

Failure Mode And Reliability Analysis Of Frame Structure

Kang Yumei

Associate Professor, School of Resources and Civil Engineering, Northeastern University, Shenyang, China

Di Xue

Graduate Student, School of Resources and Civil Engineering, Northeastern University, Shenyang, China

ABSTRACT: In reliability analysis, the correlation between the primary failure mode of a frame structure and other failure modes is very complicated. At present, there are a number of issues associated with the implementation of reliability analysis, including the complex calculation procedure, the low calculation efficiency, and the difficulty in identifying the primary failure mode, etc. Following the framework of reliability analysis for hybrid systems, the reliability of a two-story frame structure is investigated. The internal force distribution in all structural members is analyzed using the numerical method of Finite Element Method (FEM). The critical structural element is calculated, which can help to evaluate the primary failure mode of the frame structure. Once local failure occurs, the internal force redistributes in the system. The pattern of progressive failure of frame structure is then analyzed. The developed numerical model can provide the real-time performance for the structure, which is further employed to conduct reliability analysis. This study demonstrates that the reliability analysis method for hybrid system can reflect the true performance of the structure more realistically.

1. INTRODUCTIONS

Reliability theory has become one of the main methods of structural safety design in recent years. Most structural reliability analysis is based on structural members, and the structure in actual engineering is a structural system composed of many members. Members are interrelated with each other, interacting and sharing the load together. When one or more members fail, the structure as a whole can continue to bear load and complete the scheduled function. Only when the number of member failure reaches a critical value, the integrity of the structure is destroyed. As a result, the overall bearing capacity of the structure is insufficient, which causes to the failure of the entire structure, and there are a wide variety of failure modes. From this point of view, it is impossible to analyze the overall stability and bearing capacity of the structure accurately by simply studying the member reliability. Therefore, it is necessary to study the reliability of the system for the analysis of the overall stability of the

structure. In the initial stage of reliability research, the research level is limited to the reliability analysis of member level (Tang, 2013). Ang is one of the founding members of structural reliability, he introduces the reliability problem of members into the system for the first time (Ang and Ma, 1981). It is a major breakthrough in research of system reliability. Since then, research on the reliability of structural systems has gradually developed. The generalized maximum bearing capacity ratio criterion proposed by Moses (1983,1990) can clearly represent the failure element of the structure. The structural force is changed by the load increment method, when the $k+1$ stage of the failure analysis is entered, then the failure conditions of each element are analyzed one by one. This method is very conducive to the search for the main failure mode of system reliability, and finally the system reliability is obtained according to the series-parallel relationship between components. Thoft-Christensen (1982) proposed β -unzipping

method for finding the main failure modes of structural systems. The method is to remove the invalid element that matches the condition. After adding the virtual load to the structure, and then search for the next stage of failure mode in the form of virtual loads. Murotsu (1986) proposed a joint failure probability bounding method on the basis of the β unzipping method. The research object of this method is the joint failure probability of each structural element in each failure stage. The efficiency of finding the failure mode is higher and the data is more accurate. It is also one of the most commonly used methods in the calculation of structural system reliability.

China's research on the reliability theory of structural systems has also made rapid progress. Feng and Dong (1988) proposed a modified β unzipping method on the basis of the β unzipping method, which improved computational efficiency and accuracy in calculating the reliability index of the ten-bar truss structure; Li (1993) and Zhao (1996) proposed a point estimation method based on mathematical theory, such as numerical analysis and conditional probability. This method adds the linear correlation effect between the failure mechanisms to the range of conditions considered, and obtained good results; Zhang (2003) proposed an improved differential equivalence recursive algorithm and a generalized unzipping method. Gu (2007) proposed a reliability evaluation method for existing building structure systems based on substructures; "Monte Carlo finite element method for structural reliability analysis" was published by Dong and Zhang (2009). Dong and Tian (2010) published "Monte Carlo Method Structural Reliability Analysis" and so on.

