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Non-Convex Bargaining With Capabilities

역량을 고려한 비볼록 협상 문제

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Abstract

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We consider the problem of allocating social resources to individual agents to achieve a socially optimal profile of individual capabilities. Capability indices are used as an indicator of each agent's well-being in this allocation problem. We drop the concavity assumption in the model of Herrero(1996) for the capability index, thus admitting the best known index, Human Development Index (HDI) as an example. Without concavity, our bargaining set, namely the capability possibility set is not necessarily convex. We apply the main results by Xu and Yoshinara(2006) for non-convex bargaining and axiomatize the Nash solution and the Kalai-Smorodinsky solution in our bargaining model with capability possibility sets.

Keywords: Bargaining on economic environment, Capability index, Non-convex bargaining, Human development index(HDI), Capability possibility set, Nash solution, Kalai-Smorodinsky solution

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1 Introduction

Since Amartya Sen (1980) has introduced the capability approach, it has been developed by many scholars and widely used recently at various disciplines such as development studies, social works, political philosophy, etc. The capability approach can provide an alternative for the existing measure of personal well-being in mainstream economics, and of equality in the space of resource or primary goods in distributive justice. We can think of an achieved life of a person as a combination of human functionings:

> "What the person succeeds in doing with the commodities and characteristics at his or her command." (Sen, 1985a)

The capability of the person is the set of achievable n-tuple functioning vectors. Hence the capability is interpreted as opportunity or freedom for achieving a valuable life. Even if two different persons consume the same commodity bundle, there can be a huge gap of capability between them due to their ability of converting commodity to capability. The capability approach argues that, to wit, well-being of a person should be measured by capability terms.

Building on the capability approach, we consider a social planner's allocation problem as maximizing the aggregate index of individual capabilities. This problem is suggested by Herrero (1996) and the level of capability (or capability set) here can be measured by a certain class of capability indices. Once the capability indices of the agents are determined, the planner determines the level of individual capabilities through allocating economic resources.

However, some of the conditions for capability index in Herrero (1996) are too restrictive and thus the set of admissible indices misses important examples. The 2010 revised Human Development Index (HDI) uses geometric average of three functionings, education, health, and income. The HDI capability index is the maximum level of the geometric average over each agent's capability set. However the HDI index does not satisfy concavity, which is a key condition in Herrero (1996). When the capability index is not concave, the capability possibility set (the constraint or bargaining set of the social planner's problem) can be non-convex. Therefore to admit the HDI index there is a need for adopting the framework of non-convex bargaining on economic environment.

The issue to be discussed is a combination of three existing theories: axiomatic bargaining on economic environment, the capability index, and non-convex bargaining. Roemer (1988) extended the classical bargaining theory with additional use of economic information, resource allocation and utility functions of the agents. He argued that the axioms in classical bargaining theory is too strong because there is only consequential utility possibility set (or bargaining set) and no consideration about resource and utility function. The point is that if we do not treat identically any two different economic environment which have the same utility possibility set, then we can characterize the solutions with weaker axioms. Roemer suggested the characterizations of five classical solution, namely the Nash solution, the Kalai-Smorodinsky solution, the egalitarian solution, the family of monotone utility path solutions, and the family of proportional solutions.

Using the same framework of Roemer (1988) except that utility function is replaced with the capability index, Herrero (1996) suggested the social planner's allocation problem as we considering here. In her model, the planner allocates social endowment to the agents, which determines the set of achievable functionings of an agent, i.e., the capability set is determined by the commodity bundle she consumes. A capability index measures the level of opportunity or freedom the capability set provides for the agent by a real number. After defining the index, we can get the capability possibility set: the set of agent-wise profile of the capability indices for every feasible allocation. Then Herrero characterized the allocation mechanism which is associated capability indices are leximin suprema, with similar axioms in Roemer (1988). For assuring convexity of bargaining sets or the capability possibility sets, Roemer (1988) and Herrero (1996) used concavity assumption for utility functions and capability indices, respectively. But this assumption seriously reduces the pool of admissible capability index, and there is no plausible justification for concavity of the capability index.

