

Bayesian Damage Detection for Bridges under Noisy Condition

Yoshinao Goi

Assistant Professor, Dept. of Civil and Earth Resources Engineering, Graduate School of Engineering, Kyoto University, Kyoto, Japan

Chul-Woo Kim

Professor, Dept. of Civil and Earth Resources Engineering, Graduate School of Engineering, Kyoto University, Kyoto, Japan

ABSTRACT: This study is intended to verify validity of an efficient damage detection method by means of a Bayesian approach especially for noisy operational condition. A Bayesian inference was adopted to the regressive model representing bridge vibration. The posterior distribution for the regressive coefficients provides reasonable damage-sensitive features. Bayesian hypothesis testing is formulated using a Bayes factor, which is defined as a ratio of marginalized likelihoods to detect anomaly in the damage-sensitive features. Feasibility of the proposed method under noisy condition is examined via a field experiment on a continuous Gerber-truss bridge whose truss member was artificially severed. The proposed method robustly detected a damage considered in the experiment even under the varying traffic loadings.

1. INTRODUCTION

Management of aging infrastructure is a crucially important issue confronting civil engineering professionals. To reduce the potential risk of structural failure as well as life cycle costs, an efficient inspection method is desirable for preventive maintenance. Techniques of structural health monitoring (SHM) based on vibration measurements have been attracting bridge authorities. Changes in structural integrity of bridges engender changes in their modal properties that are identifiable from vibration data (Deramaeker et al. 2007, Zhang, 2007). Consequently, the vibration-based SHM is a useful technique if the modal properties of a bridge can be identified effectively. For bridge health monitoring, the output-only modal identification methods using traffic-induced vibration is an effective way to monitor the bridge since it requires no traffic control.

In the output-only methods conducted on actual bridges, however, noises caused by unknown environmental influences often contaminate identified modal properties. To avoid influences of the noise to the identified modal

properties, existing studies have developed damage indicator that is directly defined from a mechanical system model representing the bridge vibration. Nair et al. (2006) investigated damage sensitive-feature consisting of univariate autoregressive (AR) coefficients for a model building. To enable reliable decision-making for bridge maintenance, Goi and Kim (2017a) investigated a hypothesis-testing-based damage detection method using a vector AR (VAR) model through a field experiment on a truss bridge. Thereafter Goi and Kim (2017b) improved the hypothesis testing procedure by means of Bayesian statistics. For the proposed Bayesian damage detection, the AR model provides likelihood function for observed bridge acceleration, and thus the Bayesian inference for the AR model provides the posterior distribution of the regressive coefficients. Based on the posterior distribution, damage-sensitive features of the bridge are extracted. The proposed method adopts ratio of marginal likelihood called Bayes factor (Kass and Raftery, 1995), as a damage indicator for hypothesis testing.

The feasibility of the proposed methods has been verified through field experiments for a simply supported steel truss bridge (Goi and Kim 2017b) and a simply supported steel plate girder bridge (Goi and Kim 2018). For those simple structures, the noise disturbance caused by operational traffic loading slightly affects to the result of the damage detection. The feasibility investigation under the noisy operational condition is accordingly desired as the next step. This study therefore investigates feasibility of the Bayesian damage detection under the noisy condition through a field experiment on a continuous Gerber-truss bridge. The feasibility of the proposed method is examined by means of a damage experiment which introduces artificial damage on a truss member of the bridge. A concise damage indicator that evaluates global changes in the damage sensitive features is also proposed for an easy decision-making.

2. METHODOLOGY

The proposed damage detection method comprises two steps: First, the posterior distribution for model parameters composing a VAR model from a data set of the intact bridge is determined by means of the Bayesian inference (Bishop, 2006). Second, Bayes factors calculated from newly observed data provide indicators for damage detection according to the Bayesian hypothesis testing. Figure 1 shows the flow of the proposed damage detection method.

