Stochastic Modeling of Long-Term Degradation over Structural Lifetimes

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ABSTRACT: As structures and civil systems age, decision-making over structural lifetimes requires an understanding of the long-term degradation of these systems. Due to the extensive uncertainties in degradation processes, including the number of possible degradation modes acting on a structure, their effects, external factors, and the long time duration, estimating degradation is complex. Many previous studies investigate the impacts of individual degradation modes on reliability, providing insights into the forms of functions that can be used to estimate degradation due to these mechanisms. These have been used as a basis for more general structural degradation models through the use of, e.g., linear, polynomial, or exponential functions. These methods provide an estimate for degradation that can be used to estimate long-term reliability. However, these models can be limited in terms of the number of degradation mechanisms accounted for and often do not match known physical constraints of degradation. We propose a new stochastic model for long-term structural degradation. This model is based on the mechanical properties of individual degradation modes. The degradation at any instance is calculated as a random sum of a random number of degradation modes acting on the structure. The effect of a degradation mode is modeled as a stochastic function based on its mechanical effect. Individual degradation modes include those that start at time of initial construction and those beginning later in the lifetime of the structure. The effect of one degradation mode on the rate of another is also considered. We apply the proposed model to three years of field monitoring data and compare the resulting analyses with estimations from existing functions. How structural inspection data can be used to learn or update the parameters of the model is also described. The proposed model results in a more accurate estimation of long-term degradation, leading to improved predictions of system responses and supporting reliability-based decision-making over structural lifetimes.

As structures and civil systems age, decision-making over structural lifetimes requires an understanding of the long-term degradation of these systems. Due to the extensive uncertainties in degradation processes, including the number of possible degradation modes acting on a structure, their effects, external factors, and the long time duration, estimating degradation is complex. There have been many previous studies on the effect of individual degradation modes on civil structures. For example, Mu et al. (2002) and Park et al. (1999) study the effects of chloride attack and sulfate attack, respectively, on concrete structures. Swamy and Al-Asali (1988) study the effect of alkali-silica reaction, Roy et al. (2001) acid attack, and Ueda et al. (2004) freeze-thaw cycles. For steel structures, Koh and Stevens (1991) and Wang et al. (2013) study the effects of fatigue and increased temperatures, respectively. These studies provide insights into the forms of functions that can be used to estimate degradation in structures due to these mechanisms. For the studies listed, the functional forms suggested are, in order, shifted exponential, linear, linear, exponential, linear, and polynomial form estimator functions, or as a pre-defined function of the modulus of elasticity.

When considering structural reliability overall, these studies have been used as a basis for the form of degradation functions, e.g., through the use of linear or exponential functions as in Li et al. (2015). With only a few terms in these functions, they can fail to capture the multiple varying mechanisms and rates of degradation for the different modes of degradation acting on a structure at any given time. The functions can also not match physical constraints and bounds of degradation. To account for the effect of different degradation modes, Saini and Tien (2017a) separate out terms of different forms in a structural degradation model. However, the model does not yet match expected properties of the degradation function as the proposed model in this paper seeks to do. The objective is for the degradation function to account for the mechanisms of different degradation modes while matching known physical constraints and bounds of degradation.

Here, the proposed new stochastic model for long-term structural degradation is constructed based on combining several terms representing the mechanical properties of individual degradation modes. The degradation at any instance is calculated as a random sum of a random number of degradation modes acting on the structure. The model provides an estimate for structural degradation at any point in time during the structure's lifetime. Predictions of degradation can then be used to estimate long-term system reliability to support maintenance and retrofit decisions over structural lifetimes.

The following section describes the derived structural degradation model in detail. Individual degradation modes considered include those that start at time of initial construction and those beginning later in the lifetime of the structure. The effect of one degradation mode on the rate of another is also considered. We describe how structural inspection data can be used to learn or update the parameters of the model. We apply the proposed degradation model to three years of field monitoring data and compare the resulting analyses with estimations from existing functions. Finally, we discuss use of the model to support reliability-based decision-making, particularly for aging structures.

