Information acquisition in common value auction

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February 4, 2014

Abstract

This paper deals with the information acquisition in a common value auction model. In the simple case of two bidders, we reach the conclusion that the conventionally known statement that the auctioneer's revenue is higher in the case of English auction than in the second price auction at equilibrium is not quite true, but the contrary is correct.

Keywords: common value auction; information acquisition; auctioneer's revenue.

*Thanks to Mother , Hye Rim Lee , Professor Kim , Jun Seok and other office collegues

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1 Introduction

Compte and Jehiel(2007) has drawn a conclusion that the auctioneer's revenue is higher in English auction(Open ascending auction) than in the second price auction when there exists an uninformed bidder. This was a discovery of a new condition that can strengthen the argument of Milgrom and Weber(1982). In addition, an analysis that the above statement still holds when there is negative externality resulted in the prevalence of the thought that dynamic-form auction rules are superior to static-form auction rules, and many dynamic auction rules are devised for practical purposes.

This paper insists that the above statement should be reconsidered. Milgrom argues when the information of the value of the auction item is known to the bidders, the expected revenue under English auction rule is superior to the expected revenue under the second price auction rule, which is equivalent to the denial of the revenue equivalence in private value model. Compte argues such bidders earn higher revenue than under the static rule through 'wait and see strategy' and the acquisition of information in the private value model when there exists a bidder who is unaware of the value of the auction item. Our analysis starts from the question about the condition of increasing the revenue in the two models, that is, considering the combined model of affiliation and information acquisition.[1] It is interesting to note that the second price auction revenue can be higher under the situation where the two conditions are combined. The intuition behind this is the following. In English auction, the bidders with no information have less incentives to acquire information because they can make inferences by observing the behaviors of other bidders.

The answer to the question whether the model of this paper possesses practical meaning is given in the following. Over the last 10 years, the problems regarding the

[1]But it is acknowledged that on of the shortcomings of this analysis is the usage of common value model which is an specific interdependent value model.

information acquisition of the values of auction items have been studied. It is a try to better reflect the reality with criticism and improvements on the previous studies which assumed exogenously given information, considering the fact that information are not naturally given but should be acquired with cost. This paper can be understood in the same context. Especially the significance of the information acquisition in the common value model, not in the private value model, can be found in the distinct relationship among the pieces of information. In the private value model, the channel through which the information acquisition influences other bidders is as simple as the change of bidding price of the bidder who acquired the information whereas in the common value model, it influences through a more complicated channel and the bidders are put in a game situation. The details will be presented in the main body of this paper.

1.1 Related literature

Matthews (1984) studied information acquisition and discussed about a symmetric equilibrium in a first price auction. The first price auction defined as each bidder shared a common, yet unknown value. He also analyzed if the value of the winning bid converged to the true value of the object once the pool of bidders became larger.

Hausch and Li (1993) developed a common-value model that very similar with the model that made by Lee. The objective had two potential valuations that both of them could be rejected by a signal with a cost. The accuracy of the signal was positively related to the amount of cost spent on it. The authors characterized a symmetric equilibrium and the analysis that could be later extended to the private-value case by Hausch and Li. (1993a)

Persico (2000) conducted research on the incentive to acquire information and the value of information through introducing the notion of risk-sensitivity. He found that the value of information played a more critical role in decision problems where bidders were more risk-sensitive. The comparison between incentives for information acquisition in the first-price and second-price auctions in the affiliated value setting was established by him.

Bergemann and Valimaki (2002) researched about a general interdependent value context and the impact of ex-post efficiency on the ex -ante incentives for information acquisition.

Rezende(2005) analyzed the moment that bidders could have more information during the bidding process. The heterogeneity in the prior information, the information acquisition cost, and the existence of equilibrium should be allowed in the process. He figured that the dynamic auctions generated more revenue than one-shot auctions for many bidders.

Compte and Jehiel (2007) competition appears strong ex ante, it may turn out to be weak along the dynamic format, thereby making the option to acquire information valuable.

2 The model

There are two bidders($i = 1, 2$) to whom a single object is sold, and the valuation of the object is the same for the both bidders.($v_1 = v_2 = V$) The value of the object is a random variable that takes on the value of v_l with the probability of $\frac{1}{2}$ and v_h with the probability of $\frac{1}{2}$ [2] and if the cost c is paid, the exact value of the object can be known.[3] These settings are common knowledge, the players are not aware whether the other player has acquired the information, and the condition $\frac{v_h - v_l}{4} > c$ is assumed. We'll analyze and compare bidder's strategy and equilibrium in two auction mechanism. (Second price auction, English auction) [4] And lastly the bidding strategy has researched previous studies, so the focus of this paper is strategy on information acquisition.

