Service Life Prediction of SFRC Corbels Subjected to Chloride-Induced Corrosion

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ABSTRACT: Corrosion in reinforced concrete structures due to aggressive chloride contamination, is a major contributing factor resulting in deterioration of resistance in these structures; in addition to the resistance of structures, the service life of reinforced concrete structures is also influenced by chloride-induced corrosions. Due to the importance of effect of corrosion, in this paper, an effort is made to determine the service life of steel fiber reinforced concrete (SFRC) corbels considering chloride-induced corrosion. The approach employed herein, is based upon an analytical formulation of resistance for these components; the effect of corrosion due to chloride penetration through reinforced concrete is taken into account by applying Fick's second law of diffusion. Finally, for the selected SFRC corbels, the effects of initial chloride concentration on the surface of concrete and the thickness of concrete cover on the load carrying capacity of steel fiber reinforced concrete corbels and their service lives are investigated.

1. INTRODUCTION

Many substantial research efforts have been carried out regarding the prediction of service life of reinforced concrete (RC) structures [Clifton (1993), Prezzi et al. (1996), Maage et al. (1996), Liang (1999), Thoft-Christensen (2000a), Mori and Ellingwood (1993), Li (2004)]. In this paper, service life is specifically investigated for steel fiber reinforced concrete (SFRC) corbels by considering the effect of chloride-induced corrosion in these members. In order to study SFRC corbels, an analytical methodology is employed herein [Fattuhi (1994)1: formulation is utilized to calculate the load carrying capacity of SFRC corbels.

In order to account for the effect of corrosion on reinforced concrete due to chloride ions, Fick's second law of diffusion is utilized; the solution of Fick's second law of diffusion is then incorporated with the load carrying capacity of corbels in order to obtain the service life prediction of these components [Crank (1975), Thoft-Christensen (2003)]. The SFRC corbels studied in this paper, are selected from Fattuhi (1990b). These corbels are labeled as 24, 29, 46, and 48. By considering corbel samples with different covers and by imposing changes in the initial chloride concentration on the surface of concrete, the effects of these two major contributing factors are investigated.

2. SERVICE LIFE OF REINFORCED CONCRETE

The service life of reinforced concrete structures has been investigated in various contexts and applications [Clifton (1993), Prezzi et al. (1996), Maage et al. (1996)]. Several different definitions of service life may be considered based upon the decision maker's engineering judgement and

preferential characteristics. For instance, Liang (1999) defines the service life as "the time from construction until the chloride content at the depth of reinforcement is high enough to initiate corrosion." In other words, the initiation of corrosion is defined as the service life of the structure. Similarly, Maage et al. (1996) defines service life as the time when, at the layer of reinforcement, the chloride concentration raises to a certain critical concentration. In some other studies, such as Thoft-Christensen (2000a), the service life of reinforced concrete structures is defined as the initiation of cracks in these Thoft-Christensen structures. (2000b)extended the service life of RC structures even further by considering the service life as the time which is required for corrosion cracks to grow to a certain width; therefore, even for a specific type of structure, service life could be different based upon the implemented approach.

Furthermore, Mori and Ellingwood (1993) have conducted a parametric study of factors contributing to prediction of service life of structures in a reliability context. In order to achieve a realistic service life prediction, degradation characteristics and load process statistics need to be identified for any facility for which the reliability method is going to be utilized. Li (2004) applies performance criteria within a probabilistic approach to predict the service life of reinforced concrete structures affected by corrosion; these performance criteria are cracking, deflection and strength, each of which are considered a distinct phase within the service life of reinforced concrete. For instance, when considering excessive deflection due to corrosion, once a deflection limit is determined, by incorporating probabilistic methods, prediction for service life can be obtained. A similar strategy is employed for predicting cracking and strength of these structures. A probabilistic approach is employed by Li (2004) in order to account for the uncertainty in the degradation process. Li (2004) found that cracking due to chloride corrosion in flexural members located in a marine environment transpires at approximately 18% of the total service life. It is also noted that from the point in time where these flexural components were out of service, due to deflection criteria, to the point where the structure is deemed unsafe, there was still 13% of service life available.

