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Non-manipulability and Pareto-efficiency of DASO Rules in Landing Slot Problems

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권 혁 준

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지도교수 전 영 섭

이 논문을 경제학 석사학위논문으로 제출함

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권 혁 준

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부위원장 전 영 섭 (인)

위 원 김 진 우 (인)

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Non-manipulability and Pareto-efficiency of DASO Rules in Landing Slot Problems[1]

Hyukjun Kwon

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Abstract

We consider the landing slot problem formulated by Schummer and Vohra(2013), which is concerned with reallocating landing slots to airlines in inclement weather. Schummer and Abizada(2016) introduce Deferred Acceptance with Self-Optimization(DASO) rules and show that these rules are individually rational, non-wasteful, non-manipulable via weights, and non-manipulable by slot hiding. However, they are harmfully manipulable by intentional flights delay and not Pareto-efficient. We show that non-harmful-manipulability by intentional flights delay of these rules can be achieved by restricting the priority structure. We also show that they are Pareto-efficient if and only if all flight weights are identical for any given airline.

1. Introduction

When the weather becomes inclement, airports have to reschedule the assignment of flights into the landing slots. However, after the rescheduling, it is possible that flight cancelations take place, newly vacating some of landing slots. In this situation of excess resources, a centralized planner(e.g. Federal Aviation Administration(FAA) in the U.S.) has to reallocate slots to airlines properly.

Schummer and Vohra(2013) study the current reallocating algorithm(the Compression algorithm) and show that it is weakly non-manipulable by intentional flights delay but manipulable by slot destruction. Moreover, it does not return a landing schedule in the weak core. So, they propose a new algorithm(the TradeCycle algorithm) based on the Top Trading Cycle algorithm(Shapley and Scarf(1974)). It is weakly non-manipulable by intentional flights delay and returns

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a landing schedule in the weak core. However, it is vulnerable to the intentional slot destruction.

Schummer and Abizada(2016) extend this problem to complete airline preferences by adopting the weight structure. They focus on three manipulabilities in landing slot problems and prove that deferred acceptance with self-optimization(henceforth DASO) rules which are based on the well known deferred acceptance algorithm(Gale and Shapley(1962)) are non-manipulable via weights, non-manipulable by slot hiding. Furthermore, they are individually rational and non-wasteful. However, they are manipulable by intentional flights delay and not Pareto-efficient.

Recovering drawbacks of the deferred acceptance algorithm based rules has been studied in various literatures. Ergin(2002) defines the cycle in the priority structure and prove that the deferred acceptance algorithm based rule is Pareto-efficient and group strategyproof if and only if there is no cycle in the priority structure. Kesten(2012) adopts the concept of the cycle to school choice problems. He shows that the student-optimal stable mechanism(SOSM) which is based on the deferred acceptance algorithm is immune to manipulation via capacities if and only if the priority structure is acyclical.

This paper also adopts the concept of the cycle but the acyclicity condition has to be redefined since the model of landing slot problems differs from those of Ergin(2002) and Kesten(2012). In this paper, we modify the definition of cycle(We call the priority structure *acyclical* if it has no *cycle*).

We also introduce non-harmful-manipulability which is the weakened version of non-manipulability. It requires a rule to be robust to the harmful manipulation by intentional flights delay. The word "harmful" means that while an airline profitably manipulate an earlist arrival time of its flights, it results in another airline to be ousted from a slot to a later one.

We show that DASO rules are non-harmfully-manipulable by intentional flights delay if and only if the priority structure is *acyclical*. However, *acyclicity* alone cannot fully achieve non-manipulability by intentional flights delay. To do so, a condition on the weight structure also has to be imposed. It is that the maximum weight ratios(MWR) are close enough to 1 for all airlines. We show that if the priority structure is acyclical, then DASO rules are non-manipuable by intentional flights delay if and only if MWR are close enough to 1 for all airlines.

Finally, we also show that, to achieve Pareto-efficiency, we have to impose a stricter condition to the weight structure: MWR have to be equal to 1 for all airlines, i.e., for any given airline, all flights must have an identical weight.

Section 2 explains the formal model, including definitions, properties and the DASO algorithm. Results are explained in Section 3 and summarized and debated in Section 4.