2.1. Bayesian inference

Let $\mathbf{y}(k) \in \mathbb{R}^{m \times 1}$ denote a column vector of the discrete time series of the measured acceleration whose components correspond to m measurement locations. The following VAR model approximates the time series obtained from a linear structural system excited by white noise with sufficient model order p (He & De Roeck, 1997).

$$\mathbf{y}(k) = \sum_{i=1}^p A_i \mathbf{y}(k-i) + \mathbf{e}(k) \quad (1)$$

where $A_i \in \mathbb{R}^{m \times m}$ denotes the i -th AR coefficient matrix and $\mathbf{e}(k) \in \mathbb{R}^{m \times 1}$ denotes a white noise

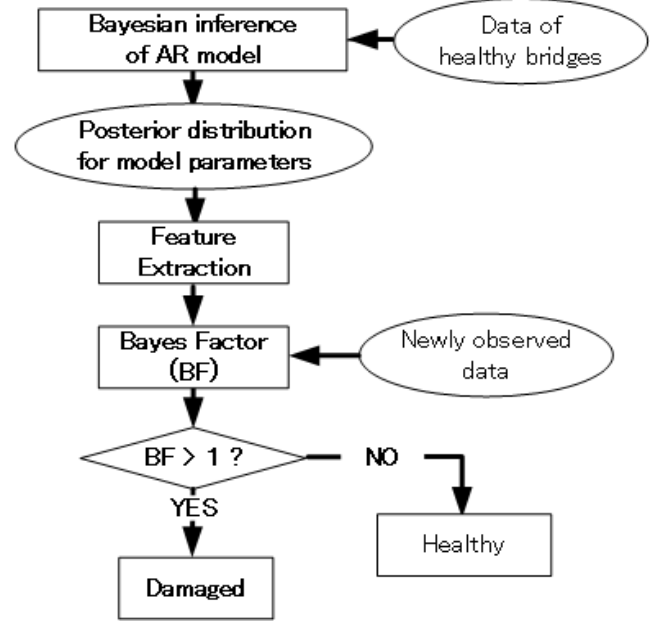


Figure 1: Flow of the Bayesian damage detection.

vector. Focusing on j -th row in Eq. (1), the following regressive model is obtained.

$$y_j(k) = \sum_{i=1}^p \mathbf{a}_{ji} \mathbf{y}(k-i) + e_j(k) \quad (2)$$

where $y_j(k)$ and $e_j(k)$ respectively represent j -th element of $\mathbf{y}(k)$ and $\mathbf{e}(k)$, and $\mathbf{a}_{ji} \in \mathbb{R}^{1 \times m}$ represents j -th row of A_i . Assuming that the elements of $\mathbf{e}(k)$ are statistically independent from each other and follow Gaussian distribution with expectation 0, then $y_j(k)$ follows Gaussian distribution with expectation $\sum_{i=1}^p \mathbf{a}_{ji} \mathbf{y}(k-i)$. For simplicity, letting $t_k = y_j(k)$, $\mathbf{w} = [\mathbf{a}_{j1}, \dots, \mathbf{a}_{jp}]^T \in \mathbb{R}^{m \times p}$ and $\boldsymbol{\phi}_k = [\mathbf{y}(k-1)^T, \dots, \mathbf{y}(k-p)^T]^T \in \mathbb{R}^{m \times p}$, probability distribution function (PDF) of t_k is given as

$$p(t_k | \boldsymbol{\phi}_k, \mathbf{w}, \beta) = N(t_k | \mathbf{w}^T \boldsymbol{\phi}_k, \beta^{-1}) \quad (3)$$

where $N(\cdot | \mu, \sigma^2)$ denotes PDF of Gaussian distribution with expectation μ and variance σ^2 . β represents the precision parameter of the regression, which is the inverse of the variance of the noise term $e_j(k)$. Assuming that n samples of t_k and $\boldsymbol{\phi}_k$ are observed, and letting $\mathbf{t} = [t_1, \dots, t_n]^T \in \mathbb{R}^{n \times 1}$ and $\boldsymbol{\Phi} = [\boldsymbol{\phi}_1 \dots \boldsymbol{\phi}_n]^T \in \mathbb{R}^{n \times m \times p}$, the likelihood function for the parameters \mathbf{w} and β is given as

$$p(\mathbf{t} | \boldsymbol{\Phi}, \mathbf{w}, \beta) = \prod_{k=1}^n N(t_k | \mathbf{w}^T \boldsymbol{\phi}_k, \beta^{-1}). \quad (4)$$