Convexity of bargaining set is moderately accepted in the bargaining literature, assuming that the agents have von Neumann-Morgenstern utility function. Then the feasible utility set can be convexified by running lotteries over allocation set when all individuals are expected utility maximizers. But non-convex bargaining is often discussed because there are some situations for which non-convexity is meaningful in economic context. For example, it is the case for existence of economies of scale in the production technology, or of externalities in production or consumption (Starrett, 1972). Kaneko (1980), Herrero (1989), Conley and Wilkie (1996), Zhou (1997), and Mariotti (1999) suggested a characterization of the Nash solution on extended domains of non-convex bargaining sets. Xu and Yoshinara (2006) extended Nash's original axioms and did not use any continuity type axiom for their characterization. They characterized the Nash solution by efficiency, symmetry, scale invariance, and contraction independence. Also they characterized the Kalai-Smorodinsky solution by weak efficiency, strong symmetry, scale invariance, and weak contraction independence.

We will characterize Nash and Kalai-Smorodinsky solutions for non-convex bargaining on economic environments with a capability index. We consider sets of axioms that are weaker than those in Xu and Yoshinara (2006). The extension can be done by defining weakened axioms appropriately so that they imply the axioms in Xu and Yoshinara (2006). This is exactly what Roemer (1988) have done, except that the bargaining set is convex.

The remaining sections of this paper are organized as follows. Basic notation and definitions are provided in Section 2. We discuss the issues with non-convex bargaining in Section 3. The main results are provided in Section 4. Finally Section 5 concludes with some remarks.

2 Capability Set and Allocation Mechanism

2.1 Basic Notation and Assumptions

We follow a large portion of the basic model of Herrero (1996) except a few settings which are to be mentioned. Let h be the number of goods, m the number of functionings and there are n agents. A functioning correspondence $C : \mathbb{R}^h_+ \to \mathbb{R}^m_+$ associates with each consumption bundle x a set of functioning vectors $C(x)$, which describes functioning vectors available under consumption bundle $x \in \mathbb{R}^h_+$. We call $C(x)$ the capability set at x. For making the set operational, we consider following assumptions:

1. No free functioning: $C(0) = \{0\}$.

2. Resource monotonicity: For all $x, y \in \mathbb{R}^h$, If $x < y$ then $C(x) \subseteq C(y)$.

3. Comprehensiveness: For all $a, b \in \mathbb{R}^m$, if $a \in C(x)$ and $b \leq a$, then $b \in C(x)$.

4. Compactness: For all x, $C(x)$ is compact in $\mathbb{R}^m$.

5. Closed graph: The graph of C, $Gr(C) = \{(x, f) \in \mathbb{R}^h_+ \times \mathbb{R}^m \mid f \in C(x)\}$ is closed.

6. Full dimension: For all $f \in \mathbb{R}^m_+$ with $f \gg 0$, there is $x \in \mathbb{R}^{\mathrm{h}}_+$ $s.t.$ $f \in C(x)$.

Assumption 1 says that we have only zero functioning bundle when there are no commodities to consume. Assumption 2 means that the capability set does not shrink when the resource increases. Assumption 3 replaces star$^+-$shapedness in Herrero (1996), and requires that if a functioning vector is achieavable in a capability set, then vectors with less functionings is also in the capability set.[1] Assumption 4 states that the capability set is closed and bounded. With Assumption 4 and closed graph theorem, Assumption 5 implies that C is upper hemicontinuous; for all x, $\{x_p\} \to x$, and $f_p \in C(x_p)$, if $\{f_p\} \to f$ then $f \in C(x)$, i.e., the limit of achievable functioning vector sequence under corresponding consumption bundle is achievable under the limit of consumption

[1] $C(x)$ is star$^+-$shaped if for all $g \in \partial C(x)$, $[0, g] \subseteq C(x)$ and $\{\lambda g \mid \lambda \geq 0\} \cap \partial C(x) = \{g\}$, where $\partial C(x)$ denotes the boundary of $C(x)$. Star$^+-$shapedness is a weakening of convexity and comprehensiveness. Herrero(1996) assumed this because "some of the functionings may well be positively correlated, in looking for some minimal regularity, convexity or comprehensiveness of the capability sets might be too restrictive."

bundle sequence. Assumption 6 says that every functioning vector which consists of strictly positive values of functionings is obtainable by some commodity bundle(s).

