The Optimal Government Shareholding Strategy and the Cost Structure

Chin-Shu Huang, Jen-Yao Lee, and Shih-Shen Chen*

This paper analyzes government’s optimal shareholding strategy within the framework of the mixed oligopoly. It is found that: (1) When both public and domestic firms have the same cost coefficient, the government’s best policy is to adopt the full mixed oligopoly. (2) When the cost coefficient of the public firm is lower than a threshold value, the government should opt for a full mixed-oligopoly policy. However, when the public firm’s cost coefficient is higher than the threshold value, the government should privatize the public firm completely and exit the market. The single mixed oligopoly is just an alternative proposal when it fails to transform all of the private firms into mixed ownership enterprises.

Keywords: Public firm, Mixed oligopoly, Privatization, Mixed ownership enterprise

JEL Classification: D43, L21, L51

I. Introduction

China had for long adopted planned economy by 1979, and it had seriously distorted her industrial structure and resource allocation. Beyond it, there were other issues waiting to be tackled with and improved upon: The loss of entrepreneurial competitiveness, insufficient labor incentive, and notorious rent seeking

* Professor, Department of Economics, National Chung Cheng University, Chia-Yi, TAIWAN, (Tel) +886-5-2720411 (ext. 34102), (E-mail) ecdah@ccu.edu.tw; Assistant Professor, Department of Finance, Kao Yuan University, Kaohsiung, TAIWAN, (E-mail) cculec@seed.net.tw; Assistant Professor, Department of International Business and Trade, Shu-Te University, Kaohsiung, TAIWAN, (E-mail) chenss@mail.stu.edu.tw, respectively. We are grateful to anonymous referees for their helpful comments. All remaining errors are ours.

problem. (Lin, Cai, and Li 1994). How to improve the production efficiency and raise the market competitiveness remains the quintessential issue for China's economic reform. Although a planned economy restrains the monopoly power of a state-owned enterprise (SOE), the SOEs are allowed to sell excess output at market price. Besides, in the early stage of the gradual reform, non-state enterprises, compared to the dominant role of SOEs, play only a cameo role. Such a market incentive induces SOEs to produce more. Literature abounds regarding the SOEs' production behavior when there is no entry of non-state enterprises.1 (See Byrd (1989), Murphy, Shleifer, and Vishny (1992), and Zhou (1994))

With the continuous implementation of the open policy, China is marching toward the socialism-market economy. Among all the factors needed in this transformation, the entry of non-state enterprises is the essential one. It is widely regarded that non-state enterprises have become one of the main ingredients of China's economy. (Noughton 1994; Zhou 1997; Sullivan 1998; Huang and Li 2000; Huang, Liu, and Wang 2001, etc.) Needless to say, China's economic reform has achieved some appreciable results. In the wake of bubbling reform of the SOE, China intends to promote the concept of a modern enterprise system in order to transform the SOEs into an independent economic unit in the market. Equity, one form of capital structure in modern enterprises, facilitates the separation of ownership from management in order to enhance both operational efficiency and capital efficiency. From 1997 to 2001, the number of share-holding enterprises (or HOEs) increases from 72,000 to 300,000, with the corresponding increase in number of employees from 6,437,000 to 27,466,000. In addition, realized annual revenue leaps form $RMB 831.1 billion to $RMB 5.6733 trillion. From the survey of PRC's national industrial union, at least 25.7% of the private enterprises are transformed from SOEs to COEs while 8% and 13.9% are ready to merge or acquire SOEs respectively. According to Workers Daily, mixed ownership enterprises account for around 40% of PRC's economy. In a half or one more decade, the percentage is expected to increase to around 80%.

1According to Qian and Xu (1995), the non-state enterprise refers to enterprises which are not owned by the state government. This includes COEs (collectively owned enterprises) and other types of enterprises such as privately owned, foreign invested and other joint venture enterprises.
Wei Li-chun, Director of Research Office of the State Council, has recently pointed out that PRC is going to make shareholding enterprises the model system of SOEs, according to the resolution of PRC's three-central committees. The government's determination to promote mixed ownership economy suggests that China's government has more comprehensive and deeper understanding of the relationship between SOEs and non-SOEs. Mixed ownership system translates into a mixed economy equipped with state, collective, and other non-state capitals. Its purpose is to assert SOEs' dominant role and exert their leadership and efficiency in production. This is comparable to the privatization of enterprises in industrialized countries. Since 1980, the major effort has been made to minimize red tape, loss of competitiveness, and the rigidity of the internal organizational structure, and as such has enhanced the efficiency of SOEs. Take British Telecommunication Corporation for example, the British government periodically sold out shares of the corporation: Releasing 50.2%, 25.9%, and 20.7% of its equity in 1984, 1991, and 1993 respectively, instead of a full-fledged privatization at one time. Thus, the process of the privatization, the role and function of the state capital in a mixed ownership economy may well adjust to the changing economic environments.

