The Effect of the Consumption Tax on Economic Growth and Welfare with Money and Endogenous Fertility

Akihiko Kaneko and Daisuke Matsuzaki

In this study, we examine the effect of consumption tax on the growth rate and welfare in a model with money and endogenous fertility. First, we show that an increase in consumption tax always reduces the economic growth rate but has an ambiguous effect on welfare. We show a numerical example based on Japanese data and demonstrate the likelihood that increasing the consumption tax decreases welfare. We also compare the effects of consumption tax financing and monetary financing on growth rate and welfare.

Keywords: Consumption tax, Endogenous fertility, Endogenous growth, Money-in-the-utility-function model

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I. Introduction

Since France first introduced the consumption tax in 1954, many countries have adopted it and increased their own consumption tax rate (*e.g.*, in Sweden, the standard tax rate increased from 11.11% to 25%

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between 1969 and 1992; in Germany from 10% to 19% between 1968 and 2006; and in Japan from 3% to 8% between 1989 and 2014). One of the main reasons for raising the consumption tax rate is to maintain the social security system. Thus, analyzing the long-term consequences of a change in consumption policy is important. One of the justifications for the increase is that consumption tax is thought to be less distortionary than other kinds of taxes. Indeed, in a representative agent model without labor-leisure choice, many theoretical arguments predict consumption tax has no distortionary effect on economic growth and welfare (see Schenone 1975; Abel and Blanchard 1983; Itaya 1991; Rebelo 1991; Pecorino 1993). However, several authors indicate situations where the neutrality of consumption tax does not hold. Matsuzaki (2003) finds that imposing a consumption tax may decrease or increase effective demand according to the ratio of poorer households to all households. Futagami and Doi (2004) also find a positive effect of commodity taxes on economic growth because an increase in commodity tax rates reduces the demand for consumption goods and reallocates labor from the production of goods to R&D activities. Chang (2006) finds that if households accumulate capital not only for future consumption but also for social status, a rise in the consumption tax enhances the steady-state level of capital stock and consumption. Kaneko and Matsuzaki (2009) examine the effect of a consumption tax on economic growth using an overlapping generations (OLG) model with money holdings and show that the neutrality of the consumption tax does not hold in the money-in-the-utility-function model, but holds in the cash-in-advance and money-in-the-production- function models. However, no paper has noted the influence of a change in consumption tax on economic growth in a monetary model with endogenous fertility.

Many studies have attempted to explain a household's fertility decision in theoretical models. Literature reveals several important roles of fertility in economic growth. For example, works by Becker *et al.* (1990), Fan (1997), Tabata (2003), Kalemli-Ozcan (2002), and Kaneko *et al.* (2016) provide theoretical explanations for the continuous decline in fertility rate in most developed countries. Other researchers investigate the relationship between economic take-off and households' fertility choices (*e.g.*, Galor and Weil 2000; Galor and Mountford 2008). In contrast to existing literature, our research points out the effect of a change in tax policy, a change in the consumption tax in this case on the change in fertility. In other words, we show the hidden consequence

of this policy change on the growth rate and welfare.

In the subsequent discussion, we first show that an increase in consumption tax increases the fertility rate and decreases economic growth. The following mechanism explains this result. As the rate of consumption tax increases, the relative price of having a child decreases, and the fertility rate increases. In the representative agent model, parents distribute assets equally among children, which deters the capital accumulation of each household. Though this result is rather straightforward, in the process of generating it, we derived the important fact that the cost of having a child includes the level of money holding.

After revealing the non-neutrality of consumption tax on economic growth, our paper considers the effect on welfare. Contrary to the result for economic growth, the effect of consumption tax on welfare is ambiguous; it depends on the current monetary policy (specifically, the monetary growth rate). This is true because two opposing effects of the increase in the consumption tax on welfare exist. The first is the growth effect, wherein an increase in the consumption tax reduces economic growth, although it works negatively on welfare. The second is the fertility effect, wherein the increase in consumption tax induces a household to have more children because the increase in the consumption tax raises the cost of consumption. Because households generate utility from the number of children, the fertility effect increases welfare. Interestingly, the magnitude of the fertility effect depends on the current monetary policy, specifically the growth rate of the money supply. The reason is explained as follows. When the monetary authority demonstrates an expansionary monetary policy (*i.e.*, the money supply has a high growth rate), the household reduces its money holdings relative to capital to avoid the cost of inflation. This reduction causes an increase in fertility because the cost of having a child becomes smaller (remember that the cost of having a child includes the level of money holding). In turn, this increase makes the negative growth effect superior to the positive fertility effect because the high fertility rate implies the marginal utility of having an additional child is low. We provide a numerical analysis based on Japanese data and show its likelihood of increasing the consumption tax decreases welfare.