The capability set of an agent reflects the ability or efficiency to convert resources into functionings. For example, when there is a bicycle some people who have paralysis of lower half of body could hardly achieve functionings both of moving freely(f_1) and achieving social life by participating in cycling club(f_2). These people cannot convert a bicycle into the two functionings compared to the non-disabled. This situation is illustrated in Figure 1, the non-disabled has the larger capability set than the disabled when they consume the same commodity bundle $\bar{x}$. If one has a high conversion factor, his capability set is relatively large. Hence the capability set of an agent shows the essential characteristic of the agent. If two agents have identical capability set at every bundle, then we see them essentially identical.

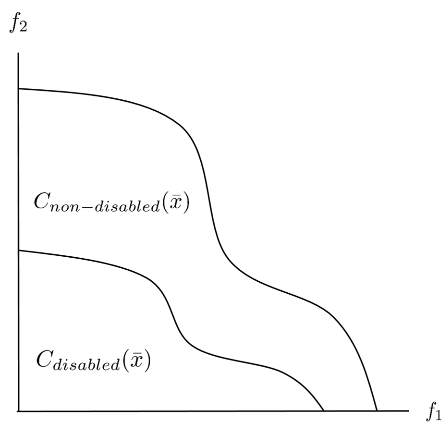

Figure 1. Capability Sets

2.2 Social Planner's Problem

Now consider the problem of social planner who has full information about the agents and allocates the social endowment $\omega \in \mathbb{R}^h_+$ to them. The planner's interest is allocating social endowment to the agents for making each agent have capabilities to maximize a social objective. It can be Rawlsian maximin or utilitarian maximized sum of capabilities, but before defining social objectives, we first quantify capability sets by an index. To measure the 'size' of capability set, a concept of index is necessary. A capability index function c for a functioning correspondence C is a continuous function $c : \mathbb{R}^h_+ \to \mathbb{R}$ with the following property: for all x, y, if $C(x) \subseteq C(y)$, $c(x) \leq c(y)$, and $c(0) = 0$. For example, Human Development Index(HDI), the maximized product of functionings available in the capability set: $\text{HDI}(x) = \max \prod_{k=1}^{m} f_k \ s.t. \ f = (f_1, \ldots, f_m) \in C(x)$.

The relevant information available to the planner is now summerized by a list $\pi = \langle n, h, \omega, c \rangle$. The list represents the socio-economic environment, and we call π a problem. Let $\Sigma^{(n)}$ be the class of n-person problems such that the capability indices are HDI, and let $\Sigma = \cup_n \Sigma^{(n)}$. Given the problem π, let $Z(\pi) = \{(x_1, \cdots, x_n), x_i \in \mathbb{R}^h_+, \mid \Sigma x_i \leq \omega\}$ denote the set of feasible allocations. The set $Z_c(\pi) = \{(c_1, \cdots, c_n) \mid \exists x \in Z(\pi) \ s.t. \ c_i = c_i(x_i) \text{ for all } i \in N\}$ is the capability possibility set. An allocation mechanism F is a correspondence which associates with each problem $\pi \in \Sigma$ a nonempty set of feasible allocations $F(\pi) \subseteq Z(\pi)$. We call the element of $F(\pi)$ a solution. If $x \in \mathbb{R}^h_+$ is a solution, we call $c(x)$ a solution-induced capability index profile, or solution index profile in short. Then without ambiguity $c(F(\pi))$ denotes the set of solution index profiles.[2] The mechanism which assigns for π the allocations maximizing the

[2]Roemer (1988) assumed that F is essentially a function: if $\bar{x} \in F(\pi)$ and $\hat{x} \in F(\pi)$ then $c_i(\bar{x}) = c_i(\hat{x})$ for all $i \in N$. With this assumption, unambiguously the notation $c[F(\pi)]$ is

product of capability indices of all agents is called the Nash solution. That is, the Nash solution is $\text{argmax}_{x \in Z(\pi)} \prod_{i \in N} c_i(x_i)$.