1. METHODOLOGY: PROPOSED STOCHASTIC MODEL OF DEGRADATION

1.1. Derivation of degradation function

Let $\theta(t)$ be the degradation function such that the resistance of a structural component decreases with time and $R(t) = R_0(1 - \theta(t))$, where $R(t)$ is the resistance at time t and R_0 is the initial resistance. Based on this, the degradation function must have the following properties. The value of $\theta(t)$ must be between 0 and 1, with $\theta(0) = 0$ and $\theta(\infty) = 1$. In addition, $\theta(t)$ should be monotonically increasing over time except for external interventions such as repairs or retrofits.

To capture the varying degradation modes, the function is formulated as a sum of various degradation modes, with weights indicating the respective contributions of each mode to overall degradation, as shown in Eq. (1).

$$0 \le \theta(t) = \sum_{i=1}^{N(t)} w_i F_i(t) \le 1 \tag{1}$$

$N(t)$ is the number of degradation modes acting at time t, and $F_i(t)$ is the estimator model and w_i the corresponding weight for the i^{th} degradation mode. All weights sum to 1. Every individual degradation mode can be modeled as an estimator function of a particular form. However, each degradation mode may not act independently of all others. An individual degradation mode can be influenced by the impact of another mode. For example, in reinforced concrete structures, carbonation or abrasion can increase the rate of corrosion. Therefore, to account for these effects, we model the estimator for the i^{th} mode as in Eq. (2).

$$F_i(t) = f_i(t) + f_{ji}(t) \tag{2}$$

$f_i(t)$ is the estimator function for the individual i^{th} mode and $f_{ji}(t)$ is the effect of the j^{th} mode on the i^{th} mode. $f_{ji}(t)$ is assumed to follow the same form as $f_i(t)$, but its arrival is the same as the arrival of the j^{th} mode. Thus, we model the influence of one mode on the rate of another while not changing the way in which the original mode affects a structural component.

Many degradation modes can be modeled with either exponential or constant rates and therefore as exponential or linear functions. Other potential functions are polynomial with degrees 0.5 or 2. Assuming that the numbers of degradation modes with these latter two forms are significantly lower compared to either linear or exponential modes, we estimate them with single terms modeling a smaller effect on the overall degradation. The degradation function then becomes

$$\theta(t) = \sum_{i=1}^{N_e(t)} w_{ei} F_{ei}(t) + \sum_{i=1}^{N_l(t)} w_{li} F_{li}(t) + w_s a_s t^2 + w_r a_r \sqrt{t} \tag{3}$$

where the subscript e denotes exponential, l linear, s square, and r square-root. a_s and a_r are the coefficients for the square and square-root terms respectively. To include the effect of individual degradation modes that start at the time of initial construction and those beginning later in the lifetime of the structure, let us look more closely at the linear terms. The linear term is expanded as in Eq. (4)

$$F_{li}(t) = a_i(t - t_i) + a_{ji}(t - t_j) \tag{4}$$

where t_i and t_j are the arrival times for modes i and j, respectively. a_i is the linear coefficient for mode i and a_{ji} is the linear coefficient for the effect of mode j on mode i. We assume that some degradation modes start acting on the structure at time $t = 0$, while the others have a later time of arrival. In Eq. (4), this indicates that the effect of mode j on mode i begins after initiation of mode i and $t_j > t_i$. Without loss of generality, we can assume that arrival times $t_1 < t_2 < \cdots$. The linear functions term then becomes

$$\sum_{i=1}^{N_l(t)} w_{li} F_{li}(t) = w_a a_a t + w_b a_b t + w_c a_c t + \cdots + \begin{cases} 0 & t < t_1 \\ w_d a_d (t - t_1) & t > t_1 \end{cases} + \begin{cases} 0 & t < t_2 \\ w_e a_e (t - t_2) & t > t_2 \end{cases} + \cdots \tag{5}$$

where terms $a, b, c, \ldots$ denote modes acting starting at time $t = 0$, and terms $d, e, \ldots$ denote modes starting at a later time. Assuming the arrival times of the later modes to be Poisson with a fixed rate, Eq. (6) represents the sum of linear terms in a compact form.

$$\sum_{i=1}^{N_l(t)} w_{li} F_{li}(t) = \sum_{i=1}^{n} P(n = k) \left[t \sum_{i=1}^{k} a_i - \sum_{i=1}^{k} a_i t_i \right] + At \tag{6}$$

