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A study on the state-contingent capital tax policy

경기 조정적인 자본세 정책의 효과성 분석

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Abstract

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This paper aims to figure out the impacts of capital tax policy on the economy where the government has a certain policy restriction; state-contingent capital tax rate. Simultaneously, households fully understand how the government behaves and make decisions based on the policy expectation. Under the framework used in this paper, I find that the state-contingent capital tax policy results in larger fluctuations of allocations as well as policies without notable differences in utility and capital accumulation compared to the economy under the state-independent capital tax policy.

Key words : Dynamic Ramsey problem; Optimal taxation; Markov-Switching parameter State-contingent capital tax; MSDSGE perturbation method

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1. Introduction

The aim of this paper is to investigate the following question. When society has a particular policy convention, state-contingent capital tax rate, how policy effects would be compared to the case in which the government implements the state-independent capital tax rate policy. To be concrete, in the economy under the state-contingent capital tax policy regime, the government announces next period's capital tax rate in advance, but this announcement is flexible and is able to be revised depending on the realized state of the economy; if the realized economic condition is adverse, the actual tax rate is decreased by specified percentage compared to the announced one. On the other hand, in the economy under the state-independent capital tax policy regime, the government can not modify its policy announcement and should implement the announced tax rate regardless of the state of the economy. The reasons that the government abides by the announced tax rate or modifies its policy announcement are not explicitly considered in this paper and just regarded as an inherent property of the economies. However, it can be interpreted as a kind of a policy restriction or a policy rule. Also, in both economies, households fully anticipate how the government acts and apply their expectation when they decide.

Clearly, policy effects would be significantly different if the government is under the different policy restrictions (regimes) and households fully know how the realized capital tax rate would be based on the announced capital tax rate. For instance, if the government fully recognizes that there may be a tax cut in the future, the announced capital tax rate may differ compared to the case that the government does not violate its policy plan. Also, if households anticipate that the policy announcement would not be revised, their saving decisions would be different. Moreover, since the capital tax cut leads to higher labor taxes in the economy without government bonds, this also engenders additional distortions in economic agent's decisions.

In this paper, I try to investigate impacts of different capital tax policy regimes on the economy. To address this problem, I use the dynamic Ramsey framework. However, there is a fundamental difference in my case that I include and apply certain policy restriction and household's expectation to the general Ramsey framework.

Although two situations described above are hypothetical cases, analogous examples are easily observable in the real world. Concretely, the periodically occurred tax cut during election times or changes in policy which is consistent with political agenda of the ruling party can be the extended examples of the situation this paper concerns.

The motivation is quite similar to the 'Time-inconsistency' issue in macroeconomics. A lot of papers about time-inconsistency of policies focus on the setting that policies the government has set become no longer optimal with time, and thus the government has an incentive to re-optimize in spite of pre-determined policies. However, individuals who rationally anticipate government behaviors already make the decision based on their policy anticipation. Many papers concentrate on finding the equilibrium or examining policy effects under this circumstance. For example, Strotz (1955) studies inconsistency in dynamic utility maximization. He concludes that the optimal plan of the present moment will be violated generally, and the individuals' future behavior will be inconsistent with their optimal plan. Kydland and Prescott (1977) point out that the selection of decision which is best at certain situation does not result in the social objective function being maximized and there is no way control theory can be made applicable to economic planning when expectations are rational. Fischer (1980) investigates time-inconsistent policy using simple 2-period model and finds that consistency prevails in his framework if the government relies solely on non-distortional taxes to finance the public good. Tesfatsion (1986) establishes necessary and sufficient conditions for consistency for a general class of dynamic Walrasian economies. Recently, time-consistent policy is actively studied using the Markov-perfect equilibrium concept. Markov-perfect optimal taxation has been studied by several researchers such as Klein and Rios-Rull (2003) and Klein et al (2008). Klein at al (2008) examine how a benevolent government that cannot commit to future policy should trade off the costs and benefits of public expenditure. Klein and Rios-Rull (2003) study the properties of the optimal fiscal policy in a stochastic growth model when the government cannot commit itself beyond the next period's capital income tax rate and compared results with those under full commitment economy. At last, Debortoli and Nunes (2010) analyse an optimal fiscal policy problem where the plans made by the benevolent government are periodically revised. They devise a 'loose commitment' concept which corresponds to the periodically revising policy commitment due to the exogenous change such as replacing of ruling party. 'Time-(in)consistent fiscal policy' is an important topic which has been actively studied. However, while this kind of view regards policy deviation as an active policy re-optimization process[1], I comprehend changes in the

1) Generally, capital tax imposed on initial asset is lump-sum tax. Thus, benevolent government has an incentive to impose heavy capital tax in upcoming periods, if there is not commitment technology, even though it already levied a heavy capital tax on initial asset. A lot of papers concerning the time-inconsistent issue deal with this situation, which is obviously different with the topic of mine.

policy discussed in this paper as a passive response to the policy restriction or a kind of the social convention. That is, the government sets subsequent announced capital tax rates at the initial period and this tax policy is committed. The only thing which is revised periodically is realized capital tax rate which follows the specific policy restriction. Also, while papers on the time-inconsistent policy consider changes in policies as a result from ability to alter the previous decision with time, I focus on policy changes caused by the state of the economy, not a decision timing. Thus, in the strict sense, the motivation of this paper is far different with a 'time-inconsistent policy' issue.

As the Ramsey framework assumes the full-commitment technology, the question of this paper is more close to the dynamic Ramsey problem with a certain policy restriction. The dynamic Ramsey problem has been studied by many researchers such as Lucas and Stockey (1983), Chamley (1986), Zhu (1992), Chari et al (1991, 1993, 1994, 1995, 1999) and Stockman (2001). Lucas and Stokey (1983) are concerned with the structure and time-consistency of optimal fiscal and monetary policy in an economy without capital. Their main finding is that with debt commitments of sufficiently rich maturity structure, an optimal policy, if one exists, is time-consistent. Meanwhile, Chamley (1986) makes a elementary framework of dynamic Ramsey problem under the deterministic economy and concludes that if an equilibrium has an asymptotic steady state, the optimal policy is eventually to set the tax rate on capital to zero. Zhu (1992) and Chari et al Chari et al (1991, 1993, 1994, 1995, 1999) analyse the Ramsey problem under a stochastic economy. Chari et al (1991, 1993, 1994, 1995, 1999) conclude that labor income tax should be essentially constant over the business cycle, Ex-ante capital tax rates should be roughly zero in each period and the return on debt and the Ex-post tax on capital income should absorb most of the shocks of the government budget constraint. Especially, Chari et al consider a few special examples such as non-contingent capital taxes with state contingent return on bond (or inverse case) and exogenous tax policies[2]. Finally, Stockman (2001) solves the Ramsey problem under the balanced budget constraint. Using the Ramsey approach which assumes full-commitment technology, I include the aforementioned restriction on the capital tax policy as an additional constraint of the general Ramsey problem. Also, I try to apply household expectations to the general Ramsey framework, while the previous studies on the Ramsey problem assume households take government policies as given.

2) They consider the economy which has a constant capital tax rate $\theta(s^t)$=0.27 and a constant labor tax rate $\tau(s^t)$=0.24.

In this paper, I focus on the comparative study of policy effects between different policy regimes ; One is the Markov-switching (MS) policy regime that the government announces the next period capital tax rate in previous period but is not obliged to keep that. In other words, it freely adjusts announced capital tax rate based on the realized economic condition. However, agents who already know the government tends to deviate from the previous policy announcement apply it to their decision. Another policy regime is the Fixed policy regime under which government abides by its policy announcement regardless of the realized state of the economy. When I solve this problem, the Markov-switching parameter term $a(s_t)$ which changes depending on the state of the economy plays an important role. To capture policy effects which result from $a(s_t)$ using the first-order approximation, I opt for a Markov-Switching DSGE perturbation method developed by Foerster et al (2010, 2011, 2013).

This paper is laid out as follows. Section2 describes the models, solves the Ramsey problem under the particular policy restrictions and discusses about determining parameters and steady state values. After that, the MSDSGE perturbation method is briefly introduced and applied to the problem. Section 3 provides numerical results ; Interpretation of policy functions, impulse-response analysis and Monte-carlo simulation results. Lastly, section 4 concludes. In the Appendixes, impulse-response figures and necessary expositions about MSDSGE perturbation method and Proposition 1 are presented.

2. The model

2.1. The economy

I postulate two different types of policy regimes.

① The MS policy[3] regime : $\theta(s^t) = \begin{cases} \bar{\theta}(s^t) & \text{if } s_t = boom \\ \bar{\theta}(s^t) - a & \text{if } s_t = recession \end{cases}$

② The Fixed policy regime : $\theta(s^t) = \begin{cases} \bar{\theta}(s^t) & \text{if } s_t = boom \\ \bar{\theta}(s^t) & \text{if } s_t = recession \end{cases}$

where $\theta(s^t)$ is realized capital tax rate at period t, $\bar{\theta}(s^t)$ is announced capital tax rate determined at period t-1 and a is the policy restriction parameter. In both economies, the government announces in period t-1 that the capital tax rate of the next period (t)

3) MS means Markov-switching. The concrete meaning is described in the later section.

would be $\overline{\theta}(s^t)$. While, in the Fixed policy case, the government keeps its policy announcement regardless of realized state of the economy, the government under the MS policy regime adjusts its policy announcement depending on the realized state of the economy. Simultaneously, households already know how the government would behave (decrease capital tax rate by a% if the state of the economy at the following period would be recession) and apply their expectation when they decide.

The model and framework used here are similar to those used in Chari et al (1991, 1993, 1994, 1995, 1999) and D. Stockman (2001). The model is a standard neoclassical real business cycle model with an exogenous growth shock which follow the Markov process and a government spending shock which is independent from a growth shock.