The classical reliability theory is based on the study of ideal elastic-plasticity to analyze the failure probability in different failure modes. It is difficult to accurately grasp the actual behavior of the structure and the structural reliability. At present, there are still many difficulties that cannot be well solved (Lu, 2015). The analysis method of structural system reliability is the evolution of structural design from the component

level to the overall structure level. This method searches the overall failure mode of the structure by analyzing the true nonlinear structure constitutive relationship of the material, and successfully avoids the failure mode. It is difficult to find the main failure mode in the identification method. At the same time, the real constitutive relation of the material is considered, and the problem of failure correlation is included. Furthermore, the overall limit state equation of the structure is constructed, and then the reliability of the structural system is approximated by the mature reliability calculation method, which is efficient and practical. However, the existing research has been mainly limited to the ultimate bearing capacity of the structure (Dong, 2009). Therefore, Chen and Li (2007) proposed that the reliability of the structural system should be grasped from the whole process of nonlinear development of the structure, and the theory of probability density evolution is put forward.

At present, the research on structural reliability is still in progress, and it is far from being completely flawless. Domestic and foreign scholars are constantly trying to explore and verify. The study of system reliability problem is not only required to improve and perfect the existing methods, but also to change the research perspective of the problem and the use of research methods for a deeper exploration.

2. RESEARCH ON RELIABILITY CALCULATION METHOD OF HYBRID SYSTEM

The failure of any member in the structural system will lead to the failure of the whole structure. Such a structural system becomes a series system. When multiple members fail simultaneously, the system that the whole structure will fail is called a parallel system. When there are both series and parallel systems in the structural system, the system is called the hybrid system.

In fact, the reliability of the series system only takes into account some members with relatively large bearing capacity, and there is no real consideration of the situation after the failure of the key member in the calculation range.

Therefore, the reliability analysis of the hybrid system in this paper is based on the reliability analysis of the series system. Extend the role of a key element to the role of a key element pairs. In other words, the reliability of the hybrid structure system consisting of two parallel key elements in series is calculated. Combined with the previous search method of structural failure mode, and aiming at the reliability of the hybrid structure system, the detailed analysis and calculation method of structural system reliability adopted in this paper is as follows:

(1) Firstly, Ansys software is used to establish the simulation model for the calculation structure. The corresponding internal forces of each structural member are calculated by the model. According to the value of internal force, the structural members are arranged in the order of "the most probably damaged members ". The internal force coefficients of the "most easily damaged section" corresponding to each member are obtained through model simulation.

(2) Based on the determination of the key elements in the reliability analysis of the series system, the reliable indicators are solved for the most probably damaged members. Based on the modified β boundary method, the members with reliable indicators in the limited interval $[\beta_{\min}, \beta_c]$ ($\beta_c = 1.29 + 0.86\beta_{\min}$) are selected as the key elements.

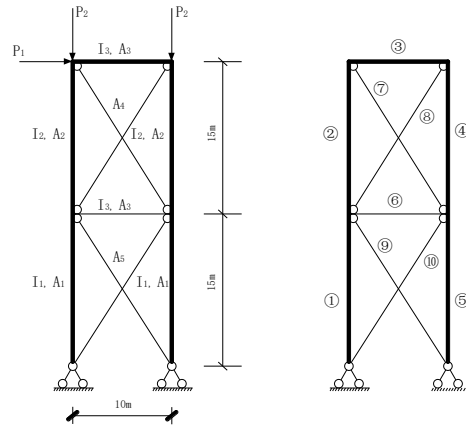
(3) A failure occurs in a key element within the defined interval. If the failure member is ductile failure, the member is removed and a virtual force equal to the magnitude of the internal force before the failure is applied to the corresponding position of the structure. If the member is brittle failure, the member is removed and not apply a virtual reaction.