A is the coefficient for all the linear terms starting at $t = 0$ and n is the number of linear terms with a non-zero arrival time. In a similar way, we expand the exponential terms beginning with the exponential estimator function as shown in Eq. (7).

$$F_{ei}(t) = \left(1 - e^{-\lambda_i (t - t_i)}\right) + \left(1 - e^{-\lambda_i (t - t_j)}\right) \tag{7}$$

λ_i indicates the exponential rate of the i^{th} exponential mode. The exponential estimator function is then expanded as shown in Eq. (8).

$$\sum_{i=1}^{N_e(t)} w_{ei}F_{ei}(t) = \sum_{i=1}^{N_e(t)} w_i + \sum_{i=1}^{n} P(n=k)\sum_{i=1}^{k} w_i + w_a e^{-\lambda_a t} + w_b e^{-\lambda_b t} + w_c e^{-\lambda_c t} \dots + \begin{cases} 0 & t < t_1 \\ w_d e^{-\lambda_d (t-t_1)} & t > t_1 \end{cases} + \begin{cases} 0 & t < t_2 \\ w_e e^{-\lambda_e (t-t_2)} & t > t_2 \end{cases} + \cdots \quad (8)$$

In Eq. (8), n is the number of exponential terms with a non-zero arrival time and the counter k in the summation is $k \in [1, n]$ such that every possible number of terms is considered. The sum of the exponential terms from Eq. (8) can then be written as Eq. (9)

$$\sum_{i=1}^{N_e(t)} w_{ei}F_{ei}(t) = c + \sum_{i=1}^{N_e(t)-n} w_i\, e^{-\lambda_i t} + \sum_{i=1}^{n} P(n=k)\sum_{i=1}^{n} w_i\, e^{-\lambda_i t} e^{\lambda_i t_i} \quad (9)$$

where $c < 1$. To simplify Eq. (9), we consider the terms on the right-hand side of Eq. (9) as the probability density of a convolution of exponentially distributed random variables with varying parameters as in Akkouchi (2008). For each term, we multiply and divide by its respective λ_i, then use the expression as shown in Eq. (10) for the probability density of an exponential distribution convolution with varying parameters.

$$S_n(t) = \sum_{i=1}^{n} \frac{\lambda_1 \dots \lambda_n}{\prod_{\substack{j=1 \\ j \neq i}}^{n} (\lambda_j - \lambda_i)} e^{-\lambda_i t} \quad (10)$$

In using Eq. (10), we assume that a subset of the degradation modes are more dominant than others in affecting the structure such that, in most cases, $(\lambda_j - \lambda_i) \sim \lambda_j$. Thus, the rate of total degradation of the structure is governed by a smaller number of dominant modes while a larger number of other modes have a relatively smaller effect. Correspondingly, the rate λ_j for the subset of dominant exponential modes j is higher than that of other modes. In this way, we use the convolutions for each term to obtain the exponential terms. We then add the estimations for the linear, square, and square-root terms to form the full function. Once all terms are expanded and simplified, we use the Taylor expansion of each term to obtain the full degradation function as shown in Eq. (11).

$$\theta(t) = 1 - e^{-\lambda t} - \frac{\log(1+(\lambda t)^{\frac{1}{n}})}{n!} \frac{e^{-\frac{\lambda}{n} t}}{\Gamma n} \quad (11)$$

In the final form of the degradation model as shown in Eq. (11), the function has two unknown parameters: n and λ. n can be viewed as the number of degradation modes and λ the Poisson rate of arrival of modes. In the derivation of this function, we have ignored the higher order terms for the square and square-root terms because of the assumption that their relative impact on the extent of degradation is lower. The final form arrives from separating the Taylor expansion into two groups, with the first group directly converging into an exponential function and the convergence for the other group estimated as in Portnoy (1988) and Cherrualt and Adomian (1993). Looking at the values of the model at $t = 0$ and $t = \infty$, this model fits the expected physical properties of a degradation function in terms of bounds and limits. It is adaptable to changes in structural properties over structural lifetimes by changing the two parameters. As there are only two parameters that need to be estimated, the function provides a way to estimate long-term structural degradation with limited data.