2. Model

2.1 Definitions

There is a finite set of landing slots $S = \{1, 2, 3, \ldots, |S|\}$ and labels of these slots are representations of its time: e.g. slot 2 is two units of time earlier than slot 4. There is a finite set of airlines $\mathcal{A}$ and each airline $A \in \mathcal{A}$ has its own finite set of flights F_A. Denote the set of all flights by F. Flights must be owned by exactly one airline, i.e. there are no two distinct airlines $A, B \in \mathcal{A}$ such that $F_A \cap F_B \neq \emptyset$. We assume that $|F| \leq |S|$ since all flights must be assigned to exactly one slot. The earliest arrival time of flight $f \in F_A$ is denoted by $e_f \in S$. All flights can be assigned to a slot s only if $e_f \leq s$. Note that f cannot be assigned to s' such that $s' < e_f$ because f will not arrive at the time of s'. We say s is feasible for f and s' is infeasible for f.

Each flight $f \in F_A$ has a weight $w_f > 0$ meaning the cost incurred by one unit of time in the air: e.g. if f is moved down from slot 2 to slot 5, then f has to stay in the air 3 units of time longer. Hence, the increase of the total cost for the airline owning f is $3w_f$. We define $w_A^{min} = min_{f \in F_A} w_f$ and $w_A^{max} = max_{f \in F_A} w_f$. We also define the maximum weight ratio of an airline A(denoted by MWR_A) as $\frac{w_A^{max}}{w_A^{min}}$. Consequently, we have $MWR_A \geq 1$.

A landing schdule is an injective function $\Pi : F \to S$ such that $\Pi(f) \geq e_f$, for all $f \in F$. A slot ownership function is $\Phi : S \to \mathcal{A}$ such that for each $A \in \mathcal{A}$ and each $f \in F_A \Rightarrow \Phi(\Pi(f)) = A$. We say a pair (Π, Φ) is an assignment, and (Π^0, Φ^0) is an initial assignment.

We now define the preference relation of airlines over landing schedules. Given a weight structure $(w_f)_F$, for all landing schedules $\Pi, \Pi' : \Pi \succeq_A^w \Pi' \Leftrightarrow \sum_{f \in F_A} w_f(\Pi(f) - \Pi'(f)) \leq 0$.

An instance $I = (S, \mathcal{A}, (F_A)_{\mathcal{A}}, (e_f)_F, (w_f)_F, \Pi^0, \Phi^0)$ summarizes our model.

A rule(rescheduling rule) φ is a function associates each instance I to a landing schedule Π. $\varphi_f(I)$ is a slot a flight f is assigned to under I.

2.2 Properties

The basic goal of any rules in landing slot problem is preventing the waste of vacant landing slots. However, it does not mean that we have to fill out all of slots. We only want to fully utilize desirable vacant slots. If there are no airlines want to use a vacant slot, we may let it unfiled. This is the non-wastefulness property. The formal definition of non-wastefulness follows:

A rule φ is **non-wasteful** if for each instance $I = (S, \mathcal{A}, (F_A)_{\mathcal{A}}, (e_f)_F, (w_f)_F, \Pi^0, \Phi^0)$ there is no flight $f \in F$ and no slot $s \in S$ such that (i) $e_f \leq s < \varphi_f(I)$ and (ii) there is no $g \in F \setminus \{f\}$ such that $\varphi_g(I) = s$.

Though non-wastefulness is a reasonable base of efficiency of rules, we might think of stronger notion of efficiency. It is Pareto-efficiency. Schummer and Abizada(2016) prove that DASO rules which will be explained in the next subsection are not Pareto-efficient. We show how to achieve it in section 3. Before that, note first the following formal definition of Pareto-efficiency.

A rule φ is **Pareto-efficient** if for each instance $I = (S, \mathcal{A}, (F_A)_{\mathcal{A}}, (e_f)_F, (w_f)_F, \Pi^0, \Phi^0)$ there is no landing schedule Π' such that (i) for all $A \in \mathcal{A}, \Pi' \succeq_A^w \varphi(I)$, and (ii) for some $A \in \mathcal{A}, \Pi' \succ_A^w \varphi(I)$.

Schummer and Abizada(2016) consider three incentives in landing slots problems. We can envision airlines misreporting earliest arrival times or weights of their flights to reduce their costs, in other words, to attain more favorable set of slots. Furthermore they can destruct or hide their vacant slots to be better off. Schummer and Abizada(2016) show that DASO rules are robust to those manipulations except the manipulation by misreporting earliest arrival times. Though there are 2 kinds of misreporting earliest arrival times, i.e. misreporting the time to be earlier of later, we only consider misreporting the earliest arrival time to be delayed. The reason for not considering the contrary misreporting is that even in the case that misreporting airline succeeds to get earlier slot, this misreporting will be revealed since the flight will not arrive at that earlier time. The formal definition of manipulability by intentional flights delay follows.