Merrill and Schneider (1966) first explore the case in which the government affects the operation of an industry via a public firm that maximizes social welfare while competes with a profit-maximizing private firm in the market. They showed that entry of a public firm into the industry could indeed improve the social welfare by reducing the market price and increasing the quantity. Such an industry characterized by competition between the public and private firms is known as mixed oligopoly. In recent years, study of the mixed oligopoly has received more and more attention. Some relevant issues such as market strategies of public firms, subsidy and tax policies of the government, privatization of public firms and strategic trade policy are widely discussed under the model of mixed oligopoly. (See De Fraja and Delbono (1989), Cremer, Marchand, and Thisse (1989), Fershtman (1990), Katsoulacos (1994), Barros (1995), White (1996), George and La Manna (1996), Fjell and Pal (1996), and Pal and White (1998)).

Some of the prior studies are on the full privatization of SOEs; that is, the transformation from pure public to pure private ownership firms. Little attention has been paid to the partial
privatization where the government still retains some shareholding. Mastumura (1998) indicates that neither the pure public nor the pure private is socially optimal in the duopoly and closed economy model. And he states that the government should indirectly control the target function of the firm through shareholding in order to achieve the optimum privatization. His paper, however, neglects how the optimum privatization is affected by the existence of foreign firms and the number of domestic firms.

Recently, Weng, Lo, and Liu (2003) have extended the models of Matsumura (1998) and Fjell and Pal (1996), and discussed the optimum privatization of the SOE within the framework of mixed oligopoly when both public and private firms have homogeneous technology. Our paper argues that the degree of optimum privatization increases with the number of domestic private firms. However, the effect of the number of the foreign firms on the optimum privatization cannot be determined: It depends on the ratio of domestic firms to foreign firms. Interestingly, De Fraja and Delbono (1989) compare the social welfare under four market structures: Oligopolistic market of nationalization, mixed oligopoly with Cournot competition, the public firm acting as a Stackelberg leader, and pure private oligopoly. It is found that the social welfare under nationalization always outperforms that under the public firm with the Stackelberg leadership, which is in turn better than that under Cournot-Nash competition in mixed and pure oligopoly. The assumption of homogeneous technology may have accounted for this result, which leads to the conclusion that the best policy of the government is to take over all the private firms: Nationalization.

In other words, the conclusion of Weng, Lo, and Liu (2003) may not lead to the best policy for the society or government. In this paper, it is suggested that under the goal of social welfare maximization, the government can not just sell off its stock shares, but should also adopt a more comprehensive optimal policy by taking the market structure and operational efficiency into consideration. That is, in the presence of great discrepancy in the operational efficiency among public and private firms, the optimum policy may be a full privatization. But if the discrepancy is not significant, by changing the shareholding of all the private firms could be the optimum policy in terms of welfare improvement.

In this research, we propose a mixed oligopoly model, and
discuss how a government determines the optimum privatization of public firms when private firms have the same and different cost structure in comparison to the public firm. We compare social welfare and the government policies under full mixed ownership, single mixed ownership and full privatization policy. In what follows, section II discusses the assumption. Section III focuses on the government's optimum shareholding policy in the case when the cost coefficient is the same. Section IV investigates the same when the cost coefficient is different. Section V explores the dynamic effect of government sell-off. Section VI provides a conclusion and suggestions for future studies.

II. The Model

We assume that there are one public firm, \( m \) domestic private firms and \( n \) foreign private firms in a market of Cournot competition with the demand function given by \( P = a - Q \), where \( Q \) is the total output, \( Q = q_s + \sum_{i=1}^{m} q_{pi} + \sum_{j=1}^{n} q_j \cdot q_s \), \( q_{pi} \) and \( q_j \) are output for the public firm, the \( i \) th private firm and the \( j \) th foreign firm respectively. The cost function of the public firm is \( C_s = F + (c_s q_s^2/2) \), where \( c_s \) is a constant \((c_s > 0)\) and \( F \) denotes the fixed cost. \((F > 0)\). The larger \( c_s \) is, the less efficient the firm will be. The cost structures for the private firm and the foreign firm are \( C_{pi} = F + (c_p q_{pi}^2/2) \) and \( C_{fj} = F + (c_f q_{fj}^2/2) \) respectively, where \( c_p > 0 \), and \( c_f \leq c_s \), indicating that the efficiency of the private firm is no less than that of the public firm. Without the loss of generality, we assume that \( F = 0 \). That is, the entry cost is not considered here.

Given the assumption above, the profit functions for the public, the private, and the foreign firms are as follows:

\[
\pi_s = P q_s - \frac{c_s q_s^2}{2} \quad (1)
\]

\[
\pi_{pi} = P q_{pi} - \frac{c_p q_{pi}^2}{2} \quad i = 1, \ldots, m \quad (2)
\]

\[
\pi_{fj} = P q_{fj} - \frac{c_f q_{fj}^2}{2} \quad j = 1, \ldots, n \quad (3)
\]

\(^2\)This research, following the previous literature, assumes that all firm’s costs (entry cost) are zero. See Weng, Lo, and Liu (2003), Pal and White (1998), White (1996), and Fjell and Pal (1996) for reference.
We assume that all firms, except for the public firm, seek maximum profit while the goal of the public firm is to maximize social welfare. The welfare maximization problem for the public firm is:

\[ \text{Max. } \sum_{i=1}^{m} \pi_{pl} \]

where \[ CS = \int_0^{\theta} P(x)dx - \varphi = \frac{\theta^2}{2} \], denotes the domestic consumer surplus.

