutrality 2. Note that $n^+(V^c) = n^-(-V^c)$, and $n^-(V^c) = n^+(-V^c)$. By Weak monotonicity,

$$n^+(V^c) + n^-(V^c) = n^+(-V^c) + n^-(-V^c)$$
$$n^+(V^c) > n^+(-V^c) \text{and } n^-(V^c) < n^-(-V^c)$$
$$\therefore C(V^c) \geq C(-V^c)$$

which contradicts to the assumption that $C(V^c) = -1$and $C(-V^c) = 1$. Hence, $C(V) = 1$ $\forall V \in \mathcal{V}$ such that $n - 1 \geq n^0(V)$ and $n^+(V) > n^-(V)$. Since $C(V) = 1$ $\forall V \in \mathcal{V}$ such that $n - 1 \geq n^0(V)$ and $n^+(V) > n^-(V')$,$\forall V \in \mathcal{V}$ such that $n - 1 \geq n^0(V)$ and $n^+(V) < n^-(V)$, $C(V) = -1$ by Weak neutrality2. □

Since the majority voting rules with quorums and status-quo bias is a family of pseudo majority voting rules with quorums, we can characterize pseudo majority voting rules with quorums by imposing a weaker axiom, Weak neutrality3, instead of Weak neutrality2. Contrary to M_q, M_p do not require social decision to choose s(maintaining the status quo) when all individuals abstain. If M_{pq} allow society to choose a when all individuals abstain, these voting rules can be considered as some cases of the pseudo majority voting rules with quorum 0.

Theorem 6. *The voting rule C satisfies Anonymity, Weak monotonicity, Nondegeneracy, and Weak neutrality3 if and only if C is a pseudo majority voting rule with quorum or a pseudo majority voting rule with $C(V) = 1$ when $N^0(V) = N$.*

Proof. Checking that any C satisfies Anonymity, Nondegeneracy, and Weak neutrality3 is straightforward. Also as in Theorem 3, any C satisfies Weak monotonicity.

For the proof of the sufficiency part of Theorem 6, assume that C satisfies Anonymity, Weak monotonicity, Nondegeneracy, and Weak neutrality3.

Since C satisfies Anonymity, $C(V)$ depends only on $n^+(V)$, $n^0(V)$, and $n^-(V)$. Let $C(V) = c(n^-(V), n^0(V), n^+(V))\ \forall V \in \mathcal{V}$.

With Anonymity, Weak neutrality3 can be represented as follows;
The anonymous voting rule C satisfies Weak neutrality3 if and only if $\forall V \in \mathcal{V}$, $c(n^-(V), n^0(V), n^+(V)) = 1$ implies $\forall V' \in \mathcal{V}$ such that $n^+(V') \neq n^-(V')$ and $n^0(V') \leq n^0(V)$, $c(n^-(-V'), n^0(-V'), n^+(-V')) = -c(n^-(V'), n^0(V'), n^+(V'))$.

Suppose $C(V) = -1$ when $N^+(V) = N$. By Weak monotonicity $C(V) = -1\ \forall V \in \mathcal{V}$ which contradicts to Nondegeneracy. Hence $C(V) = 1$.

By Weak neutrality2, $C(-V) = -C(V) = -1$when $N^-(-V) = N$. Thus, $C(V) = -1$ when $N^-(V) = N$.

If $c(0, n, 0) = -1$, since $c(0, 0, n) = 1$, there exists $p \in \{1, \ldots\ldots, n\}$ such that

$$
\begin{aligned}
&c(0,0,n) \geq c(0,1,n-1) \geq \ldots \geq c(0,p-1,n-p+1) = 1 \\
&> c(0,p,n-p) \geq c(0,p+1,n-p-1) \geq \ldots \geq c(0,n,0) = -1
\end{aligned}
\tag{3.2}
$$

Since the second row of (3.2), $C(V) = -1 \; \forall V \in \mathcal{V}$ such that $p \leq n^0(V)$ by Weak monotonicity.