(4) On the basis of step (3), the structural analysis model is rebuilt. The internal force of the structure is analyzed. The internal force coefficients of all sections of the structure are obtained by applying element force. According to the magnitude of the internal force, the structural damage members are sorted and their reliability indexes are calculated in turn. The modified β

boundary method is used to take the members of the reliable index in the limited interval $[\beta_{\min}^*, \beta_c^*]$ ($\beta_c^* = 1.29 + 0.86\beta_{\min}^*$) as the new key element, and the key element pairs are formed with the key elements identified in the process (2).

(5) Select other key elements in process (2) and repeat steps (3) and (4). The final structure can be approximately changed into a hybrid structure consisting of multiple key element pairs in series.

The calculation method of the reliability of the structural system given above is applied to the steel frame structure system with oblique support shown in Figure 1, and the solution process and results of the reliability of the hybrid system can be analyzed and explained by calculating the reliability of the frame structure system.



(a) Geometric dimensioning (b) Bar code
 Figure 1: The two-story frame structure

Assuming that the internal force of the structure is calculated using the numerical method of FEM, component j, k, l is determined to be the key element, and its corresponding reliability indices are $\beta_j, \beta_k, \beta_l$. Assuming that element j fails due to ductile failure and its internal force is F_j , the failure element i is removed and the reverse F_j force is applied to the structure at element position j at the same time. At this time, under the combined action of the original external load P_i ($i=1, n$) and the reverse F_j force, the analysis model of the structure is established again and the internal force reanalysis of the

structure can obtain the internal force S_{ij} of each member element in the structure. Among them, S_{ij} indicates the internal force effect of element j after failure of element i . At this point, the security margin of element i is:

$$M_{ij} = R_i - S_{ij} \quad (1)$$

The reliability index corresponding to element i is:

$$\beta_{ij} = \frac{\mu_{M_{ij}}}{\sigma_{M_{ij}}} \quad (2)$$

where μ_{ij} and σ_{ij} denote the mean and standard deviation of M_{ij} respectively.

Similarly, assuming that the key elements of reliability in interval $[\beta_{\min}, \beta_c]$ is m, n, p , they are connected in parallel with element j to form element $j-m$, $j-n$, $j-p$. The three groups of element pairs are connected in series to form a hybrid structure system with three groups of key element pairs. The reliability of parallel system with key element pairs is calculated separately, and the solution of reliability is illustrated by taking $j-m$ as an example.

Given that the reliability index of element j is β_j , the reliability index of element m is $\beta_{m|j}$, and the correlation coefficient between the two margins of safety is ρ . The failure probability composed of two elements is expressed as:

$$P_f = \Phi_2(-\beta_j, -\beta_{m|j}; \rho) \quad (3)$$

The reliability of other parallel systems $j-n$ and $j-p$ is calculated according to the above steps. Then return to step (3). The ductile failure of other key elements (e.g. k or l) is caused, and then the failure probability of the parallel system is calculated according to step (4) and (5) and Eq. (1) ~Eq. (2) respectively. Finally, the structure can be simplified as a series of hybrid systems composed of multiple key elements. Its logical relationship is shown in Figure 2.

Finally, the reliability of the hybrid structure system is simplified to the reliability of the system

after all the key pairs are connected in series in the logic diagram. The principle is the same as the reliability of the series system. In the calculation, it is necessary to consider the correlation between each element pair. That is to say, the correlation coefficients of each pair of elements need to be obtained by means of linear equivalent safety margin.

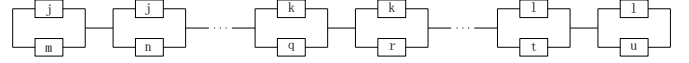


Figure 2: Logical relationship schematic of hybrid structure system

Now assuming a two-story frame structure, it is solved by the above analysis method. The frame width is 10m and the height of each layer is 15m. The sectional attribute of each member is shown in Table 1. The modulus of elasticity $E = 2.1 \times 10^8 \text{ kN/m}^2$. The upper beam of the structure has been concentrated force P_1 and P_2 function, and the specific location is shown in Figure 1. According to the finite element analysis, the axial force of each member of the structure is shown in Figure 3.