We assume that the government attempts to achieve the privatization by selling the stock, and as such enables the public firm to turn into a mixed ownership enterprise. Furthermore, we also assume that the proportion of the private shareholding is \( \theta \) (0 ≤ \( \theta \) ≤ 1), while that of the government shareholding is (1 − \( \theta \)). Being a mixed ownership enterprise, the decision of the firm rests on the goals of both government and the private sector. It is, therefore, assumed that a convex combination of the profit and social welfare can be expressed in terms of optimal shareholding. Thus, the maximization problem for the mixed ownership enterprise is:

\[ \text{Max. } S = \theta \pi_s + (1 - \theta) \sum_{i=1}^{m} \pi_{pl} \]

We assume that, the optimum decision of shareholding and the output is based on a full information sequential game. It is essentially a three-stage game. First, government chooses the best shareholding policy. Then, given the optimum shareholding policy, the government chooses the optimal number of mixed ownership firms \( h^* \) between 0 and \( m+1 \). However, we, in this study, assume that the government can issue seasoned equity offerings or acquire more equities without extra cost, and the mixed ownership firms have their fixed cost zero. These assumptions will result to corner solution to the optimal number of mixed ownership firms. On the other hand, to contrast with before literature, we only study full nationalization, single mixed oligopoly, full mixed oligopoly, and full privatization, these four strategies in this
the government decides the optimum sell-off. Finally, the mixed ownership enterprise has the Cournot competition with all other firms in the market. To achieve the subgame perfect Nash equilibrium for government’s best shareholding, we use backwards induction.

A. Cournot Competition in Single Mixed Oligopoly

To obtain the Cournot solution \((q^*_s, q^*_p, \text{and } q^*_f)\), the derivatives of Equation (2) at \(q^*_p\), Equation (3) at \(q^*_f\) and Equation (5) at \(q^*_s\) must equal zero. Solving these three first order conditions for \(q^*_s, q^*_p, \text{and } q^*_f\), we have

\[
q^*_s = \frac{a(1 + n(1 - \theta) + c_s)}{H_1} \tag{6}
\]

\[
q^*_p = \frac{a(\theta + c_s)}{H_1} \tag{7}
\]

\[
q^*_f = \frac{a(\theta + c_s)}{H_1} \tag{8}
\]

We further have \(P^*, Q^*, \text{and } CS^*:\)

\[
P^* = \frac{a(\theta + c_s)(1 + c_p)}{1 + n + \theta + m \theta + (1 + \theta)c_p + c_s(1 + m + n + c_p)} \tag{9}
\]

\[
Q^* = \frac{a(1 + n + m \theta + (m + n)c_s + c_p)}{H_1} \tag{10}
\]

\[
CS^* = \frac{a^2(1 + n + m \theta + (m + n)c_s + c_p)^2}{2H_1^2} \tag{11}
\]

where \(H_1 = 1 + n + \theta + m \theta + (1 + \theta)c_p + c_s(1 + m + n + c_p)\).

Proposition 1

When the private shareholding \((\theta)\) of the public firm (a single research.

\(^5\)The second order conditions are assumed to hold. That is, \((\partial^2 S/ \partial q^*_s) = -1 - \theta - c_s < 0, (\partial^2 \pi^*_p/ \partial q^*_p) = -1 - m - c_p < 0 \text{ and } (\partial^2 \pi^*_f/ \partial q^*_f) = -1 - n - c_p < 0.\)
mixed ownership enterprise) is greater, the market price and the output of any firms (including domestic ones and the foreign ones) will increase. The output of the public firm, the total output of the market and the consumer surplus is expected to decrease.

From the proposition 1, it is found that the reduction of public firm’s shareholding decreases its output and causes the private firms to produce more output. The influence of the former dominates the latter, leading to the reduction of the overall market output, pushing up the market price, and decreasing the consumer surplus.

Substituting the Equations (6)-(9) into the Equations (1)-(3), we get:

\[ \pi_s^p = \frac{\alpha^2(2+cp)(\theta+c_p)^2}{2H_1^2} \]  
(12)\(^7\)

\[ \pi_p^a = \frac{\alpha^2[1+n(1-\theta)+c_p][2\theta(1+c_p)+(1-n(1-\theta)+c_p)c_a]}{2H_1^2} \]  
(13)

\[ \pi_{ij}^a = \frac{\alpha^2(2+cp)(\theta+c_p)^2}{2H_1^2} \]  
(14)

**Proposition 2**

If the private shareholding of the public firm is greater, the profit of both the domestic and the foreign firms are expected to increase. Moreover, if the privatization of the public firm is not adequate (\(\theta < \theta^\circ\)), \(\theta = [(1 + c_p)(1 + n + c_p) + n(1 + m + n + c_p)c_a]/[(1 + c_p)(1 + m + n + c_p) + n(1 + m + n + c_p)c_a]\), the profit of mixed ownership enterprise will rise as well; otherwise, it will decrease.

