A Probabilistic Framework for Post-Disaster Functionality Recovery of Community Building Portfolios

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ABSTRACT: One distinguished feature of a resilient community is its ability and rapidity to recover from severe natural hazard events. The building portfolio acts as a crucial link in supporting the overall recovery of a community, as on the one hand, it relies on community’s lifeline systems to maintain functionality, and on the other hand, it interfaces with people in the most direct manner to sustain social and economic vitality in the community. In this study, we introduce a probabilistic framework for post-disaster functionality recovery of community building portfolios, which at the same time allows the time-variant dependencies among different infrastructure systems (water, power, transportation, building portfolio) to be collectively reflected in the recovery outcome of the community’s building portfolio. The post-disaster functionality restoration at individual building level is modeled as a discrete-state, continuous time Markov Chain (CTMC). To capture the functional dependency of a building on the availability of utilities (i.e. water and power), as well as its restoration dependency on the efficiency of the transportation system, the time-variant system-level performances of these lifelines are first de-aggregated (or ‘downscaled’) to each building site, then their impacts on building restoration are incorporated in the building-level CTMC. The CTMCs of all individual buildings can then be aggregated to obtain the overall building portfolio recovery trajectory and recovery time. Such coupling of physical systems of distinct topologies over a consistent spatial and temporal scale can provide a rich array of information to support community recovery planning in a systematic manner. Lastly, this framework is implemented to Shelby County, TN under a scenario earthquake event.

1. INTRODUCTION
The building portfolio is essential to the day-to-day operation of the community as it provides infrastructure that supports critical community functions such as housing, education, business, health services and government. Physical damages and functionality losses caused by past natural hazard events to a community building portfolio, as a system, has led to multi-scale social-economic impacts that cascade throughout all sectors of the community during and long after the hazard event (as manifested in Hurricane Katrina, 2005 and Sandy, 2012). The reason to such devastating consequences is twofold. Firstly, the impact of natural hazards on individual buildings has traditionally been considered by structural engineers through codes, standards and regulations in building design, construction and management (NEHRP, 2009; ASCE Standard 7, 2016). These codes and standards for individual buildings, however, were developed mainly to protect life safety against natural hazard events, without addressing buildings’ functionality concerns explicitly. Secondly, current risk managements for buildings have overlooked the functional dependences among buildings of different occupancies or between a building portfolio and other supporting infrastructure systems that together contribute to the social-
econmic stability of a community. To achieve community disaster resilience goal, the current engineering practice of design, assessment, and risk management of buildings should move beyond the life-safety focused consideration at the individual building level to a comprehensive portfolio-level approach (Lin & Wang, 2016).

Modeling post-disaster recovery processes of a community building portfolio is a crucial step towards achieving community disaster resilience (Bruneau et al., 2003; Cimellaro et al., 2010; NIST, 2015). However, it received scarce investigation at the early stage of disaster research due to its complexity and uncertainty in nature (Miles & Chang, 2003). The building portfolio is perceived as one of the most challenging and unpredictable systems as it, on the one hand, relies on the community’s lifeline systems to maintain functionality, and on the other hand, interfaces with people in the most direct manner to sustain social and economic vitality in the community (Lin & Wang, 2017a, b). Despite attention on post-disaster recovery modeling is increasingly growing, comprehensive building portfolio recovery models seldom exist that take into account various dependencies of buildings on its supporting lifelines at a community scale.

In this study, we introduce a probabilistic framework for post-disaster functionality recovery of community building portfolios, which at the same time allows the time-variant dependencies among different infrastructure systems (transportation, water, power networks and building portfolio) to be collectively reflected in the recovery outcome of the community’s building portfolio. In the next section, the study starts with an introduction of the recovery metrics it utilizes as well as a comprehensive description of the probabilistic analysis framework.

2. RECOVERY METRICS AND ANALYSIS FRAMEWORK
Prediction of building portfolio recovery should begin with clearly-defined metrics that can directly measure the functionality of a building portfolio, and at the same time, explicitly reflect the dependency of the building portfolio on other infrastructure systems in maintaining its desired functionality level. The concept of building functionality is defined as the availability of a building to be used for its intended purpose, determined by whether a building is structurally safe to occupy and whether basic utilities (e.g., water, power, etc.) are available at the building site (Almufti & Willford, 2013; Lin & Wang, 2017a). Specifically, a building’s damage is categorized into five states ($d_1$—none, $d_2$—slight, $d_3$—moderate, $d_4$—extensive, $d_5$—complete), with each symbolizing different extents of structure and non-structural damages (as listed in Figure 1); while the utility availability is categorized into three states ($uA_1$—not available, $uA_2$—partially available, $uA_3$—all available). Based on a building’s damage condition and utility availability at the building site, five different building functionality states are defined in Figure 1 (Lin & Wang, 2017a), i.e., Restricted Entry ($RE$), Restricted Use ($RU$), Re-occupancy ($RO$), Baseline Functionality ($BF$), and Full Functionality ($FF$).