I denote a history up to and including period t as $s^t = (s_0, s_1, \ldots, s_t)$ and each s_t is an element of a finite state space S. The initial realization s_0 is given and there are a large number of identical infinitely lived agents. A constant returns to scale technology is available to transform labor $l(s^t)$ and capital $k(s^{t-1})$ into output via $F(k(s^{t-1}), l(s^t), s_t)$. Note that the production function incorporates an exogenous stochastic shock as defined in Chari et al (1991, 1993, 1994, 1995, 1999). The output is used for private consumption $c(s^t)$, exogenously specified government spending $g(s^t)$ and investment $k(s^t) - (1-\delta)k(s^{t-1})$. Thus, the resource constraint is

$$c(s^t) + k(s^t) + g(s^t) = F(k(s^{t-1}), l(s^t), s_t) + (1-\delta)k(s^{t-1}) \tag{1}$$

Households

The representative agent chooses an allocation $\{c(s^t), l(s^t), k(s^t)\}$ to maximize

$$E(\sum_t \beta^t U(c(s^t), l(s^t))),$$ [4]

subject to the household budget constraint

$$c(s^t) + k(s^t) \leq (1 - \tau(s^t))w(s^t)l(s^t) + R^k(s^t)k(s^{t-1}) \tag{2}$$

where $R^k(s^t) = (1 + (1 - (\overline{\theta}(s^t) - a(s_t))(r(s^t) - \delta))$

for all s^t, with initial capital stock k_{-1} and $\{\tau(s^t), \overline{\theta}(s^t), r(s^t), w(s^t)\}$ taken as given. $R^k(s^t)$ means a gross return on capital after taxes and depreciation and $r(s^t)$ and

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4) Throughout this paper, I assume that utility function satisfies the Inada condition.

$w(s^t)$ are the before-tax returns on capital and labor.

The unique part of this paper is the existence of $a(s_t)$ term which can be interpreted as a social tax-cut convention or a policy restriction that government must follow as well as households believe it to be exactly realized. When the government implements the state-contingent capital tax policy, $a(s_t)$ is a varying parameter depending on the state of the economy s_t[5]. Conversely, when the government implements the state-independent capital tax rate, $a(s_t)$ is a fixed value, $\bar{a}$, independent of the realized state s_t. In this paper, I call the former case the MS (Markov-switching) policy regime and the latter one the Fixed policy regime.

Household's first-order conditions are as follows.

$$\frac{U_l(s^t)}{U_c(s^t)} = -(1-\tau(s^t))w(s^t) \tag{3}$$

$$U_c(s^t) = \beta E(U_c(s^{t+1})R^k(s^{t+1})|s^t) \tag{4}$$

where $U_c(s^t)$ and $U_l(s^t)$ denote the marginal utilities of consumption and leisure at state s^t.

Firms

A representative firm maximizes profit[6] by choice of $\{y(s^t), k(s^t), l(s^t)\}$ subject to $y(s^t) = F(k(s^{t-1}), l(s^t), s_t)$, where the production function F is constant returns to scale in capital and labor. The assumption of constant returns to scale and the perfect competition in the labor, capital and output markets ensures that

$$r(s^t) = F_k(k(s^{t-1}), l(s^t), s_t) \quad , \quad w(s^t) = F_l(k(s^{t-1}), l(s^t), s_t) \ . \tag{5}$$

$F_k(s^t)$ and $F_l(s^t)$ mean marginal products of capital and labor, respectively.

Government

The benevolent government takes an exogenous stream of expenditures $\{g(s^t)\}$ as

5) Since $a(s_t)$ is a varying parameter depending on the realized economic state s_t, this is not a general type parameter, but a Markov-switching parameter.

6) $\Pi(s^t) = y(s^t) - r(s^t)k(s^{t-1}) - w(s^t)l(s^t)$ where $\Pi(s^t)$ means profit in period t.

given and must satisfy its period-by-period budget constraint under the certain capital tax policy regime. The government's budget constraint is as follows.

$$g(s^t) = \tau(s^t)w(s^t)l(s^t) + (\overline{\theta}(s^t) - a(s_t))(r(s^t) - \delta)k(s^{t-1}) \quad \text{for all} \quad s^t \tag{6}$$

where the government policies are $\{\tau(s^t), \overline{\theta}(s^t)\}$.

2.2. The Ramsey Problem

A Ramsey equilibrium is defined as a fiscal policy $T = \{\tau(s^t), \overline{\theta}(s^t)\}$, an allocation rule $X(T)$, and price rules $r(T)$ and $w(T)$ which satisfy following properties.

Firstly, the policy T maximizes the agent's utility $E[\sum_t \beta^t U(c(s^t|T), l(s^t|T))]$ subject to the government's budget constraint where the allocation and prices are given as $X(T)$, $r(T)$and $w(T)$.

Secondly, the allocation $X(T')$ maximizes $E[\sum_t \beta^t U(c(s^t), l(s^t))]$ subject to the household's budget constraint evaluated at the policy $T'$and the prices $r(T')$ and $w(T')$ for every T'

Lastly, Prices satisfy $w(T') = F_l(s^t \mid T')$ and $r(T') = F_k(s^t \mid T')$ for every T'.

Because the dual approach which finds allocations and policies directly makes a problem too complex to solve under the dynamic framework, most dynamic Ramsey problem papers such as Chari et al (1991, 1993, 1994, 1995, 1999), Chamley (1986) and Stockman (2001) use the so-called primal approach. Although the typical primal approach has several advantages such as making the problem simple, it is sometimes difficult to use. For example, substituting all price and policy terms with allocation terms can be too burdensome under the certain case, or the problem which needs to consider additional constraints can be improper to use the primal approach. For a similar reason, the primal approach is no longer convenient when it comes to the problem of this paper because the additional policy constraint on the capital tax, state-contingent or state-independent capital tax policy regime, should be considered. Thus, I use the modified primal approach under which $r(s^t), w(s^t)$ and $\tau(s^t)$ are substituted with allocation terms, but $\overline{\theta}(s^t)$ is left as a variable.

Using formulas presented above and the definitions of the Ramsey problem, I obtain the following proposition.

Proposition 1

① If $\{c(s^t), l(s^t), k(s^t), g(s^t)\}$ and $\{\tau(s^t), \overline{\theta}(s^t), w(s^t), r(s^t)\}$ satisfy (2)~(6), given initial conditions $\{\theta(s^0), k_{-1}\}$ and $a(s_t)$, then $\{c(s^t), l(s^t), k(s^t), \overline{\theta}(s^{t+1})\}$ satisfy

$$c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}), l(s^t), s_t) + (1-\delta)k(s^{t-1})$$

$$U_c(s^t)(c(s^t) + k(s^t) - (1 + (1 + a(s_t) - \overline{\theta}(s^t))(F_k(s^t) - \delta)))k(s^{t-1})) + U_l(s^t)l(s^t) = 0$$

$$U_c(s^t) = \beta E[U_c(s^{t+1})(1 + (1 + a(s_{t+1}) - \overline{\theta}(s^{t+1}))(F_k(s^t) - \delta))|s^t]$$

$$U_c(s^0)(c(s^0) + k(s^0) - (1 + (1 + a(s_0) - \theta(s^0))(F_k(s^0) - \delta)))k_{-1}) + U_l(s^0)l(s^0) = 0$$

② Conversely, if $\{c(s^t), l(s^t), k(s^t), \overline{\theta}(s^{t+1}), g(s^t)\}$ satisfy above constraints given initial conditions, then there exist $\{r(s^t), w(s^t), \tau(s^t)\}$ such that $\{c(s^t), l(s^t), k(s^t), \overline{\theta}(s^{t+1}), g(s^t)\}$ and $\{r(s^t), w(s^t), \tau(s^t)\}$ satisfy resource constraint, government budget constraint, household's first-order conditions and firm's first-order conditions.

Proof) See the Appendix A.

Intuitively, the Ramsey equilibrium should be feasible and optimal under the social capital policy regime and the household's expectation. The proposition 1 states that the Ramsey equilibrium satisfy the resource constraint, the period-by-period budget constraint to which households and firms optimal conditions are applied and the Euler equation which makes the Ramsey equilibrium satisfy the policy restriction as well as a household's expectation.

We now have the Ramsey problem

$$\max_{\{c(s^t), l(s^t), k(s^t), \overline{\theta}(s^{t+1})\}} E\left[\sum_t \beta^t U(c(s^t), l(s^t))\right]$$

subject to

$$c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}), l(s^t), s_t) + (1-\delta)k(s^{t-1})$$

$$U_c(s^t)(c(s^t) + k(s^t) - (1 + (1 + a(s_t) - \overline{\theta}(s^t))(F_k(s^t) - \delta)))k(s^{t-1})) + U_l(s^t)l(s^t) = 0$$ [7]

7) When the government is able to issue debt freely, we can use the implementability constraint to greatly simplify the Ramsey problem. Without the ability to borrow and lend, it is not usable. Stockman (2001) solves the Ramsey problem under the budget balance restriction and he uses the modified period-by-period budget constraint to solve the problem. Similarly, I use

$$U_c(s^t) = \beta E[U_c(s^{t+1})(1+(1+a(s_{t+1})-\overline{\theta}(s^{t+1}))(F_k(s^t)-\delta))|s^t]$$

$$U_c(s^0)(c(s^0)+k(s^0)-(1+(1+a(s_0)-\theta(s^0))(F_k(s^0)-\delta)))k_{-1})+U_l(s^0)l(s^0)=0$$

given $\{\theta(s^0), k_{-1}\}$, $\{g(s^t)\}$ and $a(s_t)$.

Let $\phi(s^t)$, $\lambda(s^t)$ and $\mu(s^t)$ be the Lagrange multipliers on the resource constraint, household budget constraint and Euler-equation, respectively.