In the following analysis, we compare the relative effectiveness of consumption tax and monetary financing. Chang *et al.* (2013) show

that the superneutrality of money does not hold in the presence of endogenous fertility. Therefore, determining which is more efficient in terms of raising the same economic growth and welfare level is natural.

The paper is organized as follows. Section II proposes the fundamental model. Section III analyzes the effect of the consumption tax on economic growth and welfare. Section IV compares effects of consumption tax financing and monetary financing on the growth rate and welfare. The last section provides a conclusion to the study.

II. The Model

A single good can be used for consumption, investment, and raising children. Production of the good is carried out by a representative household that operates the *AK* technology a la Rebelo (1991):

$$y = Ak, \tag{1}$$

where y is the per capita real output, A is the technology level, and k is the per capita real capital stock.

The representative household decides how much to consume, how much to save, and how many children to have. Households can accumulate two types of assets: money and capital. The representative household's real budget constraint is written as

$$\dot{m} + \dot{k} = y - \pi m - (m + k)n - (1 + \tau_c)c - qn + \tau, \tag{2}$$

where m is the per capita real money holdings, π is the inflation rate, n is the rate of fertility, τ_c is the consumption tax rate, c is the per capita consumption, q is the cost of child bearing per a child, and τ is the per capita lump-sum transfer. A dot indicates a time derivative. We assume that $q = \bar{q}k$, where $\bar{q}$ is a fixed value to keep the cost of child bearing meaningful in a growing economy, as in Chang *et al.* (2013). The term on the right-hand side (RHS) of (2), $-(m + k)n$, shows the wealth reduction due to having new children because the representative household should reallocate its assets for new-born children. Chang *et al.* (2013) call this reduction in total assets the wealth-narrowing effect of newborn children.

The representative household gains utility from consumption, the amount of money holdings, and the number of children. The household

maximizes the discounted sum of future utility, as given by

$$\int_0^\infty \left[\ln c + \alpha \ln n + \beta \left(\frac{m}{1+\tau_c} \right) \right] e^{-\rho t} dt, \tag{3}$$

where α is a preference parameter measuring the desire for fertility relative to consumption, β is preference parameter for money holdings, and ρ is the subjective discount rate. We assume $\alpha > 0$ and $\beta > 0$. Note that the utility of money holdings is formed by the purchasing power of money, which is represented by $m/(1 + \tau_c)$. The representative agent derives its utility from raising children emotionally in line with Palivos (1995), Yip and Zhang (1997), and Chang *et al.* (2013). We employ a representative agent model with a fertility choice to exclude any effects from heterogeneity among agents. The specification of utility function is compatible with balanced growth with a constant fertility rate.

Let λ be the co-state variable and using (2) and (3), we have the necessary conditions for our optimal control problem as follows:

$$\frac{1}{c} = \lambda(1 + \tau_c), \tag{4}$$

$$\frac{\alpha}{n} = \lambda(m + k + \bar{q}k), \tag{5}$$

$$\frac{\beta(1+\tau_c)}{m} \frac{1}{1+\tau_c} = -\dot{\lambda} + \lambda(\pi + n) + \lambda\rho, \tag{6}$$

$$\lambda(A - n - \bar{q}n) = -\dot{\lambda} + \lambda\rho. \tag{7}$$

(4)–(6) represent the equality of the marginal utility of consumption, having a child, and money holding to the cost of each. It is worth noting that the term in the brackets on the RHS of (5), $m + k$, reflects the wealth-narrowing effect. (7) is a standard Euler equation that dictates the growth rate of the shadow value of wealth, $\dot{\lambda}/\lambda$. The transversality condition must also be satisfied as the following necessary condition:

$$\lim_{t\to\infty} \lambda k e^{-\rho t} = 0, \ \lim_{t\to\infty} \lambda m e^{-\rho t} = 0. \tag{8}$$

Substituting λ in (4) into (5) gives

$$\frac{1/c}{\alpha/n} = \frac{(1+\tau_c)}{m+(1+\bar{q})k}. \tag{9}$$

The left-hand side of (9) indicates the marginal rate of substitution of consumption for having a child. The RHS shows the relative price of goods in terms of the number of children. Note that the price of having a child includes the level of money holdings. It reflects the wealth-narrowing effect of newborn children. Applying λ given by (4) and $\dot{\lambda} + \lambda\rho$ in (6) to (7) yields

$$\pi = \frac{\beta(1+\tau_c)c}{m} - A + \bar{q}n. \tag{10}$$

The consolidated government can determine the growth rate of the money supply, μ, and the consumption tax rate independently. Newly issued money and tax revenue are transferred to each household equally in a lump-sum. Thus, the per capita lump-sum transfer is expressed as follows:

$$\tau = \mu m + \tau_c c. \tag{11}$$

We write the money market equilibrium condition as $M^s \equiv pmN$ where p is the general price level, M^s is the nominal money supply, and N is the total population. Thus, the money market equilibrium condition requires that

$$\frac{\dot{m}}{m} = \mu - \pi - n. \tag{12}$$

Introducing π given by (10) into (12), we obtain

$$\frac{\dot{m}}{m} = \mu - \frac{\beta(1+\tau_c)c}{m} + A - \bar{q}n - n. \tag{13}$$