The building functionality states $S_j, j \in (RE, RU, RO, BF, FF)$ introduced above serves as the functionality metric for individual buildings. To track functionality recovery of a building portfolio as a whole, we further introduce a portfolio recovery index ($PRI$), defined as the percentage of buildings in a portfolio that are in each of the five functionality states, i.e.,

$$PRI_j(t) = \frac{1}{N} \sum_{n=1}^{N} I^n_j(t), \quad j \in RE, RU, RO, BF, FF$$

(1)

where $N$ is the total number of buildings in a portfolio, and $I^n_j(t)$ is the functionality state indicator of the building $n$, i.e.,
\[ I_n^n(t) = \begin{cases} 0, & S_n^n(t) \neq S_j^j; \ n \in 1, 2, \ldots, N \\ 1, & S_n^n(t) = S_j^j \end{cases} \quad (2) \]

in which \( S_n^n(t) \) is the functionality state of building \( n \) at time \( t \).

### Table: Functionality States (ATC Placard) vs Damage Condition vs Utility Availability

<table>
<thead>
<tr>
<th>State</th>
<th>Condition</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF (Green )</td>
<td>Full Functionality</td>
<td>All available</td>
</tr>
<tr>
<td>BF (Green )</td>
<td>Baseline Functionality</td>
<td>Partially available</td>
</tr>
<tr>
<td>RO (Green )</td>
<td>Re-Occupy</td>
<td>N/A</td>
</tr>
<tr>
<td>RU (Yellow)</td>
<td>Restricted Use</td>
<td>N/A</td>
</tr>
<tr>
<td>RE (Red)</td>
<td>Restricted Entry</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**Figure 1:** Building functionality state definition (Lin & Wang, 2017a)

The probabilistic analysis framework to quantify the PRI(t) can be expressed in Eq. (3) (Lin & Wang, 2017b):

\[
\begin{align*}
    f[PRI(t) | H] & = \\
    & \iint f[PRI(t), S(t_0)]dF[S(t_0)]dF[DS]dF[IM]dF[IM | H] 
\end{align*}
\]

where \( f(\cdot) \) and \( F(\cdot) \) denote the probability density function and cumulative distribution function (CDF) of random variables, respectively; light-face letters represent one-dimensional variables while bold-face letters represent multi-dimensional variables; from left to right, \( H \) denotes the extreme hazard event investigated; \( IM \) denotes the hazard intensity (e.g. spectral acceleration, spectral displacement for earthquake events) at all building sites; \( DS \) denotes the damage states of all buildings; \( S(t_0) \) denotes the functionality states of all individual buildings at \( t_0 \) (the time of hazard occurrence, also the starting time of recovery); the dimension of the random vectors - \( IM, DS, S \) - is consistent with the total number of buildings in the portfolio, \( N \); for each of these random vectors, correlation exists among its \( N \) random variables associated the \( N \) spatially distributed buildings (Vitoontus & Ellingwood, 2013, Lin & Wang, 2017b).

**Figure 2:** Flowchart of the building portfolio analysis framework

In Eq. (3), the probabilistic characterization of the PRI(t) requires four steps of analysis — probabilistic hazard modeling at community scale \( [IM|H] \), damage fragility analysis for spatially distributed buildings \( [DS|IM] \), functionality loss assessment (damage to functionality mapping) for all buildings in the portfolio \( [S(t_0)|DS] \), and building portfolio functionality recovery modeling (BPRM) \( [PRI(t)|S(t_0)] \) - with each step conditional on its previous step and performed at the community scale.

In addition, the building portfolio analysis requires taking into consideration building dependences on its supporting lifelines, namely, functional dependencies on utilities (as buildings rely on power and water networks to provide utility service, which affects step 3 and step 4) and restoration dependencies on transportation (building’s repair/reconstruction activities rely on the travel efficiency of the transportation network, which affects step 4), as shown in Figure 2.