We consider the following basic axioms allocation mechanism satisfying:

Fullness (FN): If $x \in F(\pi)$ and there exists $y \in Z(\pi)$ *s.t.* $c(x) = c(y)$, then $y \in F(\pi)$.

Capabilitism (C): Let $\pi, \pi' \in \Sigma$ be two problems with $Z_c(\pi) = Z_c(\pi')$. Then $c(F(\pi)) = c(F(\pi'))$.

Pareto Optimality (PO): Let c be a solution index in $c(F(\pi))$. Then there is no $c' \in \mathbb{R}^n_+$ *s.t.* $c'_i \geq c_i$ for all $i \in N$ and $c'_i > c_i$ for some $i \in N$.

Axiom FN says that the mechanism should choose all the allocations corresponding to the solution index profile. For two problems which have the same capability possibility set, axiom C requires the mechanism to choose solutions whose induced indices are same. Axiom C is so named because it asks for the mechanism to consider the capability-related information only, and ignore the other information which are reduced in the capability possibility set. Axiom PO stands for the ordinary efficiency concept.

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singleton. But we do not assume this for understanding $c[F(\pi)]$ as a set of solution index profiles.

3 Issues With Non-Convexity

3.1 Domain of Mechanisms

In Section 2, there is no assumption for concavity of the capability index of the agents and we used HDI for the capability index. In fact, concavity does not hold for many other indices as well as HDI. Concavity of an index does not depend much on property of the index itself, but propensity of change in capability set as consumption bundle changes. It is natural in a sense because an capability index is a kind of measured value of associated capability set. Thus we can say that assuming concavity of an capability index is making an concavity type assumption for associated capability set, which is unwanted.

When the capability indices of the agents are not necessarily concave, the capability possibility set can be non-convex, which causes some difficulty in applying the classical bargaining theory developed by Nash (1950) and Kalai-Smorodinsky (1975). They define a bargaining problem by a closed, comprehensive and convex bargaining set $S \in \mathbb{R}^n_+$ and a threat point $d \in S$. To apply this bargaining problem to deal with resource allocation in economic environments, Roemer (1988) showed that any such bargaining set S is in his domain for a mechanism on economic environments, that is, S is a utility possibility set of a problem. But now we use capability indices rather than utility functions, and do not assume concavity for the index. We establish the same result as Roemer's when there are two agents.

Proposition 1. *Let $n = 2$. For any compact and strictly comprehensive set $S \in \mathbb{R}^2_+$, there is a problem π in our domain with two agents such that the* *capability possibility set at π is identical with S.*

Proof. First we express the set S with its Pareto surface. Let the equation for Pareto surface of S be $c_2 = s(c_1)$. Note that s is decreasing since S is strictly comprehensive. Then we can write that $S = \{(\bar{c_1}, \bar{c_2}) \mid \bar{c_2} \leq s(\bar{c_1})\}$. Now we try to find an example of functioning correspondences that induces capability possibility set which is identical with S.

Let $C_i(x_i) = \{(f_k)_{k=1,\ldots,m} \mid f_k \leq g_i^k(x_i)\}$ for $i = 1, 2$, where $g_i^k : \mathbb{R}_+^l \to \mathbb{R}_+$. By solving the problems:

$$\max \prod_k f_k \ s.t. \ (f_1, \ldots, f_m) \in C_i(x_i) \text{ for } i = 1, 2$$

we get the HDI capability indices of agents:

$$c_i(x_i) = \prod_{k=1}^{m} g_i^k(x_i) \text{ for } i = 1, 2.$$

Now assume $l = 1$ and let $g_1^k(x_1) = (\|x_1\|)^{\frac{1}{m}}$ and $g_2^k(x_2) = (s(\|\omega - x_2\|))^{\frac{1}{m}}$ for all $k = 1, \ldots, m$. Then using HDI, we get $c_1(x_1) = \|x_1\|$ and $c_2(x_2) = s(\|\omega - x_2\|)$, where $\|x_i\|$ is a Euclidean norm of x_i. Then with the feasibility constraint $x_1 + x_2 \leq \omega$, we have $c_2 = s(\omega - x_2) \leq s(x_1) = s(c_1)$. Therefore we get the capability possibility set $\{(\bar{c_1}, \bar{c_2}) \mid \bar{c_2} \leq s(\bar{c_1})\}$, which is identical with S.