A rule φ is **manipulable by intentional flights delay** if, there is

$I = (S, \mathcal{A}, (F_A)_{\mathcal{A}}, (e_f)_F, (w_f)_F, \Pi^0, \Phi^0), A \in \mathcal{A}, F'_A \subset F_A$ and $(e'_f)_{F'_A} > (e_f)_{F'_A}$ such that A gains from delaying $f \in F'_A$ to e'_f, i.e., $\varphi(I_{(e_f)_{F'_A} \to (e'_f)_{F'_A}}) \succ_A^w \varphi(I)$ where $I_{(e_f)_{F'_A} \to (e'_f)_{F'_A}} \equiv (S, \mathcal{A}, (F_A)_{\mathcal{A}}, ((e'_f)_{F'_A}, (e_g)_{F \setminus F'_A}), (w_f)_F, \Pi^0, \Phi^0)$.

A rule φ is **non-manipulable by intentional flights delay** if it is not manipulable by intentional flights delay.

Since the FAA explicitly granted the right of slots for airlines(*JO 7210.3Y, Facility Operation and Administration, Section 17-9-5*), they can swap their flights among the set of slots they attained under the rules. Given two flights with same earlist arrival time, it is plausible that the owner airline wants to move up the flight with bigger weight and move down the flight with less weight to reduce the total cost. Our model also assumes that airlines reallocate their fights within the attained set of slots to minimize the total cost and it is formally defined as below.

An assignment (Π, Φ) is **self-optimized** for instance I if there exists no airline A and no landing schedule Π' such that both (i) $\Pi' \succ_A^w \Pi$ and (ii) $\Pi'(f) \in \Phi^{-1}(A)$ for all $f \in F_A$. We say a landing schedule Π is self-optimized if it is a part of self-optimized assignment (Π, Φ) for some Φ. A rule φ is **self-optimized** if it always outputs a self-optimized landing schedule.

2.3 DASO rules

We need to introduce a choice function before defining Deferred Acceptance with Self Optimization(DASO) rules. At each step of DASO rules, airlines propose to a set of slots. Choice functions determine exactly to which set of slots airlines will propose.

Definition 1 Fix an instance I, airline A, and set $T \subseteq S$ such that A's flights can be feasibly assigned within T. A's **choice function** $C_A(T)$ over such sets $T \subseteq S$ is the output of the following algorithm

- Following the initial order, assign flights sequentially to the earliest slot in T that each flight can feasibly use.(break ties randomly)
- Denote the set of occupied slots $C_A(T) \subseteq T$.

Schummer and Abizada(2016) introduce a class of rules based on the deferred acceptance algorithm. In this class, each rule exploits a priority structure. A priority structure gives each slot s a linear priority over $\mathcal{A}$. We denote a priority of each slot and a priority structure by $\succ_s$ and $(\succ_s)_{s \in S}$, respectively. To guarantee the initial slot ownership of airlines, we assume that a priority structure satisfie the following condition: each slot assigns the top priority to the initial owner of it, i.e. for each $s \in S$, $\Phi^0(s) \succ_s A$ for all $A \in \mathcal{A} \setminus \{\Phi^0(s)\}$.

Definition 2 For a priority structure $\succ$ over $\mathcal{A}$, the **DASO rule with respect to** $\succ$ associates with every instance the landing schedule computed with the following algorithm.
Step $k = 1$, : Each airlne A proposes to $C_A(S)$. Each slot s tentatively accepts the offer of its highest ranked proposer under $\succ_s$ and rejects others.
Step $k \geq 2$, : If there were no rejections in the previous step, proceed to Self-optimization step. Otherwise, each airline A proposes to $C_A(S \setminus T^{k-1})$ where T^{k-1} is a set of slots which had rejected A until step $k-1$. Each slot s tentatively accepts the offer of its highest ranked proposers under $\succ_s$ among the airlines it accepted at step $k-1$ and new proposers.
Self-optimazation step : For each airline A, assign A's flights to the slots who accepted its proposal in the previous step so that the resulting landing schedule is self-optimized. (Break ties following the initial relative order.)

We will denote the rule that associates to each I the DASO algorithm outcome by φ^D and call them DASO rules.

3. Results

There are two more concepts to be defined before we move to results. The first one is about how to compare sets of slots. It is straightforward that a slot with smaller number is earlier than one with bigger number and a slot with bigger number is lagged than one with smaller number. Then, how can we compare two sets of slots, not two slots? The answer for this question is the definition 3. The second one is *cycle*. If the priority structure has no cycle, then we say that it is *acyclical*. *Acyclicity* is needed to be satisfied to prevent airlines from getting some earlier slot by intentionally delaying their flights. We show that if there is *cycle*, then airlines merely get the same or lagged set of slots by intentional flights delay.