3. ANALYTICAL FORMULATION FOR THE LOAD CARRYING CAPACITY OF SFRC CORBELS

Corbels are structural elements typically constructed using precast reinforced concrete. These structural elements are used to transfer applied loads from slabs or beams to either columns or walls. The reason that corbels are typically built using the pre-cast method, is due to the fact that pre-cast elements, in general, are constructed with better concrete quality compared to those constructed by cast-in-place methods [Strauss et al. (2006), Fattuhi (1990a)]. Depending upon the type of structure, position of the column, and the applied loading on the structure, up to four corbels may be utilized alongside the column height [Fattuhi (1990a)]. For steel fiber reinforced corbel samples in which flexural failure has occurred during the performed experiments, the following equation is derived for calculation of the load carrying capacity of SFRC corbels, V_n (N) [Fattuhi (1994)]:

$$V_{n} = \frac{f_{y}A_{s}}{a} \left(d - \frac{a_{I}}{2} \right) + \frac{k_{0}f_{ct}b}{2a} \left(h - \frac{a_{I}}{\beta_{I}} \right) \left(h + \frac{a_{I}}{\beta_{I}} - a_{I} \right)$$
 (1)

In equation (1), f_y (MPa) and A_s (mm²) are yield stress and area of main tensile reinforcement, respectively; h (mm) and d (mm) are the total and effective depth of the corbel, respectively. Parameter a (mm) denotes the distance between the center of loading to the support and b is the width of the sample. f'_c (MPa) and f_{ct} (MPa) are the compressive and tensile strength of concrete. a_1 is obtained considering the following expression [Fattuhi (1994)]:

$$a_{I} = \frac{f_{y}A_{s} + k_{0}f_{ct}bh}{0.85 f_{c}^{'}b + k_{0}f_{ct}\frac{b}{\beta_{I}}}$$
(2)

In order to account for the contribution of steel fibers in the concrete, k_0 is implemented within equations (1) and (2). Based on the regression analyses performed on the results of the tested steel fiber reinforced corbels, the following equation is proposed for k_0 [Fattuhi (1994)]:

$$k_0 = \frac{9.519}{f_c^{(0.957)}} \tag{3}$$

 β_1 in equation (2) is calculated based on ACI code [ACI (2014)]; for values of f'_c between 2500 psi and 4000 psi, β_1 is equal to 0.85; for concrete strengths between 4000 psi and 8000 psi, β_1 is computed using equation (4):

$$\beta_I = 0.85 - \frac{0.05 \left(f_c' - 4000 \right)}{1000} \tag{4}$$

Also for concrete with $f_c' > 8000$ psi, β_1 has a value of 0.65 [ACI (2014)]. The SFRC corbel samples used in this study are selected from Fattuhi (1990b); these corbels correspond to corbel numbers 24, 29, 46, and 48. It should be noted that corbels 24 and 48 are the same, except the cover provided for their main reinforcement; additionally, corbels 29 and 46 are also the same but possess different concrete covers for the main bars. The concrete covers provided in corbels 24 and 29 are 20 mm, whereas in corbels 46 and 48, the covers are 50 mm. In general, increasing the cover leads to a decrease in the effective depth to main bars of the corbel; this reduction of effective depth to main bars for corbels 46 and 48 is equal to 25%. The material

properties and dimensions of these corbels are presented in Table 1. In Table 1, V_{Exp} and $V_{Analytic}$ denote the corbel resistances obtained through experiments and proposed analytical approach, respectively.

Flexural failure has been observed in all selected samples herein (i.e., corbels 24, 29, 46, and 48). Even though flexural/shear failure is reported in Fattuhi (1990b) for corbel 24, applying the analytical formulation for predicting the load carrying capacity of corbels (i.e. equation (1)) shows agreement between the load carrying capacity obtained in the experiments [Fattuhi These load carrying (1990b)]. capacities correspond to 131.5 kN for the experimental result, and 132.025 kN computed by equation (1). The difference between these values is 0.399%; therefore, when calculating load carrying capacity, equation (1) is sufficient for corbel 24.