Because of the first row of (3.2), $c(n,0,0) = c(n-1,1,0) = \ldots c(n-p+1,p-1,0) = -1$ by Weak neutrality3.

We prove by contradiction that $C(V) = 1 \; \forall V \in \mathcal{V}$ such that $p-1 \geq n^0(V)$ and $n^+(V) > n^-(V)$.

Suppose there exists $C(V^c) = -1$ such that $p-1 \geq n^0(V^c)$ and $n^+(V^c) > n^-(V^c)$. Since $c(0,p-1,n-p+1) = 1$ and $p-1 \geq n^0(V^c)$, $1 = -C(V^c) = C(-V^c)$ by Weak neutrality 2. Note that $n^+(V^c) = n^-(-V^c)$, and $n^-(V^c) = n^+(-V^c)$. By Weak monotonicity,

$$
\begin{gathered}
n^+(V^c) + n^-(V^c) = n^+(-V^c) + n^-(-V^c) \\
n^+(V^c) > n^-(V^c) = n^+(-V^c) \text{and } n^-(V^c) < n^+(V^c) = n^-(-V^c) \\
\therefore C(V^c) \geq C(-V^c)
\end{gathered}
$$

which contradicts to the assumption that $C(V^c) = -1$and $C(-V^c) = 1$. Hence, $C(V) = 1 \; \forall V \in \mathcal{V}$ such that $p-1 \geq n^0(V)$ and $n^+(V) > n^-(V)$. Since $C(V') = 1 \; \forall V' \in \mathcal{V}$ such that $p-1 \geq n^0(V')$ and $n^+(V') > n^-(V')$,$\forall V \in \mathcal{V}$ such that $p-1 \geq n^0(V)$ and $n^+(V) < n^-(V)$, $C(V) = -1$ by Weak neutrality3.

If $c(0,n,0) = 1$, since $c(0,0,n) = 1$,$\forall V \in \mathcal{V}$ such that $n^-(V) = 0$, $C(V) = 1$ by Weak monotonicity.

We prove by contradiction that $C(V) = 1$ $\forall V \in \mathcal{V}$ such that $n^+(V) > n^-(V)$.

Suppose there exists $C(V) = -1$, $\forall V \in \mathcal{V}$ such that $n^+(V) > n^-(V)$. Since $c(0, n, 0) = 1$, $C(-V) = 1$ by Weak neutrality3. Note that $n^+(V) = n^-(-V)$, and $n^-(V) = n^+(-V)$. By Weak monotonicity,

$$n^+(V) + n^-(V) = n^+(-V) + n^-(-V)$$
$$n^+(V) > n^-(V) = n^+(-V) \text{and } n^-(V) < n^+(V) = n^-(-V)$$
$$\therefore C(V) \geq C(-V)$$

which contradicts to the assumption that $C(V) = -1$and $C(-V) = 1$. Hence, $C(V) = 1$ when $n^+(V) > n^-(V)$ and by Weak neutrality3, $C(V) = -1$,when $n^+(V) < n^-(V)$ □

The following proposition presents that the characterization of $M_q$is minimal in the sense that all the axioms used for Theorem 6 are independent. Also, since Strong pareto implies Nondegeneracy and Weak neutrality2 implies Weak neutrality3, all the group of axioms used for Theorem3, Cororally5, and Theorem6 are independent.

Proposition 7. *Anonymity, Weak monotonicity, Nondegeneracy, and Weak neutrality3 are independent.*

Proof. Weak monotonicity, Nondegeneracy, Weak neutrality2 $\nrightarrow$Anonymity.

Let us have $i \in N$. Let us define the voting rule C_{-A} by $\forall V \in \mathcal{V}$,

$$C_{-A}(V) = \begin{cases} V_i & \text{if } V_i \neq 0 \\ 1 & \text{if } V_i = 0 \text{ and } n^+(V) > n^-(V) \\ -1 & \text{if } V_i = 0 \text{ and } n^+(V) \leq n^-(V). \end{cases}$$

Obviously, C_{-A} satisfies Nondegeneracy, while C_{-A} violates Anonymity. C_{-A} also satisfies Weak monotonicity as we showed in proposition 2.