Definition 3 Let $T, U \subset S$ be subsets of S such that $T = \{s_1, s_2, \ldots, s_n\}$, $s_1 < s_2 < \ldots s_n$ and $U = \{s'_1, s'_2, \ldots, s'_n\}$, $s'_1 < s'_2 < \ldots s'_n$. If $s_i \leq s'_i$ for all $i \in \{1, 2, \ldots, n\}$ and $s_j < s'_j$ for some $j \in \{1, 2, \ldots, n\}$, then we say T is earlier than U(U is lagged than T) and denote it by $T < U$. If $T < U$ or $T = U$, then denote it by $T \leq U$. If neither $T \leq U$ nor $U \leq T$ holds, then we say $T \nleq U$ or $U \nleq T$.

Definition 4 A *cycle* consists of $A_1, \ldots, A_m, \bar{A}_1, \bar{A}_2, \ldots, \bar{A}_l \in \mathcal{A}$ and distinct $s_1, \ldots, s_m, \bar{s}_1, \bar{s}_2, \ldots, \bar{s}_l \in S$ such that

i) $A_1 \succ_{s_m} A_m \succ_{s_{m-1}} A_{m-1} \succ_{s_{m-2}} \cdots \succ_{s_2} A_2 \succ_{s_1} A_1$, $s_m < s_{m-1} < \cdots < s_2 < s_1$ for some $m \geq 2$,

ii) $A_1 \succ_{\bar{s}_1} \bar{A}_1 \succ_{\bar{s}_2} \bar{A}_2 \succ_{\bar{s}_3} \cdots \succ_{\bar{s}_l} \bar{A}_l \prec_{s_1} A_1$, $s_k < \bar{s}_1 < \bar{s}_2 < \cdots < \bar{s}_l < s_1$ for some $2 \leq k \leq m$,

iii) if $\bar{s}_i < s_j < \bar{s}_{i+1}$ for some $i \in \{1, \ldots, l-1\}$, then $A_j \succ_{s_j} \bar{A}_i$ and if $s_j > \bar{s}_l$, then $A_j \succ_{s_j} \bar{A}_l$, and

iv) if $s_j < \bar{s}_i < s_{j-1}$ for some $j \in \{2, \ldots, m\}$, then $\bar{A}_i \succ_{\bar{s}_i} A_j$.

We write a priority structure is *acyclical* if it has no *cycle*.

As Schummer and Abizada(2016) prove, DASO rules are non-wasteful. This result is essential for proving the later theorm.

Lemma 1 DASO rules are non-wasteful.

We now show if the priority structure is *acyclical*, then any intentional flights delay under the DASO rules result in merely the lagged set of slots. Furthermore, the converse also holds.

Lemma 2 Let $S_A = \{s \in S : s = \varphi_f^D(I)$ for some $f \in F_A\}$ and $S'_A = \{s \in S : s = \varphi_f^D(I')$ for some $f \in F_A\}$. The priority structure is *acyclical* if and only if for all I, $A \in \mathcal{A}, F'_A \subset F_A$ and $(e'_f)_{F'_A} > (e_f)_{F'_A}$, $S_A \leq S'_A$.

Proof. ($\Rightarrow$) We show the claim by contraposition. Suppose that for some I, $A \in \mathcal{A}, F'_A \subset F_A$ and $(e'_f)_{F'_A} > (e_f)_{F'_A}$, $S_A \nleq S'_A$. In other words, for some I, A can attain $s_i > s'_i$ by intentional flights delay. Since s'_i is still the ith slot in S'_A, it is not attained by delaying an flight which was assigned earlier than s'_i. Hence it is attained by a flight f which was assigned later than s'_i in S_A. Without loss of generality, say $A = A_1$ and $s'_i = s_1$. Since f could not attain s_1 under I but could under I', there exists another airline $A_2 \in \mathcal{A} \setminus \{A_1\}$ such that $A_2 \succ_{s_1} A_1$. A_1 attaining s_1 under I' implies that A_2 do not propose to s_1 under I'. Hence A_2 attains s_2 such that $s_2 < s_1$ instead of s_1 under I'. However, A_2 could not attain s_2 under I which implies that there exists $A_3 \in \mathcal{A} \setminus \{A_2\}$ such that $A_3 \succ_{s_2} A_2$.

Since the number of slots and flights are finite, this process has to end at some finite step. So, we may assume that this process end at step $m < \infty$. Now, let us consider s_m and A_m. Since A_m could not attain s_m under I, there exists $A^* \in \mathcal{A} \setminus \{A_m\}$ such that $A^* \succ_{s_m} A_m$.