4. CORROSION OF REINFORCED CONCRETE

One of the major factors contributing to the deterioration of reinforced concrete structures is corrosion of reinforcements. Corrosion in reinforced concrete structures may have various effects. Loss of bond between reinforcement steel and concrete, loss of reinforcement cross section, crack initiation and evolution and spalling are some of these impacts [Zhang and Lounis (2006)]. In Thoft-Christensen (2003), the deterioration exhibited in reinforced concrete structures, is categorized in six steps; these steps are: 1) penetration of chloride in concrete, 2) initiation of corrosion of steel reinforcement, 3) evolution of corrosion in reinforcement, 4) initiation of

Table 1: Properties of selected SFRC corbels

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Corbel #	a (mm)	b (mm)	d (mm)	h (mm)	f'c (MPa)	f _{ct} (MPa)	f _y (MPa)	$A_s (\mathrm{mm}^2)$	V _{Exp} (kN)	V _{Analytic} (kN)	βΙ
46	75	154.5	92.00	146.0	28.19	4.37	451	100.531	74.50	81.196	0.8455
29	75	151.5	122.5	148.5	30.21	4.42	451	100.531	100.0	99.205	0.8309
48	80	155.5	93.20	148.2	28.92	5.16	454	157.080	100.0	104.36	0.8402
24	80	153.0	124.0	150.0	27.38	5.12	454	157.080	131.5	132.03	0.8500

cracking in concrete, 5) evolution of cracks in concrete, and finally 6) spalling of concrete.

Corrosion in reinforced concrete structures is mostly due to chloride-induced or carbonationinduced corrosion [Roberge (1999)]. In this study, the calculations are focused on the effect of corrosion caused by the penetration of chloride ions through concrete. The corrosion initiation time is defined as the time required for the concentration of chloride on the surface of reinforcement to exceed a certain threshold concentration [Zhang and Lounis (2006), Arora (1997)]. In order to compute the time of initiation of corrosion, Fick's second law is applied; Fick's second law is a relation by which chloride concentration is determined for any time at any location of the reinforced concrete [Crank (1975)].

$$\frac{\partial C(x,t)}{\partial t} = D_c \frac{\partial^2 C(x,t)}{\partial x^2} \tag{5}$$

In equation (5), C(x,t), which has units of g/mm³, denotes the time-dependent chloride concentration at time t (years) and at depth x (mm) from the surface of concrete. D_c denotes the effective chloride diffusion coefficient and has units of mm²/years. The solution for equation (5) may be determined assuming that D_c and surface chloride concentration are both constant in time and using the boundary conditions $C(x=0, t>0) = C_0$, initial condition C(x>0, t=0) = 0 [Crank (1975)]:

$$C(x,t) = C_0 \left[1 - erf\left(\frac{x}{\sqrt{4D_c t}}\right) \right]$$
 (6)

In equation (6), C_0 is the chloride concentration on the concrete surface, which has units of percent weight of concrete, and *erf* is the error function. Error function is computed by the following integral:

$$\operatorname{erf}(t) = \frac{1}{\sqrt{\pi}} \int_{-t}^{t} e^{-x^{2}} dx = \frac{2}{\sqrt{\pi}} \int_{0}^{t} e^{-x^{2}} dx \tag{7}$$

In order to find the time of initiation of corrosion, equation (7) needs to be rewritten in terms of time, and the depth of corrosion. *x* should be assumed to be equal to the thickness of concrete cover provided for the investigated corbel samples. Using this approach, the time of corrosion can be obtained as follows [Rafiq (2005)]:

$$T_{corr} = \frac{d^2}{4D_c} \left[erf^1 \left(\frac{C_0 - C_{cr}}{C_0} \right) \right]^{-2}$$
 (8)