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2.1 Information Acquisition in Second Price Auction

Second price auction is the auction where each bidder submits a sealed bid and the bidder with the highest bid obtains the object at a price equal to the second highest bid. And each bidder decides prior to the auction whether or not to acquire information. I use a Bayes-Nash Equilibrium concept.

Lemma 1 *Neither the state where the both bidders acquire information nor do not acquire information is an equilibrium.*

[2]Even though it is a simple model, as the number of bidders increases and the distribution of the value of the auction object gets complicated, it is conjectured that the offsetting force of the two effects would produce a similar result.

[3]This can be interpreted as a model in which the two bidders who already possess information obtain more accurate information, and perfect information can be thought of as the exactly same information given by a certain information-providing agent preventing the acquisition of different information.

[4]In my model, Second price auction can be interpreted as an English auction with rapidly rising price, so that information cannot be acquired during auction.

	info acq	not acq
info acq	$-c$, $-c$	$\frac{1}{2}(v_h - \frac{v_h+v_l}{2}) - c$, 0
not acq	0 , $\frac{1}{2}(v_h - \frac{v_h+v_l}{2}) - c$	0 , 0

Table 1: payoff matrix

Proof. It can be intuitively understood from the model because it only causes the cost for the information acquisition, but does not bring any profit. First, this is a two-stage game where in the first stage, whether they would acquire information is decided and in the second stage, the bidding is carried out. Since the cost for information in the first stage is a sunk cost, it does not affect the decision in the second stage, and the nature of second price auction forces both bidders to bid the true value which brings both bidders the pay off of $-c$, so it is not an equilibrium.

The state where the both bidders do not acquire information is also not an equilibrium. Because when one bidder does not acquire information, the other bidder has an incentive to acquire information. Let us elaborate this with the following calculation. First, the bidder without the information would bid $\dfrac{v_h + v_l}{2}$ and the other bidder gets better off $\dfrac{1}{2}(v_h - \dfrac{v_h + v_l}{2}) - c$ once he acquires information. Certainly, this is an asymmetric equilibrium. ■

Proposition 1 *A mixed strategy that the two bidders acquire information with probability* $1 - \frac{4c}{v_h - v_l}$ *is an unique symmetric Nash equilibrium.* [5]

Proof. See appendix. ■

[5] It's meaning is probability α is decreasing function of cost and increasing function of information's value$(v_h - v_l)$.

2.2 Information Acquisition in English Auction

The rules of English auction is the following. Since the price increases continuously from 0 to v_h and there are only two bidders, if one drops out, the remaining bidder obtains the object at the price of at that point of time. Also, we assume the information is acquired instantly.[6]

In the English auction, each uninformed bidder may decide to learn his valuation not only before the auction starts but also during the auction. In other words, in the English auction, the strategy is to decide whether to acquire the information, at which point the information should be acquired, and if the information is not to be acquired, at which point they should drop out. I use a Perfect Bayesian Equilibrium concept.

Having thought of the structure in game, the pure strategy of each bidder is the information acquisition at each stage and the drop point.

Lemma 2 *Neither the state where the both bidders acquire information nor do not acquire information is an equilibrium.*

Proof. From the same reasoning of the second price auction, if both bidders acquire information, they wait until the price reaches the true value and then drop out, so each of them receives the pay off of $-c$.

If one bidder does not acquire information, his best strategy is to drop out when the price is $\frac{v_h + v_l}{2}$, and based on this strategy, if the other bidder acquires information, it is better for the other bidder to acquire the information because the pay-off is $\frac{1}{2}(v_h - \frac{v_h + v_l}{2})$. ■

[6] Since the value is discretely distributed only at two points, price rises discretely in the order of $0 \Rightarrow v_l \Rightarrow \frac{v_h + v_l}{2} \Rightarrow v_h$, and it can be thought that the information is acquired at each time points which are assigned to each situation.

Proposition 2 *There is no symmetric equilibrium in the English auction.*

Let us see the intuitive reason for this first.[7] We divide the price range into three intervals I_1, I_2, I_3 which are $(0, v_l]$ $(v_l, \frac{v_h + v_l}{2}]$ $(\frac{v_h + v_l}{2}, v_h]$ respectively, and let the probability of information acquisition in each interval be α_1, α_2, α_3.[8] Then the information acquisition in the interval I_3 is a pure strategy which is inferior to the information acquisition in the interval I_1 and I_2. Therefore α_3 is necessarily 0. Now, neither $\alpha_1 > 0$, $\alpha_2 = 0$ nor $\alpha_1 = 0$, $\alpha_2 > 0$ can be a symmetric equilibrium. First, the reason that $\alpha_1 = 0$, $\alpha_2 > 0$ cannot be an equilibrium is simply that when the value is v_l , the pay off from acquiring information in interval I_2 is less by $-\epsilon$ than the pay off from acquiring information in interval I_1. Finally, the reason that $\alpha_1 > 0$, $\alpha_2 = 0$ cannot be an equilibrium is that when there is a possibility that a bidder could acquire information in interval I_1, if the other bidder acquires information in interval I_2 instead of interval I_1 with a positive probability, he gains $-\frac{1}{2}\alpha_1 c$. It is the key point of this paper that when the value is low(v_l) there exists an incentive to use the information of the other while observing whether the other player drops out or not.