The first-order conditions are [8)]

$$\frac{\partial L}{\partial c(s^t)} : U_c(s^t) - \phi(s^t) + \lambda(s^t)(U_{cc}(s^t)(c(s^t)+k(s^t)-R^k(s^t)k(s^{t-1})) + U_c(s^t) + U_{cl}(s^t)l(s^t))$$
$$- \mu(s^t)U_{cc}(s^t) + \mu(s^{t-1})U_{cc}(s^t)R^k(s^t) = 0$$

$$\frac{\partial L}{\partial l(s^t)} : U_l(s^t) + \phi F_l(s^t) + \lambda(s^t)(U_{cl}(s^t)(c(s^t)+k(s^t)-R^k(s^t)k(s^{t-1})) + U_{ll}(s^t)l(s^t) + U_l(s^t)$$
$$- U_c(s^t)R^k_l(s^t)k(s^{t-1})) - \mu(s^t)U_{cl}(s^t) + \mu(s^{t-1})(U_{cl}(s^t)R^k(s^t) + U_c(s^t)R^k_l(s^t)) = 0$$

$$\frac{\partial L}{\partial k(s^t)} : -\phi(s^t) + \beta E[\phi(s^{t+1})(F_k(s^{t+1})+1-\delta)|s^t] + \mu(s^t)\beta E[U_c(s^{t+1})R^k_k(s^{t+1})|s^t]$$
$$+ \lambda(s^t)U_c(s^t) - \beta E[\lambda(s^{t+1})U_c(s^{t+1})(R^k_k(s^{t+1})k(s^t) + R^k(s^{t+1}))|s^t] = 0$$

$$\frac{\partial L}{\partial \overline{\theta}(s^{t+1})} : -E[\lambda(s^{t+1})U_c(s^{t+1})R^k_{\overline{\theta}}(s^{t+1})|s^t]k(s^t) + \mu(s^t)E[U_c(s^{t+1})R^k_{\overline{\theta}}(s^{t+1})|s^t] = 0$$

$$\frac{\partial L}{\partial \phi(s^t)} : F(k(s^{t-1}), l(s^t), s_t) + (1-\delta)k(s^{t-1}) = c(s^t) + g(s^t) + k(s^t)$$

$$\frac{\partial L}{\partial \lambda(s^t)} : U_c(s^t)(c(s^t)+k(s^t) - (1+(1+a(s_t)-\overline{\theta}(s^t))(F_k(s^t)-\delta))k(s^{t-1})) + U_l(s^t)l(s^t) = 0$$

$$\frac{\partial L}{\partial \mu(s^t)} : U_c(s^t) = \beta E[(1+(1+a(s_{t+1})-\overline{\theta}(s^{t+1}))(F_k(s^{t+1})-\delta))U_c(s^{t+1})|s^t]$$

for $t \geq 1$. The first-order conditions at t=0 are slightly different due to the initial condition.

There are two important features. Firstly, unlike the typical dynamic Ramsey problem, the household's Euler equation is included as a constraint of the Ramsey problem to explicitly capture effects of the policy restriction on capital taxation. In most dynamic Ramsey problem papers, all price and policy terms disappear behind an implementability constraint and, after deriving optimal allocations, are just backed out using household and firm's first-order conditions. However, in this paper, it is more convenient to directly find period-by-period optimal announced capital tax rate since not only the substitution process is far more difficult, but the problem becomes too complex to solve

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the modified period-by-period budget constraint since I assume the economy without bond.

8) $R^k(s^t) = 1+(1+a(s^t)-\overline{\theta}(s^t))(F_k(s^t)-\delta)$ is a function of $l(s^t), k(s^{t-1})$ and $\overline{\theta}(s^t)$. Also, $R^k_x(s^t)$ means the partial derivative of $R^k(s^t)$ with the variable $x(s^t)$

if we stick to the general primal approach.

Secondly, as I already mentioned, what the government determines is the announced capital tax rate, not the realized one and the announced capital tax rate $\overline{\theta}(s^t)$ is determined in the previous period t-1[9]. If one interprets that $\overline{\theta}(s^t)$ is an announced capital tax rate and $\overline{\theta}(s^t) - a(s_t)$ as a realized one, the meaning would be more clear.

2.3. Numerical analysis

2.3.1. Parameter values

Basically, I borrow parameter values and functional forms from Chari et al (1991, 1993, 1994, 1995, 1999)[10]. The utility function and the production function are as follows.

Utility function : $U(c(s^t), l(s^t)) = (1-\gamma)\ln(c(s^t)) + \gamma\ln(1-l(s^t))$

Production function : $F(k(s^{t-1}), l(s^t), s_t) = k(s^{t-1})^{\alpha}(z(s_t)l(s^t))^{(1-\alpha)}$

$z(s^t)$ is a labor-augmenting technology shock and the technology shock has the law of motion $z(s^t) = e^{\eta(s^t)}$, where $\eta(s^t)$ follows a two state Markov process (Boom (Good) / Recession (Bad))[11]. The transition probabilities are $P(s^{t+1} = s_i | s^t = s_j) = \begin{cases} 0.9 & \text{if } i = j \\ 0.1 & \text{if } i \neq j \end{cases}$.

$a(s_t)$ which follows the same markov process with $\eta(s^t)$ is 0 when the state is a boom, and 0.04 in a recession [12] under the MS policy regime. Also, I consider the economy where $a(s_t)$ is fixed at zero, which means the Fixed policy regime. Finally, the law of motion for $g(s^t)$ is given as $g(s^t) = Ge^{\epsilon_t}$ where ϵ_t is an I.I.D shock

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9) It is easily observable that

$$\overline{\theta}(s^{t+1}) = \frac{\beta E[U_c(s^{t+1})(1+(1+a(s_{t+1}))(F_k(s^{t+1})-\delta))|s^t] - U_c(s^t)}{\beta E[U_c(s^{t+1})(F_k(s^{t+1})-\delta)|s^t]}$$

10) To avoid the computational complexity related with detrending, I define shocks differently with Chari et al (1993). So, in principle, parameter values should be modified in accordance with the newly defined shock. However, since it is not a critical part of the research, I skip the calibration process and just borrow values from Chari et al (1993)

11) $\eta(s_t) = \begin{cases} 0.04 & \text{if } s_t = boom \\ -0.04 & \text{if } s_t = recession \end{cases}$

12) In the MS policy case, the government do not change its policy announcement when the realized state is a boom, but decrease capital tax rate by 4% in a recession. On the contrary, the fixed policy case can be interpreted as if the government keeps its policy announcement regardless of the realized state of the economy.

following $N(0,\sigma^2)$.

The definition of the steady-state for models with Markov switching parameters is more complicated than that for standard DSGE models with constant parameters. To be consistent with the definition of the steady-state in standard DSGE models with constant parameters, the definition of the steady-state for MSDSGE model should be independent of the realization of the discrete Markov process. So I fix Markov switching parameters which influence steady state values ($\eta(s_t), a(s_t)$) on its ergodic means and derive steady states[13]. The specific parameter and steady state values are summarized in Table 1. I find one notable property of the steady state value; the steady-state value of $\overline{\theta}$ is $\overline{a}$, the ergodic mean of $a(s_t)$. Since $\overline{\theta}(s^t)-a(s_t)$ is an actual (realized) capital tax rate, $\overline{\theta_{ss}}=\overline{a}$ means that the steady state value of the capital tax rate is zero. This finding corresponds to the Chamley (1986) and Chari et al (1991, 1993, 1994, 1995, 1999) results.

Table 1

Parameter values used in the numerical example

Parameters	
β	0.98
γ	0.75
α	0.34
δ	0.08
G	0.07
σ	0.07
$\eta(s_t)$	$\begin{cases} 0.04 & \text{if } s_t = boom \\ -0.04 & \text{if } s_t = recession \end{cases}$
$a_{MS}(s_t)$ / $a_{Fixed}(s_t)$	$a_{MS}(s_t) = \begin{cases} 0 & \text{if } s_t = boom \\ 0.04 & \text{if } s_t = recession \end{cases}$ / $a_{Fixed}(s_t) = 0$

13) Because the transition matrix is symmetric, the ergodic mean is just the arithmetic mean. Meanwhile, not all Markov switching parameters influence the steady state values. If we consider the economy that policy restriction takes this form ; $a(s_t)\times\overline{\theta}(s^t)$, then $a(s_t)$ doesn't affect steady state values since $\overline{\theta_{ss}}=0$ in this case. Whether the Markov switching parameter affects steady-state value is important when we apply MSDSGE perturbation method.

Steady state values

c_{ss}	0.2393
l_{ss}	0.2263
k_{ss}	1.4368
ϕ_{ss}	1.0447
μ_{ss}	0.3704
λ_{ss}	0.2578
$\overline{\theta_{MS}}$ / $\overline{\theta_{Fixed}}$	0.02 / 0

Note : I assume that necessary initial values such as initial capital stock are the same with steady state values.

2.3.2. MSDSGE perturbation method

Chari et al (1991, 1993, 1994, 1995, 1999) is often cited as an evidence showing that log-linearization is dangerous in the context of optimal tax policy problems[14]. Benigno and Woodford (2006) also point out a problem concerning the 'naive' linear-quadratic approximation of model in Chari et al (1991, 1993, 1994, 1995, 1999) and suggest 'correct' linear-quadratic (LQ) approximation method.

As pointed out by Schmitt-Grohe and Uribe (2004), the certainty equivalence of the first-order approximation is a main result of a constant parameter model. This implies that the first-order approximation of a constant parameter model is inadequate for analyzing behaviors of agents who response to uncertainty. Because I concentrate on the policy effects affected by Markov-switching parameters, the approximation methods such as log-linearization, 'typical' first-order perturbation method or LQ approximation are inappropriate approaches. To derive policy functions that are able to capture the effects which stem from the MS parameter, I adopt a MSDSGE perturbation method[15]. One powerful and interesting feature of the MSDSGE perturbation method is that this method can capture policy effects caused by Markov-switching parameters even with the first-order approximation. In this paper, I only briefly and intuitively introduce MSDSGE perturbation method. I recommend readers who want to know the detailed approximation process to read Foerster et al (2013).