With the observed \mathbf{t} , Bayesian theorem provides a posterior joint PDF for \mathbf{w} and β as the following conditional PDF.

$$p(\mathbf{w}, \beta | \mathbf{t}) = p(\mathbf{t} | \mathbf{w}, \beta) p(\mathbf{w}, \beta) p(\mathbf{t})^{-1} \quad (5)$$

where Φ is omitted from above equation for simplicity. $p(\mathbf{w}, \beta)$ stands for a prior joint PDF for \mathbf{w} and β , i.e., a PDF predefined before the observation of \mathbf{t} . $p(\mathbf{t})$ is a constant in manner of the Bayesian inference, and therefore the posterior PDF $p(\mathbf{w}, \beta | \mathbf{t})$ is obtained only from the observed data \mathbf{t} and the prior PDF $p(\mathbf{w}, \beta)$. Adopting a non-informative prior proposed by Jeffreys (1964), the posterior distribution is obtained as follows (Goi and Kim, 2017b).

$$p(\mathbf{w}, \beta | \mathbf{t}) = \frac{1}{N(\mathbf{w} | \mathbf{m}, \beta^{-1} L^{-1}) \text{Gam}(\beta | a, b)} \quad (6)$$

$$\mathbf{m} = L^{-1} \Phi^T \mathbf{t} \quad (7)$$

$$L = \Phi^T \Phi \quad (8)$$

$$a = n/2 \quad (9)$$

$$b = \frac{1}{2} \|\mathbf{t} - \Phi \mathbf{m}\|^2 \quad (10)$$

where $\text{Gam}(\beta | a, b)$ denotes PDF of β following Gamma distribution. The parameters \mathbf{m} , L , a and b are hyperparameters of the posterior PDF, which determine the functional properties of the posterior PDF.

2.2. Feature extraction and hypothesis testing

Letting D_r denotes a reference dataset of acceleration that is observed from a bridge under healthy condition, the hyperparameters \mathbf{m}_r , L_r , a_r and b_r of the posterior distribution $p(\mathbf{w}, \beta | D_r)$ are obtained by Eq. (7) to Eq. (10). Since Eq. (8) produces real, symmetric and positive definite matrix L_r , the singular value decomposition of the hyperparameter L_r is given as

$$L_r = U \tilde{L} U^T = [U_1 \ U_2] \begin{bmatrix} \tilde{L}_1 & O \\ O & \tilde{L}_2 \end{bmatrix} \begin{bmatrix} U_1^T \\ U_2^T \end{bmatrix} \quad (11)$$

where $\tilde{L} \in \mathbb{R}^{mP \times mP}$ is a diagonal matrix consisting of the singular values and $U \in \mathbb{R}^{mP \times mP}$ is an orthogonal matrix consisting of the singular vectors. $\tilde{L}_1 \in \mathbb{R}^{q \times q}$ and $U_1 \in \mathbb{R}^{mP \times q}$

respectively represent the q largest singular values and the corresponding singular vectors. O represents a null matrix with a proper size. Let $\tilde{\mathbf{w}}$ denotes the orthogonal transformation of \mathbf{w} such that $\tilde{\mathbf{w}} = U^T \mathbf{w}$. Eq. (6) leads the posterior distribution for $\tilde{\mathbf{w}}$ and β as

$$p(\tilde{\mathbf{w}}, \beta | D_r) = \frac{1}{N(\tilde{\mathbf{w}} | \tilde{\mathbf{m}}, \beta^{-1} \tilde{L}^{-1}) \text{Gam}(\beta | a_r, b_r)} \quad (12)$$

where $\tilde{\mathbf{m}} = U^T \mathbf{m}_r$. This study presumes the first q elements in $\tilde{\mathbf{w}}$ are damage sensitive features related to modal properties of bridges and investigates hypothesis testing to detect changes in those features.