We claim that A^* is the intentional flights delaying airline, i.e., $A^* = A_1$. If A^* truly reports, the only way to prevent A^* from proposing to s_m is that A^* attains a slot earlier than s_m instead of it. But it contradicts that m is the last step of the process. Hence A^* has to propose to s' such that $s' > s_m$ instead of s_m under I'. The only possible explanation for A^* to do so is that A^* is not truly reporting i.e., intentional delaying, hence $A^* = A_1$. Therefore we have $A_1 \succ_{s_m} A_m$. Now we have $A_1 \succ_{s_m} A_m \succ_{s_{m-1}} A_{m-1} \ldots \succ_{s_2} A_2 \succ_{s_1} A_1$ and $s_m < s_{m-1} < \cdots < s_2 < s_1$(the first condition of *cycle*).

A_1 intentionally delays a flight from s_m to a later slot, say $\bar{s}_1$. If $\bar{s}_1 > s_1$, then s_1 becomes the $i-1$th slot of airline A_1 and it contradicts that $s_1 = s'_i$ is ith slot of A. Thus, we have $s_{k-1} < \bar{s}_1 < s_k$ for some $2 \leq k \leq m$. Denote an airline which receives $\bar{s}_1$ under I as $\bar{A}_1$. Since A_1 attain $\bar{s}_1$ under I', we have $A_1 \succ_{\bar{s}_1} \bar{A}_1$. Then $\bar{A}_1$ has to propose to and will attain a later slot, say $\bar{s}_2$. Denote an airline which receives $\bar{s}_2$ under I as $\bar{A}_2$. Since $\bar{A}_1$ attain $\bar{s}_2$ under I', we have $\bar{A}_1 \succ_{\bar{s}_2} \bar{A}_2$. At some finite step l, $\bar{A}_l$ would be kicked out of $\bar{s}_l$ and will propose to s_1. Since A_1 attains s_1 under I', $\bar{A}_l \prec_{s_1} A_1$. Thus we have $A_1 \succ_{\bar{s}_1} \bar{A}_1 \succ_{\bar{s}_2} \bar{A}_2 \succ_{\bar{s}_3} \cdots \succ_{\bar{s}_l} \bar{A}_l \prec_{s_1} A_1$, $s_k < \bar{s}_1 < \bar{s}_2 < \cdots < \bar{s}_l < s_1$ for some $2 \leq k \leq m$(the second condition of *cycle*).

If the third condition is not satisfied, A_j cannot obtain s_j under I' since $\bar{A}_i$ would propose to and obtain s_j after being kicked out of $\bar{s}_i$ which contradicts that A_j attains s_j under I'.(If A_j cannot attains s_j under I', then A_1 cannot attains s_1 under I' which is a contradiction.)

Finally, If the fourth condition is not satisfied, $\bar{A}_i$ cannot obtain $\bar{s}_i$ under I since A_j would propose to and obtain $\bar{s}_i$ after being kicked out of s_j which contradicts that $\bar{A}_i$ attains $\bar{s}_i$ under I.(If A_i cannot attains s_i under I', then A_1 cannot attains s_1 under I' which is a contradiction.)

We now have a *cycle* in the priority structure.

($\Leftarrow$) We also show the claim by contraposition. Suppose that the priority structure has a *cycle*. It suffices to show that for some I, $A \in \mathcal{A}$, $f \in F_A$, it is possible to attain a set of slots S'_A such that $S_A \not\leq S'_A$ by intentionally delay a flight f from e_f to some e'_f such that $e'_f > e_f$.

Suppose that we have the following schedule as an outcome of DASO rules. Note that the priority structure has a *cycle*.

For omitted slots earlier than s_0, suppose that there exists a distinct airline B which initially owns the slots(thus has the top priority of them) with flights having exactly same earlist arrival time as each slot's number. Then these slots will be assigned to airline B and any other airlines would not be able to attain these slots under the DASO rules. Note that there can be different examples we can show for the counterexample. For example, $\bar{s}_l$ can be placed lower than s_2. However, we can easily see that this kind of variation does not change the result.