The present study will focus on the formulation of the fourth step, i.e. BPRM, which includes building-level restoration and portfolio-
level recovery. At the building level, an individual building’s post-disaster functionality restoration is modeled as a discrete-state, continuous time Markov Chain (CTMC) (Lin & Wang, 2017a). To capture aforementioned functional dependency and restoration dependency, the time-variant system-level performances of these lifelines are first de-aggregated (or ‘downscaled’) to each building site, then their impacts on building restoration are incorporated in the CTMC (introduced in Section 3). At the portfolio level, the CTMCs of all individual buildings are aggregated to obtain the overall building portfolio recovery trajectory and recovery time (introduced in Section 4).

3. BUILDING-LEVEL RESTORATION
For an individual building, n, the time-variant functionality metric $S^n(t)$ (which takes one of the five functionality states $RE, RU, RO, BF, FF$ defined in Figure 1) can be modeled as a discrete state, CTMC, with its time-variant functionality state probability vector defined as (Lin & Wang, 2017a),

$$\pi^n(t) = [\pi^n_1(t), ..., \pi^n_5(t)]$$

(4)

Next, we develop theoretical approach and mathematical formulation in order to incorporate functional dependency and restoration dependency in the CTMC modeling building-level restoration, as detailed in section 3.1 and 3.2, respectively.

3.1 functional dependency modeling
Examining building’s functionality definition, the $S^n(t)$ is determined by the building’s physical damage state $DS^n(t)$, as well as utility availability state $UA^n(t)$. Accordingly, the element of $\pi^n(t)$ is calculated by

$$\pi^n_j(t) = \sum_{k=1}^{5} \sum_{l=1}^{3} I_{k,l} \cdot \text{Prob}[DS^n(t) = ds_k, UA^n(t) = ua_l]$$

(5)

in which $I_{k,l} = 1$ only if joint state $(ds_k, ua_l)$ belongs to $S_j$, otherwise 0 (cf. Figure 1); $ds_k, k = 1, 2, 3, 4, 5$ and $ua_l, l = 1, 2, 3$ are accordance with the definition of building damage states and utility availability states in Section 2.

Define building n’s time-variant damage state probability vector as $b^n(t) = [b^n_1(t), ..., b^n_5(t)]$, in which $b^n_k(t) = \text{Prob}[DS^n(t) = ds_k]$; and utility’s time-variant availability state probability vector (at the building site) as $u^n(t) = [u^n_1(t), u^n_2(t), u^n_3(t)]$, in which $u^n_i(t) = \text{Prob}[UA^n(t) = ua_i]$ .

Neglecting statistical correlation between building and utility’s performances, Eq. (5) can be calculated by

$$\pi^n(t) = U^n(t) \cdot b^n(t)$$

(6)

in which $U^n(t)$ is defined as utility dependency matrix, with the following form

$$U^n(t) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & u^n_1(t) & u^n_2(t) \\ 0 & 0 & 0 & 1 - u^n_1(t) & u^n_2(t) \\ 0 & 0 & 0 & 0 & u^n_3(t) \end{pmatrix}$$

(7)

In Eqs. (6) and (7), the probability vector with respect to utility’s availability restoration process $u^n(t)$ is obtained by performing interdependent utility network recovery analysis, through which the time-variant performance of utility network are downscaled to each building site (more details can be found in Zhang et al., 2018); the building’s damage restoration process $b^n(t)$ is determined by that building’s own repair/reconstruction activities, which can also be modeled as a discrete, CTMC. The element of $b^n(t)$ is calculated by

$$b^n_j(t) = \sum_{i=1}^{5} p^n_{i,j}(t) \cdot b^n_i(t_0), \quad j = 1, ..., 5$$

(8)

in which $b^n_i(t_0), i = 1, ..., 5$, is the initial damage state probabilities derived in step 2 of the building portfolio analysis framework in Eq (3); $p^n_{i,j}(t)$ is the transition probability of building n upgrading to damage state $ds_j$ at time $t$ given initial building damage state $ds_i$ at time $t_0$, i.e.,

$$p^n_{i,j}(t) = \text{Prob}[DS^n(t) = ds_j | DS^n(t_0) = ds_i]$$

(9)

Further, we define building’s damage restoration time $DRT^n_{i,j}$ as the time takes to restore
the damage of building $n$ from $dS_i$ at time $t_0$ to $dS_j$ at time $t$. Accordingly, the transition probability $p_{n}^{i,j}(t)$ is estimated by (Lin & Wang, 2017a):

$$p_{n}^{i,j}(t) = \begin{cases} F_{DRT_{i,j}}(t) - F_{DRT_{i,j+1}}(t), & j = 1,2,3,4 \\
F_{DRT_{i,j}}(t), & j = 5 \end{cases}$$

(10)

Generally, the building damage restoration time $DRT_{i,j}$ includes two major time components: (1) delay time ($T_{Delay,i}$), the time takes to initiate a repair given building’s initial damage state $dS_i$, (e.g., time for inspection, secure financing, engineering/contract mobilization, permitting, etc.); (2) repair time ($T_{Repair,i,j}$), the time takes to complete all required repair items in order to restore to less severe damage state $dS_j$ given initial damage state $dS_i$. Specific methodologies to derived statistical distribution of $T_{Delay}$ and $T_{Repair}$ can be found in Lin & Wang (2017a, b).