Letting D_t denotes a newly observed test dataset for the hypothesis testing, the Bayes factor for a null hypothesis H_0 and an alternative hypothesis H_1 is defined as a ratio of their marginal likelihoods as follows (Kass and Raftery, 1995).

$$B = \frac{\iint p(D_t | \tilde{\mathbf{w}}, \beta) p(\tilde{\mathbf{w}}, \beta | H_1) d\tilde{\mathbf{w}} d\beta}{\iint p(D_t | \tilde{\mathbf{w}}, \beta) p(\tilde{\mathbf{w}}, \beta | H_0) d\tilde{\mathbf{w}} d\beta} \quad (13)$$

The null and alternative hypotheses respectively provide a stochastic model representing healthy and damaged condition of the bridge. The null hypothesis representing the healthy condition is merely modeled as the posterior distribution for the reference dataset D_r as Eq. (12). That is, $p(\tilde{\mathbf{w}}, \beta | H_0) = p(\tilde{\mathbf{w}}, \beta | D_r)$. The alternative hypothesis represents that the damage sensitive features are altered due to damage on the bridge. This study adopts the following PDF presuming that the damage sensitive features are uncertain and that the other parameters follow the reference model.

$$p(\tilde{\mathbf{w}}, \beta | H_1) = \frac{1}{N(\tilde{\mathbf{w}} | \tilde{\mathbf{m}}, \beta^{-1} \tilde{L}_{alt}^{-1}) \text{Gam}(\beta | a_r, b_r)} \quad (14)$$

where \tilde{L}_{alt} is a hyperparameter defined as Eq. (15) for the alternative hypothesis representing that the damage sensitive features are uncertain.

$$\tilde{L}_{alt} = \begin{bmatrix} \frac{1}{n_r} \tilde{L}_1 & O \\ O & \tilde{L}_2 \end{bmatrix} \quad (15)$$

where n_r is the total data length of the reference dataset.

Kass and Raftery (1995) suggested interpreting the Bayes factor on the logarithm scale. For example, if $2\ln B$ is over 10, then the evidence of the alternative hypothesis H_1 against the null hypothesis H_0 is interpreted to be ‘very strong’. This study thus investigates the logarithm-scaled Bayes factor $2\ln B$ as a damage indicator.

The marginal likelihoods for the whole observation is given as the product of the Bayes factor obtained from all measurement locations. Accordingly, letting $B^{(j)}$ represent Bayes factors obtained from the j -th measurement location, the Bayes factor for whole observation B_w is given as

$$2 \ln B_w = \sum_{j=1}^m 2 \ln B^{(j)}. \quad (16)$$

Hereafter B_w given in Eq. (16) is named as the “global Bayes factor”, and let $B^{(j)}$ relevant to each of the measurement locations is named as the “local Bayes factor” to avoid confusion.

3. FIELD EXPERIMENT

The target bridge is a continuous steel Gerber-truss bridge, as shown in Figure 2. The bridge comprises 9 spans, among which the 6th span is selected as the test span. The span was about 65.5 m in length and 8.5 m in width. The experiment was conducted during daytime over two days. The experiment truck remained the same, but its total weight varied slightly from 253 kN on the first day to 258 kN on the second. The variation in the weight of the truck can be reasonably neglected herein. 12 accelerometers were installed on the bridge deck nearby the truss nodes. 9 of those accelerometers were at the damage side and the other three were at the opposite side, as shown in Figure 3. The sampling rate for all accelerometers was 200 Hz.

A diagonal member was fully severed as the damage scenario. The artificial damage was applied at the fourth diagonal member which is marked in red in Figure 3. Figures 4a and 4b



Figure 2: Photo of the target bridge.