Here I present a rough example that illuminates the intuition behind this method. Let's

14) Kim and Kim (2007)
15) Foerster et al (2010,2011,2013)

assume a simple function $f(x,y;a)=axy$ where a is a Markov-switching parameter. In the general approach, one pins a down at its ergodic mean, $\bar{a}$, and performs taylor (first-order) approximation. After that, one would get the below function.

$f \fallingdotseq \bar{a}x_{ss}y_{ss}+\bar{a}y_{ss}(x-x_{ss})+\bar{a}x_{ss}(y-y_{ss})$

On the other hand, in the MSDSGE perturbation method, it assumes $a=\bar{a}+\chi a(s_t)$ where χ is a perturbation parameter which takes 0 in the (Long-run) steady state and $a(s_t)$ is a deviation from ergodic mean in each state. Then, one would acquire the following function.

$f(x,y,\chi) \fallingdotseq \bar{a}x_{ss}y_{ss}+\bar{a}y_{ss}(x-x_{ss})+\bar{a}x_{ss}(y-y_{ss})+a(s_t)x_{ss}y_{ss}\chi$ where the last term captures MS effects.

As noted above, one can derive several policy functions which correspond to the multiple states of the economy respectively and these functions capture Markov-switching effects even with the first-order approximation. In addition, by assuming the Markov-switching parameter in this way, one can also address the economy that the MS parameter value changes over time. The more detailed process is described in the Appendix B.

2.3.3. Deriving policy functions[16)]

It is easily observable that control (non-predetermined) variables of the Ramsey problem in this paper are $\{c(s^t),l(s^t),\phi(s^t),\lambda(s^t)\}$, predetermined variables are $\{\bar{\theta}(s^t),k(s^{t-1}),\mu(s^{t-1})\}$, an I.I.D exogenous shock is ϵ_t, $s_t \in S=\{s_g,s_b\}$ and Markov-switching parameters are $\{\eta(s_t),a(s^t)\}$ under the MS policy regime and $\{\eta(s_t)\}$ under the fixed policy regime[17)]. Since there are two states, 3 predetermined variables and 4 control variables, I have to solve the system of 42 quadratic equations[18)]. After solving equations, I obtain 9 solutions[19)], but the only one satisfies the mean square

16) Since some notations used below follow those presented in the Appendix, I recommend readers read that part first.

17) Since the steady state value of $\bar{\theta}(s^t)$ is the ergodic mean of $a(s_t)$, all markov-switching parameters influence steady state values and, thus, is a function of the perturbation parameter χ.

18) In each state, all variables should be expressed as a linear function of predetermined variables $\{\bar{\theta}(s^t),k(s_{t-1}),\mu(s_{t-1})\}$. Therefore, we should solve 3×7×2 equations. Foerster et al (2013) suggest Mathematica which works well when solving the complex polynomial system and I use Matlab application program Mupad.

stability criterion. Policy functions under the MS policy regime are summarized in Table 2. Also, I find policy functions where $a(s_t)$ is pinned down to zero, the Fixed policy regime, and record those in Table 3.

It is notable that slopes of policy functions of each regime in the same economic state are the same and the only differences across them are constant terms which capture the corresponding Markov-switching effects.

Table 2

Policy functions under the MS policy regime

Boom $(s_t = s_g)$

$c(s^t)$=0.0157($\overline{\theta}_t$-0.02)+0.1116(k_{t-1}-1.4368)-0.0691(μ_{t-1}-0.3704)-0.0030ϵ_t+0.2439

$l(s^t)$=0.0404($\overline{\theta}_t$-0.02)-0.0827(k_{t-1}-1.4368)+0.1584(μ_{t-1}-0.3704)-0.0060ϵ_t+0.2365

$\phi(s^t)$=-0.3778($\overline{\theta}_t$-0.02)+0.1305(k_{t-1}-1.4368)-2.3139(μ_{t-1}-0.3704)+0.0631ϵ_t+0.9727

$\lambda(s^t)$=-0.3725($\overline{\theta}_t$-0.02)+0.4266(k_{t-1}-1.4368)-1.8881(μ_{t-1}-0.3704)+0.0605ϵ_t+0.2011

$\overline{\theta}(s^{t+1})$=0.4099($\overline{\theta}_t$-0.02)+0.7998($k_{t-1}$-1.4368)-3.3490($\mu_{t-1}$-0.3704)-0.0660$\epsilon_t$-0.0343

$k(s^t)$=0.0343($\overline{\theta}_t$-0.02)+0.8064(k_{t-1}-1.4368)+0.2651(μ_{t-1}-0.3704)-0.0094ϵ_t+1.4560

$\mu(s^t)$=-0.0510($\overline{\theta}_t$-0.02)+0.0738(k_{t-1}-1.4368)+0.5449(μ_{t-1}-0.3704)+0.0073ϵ_t+0.3652

Recession $(s_t = s_b)$

$c(s^t)$=0.0157($\overline{\theta}_t$-0.02)+0.1116(k_{t-1}-1.4368)-0.0691(μ_{t-1}-0.3704)-0.0030ϵ_t+0.2347

$l(s^t)$=0.0404($\overline{\theta}_t$-0.02)-0.0827(k_{t-1}-1.4368)+0.1584(μ_{t-1}-0.3704)-0.0060ϵ_t+0.2161

$\phi(s^t)$=-0.3778($\overline{\theta}_t$-0.02)+0.1305(k_{t-1}-1.4368)-2.3139(μ_{t-1}-0.3704)+0.0631ϵ_t+1.1167

$\lambda(s^t)$=-0.3725($\overline{\theta}_t$-0.02)+0.4266(k_{t-1}-1.4368)-1.8881(μ_{t-1}-0.3704)+0.0605ϵ_t+0.3145

$\overline{\theta}(s^{t+1})$=0.4099($\overline{\theta}_t$-0.02)+0.7998($k_{t-1}$-1.4368)-3.3490($\mu_{t-1}$-0.3704)-0.0660$\epsilon_t$+0.0743

$k(s^t)$=0.0343($\overline{\theta}_t$-0.02)+0.8064(k_{t-1}-1.4368)+0.2651(μ_{t-1}-0.3704)-0.0094ϵ_t+1.4176

$\mu(s^t)$=-0.0510($\overline{\theta}_t$-0.02)+0.0738(k_{t-1}-1.4368)+0.5449(μ_{t-1}-0.3704)+0.0073ϵ_t+0.3756

19) See the Appendix C.

Table 3
Policy functions under the Fixed policy regime

Boom ($s_t = s_g$)

$c(s^t)$=0.0157($\overline{\theta}_t$-0.02)+0.1116(k_{t-1}-1.4368)-0.0691(μ_{t-1}-0.3704)-0.0030ϵ_t+ 0.2436
$l(s^t)$=0.0404($\overline{\theta}_t$-0.02)-0.0827(k_{t-1}-1.4368)+0.1584(μ_{t-1}-0.3704)-0.0060ϵ_t+0.2357
$\phi(s^t)$=-0.3778($\overline{\theta}_t$-0.02)+0.1305(k_{t-1}-1.4368)-2.3139(μ_{t-1}-0.3704)+0.0631ϵ_t+0.9803
$\lambda(s^t)$=-0.3725($\overline{\theta}_t$-0.02)+0.4266(k_{t-1}-1.4368)-1.8881(μ_{t-1}-0.3704)+0.0605ϵ_t+0.2085
$\overline{\theta}(s^{t+1})$=0.4099($\overline{\theta}_t$-0.02)+0.7998($k_{t-1}$-1.4368)-3.3490($\mu_{t-1}$-0.3704)-0.0660$\epsilon_t$-0.0465
$k(s^t)$=0.0343($\overline{\theta}_t$-0.02)+0.8064(k_{t-1}-1.4368)+0.2651(μ_{t-1}-0.3704)-0.0094ϵ_t+1.455
$\mu(s^t)$=-0.0510($\overline{\theta}_t$-0.02)+0.0738(k_{t-1}-1.4368)+0.5449(μ_{t-1}-0.3704)+0.0073ϵ_t+0.3662

Recession ($s_t = s_b$)

$c(s^t)$=0.0157($\overline{\theta}_t$-0.02)+0.1116(k_{t-1}-1.4368)-0.0691(μ_{t-1}-0.3704)-0.0030ϵ_t+0.2350
$l(s^t)$=0.0404($\overline{\theta}_t$-0.02)-0.0827(k_{t-1}-1.4368)+0.1584(μ_{t-1}-0.3704)-0.0060ϵ_t+0.2169
$\phi(s^t)$=-0.3778($\overline{\theta}_t$-0.02)+0.1305(k_{t-1}-1.4368)-2.3139(μ_{t-1}-0.3704)+0.0631ϵ_t+1.1091
$\lambda(s^t)$=-0.3725($\overline{\theta}_t$-0.02)+0.4266(k_{t-1}-1.4368)-1.8881(μ_{t-1}-0.3704)+0.0605ϵ_t+0.3071
$\overline{\theta}(s^{t+1})$=0.4099($\overline{\theta}_t$-0.02)+0.7998($k_{t-1}$-1.4368)-3.3490($\mu_{t-1}$-0.3704)-0.0660$\epsilon_t$+0.0465
$k(s^t)$=0.0343($\overline{\theta}_t$-0.02)+0.8064(k_{t-1}-1.4368)+0.2651(μ_{t-1}-0.3704)-0.0094ϵ_t+1.4183
$\mu(s^t)$=-0.0510($\overline{\theta}_t$-0.02)+0.0738(k_{t-1}-1.4368)+0.5449(μ_{t-1}-0.3704)+0.0073ϵ_t+0.3746

3. Numerical results

I carry out the impulse-response analysis and the MCMC simulation. In the impulse response analysis, I find the responses of the economies under the MS and Fixed policy regime to government spending shocks in each state. Each state (Boom / Recession) continues for 30 periods and government spending shock exists during period 5~10. Results help us to understand not only responses of policies or allocations to the government spending shock, but also differences between different policy regimes. In the MCMC simulation, I simulate 10000 periods without government spending shocks and the first 1000 periods are discarded to reduce the effect of the initial transition.