Where T_{corr} (years) denotes the time at which the corrosion of the main reinforcement of the corbel commences, d (mm) is the concrete cover provided for the main reinforcement, and C_{cr} is the critical (threshold) concentration. The evolution of corrosion in steel reinforcement, could be modeled in two separate ways; general (uniform) corrosion, and pitting corrosion [Val and Melchers (1997), Marsh and Frangopol (2008)]. The general (uniform) corrosion model is utilized in this study; this model is based upon the assumption that the whole cross-section of the steel reinforcement is reduced uniformly. Assuming uniform corrosion, the total effective reinforcement area is computed by the following equation:

$$A_s(t) = \begin{cases} \frac{n\pi D_0^2}{4} & \text{for } 0 < t < T_{corr} \\ \frac{n\pi [D_0 - r_{corr}(t - T_{corr})]^2}{4} & \text{for } T_{corr} < t \end{cases}$$
(9)

Where, $A_s(t)$ (mm²) is the total reinforcement area at time t in years, n denotes the number of steel bars which are considered in corrosion effect, D_0 (mm) is the initial diameter of the reinforcement bar prior to initiation of corrosion in the steel bar, T_{corr} is the time in years at which corrosion of steel bars starts, and r_{corr} (mm/years) denotes the rate of corrosion [Marsh and Frangopol (2008), Enright and Frangopol (1998)]. In order to observe how the initial chloride concentration on the surface of concrete affects the service life of SFRC corbels, two

different values for C_0 , based on information in Akgül (2002), are used; these values are 0.1% and 0.5% of the weight of concrete. Conducting interpolation, based on information provided in Akgül (2002), and accounting for strength of concrete D_c is assumed to be 54.2 (mm²/year). Threshold chloride concentration for reinforcement bars C_{cr} is assumed to be 0.04 (% by weight of concrete) [Akgül (2002)]. In order to employ the most realistic value for r_{corr} (rate of corrosion) a value of 0.06 (mm/year) is selected [Akgül (2002), Marsh and Frangopol (2008), Kim (2011)].

5. EFFECT OF CHLORIDE CONCENTRATION ON THE PREDICTION OF SERVICE LIFE OF SFRC CORBELS

Using the equations and procedure outlined in section 4, the service life is estimated for separate corbels. Figure 1 demonstrates the load carrying capacity of corbels 24, 29, 46, and 48 for C_0 values of 0.1% and 0.5% of weight of concrete with respect to time; the time horizon is extended until the reinforcement bars are entirely corroded. Figure 2 depicts the normalized corbel resistances for corbels 46 and 48 with respect to time, considering C_0 values of 0.1% and 0.5% of weight of concrete; whereas Figure 3 shows the normalized corbel resistances for corbels 24 and 29 with respect to time, considering C_0 values of 0.1% and 0.5% of weight of concrete. Based on Figures 1 through 3, it is observed that for all selected corbels, even though the initial resistances prior to initiation of corrosion are equal, once the corrosion starts, the increase of initial chloride concentration on the surface of concrete leads to a decrease in the load carrying capacity of corbels. Additionally, Figures 1 through 3, illustrate that the initiation of corrosion of steel reinforcements decrease dramatically when there is an increase of initial chloride concentration on the concrete surface; the values demonstrating the initiation of corrosion in the steel bars are presented in Table 2. As observed from these values, by increasing the initial chloride concentration on the surface of concrete C_0 from 0.1% to 0.5%, corrosion commencement of reinforcement bars decreases by 76.89% for all selected corbel samples. As demonstrated in Figures 1 through 3, the time required for the entire cross section of steel reinforcement to be corroded by chloride ions is given in Table 3.

Table 3: Time at which corrosion of reinforcement starts T_{corr} (years)

2001 (2001 (2000 2)						
C	Corbel					
C_{θ}	24	29	46	48		
0.1	5.210	5.210	32.56	32.56		
0.5	1.204	1.204	7.526	7.526		

Table 3 shows that for corbels with higher values of initial chloride concentrations on the surface of concrete, the time span in which the entire cross section of steel reinforcements is corroded is shorter. Based on Table 3, it is observed that for corbels 24, 29, 46, and 48, by increasing C_0 from 0.1% to 0.5%, the reduction in time to total corrosion of reinforcement decreases by 2.31%, 2.90%, 15.2%, and 12.6%, respectively.