Table 1: section properties of components

section attribute	A (10^{-3} m^2)	I (10^{-6} m^4)	E(M) kN · m	E(N) kN
1	4.59	57.9	135	1239
2	2.97	29.4	76	802
3	4.59	4.25	9.8	1239
4	2.6	0	0	702
5	2.8	0	0	756

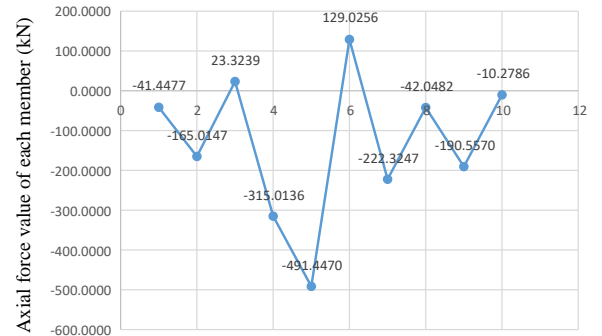


Figure 3: Schematic diagram of axial force of each member

According Figure 1, members ⑤ and ④ are the key elements, and the reliability index of the key elements is shown in Table 2.

Table 2: Reliability index of key elements in frame structure

Key element i	⑤	④
reliability index β_i	1.803	1.816

When ductile failure occurs in the key element ⑤, the member ⑤ is removed and the reaction force $F_5 = 0.5R_5$ is applied at the original position ⑤. The structure is modeled, analyzed and the internal force is calculated, as shown in Figure 4 and Figure 5. The ultimate bearing capacity of member ⑤ under compression is $0.5R_5$ because of its ductility failure under compression before failure. Element internal force coefficient after element ⑤ failure is shown in Table 3.

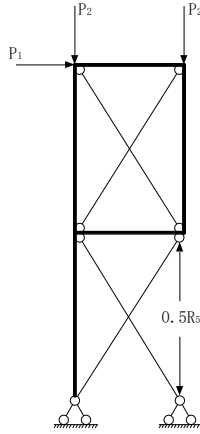


Figure 4: Internal force redistribution model after element ⑤ failure

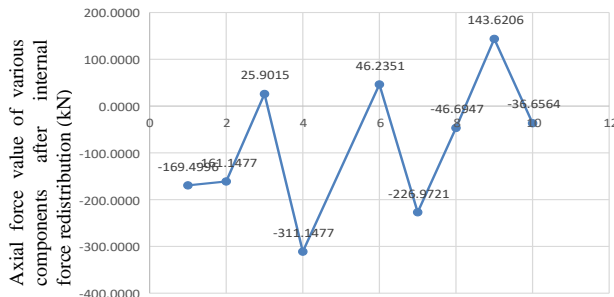


Figure 5: Element axial force after internal force redistribution

According to the data of modeling calculation, when member ⑤ fails and the internal force redistributes. At this point, member ④ bears the largest value of axial force, so section 4 becomes the most probably damaged

section after redistribution of internal force, and its corresponding reliability index is the smallest. Table 3 Element internal force coefficient after element ⑤ failure

No.	a_{i1}	a_{i2}	a_{i3}	No.	a_{i1}	a_{i2}	a_{i3}
1	4.5	-0.0	-1.0	7	-1.8	-1.2	1.2
2	0.7	-0.7	0.03	8	-3.6	-1.2	1.2
3	-0.5	0.2	0.02	9	-0.8	-0.3	-0.03
4	-0.8	-0.7	0.03	10	0.95	-0.3	-0.03
6	1.5	0.9	-0.6				

At this time, the safety margin of the member ④ can be obtained as follows:

$$M_{4|5} = \frac{1}{2} R_4 + S_{4|5} = 89.8523 \quad (4)$$

Further, the reliable index of corresponding section 4 can be obtained.

$$\beta_{4|5} = \frac{\mu_{4|5}}{\sigma_{4|5}} = \frac{89.8523}{47.9678} = 1.873 \quad (5)$$

After obtaining the minimum reliability index $\beta_{4|5} = 1.873$, the interval is defined according to the modified β unzipping method $[\beta_{4|5}, 1.29 + 0.86\beta_{4|5}]$, that is $[1.873, 2.901]$. The reliability index of the probably damaged section is calculated respectively.