Substituting τ in (11) and $\dot{m}$ in (12) into (2) gives

$$\dot{k} = Ak - (1+\bar{q})nk - c. \tag{14}$$

The above is the dynamic equation for capital accumulation. Combining (4) and (7) yields

$$\frac{\dot{c}}{c} = -\frac{\dot{\lambda}}{\lambda} = A - \bar{q}n - n - \rho, \tag{15}$$

or the Euler equation for per-capita consumption.

To derive the autonomous dynamic system, we define the transformed variable $\chi \equiv c/k$ and $z \equiv m/k$. From (13), (14), and (15), the autonomous dynamic system of this economy is described by

$$\frac{\dot{\chi}}{\chi} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = \chi - \rho, \tag{16}$$

$$\frac{\dot{z}}{z} = \frac{\dot{m}}{m} - \frac{\dot{k}}{k} = \mu - \frac{\beta(1+\tau_c)\chi}{z} + \chi. \tag{17}$$

The balanced growth path (BGP) is the situation in which all real variables grow at the same constant rates. We illustrate that the transversality condition is satisfied at the BGP equilibrium and the BGP equilibrium is totally unstable in the Appendix. Thus, no transitional process exists in this model as in the standard *AK*-type endogenous growth model.

III. Analysis

On the BGP, c, k, and m grow at the same rate. Thus, $\dot{\chi}/\chi = \dot{z}/z = 0$. This fact, (16), and (17) yield

$$\chi^* = \rho, z^* = \frac{\beta\rho(1+\tau_c)}{\mu+\rho}, \tag{18}$$

where an asterisk over a variable represents its BGP value. From (9), we know the fertility rate n also takes a constant value:

$$n^* = \frac{(1+\tau_c)\alpha\chi^*}{z^* + (1+\bar{q})}. \tag{19}$$

A. Effect of a Change in Consumption Tax on Economic Growth

Differentiating (19) with respect to τ_c gives the effect of a change in the consumption tax on the fertility rate:

$$\frac{dn^*}{d\tau_c} = \frac{\alpha\rho}{(z^* + 1 + \bar{q})}\left[1 - \frac{(1+\tau_c)}{(z^* + 1 + \bar{q})}\frac{dz^*}{d\tau_c}\right]. \tag{20}$$

The terms in the large bracket on the RHS of (20) show two effects of an increase in consumption tax rate on the fertility rate. The first term in bracket, 1 shows the direct effect of the tax on the price of consumption, which is households always prefer having children to consumption. The second term in the bracket is the indirect effect of consumption tax through a change in money holdings. As mentioned in the argument below (9), the cost of having a child includes the level of money holdings.[1] Differentiating z^* in (18) with respect to τ_c shows that

$$\frac{dz^*}{d\tau_c} = \frac{\beta\rho}{\mu + \rho} > 0. \tag{21}$$

This result indicates that an increase in consumption tax increases z^*, the ratio of money holdings to capital stock and reflects the fact that households increase money holdings in response to the reduction in purchasing power caused by an increase in the consumption tax. Thus, the indirect effect in (20) works negatively on the fertility rate.

Applying z^* in (18) and $dz^*/d\tau_c$ in (21) to (20) gives

$$\frac{dn^*}{d\tau_c} = \frac{\alpha\rho(1+\bar{q})}{(z^* + 1 + \bar{q})^2} > 0. \tag{22}$$

Though the direct and indirect effects affect the fertility rate in opposite directions, (22) means the direct effect is always superior to the indirect effect in our model.

[1] Specifically, it is a change in z, the ratio of money holdings to capital stock, not a change in m that influences the fertility rate, as in (20). However, because of the usual property of the endogenous growth model, the ratio of consumption to capital stock is fixed, as in the first equation (18). Thus, we can express the indirect effect by z.

Lemma. An increase in the consumption tax leads to an increase in fertility rate.

We move on to investigate the effect of consumption tax on economic growth. In the BGP equilibrium, the level of capital, money holdings, and consumption grow at the same constant rate. Let g denote the steady growth rate. The growth rate of consumption itself can represent the BGP growth rate, g. Differentiating the entire RHS of (15) on the BGP and applying (22) to the result provides the following:

$$\frac{dg^*}{d\tau_c} = -(1+\bar{q})\frac{dn^*}{d\tau_c} < 0. \tag{23}$$

The inequality directly follows from the lemma. Formally, we state that

***Proposition* 1.** An increase in consumption tax decreases the growth rate in monetary economy under the endogenous fertility model.