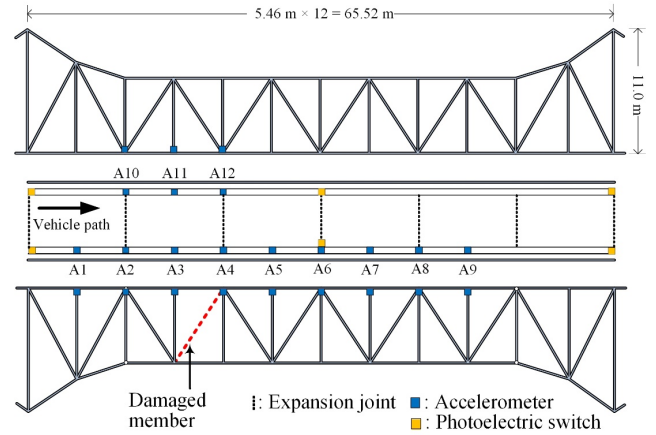


Figure 3: Sensor layout and damage location.

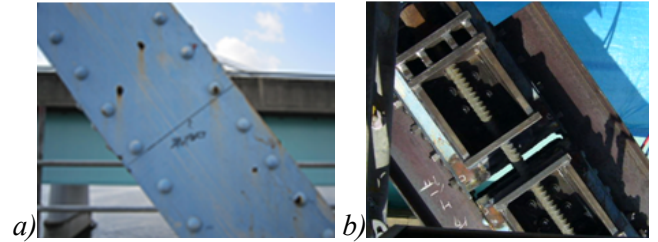


Figure 4: Photos of the damaged member: a) before the damage; and b) after the damage.

Table 1: Scenarios considered in the field experiment.

	Description	Vehicle Speed (km/h)	Number of Sample
INT10	Intact bridge	10	4
INT20	Intact bridge	20	7
INT40	Intact bridge	40	7
DMG10	Full cut in a truss member	10	3
DMG20	Full cut in a truss member	20	6
DMG40	Full cut in a truss member	40	6

respectively show photos of the element before and after it was severed. The scenario before the artificial damage is referred as INT and the scenario after the damage is as DMG. The truck passed the bridge with three constant speeds: 10, 20 and 40 km/h both for INT and DMG scenarios. According to the vehicle speed, the scenarios are referred as INT10, INT20, and so on as listed in Table 1. For further information of the experiment, see also (Kim et al. 2016).

4. FEASIBILITY INVESTIGATION

4.1. Preliminary analysis

Figure 5 and Figure 6 respectively present the time series and PSD curves of the acquired acceleration at sensor A6 for the INT10, INT20 and INT40. Here the PSD curve is estimated as the modified periodograms adopting the Hamming window. These figures describe the

noisy condition. The noise in the time series is possibly because of the vibration induced on the adjoining spans. In the frequency domain, it is hard to distinguish the dominant modes merely from the PSD curves listed in Figure 6. This observation suggests that modal analysis on the target bridge is hard because of the noisy condition. In the previous study (Kim et al. 2016),

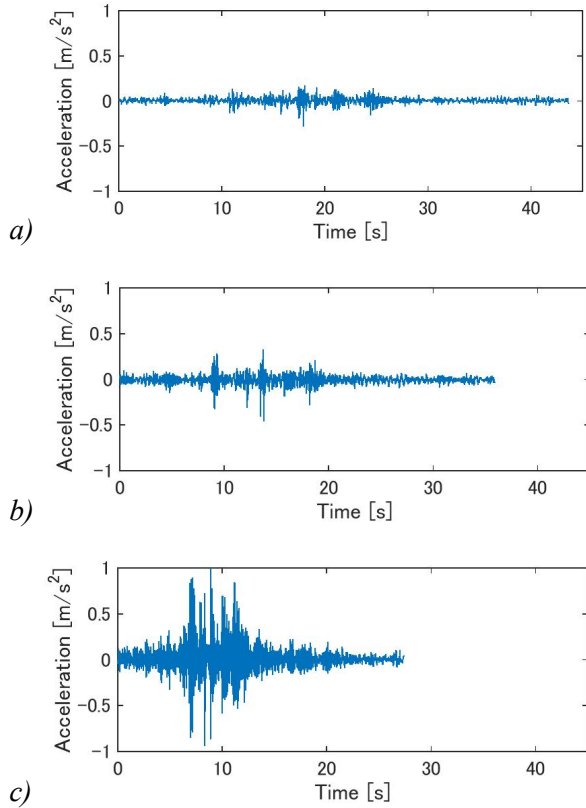


Figure 5: Acquired time series: a) for INT10; b) for INT20; and c) for INT40.