3.1. The interpretation of policy functions

By comparing policy functions, one can get a rough idea on differences between regimes. Since it is observable that slopes of policy functions are the same in both regimes, constant terms imply policy effects occurred by the difference in the policy regime. Ceteris paribus, economic agents under the MS policy regime consume more, supply labor more, save more and impose capital tax higher than the fixed policy case in a boom. In a recession, all trends are reversed. Furthermore, since absolute differences of constant term between policy functions are bigger under the MS policy regime, I guess that fluctuations of allocations and policies would be greater under the MS policy regime.

3.2. Impulse-Response analysis[20)]

Investigating figures in the Appendix D, I find a few interesting characteristics. Especially, an increase in the government spending[21)] brings about a decrease in consumption, labor supply and capital tax rate, while an increase in labor tax rate. It is notable that capital tax rate decreases if there exists positive government spending shock.

The more interesting features are found through examining differences between the economy under the different policy regimes; The MS policy and the Fixed policy in each state.

At first, capital accumulation is almost similar in both cases. In both states, the MS capital tax policy regime does not bring about notable differences in the capital accumulation compared to the state-independent capital tax policy regime.

Secondly, it is observable that capital tax rates under the MS policy regime is relatively high in a boom compared to the fixed case. If the economic condition is bad, the government under the MS policy restriction has no choice but to impose lower capital tax rate. This makes the government announce higher tax rate to prepare against

20) Figures are presented in appendixes

21) The government spending shock impact is more clear when the magnitude of shocks is larger. In fact, the capital tax rate shows explicit decreasing path during periods of shock when we increase the magnitude of shock.

the possible future tax cut. Meanwhile, labor tax rate shows reversed results; labor tax rate is higher (lower) in the fixed policy case in a boom (recession). Since we assume the economy without government bonds, this result is reasonable.

Thirdly, households under the MS policy regime work more (less) in a boom (recession) than the Fixed one. One plausible interpretation of this result is as follows. Since the labor tax rate is lower (higher) in the MS case due to the high capital tax in a boom (recession), people have a disincentive to work and decrease their supply of labor.

Lastly, one of the most interesting traits is found in the consumption. Consumption under the MS policy regime is higher in a boom and lower in a recession than the Fixed policy regime. Because the state-contingent capital tax policy is seen as an attempt to stabilize the economy against economic fluctuations, this is counter-intuitive at a glance. One may expect relatively low consumption with the high capital tax rate in a boom and high consumption with the low capital tax rate in a recession compared to the fixed policy case. I find the clue to this strange result by considering a secondary effect of the state-contingent capital tax policy. Since I consider the economy without government bonds, the decrease in the capital tax revenue should be replenished through the increase in the labor tax revenue. An increase in the labor tax rate influences not merely agent's labor supply decision but disposable income they can spend. The larger proportion of labor income in total income is or the higher labor tax rate is, the lower consumption people can enjoy.

The purpose of the state-contingent capital tax rate is generally considered to foster investment in the bad state and stabilize economy. However, I find contrary results; when the state-contingent capital tax rate is perfectly expected by households and implemented without government bonds, the state-contingent capital tax policy does not cause notable differences in the capital accumulation, but result in larger fluctuations in the economy. I summarize results in Table 4.

Table 4
The comparisons of the economy under different regimes in each state

	Boom	Recession
Consumption	MS $>$ Fixed	MS $<$ Fixed
Labor	MS $>$ Fixed	MS $<$ Fixed

Capital	MS ≒ Fixed	
Capital tax rate	MS > Fixed	MS < Fixed
Labor tax rate	MS < Fixed	MS > Fixed

3.3. Simulation results

I record Monte-Carlo simulation results in the table 5 and 6. First of all, I sort data by state of the economy and calculate elementary statistics, mean and standard deviation, of allocations and policies in each state. Results are presented in the Table 5.

As I already guessed in section 3.1 and 3.2, households averagely consume more, supply labor more, save more and the government levies higher capital tax rate and lower labor tax rate in a boom under the MS policy regime compared to the fixed policy one. Results are reversed in a recession. It is remarkable that households slightly save more (less) despite higher (lower) capital tax rate in a boom (recession) under the MS policy regime. As, in the real world, people tend to save less when they expect an increase in capital tax rate in the near future and save more in the contrary case, this finding accords with these actual tendencies.

In the table 6, I record simulation results from the whole data. It is interpretable as a long-run statistic. In the long run, differences between the MS policy and the Fixed policy disappear averagely. However, standard deviation of the economy under the MS policy regime is bigger than the fixed policy case because policies fluctuate more in the MS case and this leads to bigger fluctuations throughout the economy. People who rationally recognize the policy restriction respond actively by changing allocations a lot partly due to anticipation of policy revision and partly due to policy fluctuations. I also calculate life time utilities of each regime and find that utility under the fixed policy case is even 0.1% higher than the MS policy case[22]. That is, if agents rationally anticipate how the capital tax would be imposed and actively respond to that, effects of state-contingent capital tax policy are diminished. Specifically, if the government is not able to issue bonds or is obliged to keep the balanced budgets, state-contingent capital

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22) $\dfrac{\sum^{\infty} \beta^t U_{fixed}}{\sum^{\infty} \beta^t U_{MS}} = 1.001$

tax policy results in larger fluctuations in the economy over time without any increase of utility in the long run and policy attempts to stabilize the economy and stimulate investment would become almost ineffective.

Table 5
Basic statistics on the economy under different policy regimes in each economic state

Boom	Mean		Standard deviation	
	MS	Fixed	MS	Fixed
Consumption	0.2493	0.2489	0.0058	0.0057
Labor supply	0.2310	0.2307	0.0012	0.0013
Capital	1.4868	1.4862	0.0468	0.0469
Capital tax rate	-0.0005	-0.0191	0.0560	0.0502
Labor tax rate	0.4971	0.4983	0.0048	0.0045
Recession	**Mean**		**Standard deviation**	
	MS	Fixed	MS	Fixed
Consumption	0.2292	0.2296	0.0059	0.0058
Labor supply	0.2216	0.2219	0.0012	0.0013
Capital	1.3862	1.3868	0.0474	0.0475
Capital tax rate	-0.0003	0.0183	0.0558	0.0507
Labor tax rate	0.5137	0.5125	0.0052	0.0049

Table 6
Basic statistics on the economy under the different policy regimes

	Mean		Standard deviation	
	MS	Fixed	MS	Fixed
Consumption	0.2391	0.2391	0.0116	0.0111
Labor supply	0.2263	0.2263	0.0049	0.0046
Capital	1.4355	1.4355	0.0685	0.0681
Capital tax rate	-0.0009	-0.0006	0.0558	0.0538
Labor tax rate	0.5055	0.5055	0.0096	0.0084

4. Conclusion

In this paper, I investigate effects of the state-contingent capital tax policy compared to the state-independent capital tax policy using the dynamic Ramsey framework. To address this problem, I use the MSDSGE perturbation method that is able to capture the Markov-switching effects even with the first-order approximation. The main findings of this paper are as follows.

In the economy without government bonds,

- In the MS policy regime, the government announces higher capital tax rate than the economy under the fixed policy regime to prepare against the possible future tax cut. Furthermore, in the economy without government bonds, fluctuations in the capital tax rate result in the opposite trend of the labor tax rate which also significantly influences agent's decisions on allocations.
- The fluctuation of allocations and policies would be greater under the MS policy regime than the fixed policy regime. To be concrete, households averagely consume more, supply labor more, save more and the government levies higher capital tax rate and lower labor tax rate in a boom under the MS policy regime compared to the fixed policy one. Results are reversed in a recession. It is notable that households invest less (more) despite lower (higher) announced capital tax rate because they know the announced capital tax rate would be adjusted depending on the realized state of the economy in the next period.
- Contrary to the widespread belief that state-contingent capital tax policy would be helpful to stabilize the economy, the state-contingent capital tax policy in my framework only results in larger fluctuations of allocations and policies without any notable differences in the capital accumulation. Furthermore, the total utility is even higher under the fixed policy regime. This result implies that if the government cannot issue bonds freely or should keep balanced-budget, the state-contingent capital tax policy is by and large inefficient.

In this paper, I figure out effects of the state-contingent capital tax policy. While previous papers on 'Time-inconsistent policy' generally focus on finding out the inefficiency of the active re-optimization attempt by the government, I place major emphasis on examining ineffectiveness of the periodic policy revision depending on the state of the economy. Also, I try to apply household's policy expectations to the typical dynamic Ramsey problem unlike previous Ramsey problem studies that assume households just take policy as given.

In a broad sense, this paper supports the famous argument 'Rules rather than discretion'. That is, the policy which adhere to the planned capital tax rate can be better than the policy that frequently revise announced capital tax rate. Furthermore, this paper implies that the tax policy should be independent not merely from the realized state of the economy, but also from the outside pressures such as public opinion or political incentives. Before finishing, it is worth to mention that considering results under the economy with government bonds would be an interesting further research topic.

References

Benigno, P., & Woodford, M. (2006). Optimal taxation in an RBC model: A linear-quadratic approach. Journal of Economic Dynamics and Control, 30(9), 1445-1489.