3.2 Restoration dependency modeling

A building’s post-disaster repair/reconstruction activities are very likely to be influenced if the community’s travel efficiency is impaired due to its disrupted transportation network. To take into account this effect, we introduce a scale factor, $\gamma = \frac{T_{Repair,i}}{T_{Repair}}$, in which $T_{Repair}$ and $T_{Repair,i}$ are repair times with and without considering the effect of decreased traffic efficiency during a building’s repair phase. We assume that the ratio of post-hazard travel time (that transports crews and construction materials to the construction site) to pre-hazard travel time is inversely proportional to ratio of post-hazard traffic efficiency to pre-hazard traffic efficiency averaged over building’s post-hazard repair phase [$T_{Delay,i}, T_{Delay,i} + T_{Repair,i,j}$] (as shown in Figure 3). Accordingly,

$$\gamma = (1 - \delta) + \delta \cdot \frac{T_{Repair,i,j}}{\int_{T_{Delay,i}}^{T_{Delay,i}+T_{Repair,i,j}} TE(t) \, dt}$$

(11)

in which $\delta$ is the proportion of (crew and construction materials) travel time to the total repair time; $TE(t)$ is the time-variant traffic efficiency associated with the building site, which is obtained from transportation network recovery model (Zhang, et al., 2017); again, the performance outcome of this transportation system analysis should be downscaled to each building site.

Apparently, the scale factor $\gamma$ is dependent on $T_{Delay,i}$ and $T_{Repair,i,j}$. Therefore, the building damage restoration time, DRT, is obtained by,

$$DRT_{i,j} = T_{Delay,i} + \gamma(T_{Delay,i}, T_{Repair,i,j}) \cdot T_{Repair,i,j}$$

(12)

with its distribution

$$F_{DRT_{i,j}}(t) = \int \int f_{T_{Delay,i}}(x)f_{T_{Repair,i,j}}(\gamma)x\, dx\, dy$$

(13)

![Figure 3: Averaged travel efficiency during a building’s repair phase](image)

Note that Eqs. (11)-(13) are specific to building $n$ (for simplicity the superscript $n$ is not written). With the probability distributions of delay time and repair time available, as well as the time-variant travel efficiency obtained from transportaion system analysis, the $p_{i,j}^{n}(t)$ can be derived using Eq. (10). Further, the CTMC modeling building-level restoration process is calculated using Eqs. (5)-(8).
4. PORTFOLIO-LEVEL RECOVERY
The portfolio-level recovery is then obtained by aggregating the CTMC restoration processes of individual buildings, \( S^n(t), n = 1, ..., N \), across the geographic domain of the community and over the entire recovery time horizon. Based on the definition of \( PRI_j(t) \), i.e., Eqs. (1) & (2), we further calculate its mean value and variance by (Lin & Wang, 2017a):

\[
E[PRI_j(t)] = \frac{1}{N} \sum_{n=1}^{N} \pi^n_j(t) \tag{14}
\]
\[
\sigma^2_{PRI_j}(t) = \frac{1}{N^2} \sum_{n=1}^{N} \sum_{m=1}^{N} \rho^{mn}_j(t) \sigma^n_j(t) \sigma^m_j(t) \tag{15}
\]

in which \( n, m \in (1, ..., N) \) denote building \( n \) and \( m \); \( \sigma^n_j(t) = \sqrt{\pi^n_j(t) [1 - \pi^n_j(t)]} \) is the standard deviation of \( \pi^n_j(t) \) and \( \rho^{mn}_j(t) \) is the correlation matrix describing correlations between functionality states of building \( n \), \( \pi^n_j(t) \), and that of building \( m \), \( \pi^m_j(t) \), at any time \( t \).