From the calculation results, it can be concluded that only $\beta_{4|5}$ is the reliable index within the limited interval. So the element ④ and the element ⑤ are selected as the element pairs.

The covariance between M_5 and $M_{4|5}$ can be calculated as follows:

$$\begin{aligned} \text{cov}[M_5, M_{4|5}] &= a_1 b_1 \text{Var}[R_5] + a_2 b_2 \text{Var}[P_1] \\ &+ a_3 b_3 \text{Var}[P_2] = 960.4732 \end{aligned} \quad (6)$$

Therefore, the correlation coefficient between them is:

$$\rho_{5,4|5} = \frac{\text{cov}[M_5, M_{4|5}]}{\sigma_{M_5} \sigma_{M_{4|5}}} = 0.282 \quad (7)$$

The failure probability p_f of structural system can be obtained by MATLAB operation results. The lower bound is 0.0011, and the upper bound is 0.0305 and 0.0035, respectively.

Therefore, the average value of p_f can be obtained:

$$P_f = \Phi_2(-\beta_5, -\beta_{4|5}; \rho_{4|5}) = 0.0023 \quad (8)$$

The reliability and reliability index of the parallel key element pair are:

$$P_r = 1 - P_f = 1 - 0.0628 = 0.9977$$

$$\beta_s = -\Phi^{-1}(P_f) = -\Phi^{-1}(0.0023) = 2.834 \quad (9)$$

Similarly, the failure probability of other key element pairs can be calculated separately and the reliability index and reliability can be obtained. However, each pair of elements is also related to each other. In order to find the correlation coefficient between the key element pairs, it is necessary to calculate the linear equivalent safety margin between the key pairs, that is to calculate a linear equivalent safety margin between M_5 and $M_{4|5}$. Assuming that all the random variables appearing in the calculation obey normal distribution, a new random variable is obtained by standardized method.

$$X_i = \frac{Y_i - \mu_{Y_i}}{\sigma_{Y_i}} \quad (10)$$

The covariance between the variables X_i and X_j is the correlation coefficient of variables Y_i and Y_j .

Taking into account the action of the random variables R_4 、 R_5 、 P_1 and P_2 in M_5 and $M_{4|5}$, respectively, can get the safety margin expressions of M_5 and $M_{4|5}$:

$$M_5 = \frac{1}{2}R_5 + S_5 = \frac{1}{2} \times 1239 + (-2.25 \times 100 - 0.761 \times 350)$$

$$M_{4|5} = \frac{1}{2}R_4 + S_{4|5} = \frac{1}{2} \times 802 + \frac{1}{2} \times 0.02998 \times 1239$$

$$+ (-0.795 \times 100 - 0.715 \times 350)$$

By normalizing the coefficients of each random variable, the corresponding expression of the equivalent safety margin can be obtained as follows:

$$M_5^* = 0.873X_2 - 0.317X_3 - 0.375X_4 + 1.803 \quad (11)$$

Similarly, the equivalent safety margin of $M_{4|5}$ can be obtained as follows:

$$M_{4|5}^* = 0.835X_1 + 0.0387X_2 - 0.166X_3$$

$$- 0.521X_4 + 1.873 \quad (12)$$

A perturbation of $\varepsilon = -0.1$ is applied to the four random variables X_1 、 X_2 、 X_3 、 X_4 , and then the values of β_5^* and $\beta_{4|5}^*$ under the perturbation change are obtained by calculation.

$$\begin{bmatrix} -1.803 \\ -1.873 \end{bmatrix} - \begin{bmatrix} 0 & 0.873 & -0.317 & -0.375 \\ 0.835 & 0.0387 & -0.166 & -0.521 \end{bmatrix} \begin{bmatrix} -0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.8030 \\ -1.7895 \end{bmatrix} \quad (13)$$