As the lemma shows, an increase in the consumption tax always increases the fertility rate. This increases the child bearing cost and wealth-narrowing effect. Therefore, capital accumulation and economic growth are deterred.[2]

B. Effect of a Change in the Consumption Tax on Welfare

In the following analysis, we consider the effect of a change in consumption tax on welfare. Our model has no transitional process and we need to focus only on a BGP to calculate welfare change caused by changes in the tax rate. On the BGP, (18) holds, and the values of c and m can be related to the value of k, as below:

[2] Some papers point out the positive relationship between the population growth rate and economic growth (*e.g.*, Jones 2001). In these studies, a larger population stimulates idea production and enhances economic growth. In our paper, we employ *Ak*-type technology and such a positive mechanism does not emerge. In other words, the time span in our study is shorter than those of previous works.

$$c(t) = \rho k(t),\ m(t) = \left[\frac{\beta\rho(1+\tau_c)}{\mu+\rho}\right]k(t). \tag{24}$$

Using (14), (18), and the fact that the fertility rate is constant on a BGP, we can rewrite the dynamic equation for capital accumulation as

$$\dot{k} = Ak - (1+\bar{q})n^*k - \rho k. \tag{25}$$

Defining the initial value of capital stock as $k(0)$, the solution of the above equation is

$$k(t) = k(0)e^{g^*t}, \tag{26}$$

where $g^* \equiv A - (1+\bar{q})n^* - \rho$.[3]

Substituting (24) and (26) into (3) gives the indirect utility function, U:

$$U = \int_0^\infty \left[\ln(\rho k(0)e^{g^*t}) + \alpha \ln n^* + \beta \ln\left(\frac{\beta\rho}{\mu+\rho}k(0)e^{g^*t}\right)\right]e^{-\rho t}dt.$$

All variables except for time inside of the integral are constant and we can simplify the above equation as follows.

$$U = \frac{(1+\beta)g^*}{\rho^2} + \frac{\alpha}{\rho}\ln n^* + \frac{\beta}{\rho}\ln\left(\frac{\beta}{\mu+\rho}\right) + \frac{(1+\beta)\ln \rho k(0)}{\rho}. \tag{27}$$

By differentiating the above equation with respect to τ_c, we derive a formula for welfare change caused by a consumption tax change:

$$\frac{dU}{d\tau_c} = \frac{1}{\rho^2}(1+\beta)\frac{dg^*}{d\tau_c} + \frac{\alpha}{\rho n^*}\frac{dn^*}{d\tau_c}. \tag{28}$$

The first term on the RHS of (28) shows the growth effect of a household's consumption and money holdings on the household's utility level. The second term indicates the effect through a change in fertility, which we call the fertility effect. From the lemma and Proposition 1, we

[3] On a BGP, the growth rate of consumption per capita is equal to that of capital stock per capita, and thus we use the same notation, g^*, for the growth rate of k on the BGP.

know these effects work in opposite directions on welfare.

Applying n^* in (9), z^* in (18), and $dn^*/d\tau_c$ in (22) to (28) yields

$$\frac{dU}{d\tau_c} = \frac{\alpha}{\rho}\left[-(1+\beta)(1+\bar{q}) + \frac{\beta\rho}{\mu+\rho} + \frac{1+\bar{q}}{1+\tau_c}\right]\frac{1+\bar{q}}{(z^*+1+\bar{q})^2}. \tag{29}$$

From the terms in the large brackets on the RHS of (29), we have the following proposition.

***Proposition* 2.** An increase in consumption tax rate decreases (increases) welfare when $\mu > (<) - \rho\{\bar{q}\beta + [1+\bar{q}(1+\beta)]\tau_c\}/(1+\bar{q})[\beta + (1+\beta)\tau_c]$.

The sign of $dU/d\tau_c$ is negative (positive) if and only if

$$\mu > (<) - \frac{\rho\{\bar{q}\beta + [1+\bar{q}(1+\beta)]\tau_c\}}{(1+\bar{q})[\beta + (1+\beta)\tau_c]}. \tag{30}$$

As μ can be negative or positive, the condition above suggests whether an increase in the consumption tax increases depends on the value of μ (*i.e.*, monetary policy). The effect of consumption tax depends on the money growth rate for the following reason. A higher level of μ causes the household to reduce its money holdings relative to capital to avoid the opportunity cost of holding money and the nominal interest rate. This reduction causes an increase in fertility as the relative price of having a child decreases. When the fertility rate is high, the marginal utility of the fertility rate is low.[4] Therefore, the positive fertility effect is inferior to the negative growth effect on welfare in (28) and the increase in consumption tax decreases a household's welfare level.