Table 2: Time to complete corrosion of main reinforcement (years)

	reinjoreement (years)							
C		Corbel						
(C_{θ}	24	29	46	48			
	0.1	171	138	165	199			
	0.5	167	134	140	174			

In order to compare the service life of SFRC corbels with respect to different values of C_0 , it is assumed that the service life of the selected corbel samples is determined by a specific percentage drop in load carrying capacity of these components; this criterion is solely based on engineering judgment and could be changed based upon the decision maker's criterion. In the present study, the associated percentage drop is assumed to be 15% (i.e., once the load carrying capacity of a corbel is 85% of its initial value, the achieved). service life is The corresponding to 85% of initial load carrying capacities for selected corbel samples in this

study, considering different values for C_0 are presented in Table 4. In this table, t=0 and t= t_L are year zero and the corresponding service life, respectively. Table 5 illustrates the service life of corbels for C_0 = 0.1 and 0.5.

As it is observed from Table 5, by increasing the initial chloride concentration on the surface of concrete from 0.1% to 0.5% of weight of concrete, as it is expected, the time at which the corbel capacity reaches to 85% of its initial value at year zero, decreases. This decrease is 15.5%, 18.14%, 47.59%, and 44.71% for corbels 24, 29, 46, and 48, respectively.

EFFECT OF COVER ON SERVICE LIFE OF SFRC CORBELS

Figure 4 shows the resistance of corbels 24 and 48 with respect to time for $C_0 = 0.1$ and 0.5. Figure 5

Table 4: Initial (t=0) and end of life $(t=t_L)$ resistance of corbels for $C_0=0.1$ and 0.5 (kN)

	Corbel #				
	24	29	46	48	
t = 0	132.02	99.204	81.196	104.36	
$t = t_L$	112.21	84.324	69.016	88.707	

Table 5: Service life of corbels t_L (years)

C		Corl	Corbel #			
C_{θ}	24	29	46	48		
0.1	25.88	22.05	52.62	56.00		
0.5	21.87	18.05	27.58	30.96		

portrays the corbel resistances for corbels 29 and 46 with respect to time considering $C_0 = 0.1$ and 0.5. Figures 6 and 7 correspond to Figures 4 and 5, respectively and demonstrate the normalized load carrying capacities of corbels. Similar to observations made regarding the effect of initial chloride concentration on the surface of concrete on corbel's service life, the effect of concrete

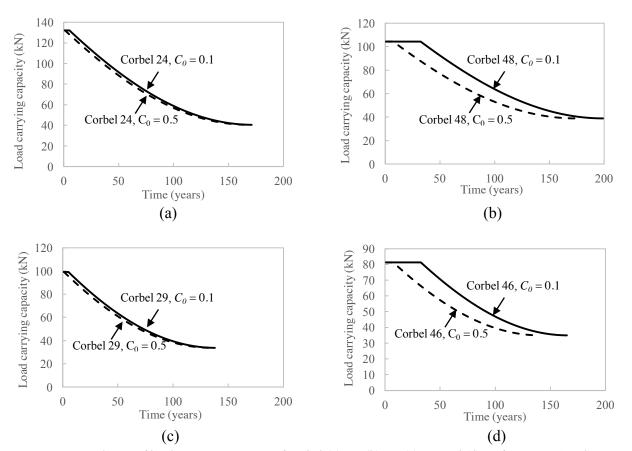


Figure 1: Evolution of load carrying capacity of corbel (a) 24, (b) 48, (c) 29, and (d) 46 for $C_0 = 0.1$ and 0.5

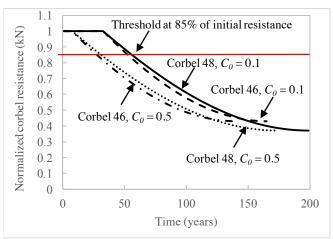


Figure 2: Normalized load carrying capacity for corbels 46 and 48 for different values of C_0