The variation on the proportion of equity sell-off certainly will affect public firm’s profit. Its profit moves in the same direction with the private shareholding \(\theta\) first, before swerving toward the opposite directions. Given that \(d\pi_s^p/d\theta = (MR_s - MC_0)dq_s/d\theta\), and

\[ d\pi_s^p/d\theta = \left| \frac{dq_s^p}{d\theta} \right| > m \left| \frac{dq_p^a}{d\theta} + n \frac{dq_{ij}^a}{d\theta} \right| . \]

\(^6\) We can express this result as \(d\pi_s^p/d\theta\).

\(^7\) Let \(\theta = (n-1-c_p)c_a/(2+2c_p+nc_a)\) and \(\theta \geq \theta^\circ\), it follows immediately that \(\pi_s^p \geq 0\). It is to be pointed out that, \(\theta < \theta^\circ\).
$dq_s/d\theta < 0$, the influence of $\theta$ on the public firm’s profit rests on the relative impact of the output variation on its total return ($TR_s$) and total cost ($TC_s$). In other words, when the difference of the marginal revenue ($MR_s$) and the marginal cost ($MC_s$) is negative, then $(d\pi_s/d\theta) > 0$; otherwise, $(d\pi_s/d\theta) < 0$. In the case when $\theta$ is near zero (in the range of $\theta < \bar{\theta}$), it can be shown that the public firm is expected to be highly concerned with the social welfare, and as such produces more output, rendering $MR_s < MC_s$ ($MR_s$ may be negative). Under such a condition, if $\theta$ increases, $\pi_s$ is expected to increase. But with the continuous increase in $\theta$, the public firm’s concern for the profit grows stronger. When $\theta > \bar{\theta}$, its output will be relatively small, making $MR_s > MC_s$. Thus, the increase of $\theta$ is to have negative impact on $\pi_s$.

With the continuous increase of the equity sell-off, the public firm may well reduce its output, giving private firms opportunities to increase their outputs. This would certainly hamper the strategic effect of profit shifting.

B. The Optimum Equity Sell-Off of the Single Mixed Oligopoly and Its Social Welfare

Now, we proceed to discuss how the government determines the optimum equity sell-off ($\theta$) for the single mixed oligopolistic public firm. In order to achieve the best equity sell-off, the government will consider the relationship between $\theta$ and the Cournot equilibrium output. Therefore, the government’s problem of seeking the optimum $\theta$ is:

$$\begin{align*}
\text{Max. } SW &= \pi_s + CS + \sum_{i=1}^{m} \pi_{pl} \\
\text{s.t. } (6)-(8)
\end{align*}$$

The public firm’s concern with the welfare (expressed as 1 $\theta$ in this paper), actually, makes its production behaviour more aggressive, helps the public firm establish its dominant role in the market, and acquire the strategic effect of shifting the profit of the private firms’ to the public firm (the profit shifting strategic effect). As long as the equity sell-off accelerates, the public firm’s production behaviour will become more conservative, and this would lower the strategic effect. See Fershtman (1985), Fershtman and Judd (1987), Gal-Or (1993), Sen (1993), and Barros (1995) for detailed discussion.
Substituting Equations from (6) to (8) into Equation (15), and assuming the derivative of Equation (15) at \( \theta \) equals zero, we can obtain the optimal equity sell-off:

\[ \theta^*_M = \frac{(m+mn+n^2)\sigma}{1+2n+nc_0(1+m+n+c_0)+c_p(2+m+2n+c_p)} \]  

(16)

**Proposition 3**

When the public firm is a single mixed oligopolist, (1) the higher the public firm's cost coefficient or a greater number of the private firms is, the higher the optimal government equity sell-off proportion \( \theta \) will be; (2) the higher the domestic private firm's cost coefficient is, the lower the optimal government equity sell-off will be.

Generally speaking, a large cost coefficient of the public firm implies the lower production efficiency and as such the public firm ought to reduce its output. Therefore, holding all other conditions unchanged, the government should increase its equity sell-off. On the contrary, a large cost coefficient of the private firm implies that the public firm's production efficiency is greater. The public firm should increase its output level. As a consequence, the government should decrease its equity sell-off, *ceteris paribus*.

Moreover, when the private firm's production efficiency is greater or there are more domestic firms in the market, government should ask domestic firms to produce more. On the other hand it should ask the public firm to decrease its output in order to increase the overall social welfare. In this scenario, the government should increase the equity sell-off of the public firm.

**Proposition 4**

When the cost coefficient of the public firm \( (c_0) \) is more than \( \bar{c} \), the number of foreign firms correlates positively with government’s optimal equity sell-off. Otherwise, it correlates negatively, where

\[ \bar{c} = \frac{m[(m+2n+c_0)c_p-1]+2n[1+(n+c_0)(1+c_p)]}{m(1+m+2n+c_p)-n^2(1+c_p)} \]

We can obtain \( \theta^*_M > \bar{\theta} \). That is, any \( \theta \) value that deviates from the equilibrium \( \theta^*_M \) will decrease the profit of the SOE.
The effect of change in the number of foreign firms on the optimal privatization is uncertain. It depends on the ratio of the cost coefficient to the total number of firms. On one hand, when the number of foreign firms increases, the government is expected to increase the proportion of equity sell-off, which is the same as the case in which the number of domestic firms increases. On the other hand, when the number of private firms increases, the government is expected to lower the proportion of equity sell-off in order to facilitate the profit shifting from the private firms to the public firm. In general, when the cost coefficient of the public firm of the mixed oligopoly, is higher \( c_s > \bar{c} \), the government tends to increase the equity sell-off. Otherwise, the government would decrease the equity sell-off.