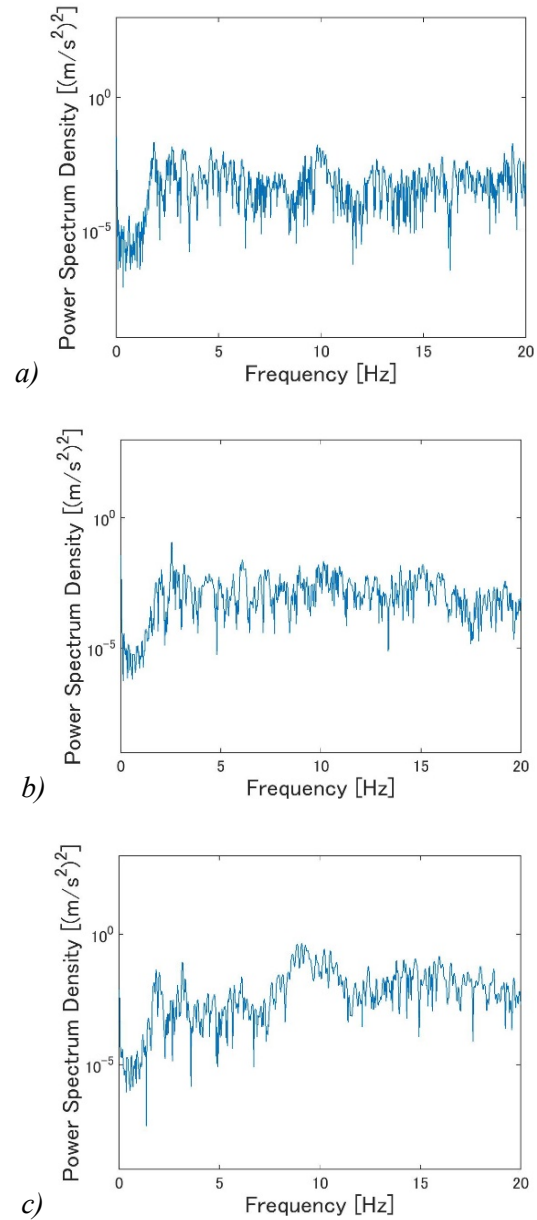


Figure 6: PSD curve estimators: a) for INT10; b) for INT20; and c) for INT40.

troublesome procedure was required to identify the modal properties of the target bridge.

Figure 5 shows that the amplitudes of the dynamic response depend on the vehicle speed. Because the AR model in Eq. (1) assumes constant noise level, the amplitudes are also presumed to be steady. Since the external force is actually not steady, it is possible that the difference in the amplitudes affects the proposed Bayes factor. Therefore in this study, the amplitudes of the time series are roughly normalized beforehand the training and testing. Since this study aims screening of numerous bridges, the normalization procedure needs to be as simple as possible. Therefore, the normalized time series are merely given as follows: calculate the root mean square of the all data contained in the raw time series, and then divide the raw time series by the root mean square.