For a more general comparison, in the case of private value model(Compte model), the only thing that is to be considered is the degree of competitiveness and the cost. Therefore there is no incentive to acquire information, and the critical reason that the revenue of English auction is higher is that the incentives for acquiring information get larger as it gets less competitive and the probability of winning becomes larger as time passes. On the contrary, in the common value model, the value of the object is the same and the information of which is correlated, so in the case of English auction where the strategies are exposed, there is less incentive for acquiring information.

[7]See Appendix for the rigorous proof.

[8]α_2 is the probability of an event given that $1 - \alpha_1$ and α_3 is the probability of an event given that $(1 - \alpha_1)(1 - \alpha_2)$.

Proposition 3 *There is an asymmetric equilibrium in the English auction that one bidder acquires information with probability 1 and drops at real value, while the other does not acquire information and drop at expected value.*

Proof. In addition to assume that each player know that other player can be irrational player with probability ϵ. Irrational player is someone who drops at random. This assumption is required to pin down the equilibrium.

The reasons that this combination of strategies can be an equilibrium are following. If an uninformed bidder (hereinafter the B_n) participates the English auction with an informed bidder (hereinafter the B_a), the B_n can ratiocinate that B_a has passed the point v_l may be known by the informed bidder with the information v_h or the irregular player with possibility of ϵ. A perfect Bayesian equilibrium is one in which the bidders play their best response at each information set given their beliefs about others signals. Hence, while the expected pay off is 0 when the dropping is made at $\frac{v_h + v_l}{2}$, [9] if take the strategy makes drop in the I_3 interval, the expected pay off is negative and thus drops at $\frac{v_h + v_l}{2}$. On the contrary, if the opposite side is the B_n, B_a's the optimal strategy is absolutely getting information certainly.

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[9] The reason that it does not drop before the point $\frac{v_h + v_l}{2}$ is if the opposite side is irregular, the expected pay off can be a positive number.

2.3 Comparing the Revenue.

Second price auctions revenue at symmetric equilibrium is

$$\alpha^2\frac{v_h+v_l}{2}+2\alpha(1-\alpha)[\tfrac{1}{2}\frac{v_h+v_l}{2}+\tfrac{1}{2}v_l]+(1-\alpha)^2\frac{v_h+v_l}{2}.$$

In case both player acquire information with probability α^2 , revenue is $\frac{1}{2}v_h+\frac{1}{2}v_l$. If one bidder acquires information but the other bidder does not with probability $2\alpha(1-\alpha)$, revenue is $\frac{1}{2}\frac{v_h+v_l}{2}+\frac{1}{2}v_l$. Lastly, if both players does not acquire information with probability $(1-\alpha)^2$, revenue is $\frac{v_h+v_l}{2}$.

English auctions revenue at asymmetric equilibrium is

$$\tfrac{1}{2}\frac{v_h+v_l}{2}+\tfrac{1}{2}v_l.$$

We can easily see Second price auctions revenue is mixture of English auctions revenue and a greater value $(\frac{v_h+v_l}{2})$. So, Second price auction's revenue is bigger than English auction.

The reason why I compare Second price auctions revenue at symmetric equilibrium and English auctions revenue at asymmetric equilibrium is that model settings are symmetric. This makes the prediction of symmetric equilibrium, if exists, more plausible. But even if the above mention cannot be accepted, Second price auction rule is not less than English rule in revenue.

3 Concluding remarks

We have shown that in the common value auction model, when there are two bidders, what strategies the bidders take in English auction and second price auction, and how the revenue in second price auction can be higher than the revenue in English auction.

As much as we see situations to which the private value model can be applicable, we encounter many situations in real life where the common value model can be applicable. Including the popular cases like oil-drilling rights auctions, art works that are frequently mentioned in private value model may also fit for the common value model if we allow for resale. Through this paper, it is cautiously argued that the superiority of English auction that has been suggested by previous studies should be reconsidered if we take into account the acquisition of information(as are the most cases in real life). The reason is that in an open dynamic model where the information on the value of the auction object has high correlation, the possibility of inferring the information based on the strategy and behaviors of the other player decreases the incentive for information acquisition, which can lead to the decline in revenue.