C. Numerical Example

Based on the Bank of Japan (2017) data, the growth rate of M2 in Japan is $\mu = 0.0269$ on average from 1995 to 2015. Thus, according to Proposition 2, an increase in a consumption tax reduces welfare.

We demonstrate a numerical example to look at how low the monetary expansion rate should be for an increase in consumption tax

[4] Note that α/n^* on RHS of (28) indicates the marginal utility of having a child.

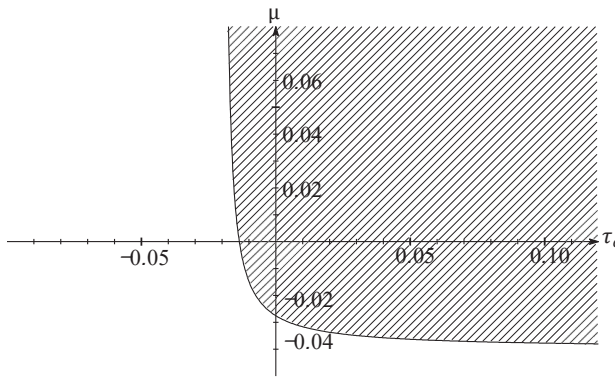

FIGURE 1
THRESHOLD VALUE OF THE GROWTH RATE OF MONEY SUPPLY

to raise welfare. To show the relationship between policy variables, μ, and τ_c in (30), we need the values of β, ρ, and $\bar{q}$. According to existing studies (*e.g.*, Holman 1998 and Chang *et al.* 2013), we set the values of preference for money holdings, β, and the subjective discount rate, ρ, as 0.02 and 0.04, respectively. To derive $\bar{q}$, we use equation (15). Once we have the growth rate of consumption, the technology level, A, and fertility rate, n, we can derive $\bar{q}$.

From the World Bank (2017) data, during 1995–2015 in Japan, we find the average annual consumption per GDP, c/y, is 0.5696, the fertility rate is approximately 0.0068, and the annual growth rate of consumption per capita is 0.0083. During most of the period, Japan set the consumption tax rate at 5%. Remember that χ defines c/k. Substituting k in (1) into the first equation of (18) gives $A = \rho/(c/y)$. We set ρ as 0.04 and we can calculate A as 0.0702. Note that the growth rate of consumption equals g. Applying the values of ρ, g, n, and A, which we derived above, to (15) yields $\bar{q}$ = 2.2206. Using the above values of τ_c, μ, n, β, ρ, and $\bar{q}$ in (19) gives α = 0.5235.

Using β = 0.02, ρ = 0.04, and $\bar{q}$ = 2.2206, we can illustrate the threshold value of the growth rate of the money supply to any value of τ_c in Proposition 2 as in Figure 1. The shaded area in Figure 1 is where the increasing consumption tax decreases welfare.

Recently in Japan, there has been a debate regarding whether the consumption tax should increase for fiscal reform or whether it prolongs the existing long-run low growth. In Japan in 2017, τ_c = 0.08.

Based on our model, the monetary policy should be extremely tight, that is, $\mu < 0.037$, for the increase in the consumption tax to increase welfare.

IV. Growth and Welfare Effects of Different Financing Methods

In the argument above, we reveal the non-neutrality of consumption tax and its effect on the growth rate and welfare through a change in the fertility rate. As Chang *et al.* (2013) confirm, the super-neutrality of money does not hold in the presence of endogenous fertility. Here, we compare the growth and welfare effects of an increase in consumption tax with those of an increase in monetary expansion rate to raise the government's revenue by the same amount. We call an increase in government expenditure using only the consumption tax, consumption tax financing and an increase in government expenditure using only monetary expansion, monetary financing. To tackle this issue, we assume that government revenue τ is represented as $\bar{\tau}k$, where $\bar{\tau}$ is a fixed value and compare the growth and welfare effects of consumption tax financing and monetary financing for a certain change in $\bar{\tau}$.

We rewrite the government budget constraint (11) as

$$\bar{\tau} = \mu z + \tau_c \chi.$$

Remember that the ratio of consumption to capital stock χ is fixed on the BGP in (18) and no transitional process exists. Thus, increased government expenditure must be financed by satisfying the following equation:

$$d\bar{\tau} = \mu dz^* + z^* d\mu + \rho d\tau_c. \tag{31}$$

In general, the responses of the BGP level of key variables in our model, such as z^*, n^*, g^* and U^*, to a change in the government expenditure ratio, $\bar{\tau}$, are calculated as follows from (15), (18), (19), and (27).