4.2. Feature extraction

In this study, the whole time series in INT10, INT20 and INT40 scenarios are regarded as the reference dataset for the hypothesis testing. In advance of the hypothesis testing, the number of the damage sensitive features q should be predefined. According to the reference dataset, this study adopts the stabilization diagram (Heylen et al. 1997) as follows to find q : first, reproduce the VAR model as a state space model consisting of the damage sensitive features for each of the singular value orders. Second, estimate modal properties from each of the reproduced state space models. Third, depict the stabilization diagram. And find the reasonable model representing actual modal response of the bridge. Aiming at practical use, the following stability criteria are adopted in this study to depict a stabilization diagram.

$$\text{abs}(f_q - f_{q-1}) < f_{th} \quad (17)$$

$$\text{abs}(\xi_q - \xi_{q-1}) < \xi_{th} \quad (18)$$

$$\text{MAC}(\phi_q, \phi_{q-1}) > \text{MAC}_{th} \quad (19)$$

where f_q, ξ_q , and $\phi_q \in \mathbb{R}^{m \times 1}$ respectively represent any of modal frequencies, damping

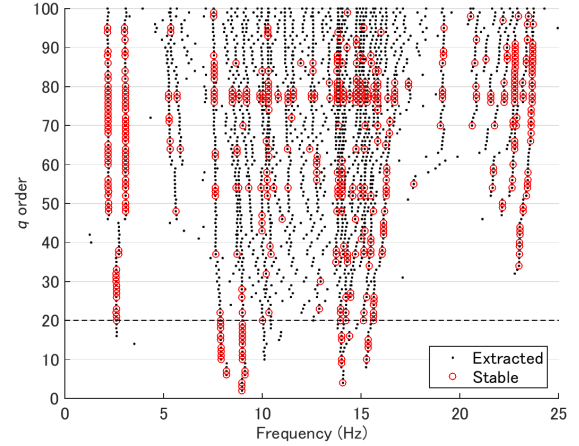


Figure 7: Stabilization diagrams with respect to the q order.

ratios, and modal vectors estimated from the feature extraction with q order. f_{th} , ξ_{th} , and MAC_{th} are predefined thresholds. $\text{MAC}(\cdot, \cdot)$ stands for the modal assurance criterion (Heylen et al. 1997).

Figure 7 shows the stabilization diagrams derived from the AR model with order $p=120$. In this case study, $f_{th} = 0.005$ Hz, $\xi_{th} = 0.005$, and $\text{MAC}_{th} = 0.99$ are respectively adopted. Figure 7 shows that the noisy condition also affects the estimated modal properties. The stabilization diagrams in time domain analysis actually likely to be affected by the spurious estimators under the noisy condition (Kim et al. 2016). In spite of the spurious estimators, the proposed feature extraction still indicates several stable modes in the stabilization diagram: for instance the frequencies 2.6, 7.9, and 9.0 Hz are stably estimated around $q=20$. For damage detection based on the existing OMA, the modal properties need to be identified both for the reference and test datasets to compare them with each other. The damage detection under noisy condition therefore requires considerable effort for engineers to detect changes in the modal properties estimated from the stabilization diagrams, PSD curves, and so on. The proposed method provides an advantage in the feature extraction since the features are not needed to be carefully selected. For instance, $q=20$, for which

the relevant state space model includes the stable frequencies, is adopted for the following damage detection.

4.3. Damage detection

For the hypothesis testing, this study adopts a pre-defined threshold as $2\ln B = 0$, where the model evidences for the null and alternative hypotheses are equivalent. Let the threshold be referred as ‘critical value’ for the hypothesis testing. Leave-one-out cross validation (CV) technique is applied to assess the validity of the Bayes factors. For example, letting a set containing l samples of time series of target variables $\{t_1, t_2, \dots, t_l\}$ and the corresponding input variables are obtained from a bridge without damage, take one of the time series (e.g., t_1) out from this set as a test data, and then refer the remaining samples $\{t_2, \dots, t_l\}$ as the reference dataset. The l Bayes factors are thus obtained from the l samples of time series. The Bayes factors calculated by the CV technique are referred as “CV samples” in this study. The validity of the Bayes factors is confirmed when the CV samples support the null hypothesis.