Getting $\beta_5^* = 1.8030$, $\beta_{4|5}^* = 1.7895$. The obtained reliable index is calculated by using MATLAB, and the failure probability of parallel element pair can be evaluated as

$$P_{f_{X_1}}^* = \Phi_2(-\beta_5^*, -\beta_{4|5}^*; \rho_{4|5}) = 0.0040 \quad (14)$$

Then the reliability and reliability indexes of the parallel element pair are

$$P_r^* = 1 - P_f^* = 0.9960$$

$$\beta_s^* = \Phi^{-1}(P_r^*) = 2.6521 \quad (15)$$

Similarly, disturbance can be applied to other random variables respectively. So as to obtain the failure probability of parallel element pairs after the disturbance $\varepsilon = -0.1$ is

$$P_{f_{X_2}}^* = \Phi_2(-\beta_5^*, -\beta_{4|5}^*; \rho_{4|5}) = 0.0041$$

$$P_{f_{X_3}}^* = \Phi_2(-\beta_5^*, -\beta_{4|5}^*; \rho_{4|5}) = 0.0032 \quad (16)$$

$$P_{f_{X_4}}^* = \Phi_2(-\beta_5^*, -\beta_{4|5}^*; \rho_{4|5}) = 0.0029$$

Then the reliable index value of the parallel element pair can be obtained as shown in Table 4.
Table 4: Reliability index of parallel element pair under disturbance variable

The disturbance variable	$X_1(R_4)$	$X_2(R_5)$	$X_3(P_1)$	$X_4(P_2)$
β_{si}^*	2.6521	2.6438	2.7266	2.7589

Again, suppose the expression of linear equivalent safety margin is:

$$M_s^* = a_1^*Y_1 + a_2^*Y_2 + a_3^*Y_3 + a_4^*Y_4 + C \quad (17)$$

The coefficients of the random variables in the equivalent safety margin can be determined by

linearization of the parallel elements under the disturbance variables in Table 3.

$$\mu_{\beta_{si}^*} = \frac{2.6521 + 2.6483 + 2.7266 + 2.7589}{4} = 2.6954$$

$$a_1 \approx \frac{\beta_{s1}^* - \mu_{\beta_{si}^*}}{\varepsilon \sigma_{Y_1}} = \frac{2.6521 - 2.6954}{(-0.1) \times 80.2} = 0.00540$$

$$a_2 \approx \frac{\beta_{s2}^* - \mu_{\beta_{si}^*}}{\varepsilon \sigma_{Y_2}} = \frac{2.6438 - 2.6954}{(-0.1) \times 123.9} = 0.00416$$

$$a_3 \approx \frac{\beta_{s3}^* - \mu_{\beta_{si}^*}}{\varepsilon \sigma_{Y_3}} = \frac{2.7266 - 2.6954}{(-0.1) \times 10} = -0.03120$$

$$a_4 \approx \frac{\beta_{s4}^* - \mu_{\beta_{si}^*}}{\varepsilon \sigma_{Y_4}} = \frac{2.7589 - 2.6954}{(-0.1) \times 35} = -0.01814$$

After normalization, the coefficients are obtained $[a_1^* \ a_2^* \ a_3^* \ a_4^*] = [0.162 \ 0.125 \ 0.936 \ 0.544]$

If $\beta_s^* = 2.6954$, the coefficient can be obtained $C=2.616$. So the linear equivalent safety margin is:

$$M_1 = M_{\varepsilon(4,5)}^* = 0.162R_4 + 0.125R_5 - 0.936P_1 - 0.544P_2 + 2.616 \quad (18)$$

Similarly, according to the key element identified by the modified β limit method earlier, the logical relationship between each link in the system can be known. As shown in Figure 6, it can be seen that there are three groups of parallel element pairs in the hybrid structure system, namely ⑤-④, ④-② and ④-⑤. At this point, each element pair is connected with each other in series. The equivalent safety margin expression of other key element pairs in the system can be obtained when considering the failure sequence of structural elements.