Slot	Flight	Airline	Earliest	Priority
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_m	f_1	A_1	s_m	$A_1 \succ_{s_m} A_m$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_{m-1}	f_m	A_m	s_m	$A_m \succ_{s_{m-1}} A_{m-1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\bar{s}_1$	$f_{\bar{1}}$	$\bar{A}_1$	$\bar{s}_1$	$A_1 \succ_{\bar{s}_1} \bar{A}_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\bar{s}_2$	$f_{\bar{2}}$	$\bar{A}_2$	$\bar{s}_2$	$\bar{A}_1 \succ_{\bar{s}_2} \bar{A}_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_3	f_4	A_4	s_4	$A_4 \succ_{s_3} A_3 \succ_{s_3} \bar{A}_{l-1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\bar{s}_l$	$f_{\bar{l}}$	$\bar{A}_l$	$\bar{s}_l$	$\bar{A}_{l-1} \succ_{\bar{s}_l} \bar{A}_l \succ_{\bar{s}_l} A_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_2	f_3	A_3	s_3	$A_3 \succ_{s_2} A_2 \succ_{s_2} \bar{A}_l$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_1	f_2	A_2	s_2	$A_2 \succ_{s_1} A_1 \succ_{s_1} \bar{A}_l$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_0	f_1	A_1	s_1	-
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Let A_1 intentionally delay its flight f_1 from s_m to $\bar{s}_1$. Then, in the resulting landing schedule DASO rules yield, we can see that A_1 attains a set of slots, $S'_{A_1} = \{\bar{s}_1, s_1\}$ in which one slot down and one slot up compared with the set of slots, $S_{A_1} = \{s_m, s_0\}$ it receives with true reporting. Hence, $S_A \nleq S'_A$. The proof completes.

$\square$

We now introduce a new type of manipulation. Needless to say, frustrating all kinds of manipulations is a desirable property. However, when it is too difficult to achieve it, we can think of a weaker alternative property of rules. In that context, we introduce non-harmful-manipulability by intentional flights delay. Rules which are non-harmfully-manipulable by intentional flights delay prevent airlines from profitably and intentionally delay its flights resulting in another airline pushed back to the later slot.

Definition 5 A rule φ is **harmfully manipulable by intentional flights delay** if, there is $I = (S, \mathcal{A}, (F_A)_{\mathcal{A}}, (e_f)_F, (w_f)_F, \Pi^0, \Phi^0), A, B \in \mathcal{A}, F'_A \subset F_A, g \in F_B$ and $(e'_f)_{F'_A} > (e_f)_{F'_A}$ such that i) $\varphi(I') \succ^w_A \varphi(I)$, ii) $\varphi_g(I') > \varphi_g(I)$ where $I' = I_{(e_f)_{F'_A} \to (e'_f)_{F'_A}}$. A rule φ is **non-harmfully-manipulable by intentional flights delay** if it is not harmfully manipulable by intentional flights delay.

Theorem 1 The priority structure is *acyclical* if and only if DASO rules are non-harmfully-manipulable by intentional flights delay.

Proof. ($\Rightarrow$) Suppose not. Then DASO rules are harmfully manipulable by intentional flights delay, i.e., there is $I = (S, \mathcal{A}, (F_A)_{\mathcal{A}}, (e_f)_F, (w_f)_F, \Pi^0, \Phi^0), A, B \in \mathcal{A}, F'_A \subset F_A, g \in F_B$ and $(e'_f)_{F'_A} > (e_f)_{F'_A}$ such that i) $\varphi^D(I') \succ^w_A \varphi^D(I)$, ii) $\varphi^D_g(I') > \varphi^D_g(I)$ where $I' = I_{(e_f)_{F'_A} \to (e'_f)_{F'_A}}$.

By lemma 2, for all I, $A \in \mathcal{A}, F'_A \subset F_A$ and $(e'_f)_{F'_A} > (e_f)_{F'_A}$, $S_A \leq S'_A$.

Case 1) If $S_A = S'_A$, then $\sum_{f \in F_A} w_f(\varphi^D_f(I) - \varphi^D_f(I')) = 0$ by self-optimization. It contradicts to $\varphi^D(I') \succ^w_A \varphi^D(I)$.

Case 2) If $S_A < S'_A$, then by definition there exists i such that $s_i \in S_A, s'_i \in S'_A$ and $s_i < s'_i$. Without loss of generality, suppose that $s'_i \in S_B$. Since A attained s'_i under I', B has to attain another slot s' instead of s'_i, i.e., B attains s' such that either $s' < s'_i$ instead of s'_i under I' or $s' > s'_i$ instead of s'_i under I'.