$\square$

Proposition 1 implies that the capability possibility sets from our domain cover all non-convex bargaining sets. Therefore axioms FN and C reduce the study of allocation mechanism F to the study of solution index profiles in $c(F(\pi))$. With FN and C, characterization results in the model of *non-convex* bargaining can be used to establish similar results in our model.

3.2 Non-Convexity and The Nash Solution

We cannot use characterization of Nash(1950) since now the capability possibility set can be non-convex. Nash characterized his solution with axioms of scale invariance, Pareto optimality, independence of irrelevant alternatives and symmetry. The problems now are in the last two axioms.

Independence of Irrelevant Alternatives (IIA): For all $\pi, \pi' \in \Sigma$, if $Z_c(\pi') \subset Z_c(\pi)$ and $c(F(\pi)) \subset Z_c(\pi')$, then $c(F(\pi')) = c(F(\pi))$.

IIA says that when capability possibility set shrinks but solution index profile is still feasible, then solution index profile should remain the same. A similar axiom is proposed in Xu and Yoshinara (2006).

Contraction Independence (CI): For all $\pi, \pi' \in \Sigma$, if $Z_c(\pi') \subset Z_c(\pi)$ and $Z_c(\pi') \cap c(F(\pi)) \neq \emptyset$, then $c(F(\pi')) = Z_c(\pi') \cap c(F(\pi))$.

Now for examining symmetry concept we need following definition. Let p be a permutation of N. The set of all permutations of N is denoted by $\mathcal{P}$. For all $c = (c_i)_{i\in N} \in \mathbb{R}^n_+$, let $p(c) = (c_{p(i)})_{i\in N}$. For all $\pi \in \Sigma$ and $\in \mathcal{P}$, let $p(Z_c(\pi)) = \{p(c) : c \in Z_c(\pi)\}$. If $Z_c(\pi) = p(Z_c(\pi))$ for all $p \in \mathcal{P}$, we call $Z_c(\pi)$ symmetric.

Symmetry (S): If $Z_c(\pi)$ is symmetric, then for all $c \in c(F(\pi))$, $c_i = c_j$ for all $i, j \in N$.

When the capability indices of all agents are identical for same resources, it follows that the capability set of all agents are same either, because the measuring method is identical for all agents. That the capability set of some agents are identical means that they are essentially identical individual. Suppose that for a problem $\pi \in \Sigma$, the capability possibility set $Z_c(\pi)$ is symmetric and a mechanism F satisfies S. Then their capability set is identical because each agent have equal division resources by S. Figure 2 (a) illustrates this. Since the agents are symmetrical, the capability possibility set is symmetric. Suppose that the capability possibility set is not convex, so Figure 2 (b) shows the symmetric and non-convex capability possibility set. By any measuring method the profile of capability indices (c_1, c_2) must be on the 45° ray in the $c_1 - c_2$ plane, but there is no Nash solution on the ray. The point is that symmetry axiom represents too strong type symmetry concept to deal with non-convex problems, so we need a weaker form of symmetry axiom. The next axiom of Xu and Yoshinara (2006) requires that when the capability possibility set is symmetric, every permutation of solution index profile is also solution index profile.

Weak Symmetry (WS): If $Z_c(\pi)$ is symmetric and $c \in c(F(\pi))$, then $p(c) \in c(F(\pi))$ for all $p \in \mathcal{P}$.