Finally, given the proportion of equity sell-off \( \delta_E^* \), the profit of each firm and the social welfare can be solved by Equation (6)-(8). In particular, the social welfare is:

\[
SW_M^* = \frac{\alpha^2[1 + 2n + c_p(2 + m + 2n + c_p) + c_s(m(2 + m) + 2mn + n^2 + mc_p)]}{2[1 + 2n + c_s(1 + m + n + c_p^2 + c_p(2 + m + 2n + c_p))]}
\]

(C) The Best Equity Sell-Off of the Full Mixed Oligopoly and Its Social Welfare

Following the previous result, we discuss the best equity sell-off in the full mixed oligopoly model, and explore the corresponding social welfare. The full mixed oligopoly implies that there are \( 1+m \) mixed-ownership enterprises. That is, in addition to the government’s equity sell-off of the public firm, it also has a role in the decision making of the private firms. To put it simply, all the domestic firms will be turned into stated own enterprises first before facing the problem of the optimal equity sell-off.\(^{10}\)

The optimal equity sell-off and the social welfare are:

\[
\delta_E^* = \frac{c_s n^2}{c_p n(1 + c_p + n) + (1 + c_p)(1 + c_p + 2n)}
\]

\[
SW_E^* = \frac{\alpha^2[(1 + m)(1 + c_p)(1 + 2n + c_p) + n^2 c_s]}{2(1 + m)(1 + c_p)(1 + 2n + c_p) + 2(1 + n + c_p)^2 c_s}
\]

\(^{10}\) The objective function of them is defined as a convex function of own profit and social surplus.
Evident from Equation (18), $\theta^e_2$ is irrelevant to $m$, implying that in the presence of $n$ private domestic firms, the government will not take the number of domestic private firms into consideration regarding the optimum equity sell-off. The optimal equity sell-off for $m+1$ domestic public and private firms is the same as that of single public firm. That is, when $m=0$, then $\theta^e_2 = \theta^e_0$.

Furthermore, when $m>0$, $\theta^e_0$ increases with $m$, and this leads to $\theta^e_2 < \theta^e_0$. In other words, the government’s optimal equity sell-off ($\theta^e_0$) for the single public firm, given the existence of $m$ domestic firms ($m>0$), will be larger than $\theta^e_2$ in the case of the full mixed oligopoly in which no private domestic firms exist ($m=0$). Under the single mixed oligopoly, the government attempts to take advantage of the domestic private firm’s low cost to secure more production, while reducing the public firm’s production. This provides explanations for $\theta^e_2 < \theta^e_0$.

We can also explain $\theta^e_2 < \theta^e_0$ from the discrepancy of production efficiency between the public firm and domestic firms. First, given the full mixed oligopoly, all domestic firms are mixed oligopoly with the same production efficiency, and thus the government has the incentive to lower the optimal equity sell-off in order to increase the domestic production. In contrast, it would discourage the production of foreign firms, and uphold the domestic firms’ competitiveness in the market. Second, under the single mixed oligopoly, there exist some domestic private firms with better efficiency than that of the public firm, and these firms’ costs are comparable to those of foreign firms. The government, therefore, can increase the public firm’s equity sell-off, and need not worry about their competitiveness.

III. The Government’s Optimal Shareholding Policy in the Case of Identical Cost Coefficient

This section makes use of the method suggested in Weng, Lo, and Liu (2003) and De Fraja and Delbono (1989). We discuss the government’s best shareholding policy in the case of identical cost coefficient. In other words, we are to compare the social welfares of different shareholding policies: Full nationalization, full mixed ownership oligopoly, single mixed ownership oligopoly, and full privatization models. As can be seen from the analysis (with the
same cost), neither the single mixed oligopoly nor the full nationalization is the best strategy for the government. The optimality involves a complete intervention in the domestic private firms' operation; that is, to turn all the private firms into the enterprises of mixed ownership.

Assuming $c_s=c_p=k$ and using Equations (16) and (17), we can obtain the optimal equity sell-off ($\theta^*_s$) and social welfare ($SW^*_s$) from Weng, Lo, and Liu (2003) as shown below:

$$\theta^*_s = \frac{k(m+mn+n^2)}{1+2n+k(1+n)(2+k+m+n)}$$

$$SW^*_s = \frac{\alpha^2[1+2n+k[(1+m)(2+k+m+2n)+n^2]]}{2(1+2n+k[3+k^2+3m+4n+(m+n)^2+k(3+2m+2n)])}$$