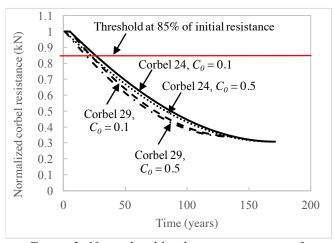


Figure 3: Normalized load carrying capacity for corbels 24 and 29 considering different values of C_0

cover for main reinforcement in corbels, as demonstrated in Figures 4 through 7, is investigated. Cover thickness for corbels 24 and 29 is 20 mm, whereas for corbels 46 and 48, cover is 50 mm; by considering the service life criterion described in previous section, it is observed that in the case of corbels with less cover thickness, the service life decreases too. Taking into account corbels 24 and 48, which are similar except for their covers (corbel 24 has 60% less cover compared to corbel 48), and based on data available in Table 5, when C_0 is 0.1% of weight of concrete, service life of corbel 24 is decreased by 53.79%. Similarly, for corbels 29 and 46, with $C_0 = 0.1$, a 60% decrease in concrete cover leads

to a 58.09% decrease in service life of the corbel. When C_0 is increased from 0.1 to 0.5, comparing corbels 24 and 48, a 60% decrease in concrete cover corresponds to a 29.36% decrease in service life of the component. Also in the case of corbels 29 and 46, which are the same except for their concrete covers, a 60% decrease in cover, causes a 34.55% drop in service life of corbels.

Hence, it is observed that by increasing the initial chloride concentration on the surface of concrete, the effect of cover thickness, even though still substantial, becomes less significant. Also as illustrated in Figures 4 through 7, it is observed that in corbels 46 and 48, by increasing the cover thickness, even though the

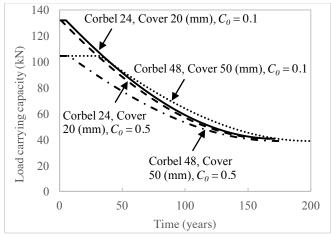


Figure 4: Resistance of corbels for different values of C_0 and different covers (corbels 24 and 48)

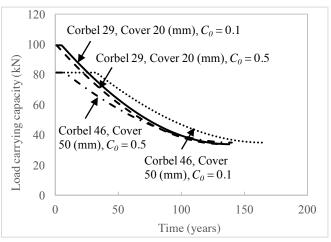


Figure 5: Resistance of corbels for different values of C_0 and different covers (corbels 29 and 46)

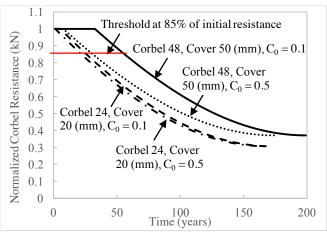


Figure 6: Normalized resistance of corbels for different values of C_0 and different covers (corbels 24 and 48)

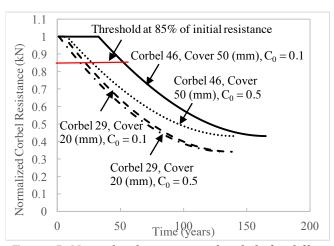


Figure 7: Normalized resistance of corbels for different values of C_0 and different covers (corbels 29 and 46)

commencement of corrosion of the reinforcement steel is delayed, the load carrying capacity of these corbels, compared to those of corbels' 29 and 24, respectively, is decreased. This reduction in resistance is due to the fact that the overall height of the corbels are not changed; increasing cover in these components is accompanied with a decrease in the effective depth of the corbels, thus leading to lower values of corbel resistance.

7. CONCLUDING REMARKS

This paper presents an analytical investigation of service life of steel fiber reinforced concrete corbels, affected by chloride-induced corrosion of main reinforcement. By introducing a load carrying capacity threshold, based upon which the

corresponding service lifetimes of corbels are determined, a study is conducted in order to investigate the impact of altering the thickness of concrete cover in corbels, as well as the effect of initial chloride concentration on the surface of concrete. It is observed that with higher values of initial chloride concentration on the concrete surface (e.g. $0.1 < C_0 < 0.5$), the effect of reduction in cover thickness on predicted service life of SFRC corbels, albeit still substantial, is not significant. Overall, the results from this study can be used by decision makers to make informed choices regarding the initial design of SFRC corbels.

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