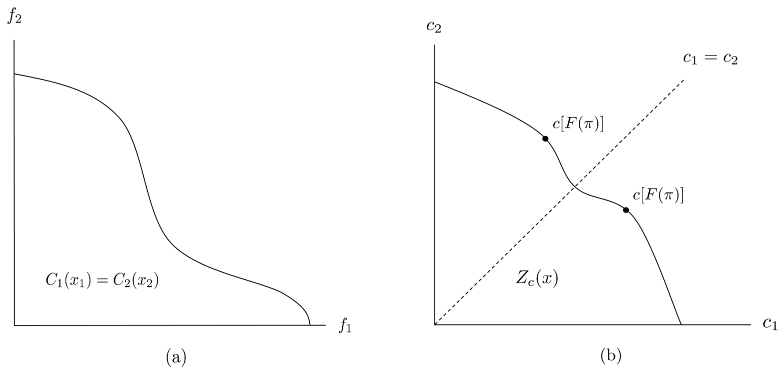

Figure 2

4 Main Results

4.1 Characterization of The Nash Solution

Now we need to note that the axiom C is quite strong because it is states with all problems in Σ; the solution index profile of two problems must be the same as long as their capability possibility set are the same, even if their numbers of commodities and capability indices of agents are different. Despite this, most of axioms in classical bargaining theory implies axiom C, which is elementally assumed in classical bargaining theory. This is why we need weaker versions of the axioms used in classical bargaining theory. We denote $\alpha \circ c \equiv (\alpha_i c_i)_{i=1}^n$.

Scale Invariance (SI): For all $\pi = \langle n, h, \omega, c\rangle$ and all $\alpha \in \mathbb{R}^n_+$, $c(F(\pi')) = \{\alpha \circ c' \mid c' \in c(F(\pi))\}$, where $\pi' \equiv \langle n, h, \omega, \alpha \circ c\rangle$.

Cardinal Non-Comparability (CNC): For all $\pi = \langle n, h, \omega, c\rangle$ and all $\alpha \in \mathbb{R}^n_+$, $F(\pi) = F(\pi')$, where $\pi' \equiv \langle n, h, \omega, \alpha \circ c\rangle$.

Weak Economic Symmetry (WES): If $c_i(\cdot) = c_j(\cdot)$ for all $i, j \in N$ and $x \in F(\pi)$, then $p(x) \in F(\pi)$ for all $p \in \mathcal{P}$.

CNC requires that solution allocations not be affected by scale change of capability indices. WES says that when the capability indices of all agents are identical, every permutation of solution is also solution. Note that SI implies CNC and WS implies WES. But with fullness and capabilitism axioms the converses hold.

Lemma 1. *(a) If F satisfies FN and C, then F satisfies CNC if and only if F satisfies SI.*
(b) If F satisfies FN and C, then F satisfies WES if and only if F satisfies WS.

Proof. (a) Let $c \in Z_c(\pi)$ and $c' \in Z_c(\pi')$. Then there exist $x \in Z(\pi)$ and $x' \in Z(\pi')$ *s.t.* $c = c(x)$ and $c' = \alpha \circ c(x')$. Now let $c \in c(F(\pi))$ and $c' \in c(F(\pi'))$. By FN, there exist x and x' *s.t.* $x \in F(\pi)$ and $x' \in F(\pi')$, and we have $x = x'$ by CNC. So $c' = \alpha \circ c(x) = \alpha c$, and thus $Z_c(\pi') = \{\alpha \circ c : c \in C(\pi)\} \equiv \alpha \circ Z_c(\pi)$. It follows that by W, $c(F(\pi')) = \alpha \circ c(F(\pi)) \equiv \{\alpha \circ c : c \in c(F(\pi))\}$.
(b) Note that π is symmetric when every agent has the same capability index

function. Now the result is straightforward.

□

Lemma 2. *(Xu and Yoshinara, 2006) A bargaining solution on the domain of non-convex bargaining problems satisfies PO, WS, SI and CI if and only if it is the Nash solution.*

Now there is our first result:

Theorem 1. *Assume $n = 2$. F satisfies FN, PO, WES, CNC and CI if and only if it is the Nash solution.*

Proof. Note that CI implies C. Suppose that F satisfies CI and $Z_c(\pi) = Z_c(\pi')$. Then by CI $c(F(\pi')) = Z_c(\pi') \cap c(F(\pi)) = c(F(\pi))$ since $c(F(\pi)) \subset Z_c(\pi) = Z_c(\pi')$. Now the theorem is an immediate result by the Proposition 1, Lemmas 1 and 2.