Case 2.1) If B attains s' such that $s' > s'_i$ instead of s'_i under I', then $A \succ_{s'_i} B$. Suppose that A assigns $f \in F_A$ such that $\varphi^D_f(I) < s'_i$ to s'_i under I'. By the assumption, $\varphi^D(I') \succ^w_A \varphi^D(I)$. By keeping the ordering of flights in $\varphi^D(I')$, reallocate flights in S_A. Then this schedule Π is better for A than $\varphi^D(I)$, contradicting the self-optimization. Hence A assigns $g \in F_A$ such that $\varphi^D_g(I) > s'_i$ and $e_g \leq s'_i$ to s'_i. It implies that A had proposed to s'_i under I but rejected. Since B attained s'_i while A failed to, $B \succ_{s'_i} A$. A contradiction to $A \succ_{s'_i} B$.
Case 2.2) Suppose that B attains s' such that $s' < s'_i$ instead of s'_i under I'. There is $C_1 \in \mathcal{A} \setminus \{A\}$ such that there is $h \in F_{C_1}$ such that $\varphi^D_h(I') > \varphi^D_h(I)$ by the assumption. Since C_1 is kicked out of $\varphi^D_h(I)(= \bar{s}_1)$ under I', there must be some other airline $C_2 \in \mathcal{A} \setminus \{C_1\}$ which attains the slot under I'. If C_2 attains a slot later than $\bar{s}_1$ under I, then there exists a contradiction between the priority C_1 and C_2 at the slot(i.e., $C_1 \succ_{\bar{s}_1} C_2$ and $C_2 \succ_{\bar{s}_1} C_1$). So, C_2 attains a slot earlier than $\bar{s}_1$ instead of it under I. Let us call that earlier slot $\bar{s}_2 < \bar{s}_1$. Similarly, we have a finite set of airlines and slots in this manner. Let the last airline and slot to be C_n and $\bar{s}_n$. Since we assume that n is the last step, $C_n = A$ and $\bar{s}_{n-1} = s'_i$. This implies that $C_{n-1} = B$. However, it contradicts to the assumption that B attains earlier slot s' instead of s'_i.

($\Leftarrow$) We show the claim by contraposition. Suppose that the priority structure has a *cycle*. By lemma 2, there exists an I and an airline A_1 which obtains a set of slots in which one slot down and one slot up by intentional flights delay. And we can see that $\bar{A}_l$ is kicked out of a slot it attains under I and obtain a later slot under I' which means that A_1's manipulation is harmful. Suppose that A_1 moves up m slots and moves down n slots. Additionally, we may suppose that moving up flights has a weight more than $\frac{n}{m}$ times bigger than the weight of moving down flight. Then A_1 harmfully manipulates by intentional flights delay thus the proof completes.

$\square$

To fully prevent manipulation by intentional flights delay, we need to restrict not only a priority structure but also a weight structure. In the following Theorem 2, we only consider when $|S| > 2$ since it is obviously impossible for airlines to manipulate by intentional flights delay when $|S| \leq 2$.

Theorem 2 Let the priority structure be *acyclical* and $|S| > 2$. Then $MWR_A \in [1, \frac{|S|-1}{|S|-2}]$ for each $A \in \mathcal{A}$ if and only if DASO rules are non-manipulable by intentional flights delay.

Proof. ($\Rightarrow$) By lemma 2, for all I, $A \in \mathcal{A}, f \in F_A$ and $e'_f > e_f$, $S_A \leq S'_A$.
Case 1) If $S_A = S'_A$, then $\sum_{f \in F_A} w_f(\varphi^D_f(I) - \varphi^D_f(I')) = 0$ by self-optimization.
Case 2) Now suppose that $S_A < S'_A$. If all flights $f \in F_A$ such that $\varphi^D_f(I) = s_i$ are assigned to s'_i under I', then $\sum_{f \in F_A} w_f(\varphi^D_f(I) - \varphi^D_f(I')) < 0$. Hence, there exists $F'_A \subset F_A$ such that for all $f \in F'_A$, $\varphi^D_f(I) = s_k$ assigned to s'_l such that $l < k$.

Then,

$$\begin{aligned}
\sum_{f \in F_A} w_f(\varphi^D_f(I) - \varphi^D_f(I')) &= \sum_{f \in F'_A} w_f(\varphi^D_f(I) - \varphi^D_f(I')) + \sum_{f \in F_A \setminus F'_A} w_f(\varphi^D_f(I) - \varphi^D_f(I')) \\
&\leq w^{max}_A(|S| - 2) + w^{min}_A(1 - |S|) \\
&\leq 0,
\end{aligned}$$

since $MWR_A \leq \frac{|S|-1}{|S|-2}$. Hence, for all cases, DASO rules are not manipulable by intentional flights delay.

($\Leftarrow$) We show the claim by contraposition. Suppose that $MWR_A > \frac{|S|-1}{|S|-2}$ and consider the following instance with *acyclical* priority structure.

Slot	Flight	Airline	Earliest	Weight
1	a_1	A	1	w^{min}_A
2	b_2	B	1	1
3	b_3	B	3	1
4	a_4	A	2	w^{max}_A

1	2	3	4
A	B	B	A
B	A	A	B

DASO rules yield the outcome which is same as the inital assginment.