$$\frac{dz^*}{d\bar{\tau}} = \frac{\beta\rho}{\mu + \rho}\frac{d\tau_c}{d\bar{\tau}} - \frac{\beta\rho(1 + \tau_c)}{(\mu + \rho)^2}\frac{d\mu}{d\bar{\tau}}, \tag{32}$$

$$\frac{dn^*}{d\bar{\tau}} = \frac{\alpha\rho(1+\bar{q})(\mu+\rho)^2}{\{\beta\rho(1+\tau_c)+(1+\bar{q})(\mu+\rho)\}^2}\frac{d\tau_c}{d\bar{\tau}} + \frac{\alpha\beta\rho^2(1+\tau_c)^2}{\{\beta\rho(1+\tau_c)+(1+\bar{q})(\mu+\rho)\}^2}\frac{d\mu}{d\bar{\tau}}, \tag{33}$$

$$\frac{dg^*}{d\bar{\tau}} = -(1+\bar{q})\frac{dn^*}{d\bar{\tau}}, \tag{34}$$

$$\frac{dU^*}{d\bar{\tau}} = \frac{(1+\beta)}{\rho^2}\frac{dg^*}{d\bar{\tau}} + \frac{\alpha}{\rho n^*}\frac{dn^*}{d\bar{\tau}} - \frac{\beta}{\rho(\mu+\rho)}\frac{d\mu}{d\bar{\tau}}. \tag{35}$$

First, we consider consumption tax financing. In this case, $d\mu = 0$ and thus (31) and (32) become

$$\left.\frac{d\tau_c}{d\bar{\tau}}\right|_{d\mu=0} = \frac{\mu+\rho}{[(1+\beta)\mu+\rho]\rho}, \quad \left.\frac{dz^*}{d\bar{\tau}}\right|_{d\mu=0} = \frac{\beta}{(1+\beta)\mu+\rho}. \tag{36}$$

Substituting (36) into (33) yields

$$\left.\frac{dn^*}{d\bar{\tau}}\right|_{d\mu=0} = \frac{\alpha\rho(1+\bar{q})(\mu+\rho)^2}{[(1+\beta)\mu+\rho]\{\beta\rho(1+\tau_c)+(1+\bar{q})(\mu+\rho)\}^2}. \tag{37}$$

By setting $d\mu = 0$ and using (34), (35), (36), and (37), we can calculate the effect on growth rate and welfare level from consumption tax financing as follows:

$$\left.\frac{dg^*}{d\bar{\tau}}\right|_{d\mu=0} = -\frac{\alpha(1+\bar{q})^2(\mu+\rho)^3}{[(1+\beta)\mu+\rho]\{\beta\rho(1+\tau_c)+(1+\bar{q})(\mu+\rho)\}^2}, \tag{38}$$

$$\left.\frac{dU^*}{d\bar{\tau}}\right|_{d\mu=0} = \frac{\alpha(1+\bar{q})(\mu+\rho)^3\left[-(1+\beta)(1+\bar{q})+\dfrac{\beta\rho(1+\tau_c)+(1+\bar{q})(\mu+\rho)}{(\mu+\rho)(1+\tau_c)}\right]}{\rho^2[(1+\beta)\mu+\rho]\{\beta\rho(1+\tau_c)+(1+\bar{q})(\mu+\rho)\}^2}. \tag{39}$$

Next, we consider monetary financing. In this case, a change in consumption tax rate should be zero, that is, $d\tau_c = 0$. Thus, based on (31) and (32), we have the following equations:

$$\left.\frac{d\mu}{d\bar{\tau}}\right|_{d\tau_c=0} = \frac{(\mu+\rho)^2}{\beta\rho^2(1+\tau_c)}, \quad \left.\frac{dz^*}{d\bar{\tau}}\right|_{d\tau_c=0} = -\frac{1}{\rho}. \tag{40}$$

Substituting (40) into (33) yields

$$\left.\frac{dn^*}{d\bar{\tau}}\right|_{d\tau_c=0} = \frac{\alpha(1+\tau_c)(\mu+\rho)^2}{[\beta\rho(1+\tau_c)+(1+\bar{q})(\mu+\rho)]^2}. \tag{41}$$

Using (31), (34), (35), and (41), we can calculate the effect on growth rate and welfare level from monetary financing as follows:

$$\left.\frac{dg^*}{d\bar{\tau}}\right|_{d\tau_c=0} = -\frac{\alpha(1+\bar{q})(1+\tau_c)(\mu+\rho)^2}{[\beta\rho(1+\tau_c)+(1+\bar{q})(\mu+\rho)]^2}, \tag{42}$$

$$\left.\frac{dU^*}{d\bar{\tau}}\right|_{d\tau_c=0} = \frac{\alpha(1+\tau_c)(\mu+\rho)^2\left[-(1+\beta)(1+\bar{q})+\dfrac{\beta\rho(1+\tau_c)+(1+\bar{q})(\mu+\rho)}{(\mu+\rho)(1+\tau_c)}\right]}{\rho^2[(1+\beta)\mu+\rho][\beta\rho(1+\tau_c)+(1+\bar{q})(\mu+\rho)]^2} - \frac{\mu+\rho}{\rho^3(1+\tau_c)}. \tag{43}$$

Now, we can compare the effects of different financing methods on the growth rate and the welfare level. First, by comparing (38) and (42), we can derive the following proposition.