At last, the additional contribution of this paper is, it has proved that the research result of if there are two bidders, Second price auction and English auction are perfectly same does not always true when information acquisition is considered.

A Appendix

A.1 mathematical analysis of Second price auction

2nd stage(bidding stage) analysis

Bidding strategy of the one who does not acquire information (b_1).

$$Max\ E[V - b_2]\,\mathbf{1}(b_1 > b_2)$$
$$b_1 = E[V] = \frac{v_h + v_l}{2}$$

Bidding strategy of the one who does acquire information (b_1).

$$Max\ [V - b_2]\,\mathbf{1}(b_1 > b_2)$$
$$b_1 = V$$

1st stage (information acquisition stage) analysis

If other bidder acquire information with probability α then player 1's pay off is

$$U_1 = \begin{cases} (1-\alpha)\frac{v_h - v_l}{4} - c & \text{if take information} \\ 0 & \text{doesnt} \end{cases}$$

so the mixed strategy is $\alpha = 1 - \frac{4c}{v_h - v_l}$.

A.2 mathematical analysis of English auction

proof of proposition 2

I had shown that neither the state where the both bidders acquire information nor do not acquire information is an equilibrium. Now, I will prove the following sequence. Assume that there is a symmetric mixed strategy $0 \leq \alpha_1 < 1$, $0 \leq \alpha_2 < 1$, $0 \leq \alpha_3 < 1$(of course one of three must be strictly positive). First, I will show α_3 should be 0. Second, α_1 also should be 0. Finally proving that $\alpha_1 = 0$, $0 < \alpha_2 < 1$ cannot be an equilibrium.

First, assume that $0 < \alpha_3 < 1$. This implied that the bidder, who had not had the information in the I_2 interval, did not drop at the point $\dfrac{v_h + v_l}{2}$. This is contradiction. The explanation of this is that the uninformed bidder will have the negative expected pay off. If the opposite side is uninformed bidder with the possibility of β, [10] his expected pay off is $(1-\beta) \times 0 + \beta \times [E(v) - P]\, \mathbf{1}[D_1 > D_2] < 0$. [11] [12]

Second, I will show α_1 should be 0. Lets assume the contrary $\alpha_1 > 0$. α_1 is calculated by $(1-\alpha_1)(1-\alpha_2) \times \frac{v_h - v_l}{4} - c = 0$. If bidder 1 is uninformed at I_2 then his pay off matrix is same as Table 1, nothing but belief system. He believes that the other player is informed with probability $\frac{\frac{1}{2}\alpha_1}{\frac{1}{2}+\frac{1}{2}(1-\alpha_1)}$, will be informed with probability $\frac{(1-\alpha_1)\alpha_2}{\frac{1}{2}+\frac{1}{2}(1-\alpha_1)}$ and keep uninformed with probability $\frac{(1-\alpha_1)(1-\alpha_2)}{\frac{1}{2}+\frac{1}{2}(1-\alpha_1)}$. Therefore if the information acquired in the I_2, the expected pay off will be $\frac{(1-\alpha_1)(1-\alpha_2)}{\frac{1}{2}+\frac{1}{2}(1-\alpha_1)} \times \frac{v_h - v_l}{4} - c$. Compare this to the equation above, the cost is same but the benefit is increased (Due to the fact that the denominator is smaller than 1). So if the α_1 is bigger than 0, the information acquisition will be certainly made in the I_2. However, this is not an equilibrium, therefore the α_1 has to be 0.

[10] Actually $\beta = \frac{(1-\alpha_1)(1-\alpha_2)}{\frac{1}{2}+\frac{1}{2}(1-\alpha_1)}$.

[11] D_i is a drop point of bidder i.

[12] Both of bidders did not have the information in the interval of I_3. So $P > E[V] = \dfrac{v_h + v_l}{2}$.

Lastly, we will prove that $\alpha_1 = 0$, $0 < \alpha_2 < 1$ cannot be an equilibrium. Given that $0 < \alpha_2 < 1$, if bidder acquire information on I_1 certainly , his expected pay off is $(1-\alpha_2) \times \frac{v_h - v_l}{4} - c$. While if he does not acquire information on I_1 , his expected pay off is $\frac{1}{2}(v_l - P) + (1-\alpha_2) \times \frac{v_h - v_l}{4} - c$. The term $\frac{1}{2}(v_l - P)$ is actually negative ($P > v_l$ in the interval of I_2). [13] So if $0 < \alpha_2 < 1$, then bidders acquire information certainly at I_1. Thus, this is contradiction.

[13]This term's meaning is loss when real value is low. Actually it should be multiplied by the probability of opponent bidder drop early when analyzed more closely. But it is not a zero probability, so the above logic is true as ever.

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