We refer the portfolio recovery index associated with \( FF, PRI_{FF}(t) \), as the portfolio recovery trajectory, which represents the percentage of buildings in the FF functionality state at any time \( t \) following hazard occurrence. We further define portfolio recovery time, \( PRT_{j,a\%} \), as the time takes for \( a\% \) (e.g. 95\%) of community buildings to regain a predetermined functionality state \( j \) (e.g. FF). Then, the CDF of \( PRT_{j,a\%} \) can be derived as (Lin & Wang, 2017a):

\[
F_{PRT_{j,a\%}}(t) = \int_{a\%}^{1} f_{PRI_j}(x, t) \, dx \tag{16}
\]

The proposed BPRM, as outlined in Section 3 and 4, provides probabilistic characterizations of both building-level restoration and portfolio-level recovery expressed in terms of portfolio recovery trajectory \( PRI_{FF}(t) \) and recovery trajectory \( PRT_{j,a\%} \). Next, we apply the BPRM framework to a testbed community – Shelby County, TN, USA.

5. CASE STUDY
The residential building portfolio in Shelby County, TN, accounts for approximately 90\% of the Shelby building inventory (nearly 300,000 buildings) and is distributed spatially across 221 census tracts as shown in Figure 4.

We subject Shelby to a scenario earthquake (\( H \)) with \( M_w = 7.7 \) and an epicenter located at 35.3N and 90.3W (the most likely event associated with 2475-year return period). The Atkinson and Boore (1995) attenuation relationship is used to calculate the ground motion intensities at the building site (\( [IM|H] \)), and the spatial correlations in \( IM \) are simulated using Wang and Takada (2005)’s model. The spatial variation in median peak ground acceleration (PGA) in Shelby is also shown in Figure 4. Fragility functions from HAZUS-MH (FEMA/NIBS, 2003) are adopted for damage evaluation (\( [DS|IM] \)). Different damage-to-functionality mapping algorithms (i.e. \( \mathbf{S}(t_q)[DS] \)) can be found in literature (e.g. FEMA, 2012; Lin & Wang, 2017b).

![Figure 4: The residential building portfolio of Shelby County (red dots) and median PGA (contour with grey shades)](image)

Statistics of delay time, repair time (as discussed in Section 3) are obtained using analytical analysis calibrated by existing empirical database (Lin & Wang, 2017b). The recovery analysis of lifelines (including power & water networks, and transpiration networks) are performed by Zhang et al. (2018), in order to obtain \( \mathbf{u}(t) \) and \( TE(t) \) at each building site. The propagations of uncertainties and spatial correlations are enforced in the case study by multiple layers of Monte-Carlo Simulation (MCS).
coupled with sampling techniques (Lin & Wang, 2016).

Figure 5 shows the mean estimate of portfolio recovery index, \( PR_j(t), j = RE, RU, RO, BF, FF \). The curve associated with \( FF \) is the mean portfolio recovery trajectory. The spatial evolution of portfolio recovery is depicted at four selected points in time — \( t = 0, 25, 50, 75 \) weeks following the event. Notable spatial disparities are observed, reflecting differences in hazard-induced damages and recovery capacities, both underlined by the social and economic disparities among different population groups.

Furthermore, the uncertainties associated with the portfolio recovery trajectory and recovery time are shown in Figure 6. The mean recovery time associated 95% residential buildings restored to their \( FF \) states is approximate 87.5 weeks.

Figure 7 shows the effect of resuming utility (blue area in the figure, which occurs at short-term phase), as well as the effect of revitalizing travel efficiency (red area in the figure, which occurs at intermediate and long-term phase) on building portfolio functionality recovery. Both effects indicate that the neglect of dependencies between building portfolio and lifeline systems will lead to unconservative estimate of portfolio recovery trajectory and recovery time.

6. CONCLUSIONS
The probabilistic analysis framework developed in this study addresses a significant gap in the building portfolio recovery modeling through investigation and incorporation of building’s dependencies on the community’ lifelines. In the literature, the building portfolio’s functionality recovery modeling is perform in two steps: building-level restoration and portfolio-level recovery (Lin & Wang, 2017a,b). To take into account functional dependency of a building on the availability of utilities (i.e. water and power), as well as its restoration dependency on the efficiency of the transportation system, performances of those lifeline systems are downscaled to each building site and incorporated
in the CTMC modeling building-level restoration. Such coupling of physical systems of distinct topologies over a consistent spatial and temporal scale can provide a rich array of information to support community recovery planning in a systematic manner. Besides the model itself, the present study contributes an illustrative example of how building portfolio recovery can be affected by its supporting lifelines, underlining the importance of considering dependences among different infrastructure systems.

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8. REFERENCES