Chamley, C. (1986). Optimal taxation of capital income in general equilibrium with infinite lives. Econometrica: Journal of the Econometric Society, 607-622.

Chari, V. V., Christiano, L. J., & Kehoe, P. J. (1991). Optimal fiscal and monetary policy: Some recent results. Journal of Money, Credit and Banking, 519-539.

Chari, V. V., Christiano, L. J., & Kehoe, P. J. (1991). Technical appendix to optimal fiscal policy in a stochastic growth model. Working Paper 158, Federal Reserve Bank of Minneapolis.

Chari, V. V., Christiano, L. J., & Kehoe, P. J. (1993). Optimal fiscal policy in a business cycle model (No. w4490). National Bureau of Economic Research.

Chari, V. V., Christiano, L.J., and Kehoe P.J. (1994). " Optimal Fiscal Policy in a Business Cycle Model. " The Journal of Political Economy, 102(4), 617-652.

Chari, V. V., Christiano, L. J., & Kehoe, P. J. (1995). Policy analysis in business cycle models. Frontiers of Business Cycle Research, 357-391.

Chari, V. V., & Kehoe, P. J. (1999). Optimal fiscal and monetary policy. Handbook of macroeconomics, 1, 1671-1745.

do Valle Costa, O. L., Fragoso, M. D., & Marques, R. P. (2006). Discrete-time Markov jump linear systems. Springer.

Debortoli, D., & Nunes, R. (2010). Fiscal policy under loose commitment. Journal of Economic Theory, 145(3), 1005-1032.

Fischer, S. (1980). Dynamic inconsistency, cooperation and the benevolent dissembling government. Journal of Economic Dynamics and Control, 2, 93-107.

Foerster, A., Rubio-Ramirez, J., Waggoner, D., & Zha, T. (2010). Perturbation methods for markov-switching models. working paper.

Foerster, A. T. (2011). Essays on Markov-Switching Dynamic Stochastic General Equilibrium Models (Doctoral dissertation, Duke University).

Foerster, A., Rubio-Ramirez, J., Waggoner, D., & Zha, T. (2013). Perturbation methods for Markov-switching DSGE models. Centre for Economic Policy Research.

Kim, J., & Kim, S. H. (2007). Two pitfalls of linearization methods. Journal of Money, Credit and Banking, 39(4), 995-1001.

Klein, P., Krusell, P., & Rios-Rull, J. V. (2008). Time-consistent public policy. The Review of Economic Studies, 75(3), 789-808.

Klein, P., & Ríos‐Rull, J. V. (2003). Time‐consistent optimal fiscal policy*. International Economic Review, 44(4), 1217-1245.

Kydland, F. E., & Prescott, E. C. (1977). Rules rather than discretion: The inconsistency of optimal plans. The Journal of Political Economy, 473-491.

Lucas Jr, R. E., & Stokey, N. L. (1983). Optimal fiscal and monetary policy in an economy without capital. Journal of monetary Economics, 12(1), 55-93.

Schmitt-Grohe, S., & Uribe, M. (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function. Journal of economic dynamics and control, 28(4), 755-775.

Stockman, D. R. (2001). Balanced-budget rules: Welfare loss and optimal policies. Review of Economic Dynamics, 4(2), 438-459.

Strotz, R. H. (1955). Myopia and inconsistency in dynamic utility maximization. The Review of Economic Studies, 165-180.

Tesfatsion, L. (1986). Time inconsistency of benevolent government economies. Journal of Public Economics, 31(1), 25-52.

Zhu, X. (1992). Optimal fiscal policy in a stochastic growth model. Journal of Economic Theory, 58(2), 250-289.

Appendixes

Appendix A

The proof of the proposition 1

① Add household budget constraint and government budget constraint to obtain $c(s^t)+g(s^t)+k(s^t)=F_k(s^t)k(s^{t-1})+F_l(s^t)l(s^t)+(1-\delta)k(s^{t-1})$.

Since we assume constant returns to scale technology, we are able to substitute $F_k(s^t)k(s^{t-1})+F_l(s^t)l(s^t)$ with $F(k(s^{t-1}),l(s^t),s^t)$. Applying this to the above formula, resource constraint is obtained.

Using (3) to substitute for after-tax wage in the household budget constraint, we would get the quasi-implementability constraint[23].

Households expect the capital tax rate based on the pre-announced capital tax rate. In addition, the benevolent government announces the next period capital tax rate in accordance with the household's optimal saving condition, Euler equation. Thus, It is trivial that household optimal decision and optimal capital tax policy should satisfy the Euler equation.

Initial condition follows from quasi-Implementability constraint evaluated at s^0, using (3) and (5) at s^0 as well.

② Set $r(s^t)=F_k(s^t)$ and $w(s^t)=F_l(s^t)$ to satisfy (5). Choose $\{\tau(s^t)\}$ to satisfy (3). The initial condition gives us government budget constraint at t=0. Subtracting government budget constraint from resource constraint gives household budget constraint.

□

23) I call this constraint quasi-implementability constraint because unlike typical implementability constraint, capital tax rate term is not substituted with allocation term and still remained in the constraint.

Appendix B
MSDSGE perturbation method

I borrow notations, formulas and some nice explanations helpful for better understanding of this method from Foerster et al (2013). As, in this section, I only summarize and describe some essential parts which is necessary to understand the approach in this paper, the explanation of this part is imperfect and I strongly recommend readers interested in this method read Foerster et al (2013).

Before initiating the detailed explanation, let's define notations first as follows

x_t: (Exogenous and endogenous) predetermined variable

y_t: non-predetermined(control) variable

ϵ_t: i.i.d exogenous shock with $E_{t-1}\epsilon_t = 0$

θ_t: Markov-switching parameters

n_x: The number of predetermined variable[24)]

χ: (MS) perturbation parameter

Given (x_{t-1} , ϵ_t , θ_t), the equilibrium conditions for MSDSGE models have the general form

$$E_t f(y_{t+1}, y_t, x_t, x_{t-1}, \chi\epsilon_{t+1}, \epsilon_t, \theta_{t+1}, \theta_t) = 0_{n_x + n_y} \tag{7}$$

Step 1 : Partition the vector θ_t of Markov-switching parameters into two sub-vectors. The first part contains the sub-vector of parameters that would influence the steady-state in the constant parameter case and thus is a function of the perturbation parameter χ. The second part contains the sub-vector of all remaining parameters that would not affect the steady-state in the constant parameter case. I denote the first sub-vector by θ_{1t} and the second sub-vector by θ_{2t}. Then we define $\theta_{1t}(\chi, s_t) = \overline{\theta_1} + \chi\, \widehat{\theta_1}(s_t)$ and $\theta_2(\chi, s_t) = \widehat{\theta_2}(s_t)$ for all s_t[25)], where $\overline{\theta_1}$ is the ergodic mean across θ_{1t} and $\widehat{\theta_1}(s_t)$ is a deviation of θ_{1t} from $\overline{\theta_1}$ in state s_t.

Step 2 : Define the steady-state of the MSDSGE model and solutions.

$f(y_{ss}, y_{ss}, x_{ss}, x_{ss}, 0_{n_\epsilon}, 0_{n_\epsilon}, \overline{\theta_1}, \widehat{\theta_2}(s_{t+1}), \overline{\theta_1}, \widehat{\theta_2}(s_t)) = 0_{n_x + n_y}$ holds for all s_{t+1} and s_t.

---

24) n_y , n_ϵ and n_θ means the number of each variables, respectively.

25) Any other functional form, so long as $\theta_1(0, s_t) = \overline{\theta_1}$ holds for all s_t, will be valid.

Let's model solutions take these form
$y_t = g(x_{t-1}, \epsilon_t, \chi, s_t)$, $y_{t+1} = g(x_t, \chi\epsilon_{t+1}, \chi, s_{t+1})$ and $x_t = h(x_{t-1}, \epsilon_t, \chi, s_t)$.

In general, we do not know the explicit form of g and h. Thus, we need to approximate them by Taylor expansions around the steady-state. In this paper, I only present first-order Taylor expansions, but this method can be easily expanded to higher orders. Our ultimate purpose is to obtain these approximated functions
$g(x_{t-1}, \epsilon_t, \chi, s_t) = y_{ss} + D_x g_{ss}(s_t)(x_{t-1} - x_{ss}) + D_\epsilon g_{ss}(s_t)\epsilon_t + D_\chi g_{ss}(s_t)\chi$ and
$h(x_{t-1}, \epsilon_t, \chi, s_t) = x_{ss} + D_x h_{ss}(s_t)(x_{t-1} - x_{ss}) + D_\epsilon h_{ss}(s_t)\epsilon_t + D_\chi h_{ss}(s_t)\chi$
, where $D_x g_{ss}(s_t)$ represents first partial derivatives of g with respect to x_{t-1} evaluated at the steady state for all s_t. Other terms are similar.

Given the steady-state, I obtain $y_{ss} = g(x_{ss}, 0_{n_\epsilon}, 0, s_t)$ and $x_{ss} = h(x_{ss}, 0_{n_\epsilon}, 0, s_t)$ for all s_t and similarly at s_{t+1}.