1.2. Parameter estimation from data

To estimate the parameters of the degradation model, we need at least two different observations of structural monitoring data at different times. An example for a system that is instrumented with strain gauges is shown in Eq. (12)-(14). The strain gauges measure the strain at a particular instance of time. Assuming that the geometric properties are constant, the mechanical strains will be directly proportional to the modulus of elasticity of the system. Assuming degradation to be the fractional decrease in the modulus of elasticity of the structure over time, we estimate the parameters of the model by

$$\varepsilon_m(t) = \frac{k}{E(t)} \tag{12}$$

where ε_m represents mechanical strains, $E(t)$ represents modulus of elasticity at time t, and k is a constant assuming the geometric properties of the structural component do not change during the time of data collection. The degradation can then be expressed as

$$\theta(t) = \frac{E(0)-E(t)}{E(0)} \tag{13}$$

which is simplified to

$$\theta(t) = \frac{\frac{1}{\varepsilon_m(0)}-\frac{1}{\varepsilon_m(t)}}{\frac{1}{\varepsilon_m(0)}} \tag{14}$$

It is assumed that different data points are collected under similar loading scenarios. However, as any measured strains are a function of the loading applied on the structure, using data from multiple strain measurements increases the confidence in our estimates of the parameters of the model. A more detailed example of the calculations used to estimate the parameters n and λ are given in the presentation of the application.

2. APPLICATION

We apply the proposed degradation function to a set of bridge monitoring field data to assess its performance. The data is from the Streicker Bridge located on the Princeton University campus. The available data are measurements of strain, prestress losses, and temperature collected continuously over three years. The data is collected at the mid-span of the bridge. The strain and temperature data is collected every five minutes, while the prestress data is collected at varying times over the three-year period.

To evaluate performance of the model, we divide the data into two sets. The first two years of data is used as a training set to estimate the parameters of the model. The data from the third year is used as the testing set. From the training data, we estimate the model parameters using the value of degradation at each time step with each time step representing a single day. We then calculate the average of the values from all estimates at each time step to obtain the value for each parameter.

For the calculations for this application, given the structure as a pedestrian bridge with data collected over three years, we assume that the loading remains on average constant. The total strain measured from the strain gauges will be a sum of the thermal strain, mechanical strain, and strain from prestress loss. We assume that the mechanical strain directly relates to the degradation. The total strain ε is given as

$$\varepsilon = \varepsilon_{thermal} + \varepsilon_m + \varepsilon_{prestress} \tag{15}$$

The thermal strain is calculated as

$$\varepsilon_{thermal} = \alpha\left(T - T_{ref}\right) \tag{16}$$

where α is the coefficient of thermal expansion, taken to be 10×10^{-6}, T is the temperature at the time of data collection, and T_{ref} is the reference temperature, taken to be 20℃, around the average of the temperature measurements in the training set. With these definitions, the total strain at a given time t becomes

$$\varepsilon(t) = \alpha\Delta T + \frac{k}{E(t)} + \frac{\Delta P}{E(t)} \tag{17}$$

where ΔP is the prestress loss until time t. For Eq. (14), we use the temperature data along with prestress data to subtract the thermal strains and strains due to prestress loss to obtain the values of mechanical strains. Eq. (13) is then used to estimate $\theta(t)$. Adjacent pairwise values of $\theta(t)$ are used to calculate n and λ, with the final values of the two parameters taken as the average values across the training dataset.

Figure 1 shows the strain and temperature data collected over the first three years from the example application. Figure 2 gives the prestressing force over the three years.

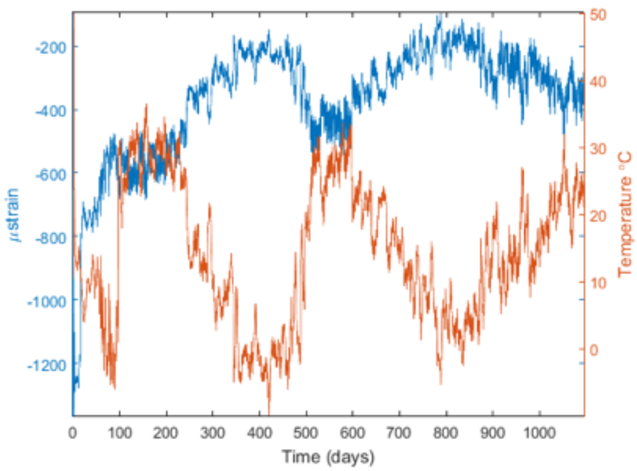

Figure 1: Strain (left axis) and temperature (right axis) data for first three years