The social welfare ($SW^*_s$) under the full nationalization ($\theta_s=0$) and the social welfare ($SW^*_p$) under the full privatization ($\theta_p=1$) are shown below:

$$SW^*_s = \frac{\alpha^2[(1+k)+(k+m)(1+m)(1+k+2n)+(4+k(3+k-m))n^2]}{2(1+m+k)^2(1+k+n)^3}$$

$$SW^*_p = \frac{\alpha^2[(1+m)(3+k+m)+2(1+m)n+n^2]}{2(2+k+m+n)^2}$$

From Equations (18) and (19), we can obtain the optimal equity sell-off ($\theta^*_N$) and the social welfare ($SW^*_N$) under the full mixed oligopoly:

$$\theta^*_N = \frac{k\alpha^2}{1+2n+k(1+n)(2+k+n)}$$

$$SW^*_N = \frac{\alpha^2[(1+k)(1+m)(1+k+2n)+kn^2]}{2(k+1)^2(1+k+m)+4(k+1)(1+k+m)n+2kn^2}$$

A comparison of these four shareholding policies and social welfare measures leads to
\[ \theta_P > \theta_S^* > \theta_N^* > \theta_S \]

Note that when \( m = 0 \), then \( \theta_S^* = \theta_N^* \); when \( m > 0 \), then \( \theta_N^* > \theta_N^* \).

In terms of social welfare measures, we have

\[ SW_N^*>SW_S^*\geq SW_N^*>SW_P^* \]

\[ SW_S^*\geq SW_K^* \text{ when } k \text{ is sufficient large.} \]

**Proposition 5**

In the case when both public firm and private firms have the same cost coefficient, the social welfare under the policy of the full mixed ownership oligopoly achieves its maximum, implying that the government's best shareholding policy is the full mixed ownership oligopoly.

From the result above, we can conclude that the government's best policy when both public firm and private firms have the same cost coefficient is to (i) nationalize all the domestic firms first, and then (ii) decides their best equity sell-off proportion. As a result, competition prevails between all mixed ownership enterprises and foreign firms in the market.

**IV. Government's Best Shareholding Policy When There Is Cost Discrepancy**

In the presence of a cost (coefficient) discrepancy between the public firm and private firms, the shareholding policy will only affect the firm's production level, rather than its the production efficiency. In the following section, we will proceed to discuss the improvement of production efficiency. For now, we investigate government's best shareholding policy. That is, we compare the social welfares under these three shareholding policies, including the full mixed oligopoly, the single mixed oligopoly, and the full privatization in order to shed light on the optimal government's policy.\(^ {11} \)

\(^ {11} \) As the full nationalization dominates the strategy of the full mixed oligopoly, we do not discuss it here.
As in prior studies, the public firm is assumed to be completely privatized, and have the same objective function and cost coefficient as other domestic private firms. To put simply, there are \( m+1 \) domestic firms and \( n \) foreign firms engaging in the Cournot competition. The social welfare under the full privatization policy can be shown as:

\[
SW^*_P = \frac{a^2(3 + m^2 + 2m(2 + n) + n(2 + n) + (1 + m)c_p)}{2(2 + m + n + c_p)^2}
\]  

Comparing the social welfares measures under different policies, or \( SW^*_P, SW^*_E, \) and \( SW^*_M, \) in Equations (17), (19), and (20), we have the following relation:

(1) When \( c_s > c_1, \) we have \( SW^*_P > SW^*_E, \) where

\[
\bar{c}_1 = \frac{(1 + c_p)(1 + 2n + c_p)(1 + 2n + c_p(3 + m + 2n + c_p))}{(1 + 2n)(3 + m + 2n + c_p(7 + 2m(1 + n) + 3n(4 + n) + c_p(5 + m + 4n + c_p))}
\]

(2) Given \( c_s > c_2, \) it can be shown that \( SW^*_P > SW^*_M, \) where

\[
\bar{c}_2 = \frac{(1 + 2n + c_p(2 + m + 2n + c_p))(1 + 2n + c_p(3 + m + 2n + c_p))}{(1 + 2n)(3 + m + 2n + c_p(7 + m^2 + 3n(4 + n) + m(5 + 4n) + c_p(5 + 2m + 4n + c_p))}
\]

(3) In the case of \( c_s > c_3, \) it follows \( SW^*_M > SW^*_E, \) where

\[
\bar{c}_3 = \frac{(1 + c_p)(1 + 2n + c_p)(1 + 2n + c_p(2 + m + 2n + c_p))}{(1 + 2n)(2 + m + 2n + c_p(5 + 2m(1 + n) + n(10 + 3n) + c_p(4 + m + 4n + c_p))}
\]

It is noteworthy that \( \bar{c}_3 > \bar{c}_1 > \bar{c}_2. \) To facilitate the comparison of \( SW^*_P, SW^*_E, \) and \( SW^*_M, \) we assume \( m = n = 2. \) In the \( c_p - c_s \) space, we have three lines \( L_1, L_2, \) and \( L_3, \) representing \( c_s \) equals \( \bar{c}_1, \bar{c}_2, \) and \( \bar{c}_3 \) respectively. The plane can be divided into four regions by the lines as shown in Figure 1. In what follows we explain implications when the coefficient set \( (c_p, c_s) \) lies in each region. Note that the area below 45° line implies that \( c_p > c_s, \) which is clearly not feasible. Thus it will not be discussed in the paper.