***Proposition* 3.** Monetary financing is more favorable than consumption tax in terms of growth rate when the consumption tax rate is relatively low. More precisely,

$$\left.\frac{dg^*}{d\bar{\tau}}\right|_{d\mu=0} < \left.\frac{dg^*}{d\bar{\tau}}\right|_{d\tau_c=0} \Leftrightarrow \frac{(\bar{q}-\beta)\mu+\bar{q}\rho}{(1+\beta)\mu+\rho} > \tau_c. \tag{44}$$

We can use the numerical values that we derived in subsection C to depict the condition in Figure 2. The horizontal axis indicates the monetary expansion rate ranging from -0.035 to 1. The current consumption tax rate in Japan is 0.08 and we can conclude that

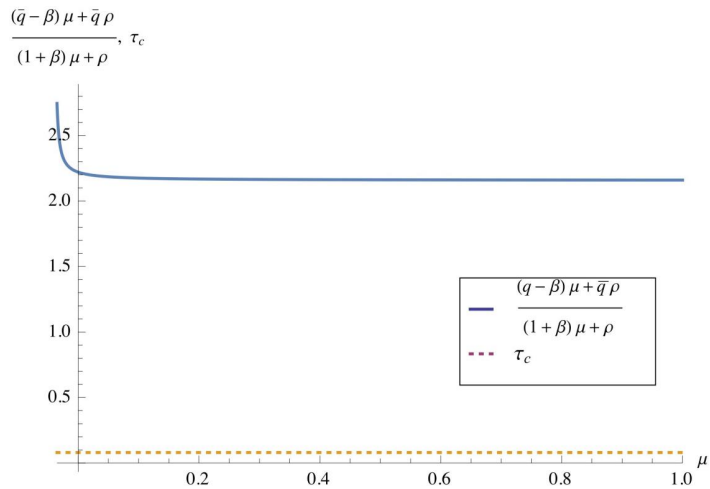

FIGURE 2

RELATIONSHIP BETWEEN THE THRESHOLD VALUE AND ACTUAL OF CONSUMPTION TAX RATE

monetary financing is more favorable than consumption tax financing in terms of the growth rate.

Basically, the condition in (44) reflects the effects of different expenditure financing on fertility rate. (34) indicates a change in growth rate is determined totally by a change in fertility rate. Indeed, the condition in (44) is exactly the same as a condition that describes the relative effect of different financing options on fertility rate. By using the responses of z^* for each financing method in (36) and (40), we can show that

$$\left.\frac{dn^*}{d\bar{\tau}}\right|_{d\mu=0} > \left.\frac{dn^*}{d\bar{\tau}}\right|_{d\tau_c=0} \Leftrightarrow \frac{(\bar{q}-\beta)\mu+\bar{q}\rho}{(1+\beta)\mu+\rho} > \tau_c. \tag{45}$$

The numerical result reflects that consumption tax financing increases the fertility rate more than monetary financing does in our numerical example.

As for welfare, we compare the change in welfare level caused by each financing method numerically. We depict the results in Figure 3 of setting $\tau_c = 0.08$. Specifically, the vertical axis indicates the level of change in welfare caused by each financing method, $dU^*/d\bar{\tau}|_{d\mu=0}$ and $dU^*/d\bar{\tau}|_{d\tau_c=0}$. The horizontal axis shows the level of monetary expansion,

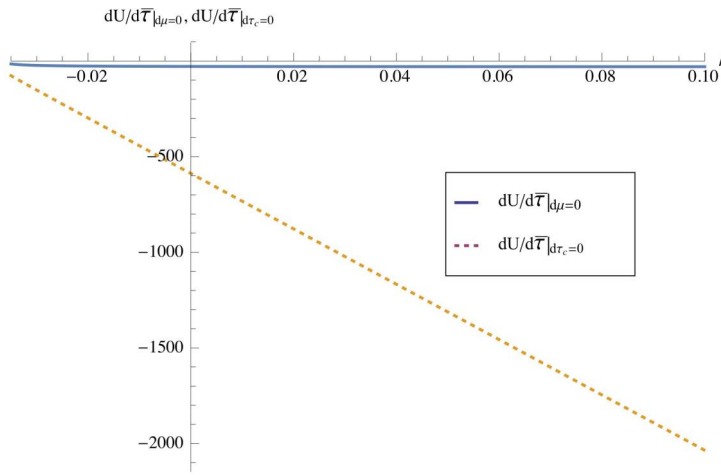

FIGURE 3
CHANGE IN WELFARE LEVEL

μ, ranging from -0.035 to 0.1. The solid line corresponds to the level of $dU^*/d\bar{\tau}|_{d\mu=0}$ and the dotted line indicates the level of $dU^*/d\bar{\tau}|_{d\tau_c=0}$. The figure indicates monetary financing decreases welfare more than consumption tax financing does in all regions.