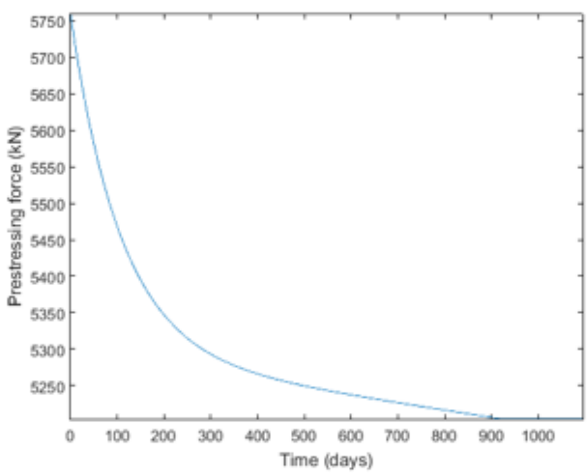

Figure 2: Prestressing force for first three years

3. RESULTS

The first two years of the data shown in Figure 1 and Figure 2 are used to estimate the parameters. From this data, the final estimated degradation model parameters are $n = 3.4$ and $\lambda = 0.07$. Using the estimated parameters, we test the proposed degradation model on the data in year three to evaluate the accuracy of the function to predict the structural degradation. The separation of the data into the training and testing sets enables us to test the generalizability of the model and assess the accuracy of the model on previously unseen data.

To evaluate performance of the proposed model, we compare the results with results from widely used exponential, gamma, and linear regression models. For the comparison models, the corresponding model parameter values for the exponential, gamma, and linear regression models are also calculated based on the training set data. The parameters for each model are estimated pairwise for adjacent dataset values and the overall average is used as the corresponding model parameter. All models are then applied to the third year testing data. Figure 3 shows the results of applying the obtained values of the parameters for the different models to the testing data to evaluate performance. The open circles represent the degradation as calculated from the collected field data at every time step, computed as the average degradation for that day.

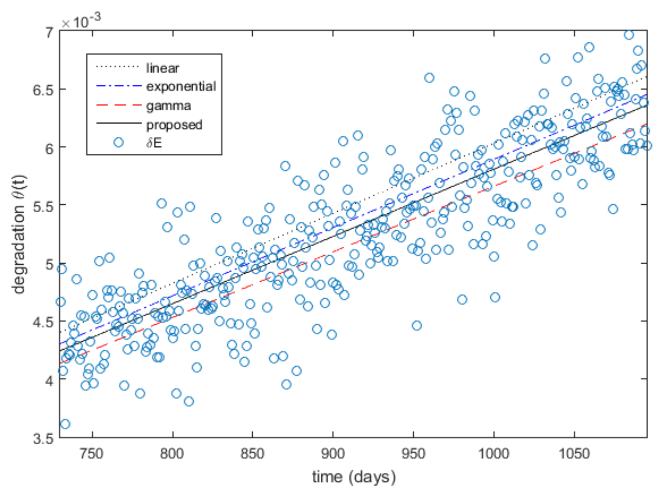

Figure 3: Fitting of varying models to testing data

In Figure 3, the proposed model outperforms the other models in terms of root mean square error (RMSE) between the estimated and field-measured values. The RMSE for the proposed model is 5.17%, while it is 10.98% for linear regression, 9.16% for exponential, and 7.44% for gamma models. In addition to the improved accuracy on the testing dataset, advantages of the proposed model include the fact that it is based on accounting for the physical mechanisms of multiple degradation modes and satisfies theoretical constraints on the degradation values.

To evaluate the proposed degradation model over longer time periods of interest for decision-making over structural lifetimes, Figure 4 shows the predicted structural degradation for the varying models over 100 years.