First, when the coefficient set \((c_p, c_d)\) lies in Region I, it implies that \(SW_F^p > SW_E^p > SW_M^p\). Besides, it suggests that the public firm's cost coefficient (or the mixed owned enterprise) is relatively larger than that of domestic private firms, signaling that the public firm is much less competitive than the private firms in terms of production efficiency. The social welfare under the full privatization, thus, is the highest, followed by that under the single mixed oligopoly. The social welfare under the full mixed oligopoly policy is the lowest. The government's best policy, therefore, is to completely privatize all firms by relinquishing all the equity holding on the firms.

Second, when the coefficient set lies in Region II, it implies that \(SW_F^p > SW_E^p > SW_M^p\). As indicated before, the government's best policy is full privatization. Notice that the social welfare under the single mixed oligopoly is the lowest followed by that under the full mixed oligopoly. Third, when the coefficient set lies in Region III, it implies that \(SW_E^p > SW_F^p > SW_M^p\). Moreover, the cost coefficient of the public...
firm (or the mixed owned enterprise) being not much higher than that of private firms, reflects that the production efficiency of the public firm is not much less than that of the private firms. The welfare under the full mixed oligopoly is the greatest, followed by that of full privatization, while that under single mixed oligopoly has the lowest value. The best policy for the government, therefore, is the full mixed oligopoly.

Last, when the coefficient set lies in Region \( IV \), it implies that \( SW^*_{p} > SW^*_{m} > SW^*_{f} \). Furthermore, the cost coefficient of the public firm is just a thread higher than that of private firms, indicating that its production efficiency is similar to that of private firms. As before, the best policy of the government is the full mixed oligopoly. What differs this from the case of Region \( III \) is that the single mixed oligopoly is the second best policy, while the full privatization remains the last choice. This result is similar to the situation when \( c_s = c_p \), as was previously discussed.

**Proposition 6**

Assuming that \( \bar{c}_1 \) is the ceiling for the cost coefficient under full privatization, when the cost coefficient of the public firm \( (c_s) \) exceeds \( \bar{c}_1 \), the government’s best policy is full privatization; when \( c_s < \bar{c}_1 \), the government’s best shareholding policy is a full mixed oligopoly; when \( c_s = \bar{c}_1 \), both full mixed oligopoly and full privatization are the best shareholding policies.

We can conclude from the above discussion that when the public firm’s cost coefficient is relatively high, government’s optimal policy is a full privatization: It ceases to hold any stocks. On the contrary, when the public firm’s cost coefficient is relatively low, the full mixed oligopoly is the optimum policy for the government. If the government, however, fails to achieve the full mixed oligopoly due to regulations or budgetary concerns, the second best policy may well be the full privatization; that is, the government should exit the market, and let the competition prevail. The plausible policy may be the single mixed oligopoly, i.e., at least holding some equity in mixed ownership enterprises.

In what follows, we discuss how numbers of domestic and foreign firms affect the ceiling cost coefficient \( \bar{c}_1 \).
Proposition 7

When the number of domestic firms grows, the ceiling cost coefficient under full privatization is expected to decrease; when the number of foreign firms grows, the ceiling cost coefficient tends to rise.

Proposition 7 suggests that with the increase in the number of domestic firms, competition tends to become fierce which improves the market efficiency. It also implies that the role for the government's intervention in the mixed owned enterprises are rather limited. In particular, it causes the ceiling cost to go down. When the number of foreign firms grows, the effect of profit shifting by the mixed oligopoly will be strengthened. This indicates that the role played by the mixed oligopoly becomes important while causing the ceiling cost coefficient to increase under full privatization. At the same time, we can also conclude that when the public firm is turned into a mixed ownership enterprise, the production efficiency is expected to improve as well, rendering its cost coefficient to be lower than the ceiling cost coefficient \( \tilde{c}_1 \). As such government's optimal policy is a full mixed oligopoly.

V. The Dynamic Effect of Government Sell-Off

In section IV, we assume that the government sell-off has no impact on the production efficiency of the public firm (or the mixed ownership); that is, \( \text{sign}[dc_s/d\theta]=0 \) (Case 1). However, considering the dynamic effect of government sell-off proportion, the public firm production efficiency may improve with the rise of this sell-off proportion, which is \( \text{sign}[dc_s/d\theta]<0 \) (Case 2). On the other hand, the effect of cost coefficient rise on the social welfare is indefinite; that is, \( \text{sign}[\partial SW/\partial c_s]=0 \). The exact effect of this rise depends on the exogenous variables. Under the conditions mentioned above, we can compare the first order condition of Case 1 with that of Case 2. In the following:

**Case 1**

\[
\frac{dSW}{d\theta} = \frac{\partial SW}{\partial c_s} \frac{dc_s}{d\theta} + \frac{\partial SW}{\partial \theta} = 0 \Rightarrow \theta^*_1
\]
TABLE 1