By looking at (35) carefully, we can see the derivation of the above numerical results for the welfare in detail. (35) shows the effect of each financing method on welfare consists of the effects through the growth rate $dg^*/d\bar{\tau}$, fertility rate, $dn^*/d\bar{\tau}$, and monetary policy change, $d\mu/d\bar{\tau}$. As described in (44), monetary financing is less harmful than consumption tax financing in the terms of the growth rate. (40) and (45) indicate that consumption tax financing is more preferable for welfare than monetary financing in terms of $dn^*/d\bar{\tau}$ and $d\mu/d\bar{\tau}$. The result in Figure 3 shows the sum of the effect through the fertility rate and the monetary policy change exceeds the former and consumption tax financing is preferable in terms of welfare.

According to the above analysis, the Japanese government should not rely on consumption tax financing or monetary financing only because economic growth is an important political goal, as is the welfare of its people. To mitigate the trade-off in choosing between consumption tax financing and monetary financing, the government should adopt of mix of these financing methods to raise a certain amount of revenue.

V. Conclusions

In this study, we examine the effects of consumption tax on endogenous growth rate and welfare under endogenous fertility in a monetary economy. Increasing the consumption tax rate lowers the growth rate on the BGP. An increase in consumption tax rate decreases (increases) welfare when the monetary expansion rate is relatively higher (lower). A higher monetary expansion rate leads to lower level of money holdings relative to capital and higher fertility rate. When the fertility rate is high, the marginal utility of the fertility rate is low. The positive fertility rate effect on welfare becomes inferior to the negative growth effect and an increase in the consumption tax rate decreases welfare when the monetary expansion rate is high.

Several extensions may be fruitful for future research. The first extension would be a more general form of utility. We use a log-linear utility function that helps us evaluate the effect of consumption tax analytically in our model. However, it is a restrictive model and worth re-investigating our argument under a more general utility function.

Second, we can analyze the government expenditure financing problem in our setting. Comparing the effects of different financing methods, such as a consumption tax and monetary financing, should reveal the optimal mix of financing methods for economic growth and welfare.

Third, a similar analysis can be conducted in an overlapping generations structure that houses a policy effect arising from heterogeneity (see Weil 1991; Kaneko and Matsuzaki 2009). To exclude these effects, we employ a representative agent model in the main analysis. If we conduct a similar analysis with an overlapping structure, the interaction of the effect we derived here with the existing effects emerges, and we suppose this interaction complicates the analysis. We would also find a richer relationship between a change in tax policy and economic growth and welfare than the one that we document in this research.

Finally, the monetary policy specification can be more realistic. We employ a simple monetary policy similar to Friedman's k-percent rule. After the global financial crisis, monetary authorities in developed countries adopted unconventional monetary policy. Examining the relationship between the new policy and the effect of a tax policy would be fruitful. Future research should aim to address these aspects.

Appendix: Transversality Condition and Stability of the Balanced Growth Equilibrium

To satisfy the transversality condition, (8), the differentiation in the present value of assets, $\lambda k e^{-\rho t}$ and $\lambda m e^{-\rho t}$, with respect to time must be negative.

Using (7), (13), and (14), the conditions can be written as

$$\frac{\dot{\lambda}}{\lambda} + \frac{\dot{k}}{k} - \rho = -\chi < 0, \tag{46}$$

$$\frac{\dot{\lambda}}{\lambda} + \frac{\dot{m}}{m} - \rho = \mu - \frac{\beta(1+\tau_c)\chi}{z} < 0.$$

Because (17) and (18) show that $\mu = \beta(1 + \tau_c)\rho/z - \rho$ in the BGP equilibrium, the transversality conditions in (46) are satisfied.

We examine the dynamic property of the equilibrium. Linearizing equations (16) and (17) around the BGP equilibrium, we obtain the following:

$$\begin{pmatrix} \dot{\chi} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \chi^* & 0 \\ -\beta(1+\tau_c) + z & \mu + \chi^* \end{pmatrix} \begin{pmatrix} \chi - \chi^* \\ z - z^* \end{pmatrix} \tag{47}$$

The trace, T, and the determinant, D, of the coefficient matrix from (47) is given by

$$T = \mu + 2x^* > 0,$$

$$D = x^*(\mu + x^*) > 0.$$

Thus, the coefficient matrix of (47) has two positive eigenvalues. Both χ and z are jumpable, the economy immediately jumps to its BGP and the BGP equilibrium is determinate.

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