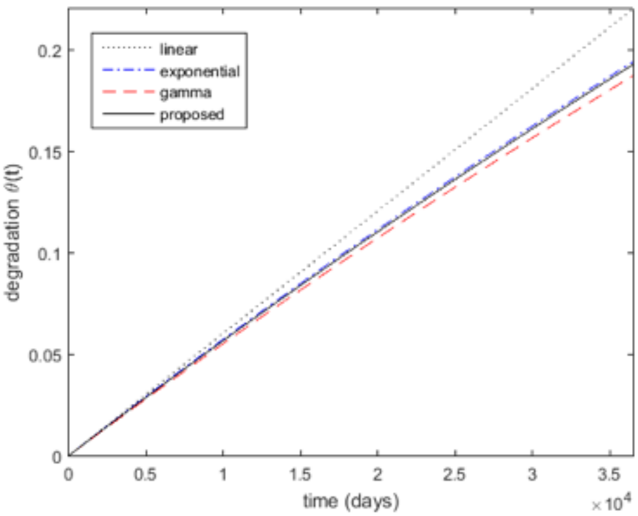

Figure 4: Predicted degradation for the varying models over 100 years

While the performance of the four models appears close in Figure 3, we see divergence in their predictions over longer time periods as seen in Figure 4. This is particularly true for the linear regression model. As the linear regression model is unbounded, it is not a realistic representation of degradation. The proposed degradation function enables an improved understanding of the long-term reliability of a structure or civil system. While the proposed model's prediction is close to that of the exponential and gamma models in Figure 4, we expect improvements in performance for the proposed model compared to previous models similar to the increases in accuracy seen for the testing data. This could be validated with additional field data as it is collected over the lifetime of a structure.

In this application example, the model is trained on a relatively short time duration dataset and the results extended to a longer time duration as shown in Figure 4. With any long-term predictions of performance, there will be potential errors and uncertainty. As the amount of training data and data available for model calibration increases, the accuracy of the predicted degradation is expected to increase. This data can be from structural health monitoring information. For example, if a structure is instrumented using accelerometers and LVDTs measuring accelerations and displacement histories over time, this information can be used to estimate the structural parameters, including mass, damping, and stiffness, e.g., as in Saini and Tien (2017b). Assuming that the geometric properties of the structure remain the same over time, the degradation can be estimated based on the fractional change in stiffness. In this scenario, the stiffness matrix would be used to estimate the parameters of the degradation function. For a system instrumented with strain gauges as in the example application, increased measurement data will update and improve the estimates of the degradation function parameters over time. This will tune the degradation function to match the measurement data, and lead to increased accuracy of the prediction of degradation when extending over longer structural lifetimes.

In evaluating the use of the proposed degradation model to support decision-making over structural lifetimes, firstly, the proposed model is computationally efficient, with a computation time of 0.8s for the full calculation over 100 years on a personal computer. This efficiency enables analyses to be easily conducted and facilitates continuous updating of predicted degradation using the model based on field data that is continuously collected for monitored structures of interest. More broadly, as civil engineering structures age, and the resources available to perform repair or retrofit activities for these structures struggle to keep pace with the increasing inventory of aging structures, particularly in the United States, the proposed degradation function provides a way to assess long-term structural reliability. While precise values for probabilities of failure can be difficult to predict, comparisons of projected degradation provide a way to prioritize structures based on predicted performance. With varying predicted reliabilities over extended time periods, this can inform the prioritization of resources to repair or retrofit the most vulnerable structures. The result will be decisions that minimize the expected probabilities of failure over time periods of interest.

4. CONCLUSIONS

The main contribution of this work is to propose a new stochastic model for structural degradation that is based on the physical mechanisms of varying degradation modes and matches theoretical constraints and bounds on the degradation function. The model includes varying degradation rates, the ability to account for the effect of one degradation mode on the rate of another, and modes that both begin at the time of construction and start later in the structural lifetime. The proposed degradation function includes two parameters that can be learned from collected field or structural monitoring data. Use of the function provides more accurate assessments of long-term reliability, supporting reliability-based decision-making in terms of maintenance, repair, or retrofit decisions over structural lifetimes. Comparing degradation predictions over longer time periods enables prioritization of structures for such repair or retrofit activities based on the predicted reliabilities.

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