THE OPTIMUM SELL-OFF PROPORTION AND CORRESPONDING SOCIAL WELFARE

| Case 2 | dSW | dSw acrebo 269  n 2  c 3 5 | d  dθ | dc  dθ | dSw/ dθ =0 \( \Rightarrow \theta^* \)  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Mixed Oligopoly</td>
<td>( \hat{c}_s = c_s )</td>
<td>( \theta = 0.7764 )</td>
<td>SW = 40.8638</td>
<td>Full Mixed Oligopoly</td>
<td>( \hat{c}_s = c_s )</td>
</tr>
<tr>
<td>Single Mixed Oligopoly</td>
<td>( \hat{c}_s = \theta c_p + (1 - \theta)c_s )</td>
<td>( \theta = 0.9099 )</td>
<td>SW = 44.5432</td>
<td>Single Mixed Oligopoly</td>
<td>( \hat{c}_s = \theta c_p + (1 - \theta)c_s )</td>
</tr>
<tr>
<td>( \alpha = 10, m = 10, n = 10, c_p = 1, c_s = 5 )</td>
<td>( \alpha = 10, m = 10, n = 10, c_p = 1, c_s = 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With the production efficiency advance, the effect on the optimum sell-off proportion is indefinite; if \( \partial SW/ \partial c_s > 0 \), then \( \theta_2^* < \theta_1^* \); on the contrary, if \( \partial SW/ \partial c_s < 0 \), then \( \theta_2^* > \theta_1^* \). However, if the sell-off proportion increase does help production efficiency advance, the increase of social welfare is definite.

There is a brief example to the point. We assume that the cost coefficient of the public firm is \( \hat{c}_s = \theta c_p + (1 - \theta)c_s \), and \( dc_s/d \theta = c_p - c_s < 0 \). With exploitation of data analysis revealing a choice of different parameters, the following table provides the simulation of the optimum sell-off proportion and corresponding social welfare.

When \( \alpha = 10, m = 10, n = 10, c_p = 1, c_s = 5 \), for both the full mixed oligopoly and single mixed oligopoly, the best sell-off proportion and social welfare increase with production efficiency advance. When \( \alpha = 10, m = 10, n = 10, c_p = 1, c_s = 2 \), the best
sell-off proportion and social welfare also increase with production efficiency advance for full mixed oligopoly, but decrease for single mixed oligopoly. This explains that when there is great production efficiency discrepancy between the public firm and the private one, considering the dynamic cost lowering effect of sell-off proportion, the government will be willing to increase the sell-off for efficiency improvement. But as with only subtle efficiency discrepancy, the government will instead lower the sell-off in order to increase strategic effect of profit shifting.

VI. Conclusion and Suggestions

Equity in the capital structure of modern enterprises plays an essential role; it facilitates the separation of ownership from management in order to enhance both the production efficiency and operation efficiency of capital. In the wake of reforms for SOEs, China expects to exert a leadership role in the stated owned economy via varying the degree of nationalization, and related nationalization methods, in the hope to establish modern ownership system via the mixed ownership approach.

However, from the prior experience of the West, the leading role and function of the state capital will have to adjust to the changing economic environments. We employ the model of a mixed oligopoly to investigate the government's optimal shareholding policy. The analysis indicates: (1) When both the public firm and domestic firms have the same cost coefficient, government's best policy is a full mixed oligopoly. (2) When the public firm's cost coefficient is lower than the ceiling cost, the government should not only sell off the public firm's shareholding, but also turn all private firms into mixed owned enterprises. In other words, the government should implement a more comprehensive shareholding policy, and/or monitoring the operation of private firms. However, when the public firm's cost coefficient is greater than the ceiling cost, the government should privatize the public firm completely to prevent deterioration in economic efficiency. The equity sell-off policy of the single mixed oligopoly is just an alternative proposal when it fails to turn all of the private firms into mixed owned enterprises.

This research mainly focuses on government's best policy in the presence of full information. Future research can be enriched
greatly if one considers the following scenarios. First what is
government's best shareholding policy when firm's cost structure
and demand structure are dominated by uncertainty. Second,
relationship between best shareholding policy and industrial
development, structure change (merger, entry and exit of firms, etc.)
in an industry are considered. In addition R&D, the production
quality, and the trade policy remain challenging topics for future
research.

(Received 20 June 2005; Revised 29 March 2006)

References

Barros, F. "Incentive Schemes as Strategic Variables: An Application
to a Mixed Oligopoly." International Journal of Industrial
Organization 13 (No. 3 1995): 373-86.

Byrd, W. "Plan and Market in the Chinese Economy: A Simple
General Equilibrium Model." Journal of Comparative Econo-
mics 13 (June 1989): 177-204.

Cremer, H., Marchand, M., and Thisse, J. F. "The Public Firm as
an Instrument for Regulating an Oligopolistic Market." Oxford

De Fraja, G., and Delbono, F. "Alternative Strategies of a Public
Enterprise in Oligopoly." Oxford Economic Paper 41 (No. 2

Fershtman, C. "Internal Organization and Managerial Incentives as
Strategic Variables in Competitive Environment." International

_______. "The Interdependence between Ownership Status and
Market Structure: The Case of Privatization." Economica 57

Fershtman, C., and Judd, K. L. "Equilibrium Incentives in Oligo-

Private Firm." Canadian Journal of Economics 29 (No. 3

Gal-Or, E. "Internal Organization and Managerial Compensation in
Oligopoly." International Journal of Industrial Organization 11


THE OPTIMAL GOVERNMENT SHAREHOLDING STRATEGY 273