Using these functions, the function f is written as

$$
\begin{aligned}
&F(x_{t-1}, \chi\epsilon_{t+1}, \epsilon_t, \chi, s_{t+1}, s_t) \\
&= f(g(h(x_{t-1}, \epsilon_t, \chi, s_t), \chi\epsilon_{t+1}, \chi, s_{t+1}), g(x_{t-1}, \epsilon_t, \chi, s_t), h(x_{t-1}, \epsilon_t, \chi, s_t), x_{t-1}, \chi\epsilon_{t+1}, \epsilon_t, \theta(\chi, s_{t+1}), \theta(\chi, s_t))
\end{aligned}
$$

for all $x_{t-1}, \epsilon_t, \epsilon_{t+1}$ and s_t. Then, (7) is written as

$$
\begin{aligned}
G(s_t) &= E_t f(y_{t+1}, y_t, x_t, x_{t-1}, \chi\epsilon_{t+1}, \epsilon_t, \theta_{t+1}, \theta_t) \\
&= \sum_{s_{t+1}} p(s_{t+1}|s_t) \int F(x_{t-1}, \chi\epsilon_{t+1}, \epsilon_t, \chi, s_{t+1}, s_t)\mu(\epsilon')d\epsilon' \; = \; 0_{n_y + n_x}
\end{aligned}
$$

Step 3 : Obtaining the derivatives of x_{t-1}, ϵ_t and χ

I only describe the process obtaining the derivatives of x_{t-1} in here, but the basic principle to derive other derivatives is similar.

Taking first partial derivatives of G with respect to x_{t-1} produces the expression for $D_x G(s_t)$

$$
D_x G(s_t) = \sum_{s_{t+1}} p(s_{t+1}|s_t) \int \begin{pmatrix} D_{y_{t+1}} f_{ss}(s_{t+1}, s_t) D_x g_{ss}(s_{t+1}) D_x h_{ss}(s_t) \\ + D_y f_{ss}(s_{t+1}, s_t) D_x g_{ss}(s_t) \\ + D_{x_{t+1}} f_{ss}(s_{t+1}, s_t) D_x h_{ss}(s_t) + D_x f_{ss}(s_{t+1}, s_t)) \end{pmatrix} \mu(\epsilon_{t+1}) d\epsilon_{t+1}
$$

for all s_t. Taking $\int \mu(\epsilon_{t+1}) d\epsilon_{t+1} = 1$ and the fact that ϵ_t is an i.i.d shock into account, one can simplify the above expression to

$$
D_x G(s_t) = \sum_{s_{t+1}} p(s_{t+1}|s_t) \begin{pmatrix} D_{y_{t+1}} f_{ss}(s_{t+1}, s_t) D_x g_{ss}(s_{t+1}) D_x h_{ss}(s_t) \\ + D_y f_{ss}(s_{t+1}, s_t) D_x g_{ss}(s_t) \\ + D_{x_{t+1}} f_{ss}(s_{t+1}, s_t) D_x h_{ss}(s_t) + D_x f_{ss}(s_{t+1}, s_t)) \end{pmatrix}
$$

for all s_t. Rearranging the above expression for each s_t leads to

$$D_x G(s_t) = \sum_{s_{t+1}} p(s_{t+1}|s_t) \begin{pmatrix} (D_{y_{t+1}} f_{ss}(s_{t+1}, s_t) D_x g_{ss}(s_{t+1}) + D_{x_{t+1}} f_{ss}(s_{t+1}, s_t)) D_x h_{ss}(s_t) \\ + D_y f_{ss}(s_{t+1}, s_t) D_x g_{ss}(s_t) \\ + D_x f_{ss}(s_{t+1}, s_t) \end{pmatrix}$$

Writing the above expression in matrix form leads to a system of quadratic equations as follows.

$$A(s_t) \begin{bmatrix} I_{n_x} \\ D_x g_{ss}(s_1) \\ \vdots \\ D_x g_{ss}(s_{n_x}) \end{bmatrix} D_x h_{ss}(s_t) = B(s_t) \begin{bmatrix} I_{n_x} \\ D_x g_{ss}(s_t) \end{bmatrix} \tag{8}$$

where $A(s_t) = [\sum p_{s_t, s_{t+1}} D_{x_{t+1}} f_{ss}(s_{t+1}, s_t) \;\; p_{s_t, s_1} D_{y_{t+1}} f_{ss}(s_1, s_t) \; \cdots \; p_{s_t, s_{n_s}} D_{y_{t+1}} f_{ss}(s_{n_s}, s_t)]$

and $B(s_t) = -\sum_{s_{t+1}} p_{s_t, s_{t+1}} [D_x f_{ss}(s_{t+1}, s_t) \;\; D_y f_{ss}(s_{t+1}, s_t)]$ for all s_t

If we are able to solve n_s systems of quadratic equations (8), we obtain $\{D_x g_{ss}(s_t), D_x h_{ss}(s_t)\}$ for all s_t. After this stage, deriving $\{D_\epsilon g_{ss}(s_t), D_\epsilon h_{ss}(s_t)\}$ and $\{D_\chi g_{ss}(s_t), D_\chi h_{ss}(s_t)\}$ for all s_t is just solving a linear equation, which is far more easy once we solve above equations.

This quadratic system has, in general, many solutions. Each solution corresponds to a different first-order approximation. Finding all the solutions to this quadratic system is the toughest part but is crucial to ascertain how many of them imply stable approximations.

Once we obtain $\{D_x g_{ss}(s_t), D_x h_{ss}(s_t)\}$ for all s_t, we should check stability of the solution. While we check whether the solution satisfies the Blanchard-Khan condition in the constant parameter case, we should consider other concepts of stability in the Markov-switching case. In Foerster et al (2010, 2011, 2013), they suggest mean square stability(MSS) criterion defined in Costa et al (2006). Specifically, the MSS requires checking whether the following matrix has all its eigenvalues inside the unit circle

$$T = (P' \otimes I_{n_x^2}) \Gamma$$

where P is the transition matrix, $I_{n_x^2}$ is an $n_x^2 \times n_x^2$ identity matrix and

$$\Gamma = \begin{bmatrix} D_x h_{ss}(s_1) \otimes D_x h_{ss}(s_1) & \cdots & 0_{n_x^2 \times n_x^2} \\ \vdots & \ddots & \vdots \\ 0_{n_x^2 \times n_x^2} & \cdots & D_x h_{ss}(s_{n_s}) \otimes D_x h_{ss}(s_{n_s}) \end{bmatrix}$$

If only one solution satisfies the MSS criterion, the model has a unique stable first-order approximation. If there is more than one solution that satisfies the MSS criterion, the model has multiple stable first-order approximations. If none of the solutions satisfies the MSS criterion or if there is no solution of (8), the model doesn't have any stable first-order approximations.

One of the powerful properties of the MSDSGE perturbation method is that the first-order approximation derived through this method is not necessarily certainty equivalent and capture Markov-switching effects. So, under some conditions, we are able to analyse economic agent's response to risk or uncertainty even with the first-order approximation.

After finishing the entire process, we finally obtain following linear functions.

$$g(x_{t-1},\epsilon_t,\chi,s_t) = g_{ss} + D_x g_{ss}(s_t)(x_{t-1}-x_{ss}) + D_\epsilon g_{ss}(s_t)\epsilon_t + D_\chi g_{ss}(s_t)\chi$$
$$h(x_{t-1},\epsilon_t,\chi,s_t) = h_{ss} + D_x h_{ss}(s_t)(x_{t-1}-x_{ss}) + D_\epsilon h_{ss}(s_t)\epsilon_t + D_\chi h_{ss}(s_t)\chi$$

The last term which does not exist in the constant parameter model is the term that captures the Markov switching effect.

Generally, we fix the MS parameter to the certain value, treat it as a constant parameter and approximate the system. While this approach is convenient, one can only analyse average effects of MS parameter. However, by assuming MS parameter as a function of the perturbation parameter χ, we can derive policy functions which capture Markov-switching effects.

The necessary and sufficient conditions to guarantee non-certainty equivalent of first-order approximations to the solution of an MSDSGE model is as follows

- $\widehat{\theta}_1(s_t) \neq 0_{n_{\theta_1}}$
- $\sum_{s_{t+1}} p(s_{t+1}|s_t)\left(D_{\theta_{t+1}} f_{ss}(s_{t+1},s_t) D\theta_{ss}(s_{t+1}) + D_{\theta_t} f_{ss}(s_{t+1},s_t) D\theta_{ss}(s_t)\right) \neq 0_{n_x+n_y}$ for

some s_t[26].

Proof) See Foerster et al (2013)

26) At first, I set up the model that $\overline{\theta}(s^t)$ is multiplied by $a(s_t)$. That is, capital tax rate take this form ; $a(s_t)\times\overline{\theta}(s^t)$. If we define capital tax rates in this way, it doesn't satisfy the second condition. Specifically, since the steady state value of $\theta(s_t)$ is zero, $D_{\theta_{t+1}} f_{ss}(s_{t+1},s_t) D_{\theta_{ss}}(s_{t+1}) + D_{\theta_t} f_{ss}(s_{t+1},s_t) D_{\theta_{ss}}(s_t)$ is zero for all s_t

Appendix C
MSDSGE perturbation method solutions

The following table presents $D_\theta h_{ss}(s_1)\ of\ \theta$ and $D_\theta g_{ss}(s_1)\ of\ c$ only. Only solution 3 satisfies MSS criterion.

Table 7
MSDSGE perturbation method solutions

	$D_\theta g_{ss}(s_1)\ of\ c$	$D_\theta h_{ss}(s_1)\ of\ \theta$
1	0.007449609918	-0.1504002545
2	-0.00002099531343	-0.0794451526
3	0.0157488537	0.4098638582
4[27)]	0.0157488537	0.4098638582
5	0.009978248628	0.8497709916
6	0.005059964943	0.07820757031
7	0.00462241618	0.834512362
8	0.03374274325	0.9681807
9	0.0002865241021	-0.06990339853

27) The recorded values are the same with (3). However, other values omitted in here are different.