The global Bayes factors provide general view to grasp feasibility of the proposed method. Figure 8 shows the global Bayes factors obtained from the normalized time series. Here, the blue numbers depicted in Figure 8 denote the vehicle speeds. Figure 8 demonstrates that the damage is effectively identified by the proposed method even under the noisy condition. Figure 9 shows the local Bayes factors. The plots depicted in Figure 9 are relevant to the measurement locations provided in Figure 3. Figure 9 shows that the local Bayes factors have significant values that are considerably higher than the Bayes factors at the other measurement locations. The significant values, which appear at A3 and A4, likely to indicate the damage location (see Figure 3). That is, the damage locally produces evidence against the healthy condition through the proposed method. This observation suggests that the local Bayes factors indicates the damage locations if the damage is considerably serious such that a member thoroughly ruptures. Therefore, the proposed local Bayes factor can be

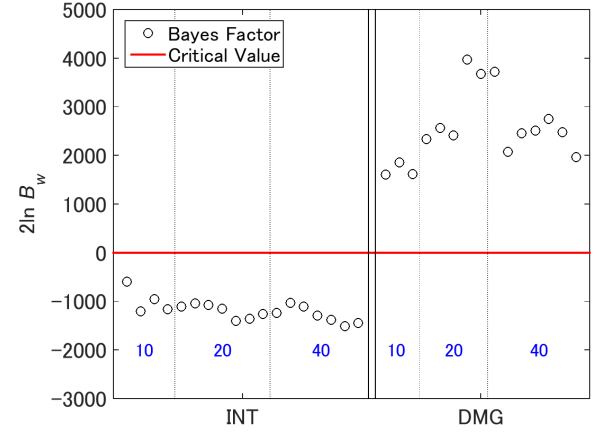


Figure 8: Global Bayes factors.

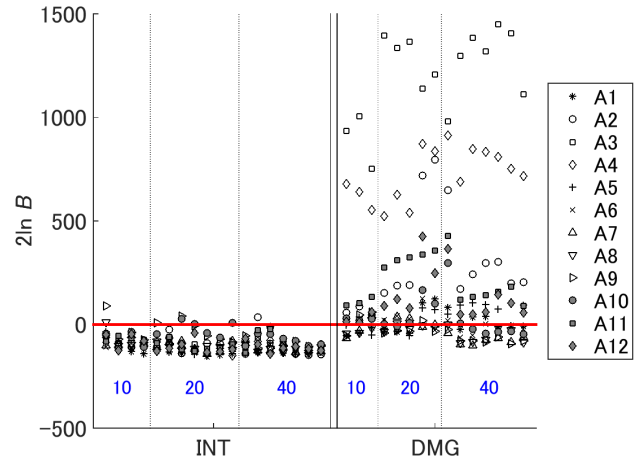


Figure 9: Local Bayes factors.

one of feasible indicators to localize damage for a severe damage.

5. CONCLUSIONS

This study investigates a damage detection method for bridges using traffic-induced vibration to cope with difficulties in decision-making for bridge maintenance. The Bayesian hypothesis testing is adopted to detect subtle changes in modal properties caused by the damage.

A time series of actually observed accelerations of a bridge provides a likelihood function of a vector autoregressive (VAR) model. The likelihood function and a non-informative prior produce the posterior distribution of the regressive parameters consisting of the VAR model. Using accelerations measured from a

bridge under healthy condition, the posterior distribution provides a stochastic reference model representing healthy bridge vibrations. Based on the posterior distribution, damage sensitive features are extracted by the singular value decomposition. Bayesian hypothesis test for damage detection is conducted utilizing a Bayes factor, which evaluates anomaly on the damage sensitive features.

The efficacy of the feature extraction are experimentally investigated. A stabilization diagram composed from the state space models reproduced by the extracted features helps reasonable choice of the features. For the noisy case considered in this study, the stabilization diagram likely to produce spuriously estimated frequencies. Although the existing modal identification under such noisy condition requires considerable effort for engineers, the proposed feature extraction enable to choose reasonable damage sensitive features without troublesome procedures.

The proposed Bayes factor stably indicates the damage even under the noisy condition. For unsteady loading condition in which acceleration amplitudes varies, data normalization can be a suitable pre-processing to enable concise decision making. The localized Bayes factors showed significant values at the measurement locations that are closest to the damaged member. This result suggests possibility of damage localization for severely damaged members.

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