We now show that C_{-A}satisfies Weak Neutrality2. When $V_i = 1$, $\forall V' \in \mathcal{V}$ such that $N^0(V') \subseteq N^0(V)$, -1=$C(-V') = -V_i = -C(V')$. When $V_i = 0$ and $n^+(V) > n^-(V)$, $\forall V' \in \mathcal{V}$ such that $V_i' = 0$ and $N^0(V') \subseteq N^0(V)$, $C(-V') = -C(V')$ since $n^+(-V') < n^-(-V')$ when $n^+(V') > n^-(V')$ and $n^+(-V') > n(-V')$ when $n^+(V') < n(V')$. When $V_i = 0$ and $n^+(V) > n^-(V)$, $\forall V' \in \mathcal{V}$ such that $V_i' \neq 0$ and $N^0(V') \subseteq N^0(V)$, $C(-V') = -V_i' = -C(V')$.

Anonymity, Nondegeneracy, Weak neutrality2 $\nrightarrow$Weak monotonicity.

Let $q \in \{0, \ldots, n\}$. Let us define the voting rule C'_{-WM} by $\forall V \in \mathcal{V}$,

$$C'_{-WM}(V) = \begin{cases} 1 & \text{if } n^+(V) + n^-(V) \geq q \text{ and } n^+(V) < n^-(V), \\ -1 & \text{otherwise.} \end{cases}$$

It is straightforward to check that C'_{-WM} satisfies Anonymity, Nondegeneracy, and Weak neutrality 2, but C'_{-WM} does not satisfy Weak monotonicity.

Anonymity, Weak monotonicity, Weak neutrality2 $\nrightarrow$Nondegeneracy.

Let us define the voting rule C_{-ND} by $\forall V \in \mathcal{V}$, $C_{-ND}(V) = -1$.

C_{-ND} satisfies Anonymity, Weak monotonicity, Weak neutrality2 but does not satisfies Nondegeneracy.

Anonymity, Weak monotonicity, Nondegeneracy $\nrightarrow$Weak neutrality2 .

Let us define the voting rule C_{-WN2} by $\forall V \in \mathcal{V}$,

$$C_{-WN2}(V) = \begin{cases} 1 & \text{if } qn^{+}(V) > n^{-}(V) \\ -1 & \text{otherwise.} \end{cases}$$

where $0 < q < 1$. It is straightforward to check that C_{-WN2} satisfies Anonymity, Nondegenercy, Weak monotonicity, but C_{-WN2} does not satisfy Weak neutrality2 . □

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국문초록

정족수 요구 시 현상유지 편향적 다수결 투표

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두 대안이 존재할 시 일반적인 투표이론의 경우 사회가 기권하는 것을 용인하며 이 사회적 기권은 사회가 무차별하다는 의미로 해석된다. 그러나 현실 정치에서는 하나의 대안만이 사회적으로 선택되어야 한다는 점에서 사회적 기권은 존재할 수 없다. 또한 현실 정치에서 저조한 투표율로 인해 낮은 지지(支持)에도 하나의 대안이 선택되는 것을 방지하고자 국민투표와 같은 경우 정족수를 요구하는데 이러한 정족수가 요구되는 투표에 대한 규범적인 특성에 관한 연구는 Houy(2007,2009)의 논문을 제외하고 거의 논의가 되지 않은 실정이다.

본 논문은 두 위의 두 가지 관점에 기초하여 사회가 기권하지 못하는 경우를 고려한 의사 다수결 투표(pseudo majority voting rules)와 정족수가 요구되는 경우 현상유지 선호적 다수결 투표(Majority voting rules with quorums and status-quo bias)를 위주로 이를 포괄하는 다양한 투표를 규정하고 이에 대한 규범적인 특성화 작업을 하는데 있다.

주요어 : 다수결투표, 사회선택이론, 정족수, 현상유지편향, 기권
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