The equivalent safety margin expression of the other two sets of key element pairs can be obtained by calculating

$$M_2 = M_{\varepsilon(2,4)}^* = 0.018R_2 + 0.210R_4 - 0.845P_1 - 0.279P_2 + 3.431$$

$$M_3 = M_{\varepsilon(5,4)}^* = 0.111R_5 + 0.105R_4 - 0.820P_1 - 0.401P_2 + 2.754$$

Repeat the previous process to calculate:

$$\begin{aligned} \mu_{M_{1,2}} &= 265.0180, & \sigma_{M_{1,2}} &= 29.3052, & \rho_{M_{1,2}} &= 0.425 \\ \mu_{M_{1,3}} &= 343.9784, & \sigma_{M_{1,3}} &= 21.2719, & \rho_{M_{1,3}} &= 0.513 \\ \mu_{M_{2,3}} &= 206.3418, & \sigma_{M_{2,3}} &= 22.8971, & \rho_{M_{2,3}} &= 0.424 \end{aligned} \quad (19)$$

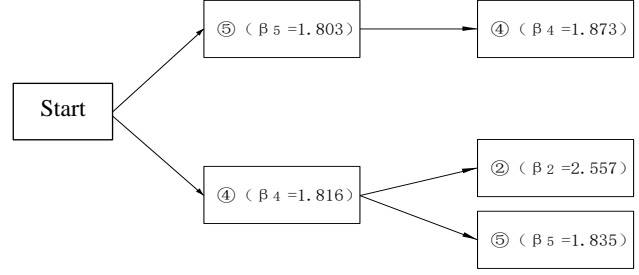


Figure 6: Logical relationship schematic of structure system

The correlation coefficients between the three groups of elements can be determined and analyzed in the correlation coefficient matrix.

$$\rho = \begin{bmatrix} 1.000 & 0.425 & 0.513 \\ 0.425 & 1.000 & 0.424 \\ 0.513 & 0.424 & 1.000 \end{bmatrix} \quad (20)$$

According to the Dunnett approximation method

$$\rho = \frac{1}{3(3-1)} (0.425 + 0.513 + 0.424) \times 2 = 0.454 \quad (21)$$

The failure probability can be obtained through MATLAB.

$$P_f = 0.007 \quad (22)$$

The corresponding reliability and reliability indexes are respectively:

$$\begin{aligned} P_r &= 1 - 0.0035 = 0.993 \\ \beta_s &= \Phi^{-1}(0.993) = 2.4573 \end{aligned} \quad (23)$$

The reliability index of the frame structure in the hybrid system is determined as $\beta_s = 2.4573$.

3. ANALYSIS OF CALCULATION RESULTS

It can be seen from the data that when the structure system is considered as a hybrid system, the reliability index of the structure is too large, and accordingly, the reliability of the structure is also large. This is because there are multiple parallel element pairs in the hybrid system. In practice, the failure of an individual member in parallel system does not necessarily result in the failure of the whole structure. In addition, the internal force redistribution occurs after the failure of a single member, and the structure becomes a statically indeterminate structure that can work normally.

Therefore, the reliability of parallel element pairs must be higher than that of individual members. In practical engineering, although the system reliability estimation of series structure is the most conservative, the designed structure is also the most secure. However, there must be a parallel element pair. Therefore, the research on the reliability of hybrid structure can better reflect the real situation of the structure reliability. In the same way, if the number of key elements in the parallel element pair is more than two, the reliability of the hybrid structure system will be closer to the real value.

4. CONCLUSIONS

According to the performance of structural members or systems, the expression of safety margin in the case of hybrid structure system is investigated. Identifying the primary failure mode in structural system reliability analysis using the numerical method of FEM is developed and applied to a two-story frame structure system reliability analysis. This study shows that the reliability analysis method for hybrid system can reflect the true performance of the structure more realistically.

5. ACKNOWLEDGMENTS

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