Appendix D
Impulse-response figures

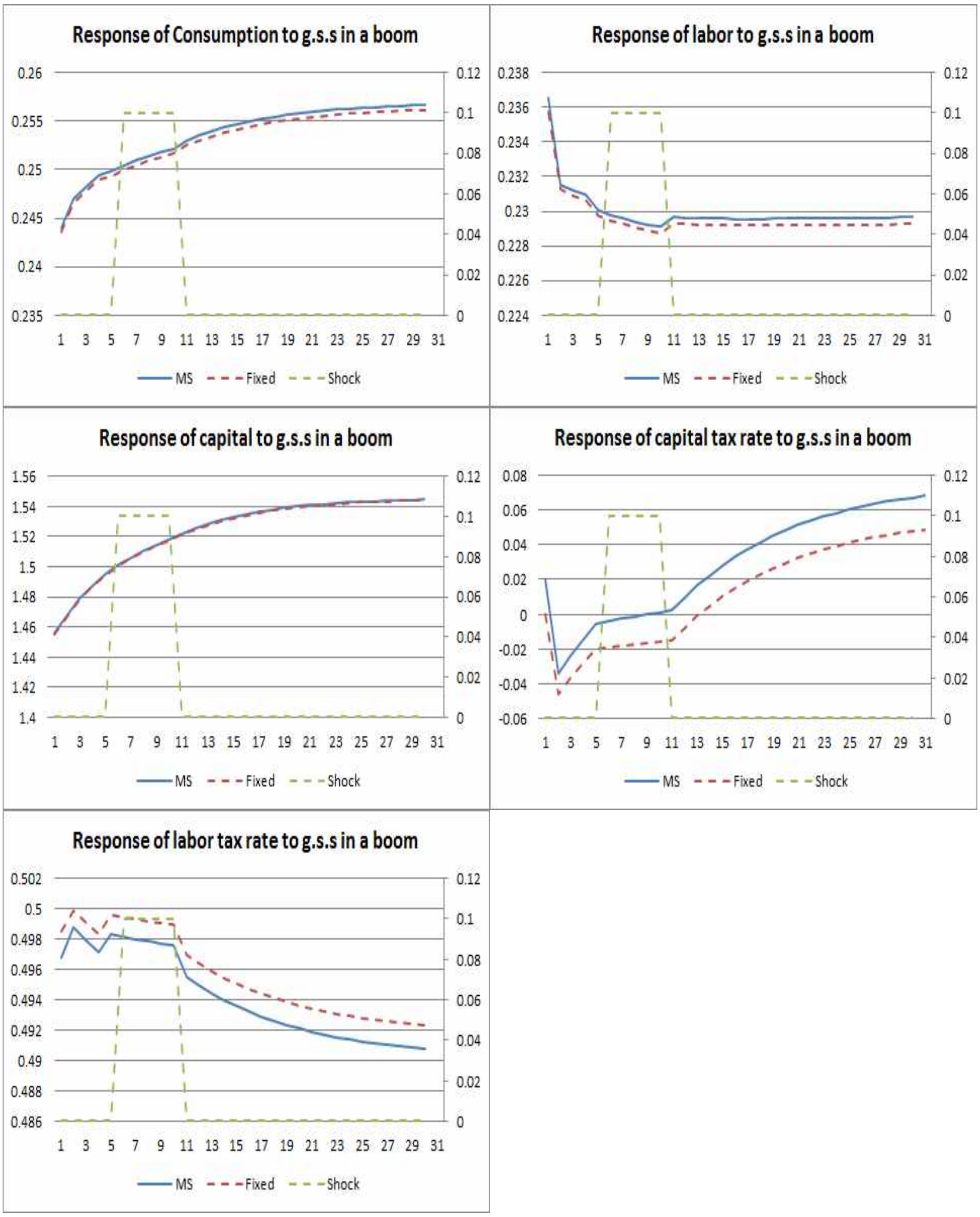

Figure 1 Impulse-response in a boom

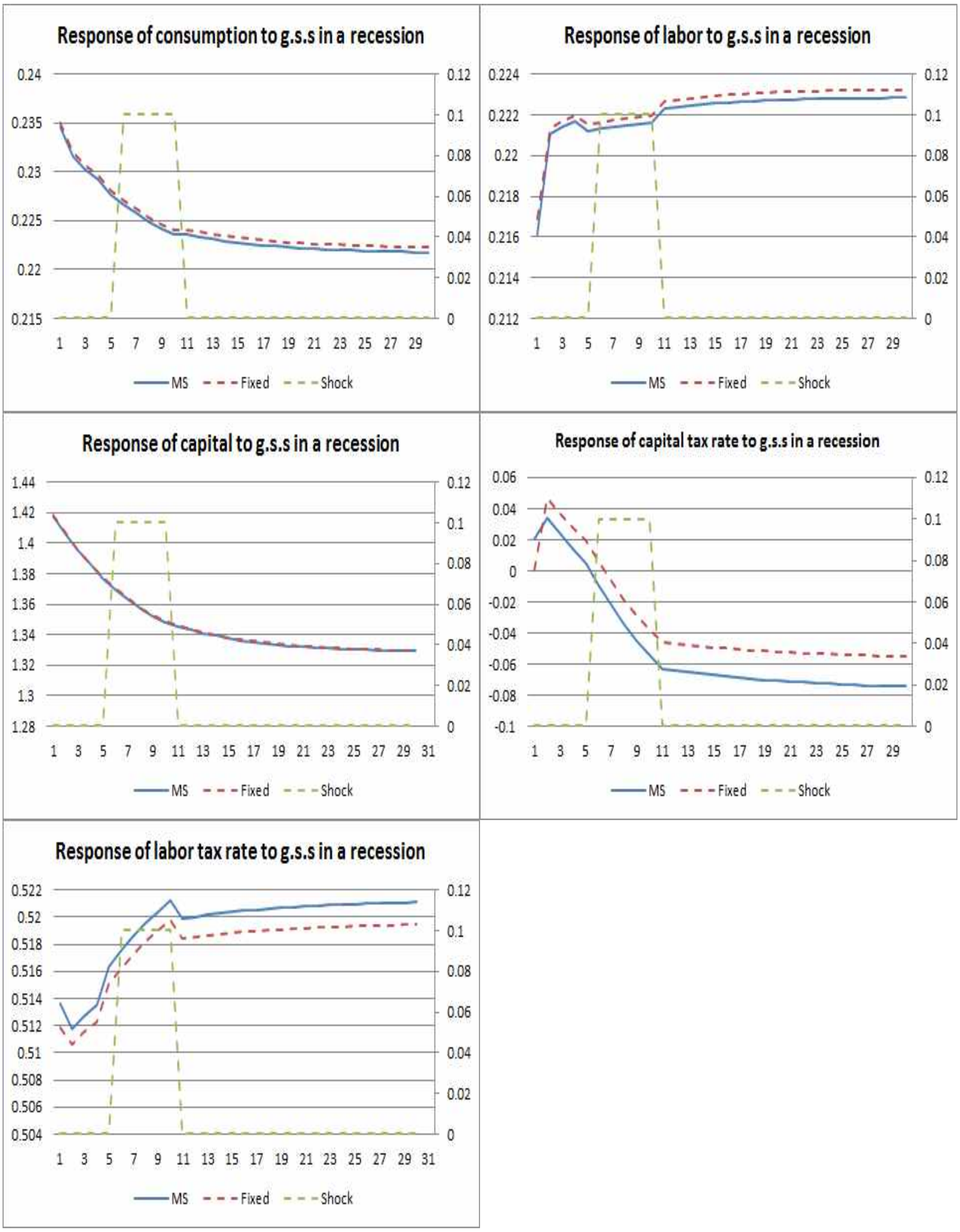

Figure 2 Impulse-response in a recession

국문초록

경기 조정적인 자본세 정책의 효과성 분석

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본 논문은 정부가 경기 동조적인 자본세 정책을 시행하고, 가계는 이러한 정부의 정책 시행 방식에 대한 인식하에서 실제 시행될 자본세율을 미리 예상하여 이를 의사결정에 반영하는 경우, 해당 경제에서의 경기 동조적 정책의 효과는 어떠할 것인지 동태적 최적 조세 문제의 방법론을 활용하여 살펴보고 있다. 구체적으로, 정부는 현재기에 다음기의 자본세율을 미리 공표하지만 다음기의 경기상황이 좋지 않은 경우 공표했던 자본세율을 그대로 시행하지 않고 특정 정도만큼 자본세율을 낮춰주는 정책 제약 혹은 일종의 정책 관행을 갖는다. 한편, 가계는 정부가 이와 같이 행동할 것임을 인식하고 있으며, 정부의 발표 세율을 바탕으로 다음기의 실제 세율을 예상하고 이를 의사결정에 반영한다. 본 논문은 이러한 상황 하에서의 정책 효과를, 정부가 실현되는 경기가 어떠한가에 관계없이 공표하였던 자본세율을 그대로 시행하고 역시 마찬가지로 가계는 이와 같은 정부의 정책 시행 방식에 대한 인식하에서 의사 결정을 내리는 경제의 경우와 비교 분석함으로써 경기 동조적인 자본세 정책의 효과를 살펴보고 있다. 본 논문은, 정부가 정부 채권을 발행하지 못하거나 균형 예산을 맞춰야 하는 등의 정책 제약 하에 있는 경우, 본 논문의 모형과 방법론 하에서는 경기 동조적인 자본세 정책이 일반의 예상과는 달리 효용증가나 투자 촉진 등의 효과 없이 단지 경제의 변동성을 더 크게 만들 뿐이고, 이로 인해 정책의 당초 의도와는 달리 장기적으로는 경제에 오히려 더 부정적인 영향을 야기할 수 있음을 보여주고 있다. 나아가, 본 논문의 결론은, 경기 상황뿐만 아니라 정치적 유인, 사회적 여론 등으로 인한 주기적인 자본 소득 감세정책이 경우에 따라서는 경제에 오히려 부정적인 결과를 발생 시킬 수 있음을 보여주고 있다.

주요어 : 동태적 램지 문제; 최적 조세; 마르코프-전환 매개변수
경기 동조적 자본세 정책; MSDSGE Perturbation 근사 방법

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