□

4.2 Characterization of The Kalai-Smorodinsky Solution

Another solution of our interest is the Kalai-Smorodinsky solution. Let $a_i(\pi) = \max\{c_i : c_i \in C(\pi)\}$ for all $i \in N$. We call $a(\pi) = (a_1(\pi), \ldots, a_n(\pi))$ the ideal point of π. A mechanism which assigns the allocations in π which choosese the maximal point of capability indices which maintains the ratio of ideal point is called the Kalai-Smorodinsky mechanism. That is, the Kalai-Smorodinsky solution is $x \in Z(\pi)$ *s.t.* $a_i(\pi)/c_i(x_i) = a_j(\pi)/c_j(x_j)$ for all $i, j \in N$ and there is no $c' \in Z_c(\pi)$ $s.t.$ $c' \gg c(x)$. Xu and Yoshinara(2006) characterized the Kalai-Smorodinsky solution without convexity assumption of bargaining set. The axioms used for characterization are following:

Weak Pareto Optimality (WPO): Let c be a solution index in $c(F(\pi))$. Then there is no $c' \in \mathbb{R}^n_+$ $s.t.$ $c'_i > c_i$ for all $i \in N$.

Economic Symmetry (ES): If $\pi = \langle n, h, \omega, c \rangle$ $s.t.$ $c_i(x) = c_j(x)$ for all $i, j \in N$, then $(\frac{\omega}{n}, \ldots, \frac{\omega}{n}) \in F(\pi)$.

Weak Contraction Independence (WCI): For all $\pi, \pi' \in \Sigma$, if $a(\pi) = a(\pi')$, $Z_c(\pi') \subset Z_c(\pi)$ and $Z_c(\pi') \cap c(F(\pi)) \neq \emptyset$, then $c(F(\pi')) = Z_c(\pi') \cap c(F(\pi))$.

ES is a strong symmetry axiom which requires equal division for same agent. Compared with normal contraction independence, WCI focuses on problems that have the same ideal point. Note that S implies ES, but converse also holds with some axioms.

Lemma 3. *If F satisfies FN and C, then F satisfies ES if and only if F satisfies S.*

Proof. This is an analogue of Lemma 8 in Roemer (1988).

$\square$

Lemma 4. *(Xu and Yoshinara, 2006) A bargaining solution F satisfies WPO, S, SI and WCI if and only if it is the Kalai-Smorodinsky solution.*

Now there is our second result:

Theorem 2. *Assume $n = 2$. F satisfies FN, WPO, ES, CNC and WCI if and only if it is the Kalai-Smorodinsky solution.*

Proof. Similar to the Theorem 1, note that WCI implies C. Then finally it is an immediate result by the Proposition 1, Lemmas 1 (a), 3 and 4.

□

The Nash solution arouses some distributional issue when the capability possibility set is not convex. In Figure 1 (b), we should choose one among the two solution-induced capability index profile for implementing allcoation. Then an agent have relatively low level of capability whereas the other agent enjoys high level. Besides, small change in conversion factor can lead a huge change in solution allocation. Monotone capability path solutions do not have these drawbacks of Nash solution. But they can choose less productive(in terms of capability) index profile than the Nash solution, both additively and multiplicatively. For example, in Figure 1 (b) the egalitarian and the Kalai-Smorodinsky solution choose a point where 45° ray meets with the sunken area of capability possibility set. For redeeming the shortcomings of the Nash solution, we may be able to consider an intertemporal bargaining model which the agents make an agreement that give away in turn large portion of allocation reciprocally.

5 Concluding Remarks

We combined the three existing theories: axiomatic bargaining on economic environment, the capability index, and non-convex bargaining. Herrero (1996) used the model which is essentially identical with Roemer (1988), and considered a social planner's problem which distribute the social endowment to the agents of economy for adjusting their capability indices. When concavity of the capability index are not assumed any longer, the capability possibility set can be non-convex. We overcome the issues with non-convexity only partially by investigating 2-agent problems. Axioms for the characterization of the Nash solution are weak economic symmetry and contraction independence instead of symmetry and independence of irrelevant alternatives in classical bargaining theory, respectively. Axioms for the characterization of the Kalai-Smorodinsky solution are weaker efficiency, weaker contraction consistency and stronger symmetry than those for the Nash solution.