Now, consider the airline A intentionally delay flight a_1 from 1 to 2. Then the possible best schedule for A is

Slot	Flight	Airline	Earliest	Weight
1	b_2	B	1	1
2	a_4	A	2	w_A^{max}
3	b_3	B	3	1
4	a_1	A	2	w_A^{min}

Then,

$$\begin{aligned}\sum_{f \in F_A} w_f(\varphi_f^D(I) - \varphi_f^D(I')) &= w_A^{max}(4-2) + w_A^{min}(1-4) \\ &= 2w_A^{max} - 3w_A^{min} \\ &> 0,\end{aligned}$$

since $MWR_A = \frac{w_A^{max}}{w_A^{min}} > \frac{4-1}{4-2} = \frac{3}{2}$.

Thus DASO rules are manipuable by intentional flights delay. The proof completes.

□

To achieve Pareto-efficiency, we need stronger condition on weight structure satisfied. It is that for all airlines, MWR has to be equal to exactly 1. As to be proven below, it is a sufficient and necessary condition.

Theorem 3 DASO rules are Pareto-efficient if and only if for all airlines $A \in \mathcal{A}$, $MWR_A = 1$.

Proof. ($\Leftarrow$) Suppose to the contrary that there is a landing schedule Π' such that (i) for all $A \in \mathcal{A}, \Pi' \succeq^w_A \varphi^D(I)$ and (ii) for some $A \in \mathcal{A}, \Pi' \succ^w_A \varphi^D(I)$. In other words, for all $A \in \mathcal{A}, \sum_{f \in F_A}(\varphi^D_f(I) - \Pi'(f)) \geq 0$ and for some $A \in \mathcal{A}, \sum_{f \in F_A}(\varphi^D_f(I) - \Pi'(f)) > 0$. Hence, by summing two inequalities, $\sum_{f \in F}(\varphi^D_f(I) - \Pi'(f)) > 0$. It is a contradiction to lemma 1, i.e. non-wastefulness of DASO rules. (Note that all non-wasteful fules are Pareto-efficient if for all airlines $A \in \mathcal{A}$, $MWR_A = 1$.)

($\Rightarrow$) We show the claim by contraposition. Consider the example below.

Slot	Flight	Airline	Earliest	Weight
1	vacant	A		
2	f	A	1	w_f
3	b_3	B	1	1
4	vacant	B		
5	b_5	B	4	1
6	g	A	4	w_g

1	2	3	4	5	6
A	A	B	B	B	A
B	B	A	A	A	B

DASO rules yield the below schedule.

Slot	Flight	Airline	Earliest	Weight
1	f	A	1	w_f
2	b_3	B	1	1
3	vacant			
4	b_5	B	4	1
5	g	A	4	w_g
6	vacant			

However, it is Pareto dominated by the below schedule for all $w_g > w_f$, hence for all $MWR_A > 1$.

Slot	Flight	Airline	Earliest	Weight
1	b_3	B	1	1
2	f	A	1	w_f
3	vacant			
4	g	A	4	w_g
5	b_5	B	4	1
6	vacant			

□

4. Conclusion

Our results show that, to fully achieve non-manipulability by intentional flights delay, we need to restrict the weight structure in addition to the priority structure. However, we have to restrict only the priority structure for achieving non-harmful-manipulability by intentional flights delay. Even though non-harmful-manipulability by intentional flights delay is weaker than non-manipulability by intentional flights delay, the condition to achieve it is much more viable. Moreover, requiring rules to be robust to only harmful manipulations is reasonable, since non-harmful-manipulations does not make any other airlines worse off. To achieve Pareto-efficiency, we have to restricte the weight structure fairly strictly.

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요약(국문초록)

기상이 악화된 상황에서는 항공기 슬랏을 항공기들에게 재분배해야 하는 문제가 있다. Schummer and Abizada(2016)은 이 문제에 대하여, DASO 룰을 제시하였다. 이 룰은 개별적 합리성, 비낭비성을 만족하며, 항공기의 중요성과 항공기 취소에 대하여 거짓을 보고할 유인이 없다. 그러나, DASO 룰에서는 항공사들이 항공기를 의도적으로 지연시킬 유인이 존재하며, 파레토효율성이 만족되지 못한다. 이 논문에서는, DASO 룰에서 항공사들이 항공기를 의도적으로 지연시킬 유인이 없을 필요충분조건과, 파레토효율성이 만족될 필요충분조건을 제시하고 있다.

주요어 : 항공기 슬랏 배분문제, 비조작성, 파레토효율성
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