The most advantageous point followed by the resolution of non-convexity problem is that we can allow more various capability indices, including the Human Development Index. When we should consider an allocation problem concerned with the capability aspect of the people, our result will be helpful if there is universally calculated and widely used data for their capability. But finally we have the ultimate agenda, that is to say, the choice of axioms and solutions. It is an issue of distribution, and therefore very sensitive topic across members of society who have great diversity of opinion. As Rawls and Sen emphasized, the open and transparent public resoning by democratic process will be needed for consensus of the society.

References

[1] Anand, S., and Sen, A. (1993). Human Development Index: Methodology and Measurement. Human Development Report Office Occasional Paper 12. New York: UNDP.

[2] Billera, L. J., and Bixby, R. E. (1973). A characterization of Pareto surfaces. Proceedings of the American mathematical Society, 41(1), 261-267.

[3] Conley, J. P., and Wilkie, S. (1996). An extension of the Nash bargaining solution to non-convex problems. Games and Economic behavior, 13(1), 26-38.

[4] Herrero, M. J. (1989). The Nash program: non-convex bargaining problems. Journal of Economic Theory, 49(2), 266-277.

[5] Herrero, C. (1996). Capabilities and utilities. Economic Design, 2(1), 69-88.

[6] Howe, R. (1987). Sections and extensions of concave functions. Journal of Mathematical Economics, 16(1), 53-64.

[7] Kalai, E., and Smorodinsky, M. (1975). Other solutions to Nash's bargaining problem. Econometrica: Journal of the Econometric Society, 513-518.

[8] Kaneko, M. (1980). An extension of the Nash bargaining problem and the Nash social welfare function. Theory and Decision, 12(2), 135-148.

[9] Nash Jr, J. F. (1950). The bargaining problem. Econometrica: Journal of the Econometric Society, 155-162.

[10] Roemer, J. E. (1988). Axiomatic bargaining theory on economic environments. Journal of Economic Theory, 45(1), 1-31.

[11] Sen, A. (1985a). Capabilities and commodities. Amsterdam and New York: North-Holland.

[12] Sen, A., (1985b), Well-being, agency and freedom: the Dewey Lectures 1984. Journal of Philosophy, 82, pp. 169–221.

[13] Sen, A. (1990) Justice: means versus freedoms. Philosophy and Public Affairs, 19, pp. 111–121.

[14] Sen, A. (2004) Elements of a theory of human rights. Philosophy and Public Affairs, 32(4), pp. 315–356.

[15] Starrett, D. A. (1972). Fundamental non-convexities in the theory of externalities. Journal of Economic Theory, 4(2), 180-199.

[16] Xu, Y., and Yoshihara, N. (2006). Alternative characterizations of three bargaining solutions for non-convex problems. Games and Economic Behavior, 57(1), 86-92.

[17] Zhou, L. (1997). The Nash bargaining theory with non-convex problems. Econometrica: Journal of the Econometric Society, 681-685.

요약(국문초록)

본 논문에서는 사회의 각 개인들에게 자원을 분배함으로써 각 개인들이 가지게 되는 역량의 수준이 사회적 최적을 달성하도록 하는 사회적 자원 배분 문제에 대해 고려해 본다. 여기서 역량지수는 각 개인들의 안녕감(well-being)의 지표로써 사용된다. 이후 Herrero(1996)의 모형에 가정된 역량지수에 대한 오목성 가정을 제외함으로써 가장 널리 이용되는 역량지수인 인간개발지수(HDI)를 본 모형에 적용시킬 수 있다. 오목성 가정이 없다면, 이 모형에서의 협상집합인 역량가능집합이 반드시 볼록하지는 않다. Xu and Yoshinara(2006)에서의 비볼록 협상 모형에서의 주요 결과를 적용하여, 내쉬 해와 칼라이-스모로딘스키 해를 우리 모형에서 특징화(공리화)한 것이 본 논문의 주요 결과이다.

핵심어: 경제환경 협상 모형, 역량지수, 비볼록 협상 모형, 인간개발지수(HDI), 역량가능집합, 내쉬 해, 칼라이-스모로딘스키 해

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