Calibration of existing semi-probabilistic design codes

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ABSTRACT: The paper addresses the reliability based calibration of the partial safety factors of semi-probabilistic design codes. The introduced methodology facilitates for the joint consideration of a general domain of design equations. An example calibration of the partial load factors is performed and results and corresponding implications are discussed. An outlook on future required research is given.

1. INTRODUCTION

Structural design can be seen as a decision problem associated with uncertainties. Structural layout and cross section dimensions are chosen in order to comply with the objective that the structures safely withstand the applied loads during the entire lifetime of the structure and at the same time natural and financial resources are spent with care. Decision making concerning design and assessment of structures is addressed in ISO2394 (2015). Here three different levels of detail of decision analysis are distinguished; risk-informed decision making, reliability based design, and semi-probabilistic design.

Risk informed decision making allows for the highest level of detail, i.e. uncertainties, costs and benefits can be considered explicitly in the analysis and optimal design solutions can be identified.

In reliability based design, the design decision is chosen such that it complies to a predefined reliability requirement. The reliability requirement is defined based on past experience, i.e. specified as the inherent reliability of traditionally accepted design solutions (Baravalle and Köhler, 2017), or it is based on formal calibration using risk-informed methods (Baravalle and Köhler, 2019), (Rackwitz, 2000).

The semi-probabilistic approach corresponds to the lowest level of detail. Here, a design decision is chosen such that it complies with the criterion that a design value of a resistance is larger than a design value of a corresponding load effect. Design values for the load bearing capacity R_d are chosen to have a sufficiently low non-exceedance probability and design values for loads S_d are chosen to have a sufficiently low exceedance probability such that the design criterion in the limit ($R_d = S_d$) corresponds to the required level of reliability.

In the so-called load and resistance factor design (LRFD) format (Ravindra and Galambos, 1978) design values are determined based on characteristic values and partial safety factors γ as e.g. $R_d = R_k/\gamma_M$ for resistance variables and $S_d = \gamma_S S_k$ for the effects of applied loads. Both, the definition of the characteristic value and the choice of partial safety factor, i.e. the reliability elements of the code, is made in order to meet the reliability requirements. This is generally referred to as code calibration. Here, the correspondence to reliability requirements is generally made for domains of design situations. For the representation of these

design situations generalised assumptions in regard to consequences and uncertainties are made. With semi-probabilistic design structural reliability on a component/failure mode level can be controlled. The explicit consideration of the interaction of failure modes in a structure, i.e. system effects, is not accommodated.
The principles of semi-probabilistic design are outlined in ISO2394 (2015). It is the method of choice for most structural design decision problems and executive guidance and standardisation is found in several national and international design standards as, e.g. the Eurocodes (CEN, 2002).

2. Reliability based code calibration of the partial safety factors of existing design codes

Systematic approaches to code calibration have been addressed e.g. in Cornell (1969), Ravindra et al. (1978), Thoft-Christensen P. (1982), Madsen H.O. (2006) and Ditlevsen O. (2007). Faber and Sørensen (2003) introduced a 7. step approach for reliability based code calibration and this was the basis for the corresponding standardisation in ISO2394 (2015).
If the partial factors of an existing code are calibrated similar principles should be followed. However, the existence of a semi-probabilistic design framework that is practically applied for years requires some adaption.

2.1. Target reliability

The choice of the target reliability is thereby a very relevant assumption. Here, different reasoning is followed in the practical context of calibration:

Definition of β_t as a overall prescriptive requirement for structural reliability: Here, a fixed requirement for structural reliability is introduced by, e.g. by authorities, and this requirement is also used in calibration. Although, this is the most common interpretation of a reliability target, it relies on a misconception and a misinterpretation. Reliability, and similarly failure probability, is falsely considered as a property a structure can display. Reliability and failure probability, however, are only attributes of the (decision) analysis on structures and should only understood as such.

Defining β_t as reliability requirement derived from optimisation: This can in principle be done following formal (normative) decision theory as introduced in Rackwitz (2000) and followed up in Baravalle and Köhler (2019). The design situations that are jointly considered in the calibration exercise would have been represented in terms of the uncertainty in the limit state and the consequences. However, so far this is practically not done in a strict manner. Instead, design situations that are jointly calibrated are considered as "regular" situations with "moderate" consequences of failure and "medium" relative cost of safety measure, according to ISO2394 (2015) arriving on the recommendation for usual design situations which is $\beta = 4.2$ for a one year reference period. This definition of the requirement for reliability based code calibration is also not unproblematic. Design equations as they are presented in the semi-probabilistic design code have evolved over a long period of time and represent the long term accumulation of engineering experience and expertise. This is very good, but during this development uncertainties in the representation of physical phenomena by the corresponding design equations have not been considered explicitly but implicitly by the introduction of conservative assumptions leading to model bias. It is in general very difficult to identify and to quantify this model bias which might be rather different in magnitude for different design equations. The biases, also referred to as "hidden safety", directly trigger the corresponding reliability of the design solutions and calibration of semi-probabilistic reliability elements to an absolute reliability requirement might not lead to the envisaged result.

Defining β_t as reliability that is represented by the design solution of a generally accepted design code: If it can be stated that the reliability that is attained by implementing a design code is acceptable and also considered as sufficiently economic, the objective of the calibration exercises might reduce to the decrease of variation of reli-

ability level among the design situations that are jointly considered. I.e. representative reliability level of the existing code (given the existing reliability elements) can be considered as the reliability target, and the minimisation of the penalty function is reducing the variability of the reliability. In Baravalle and Köhler (2017), it was discussed how such a representative value for β can be identified. As a simplification, the average reliability level of the existing code (given the existing reliability elements) can be also considered as the reliability target (Baravalle, 2017). The appeal of this relative calibration is that it is relative insensitive against modelling assumptions in regard to the uncertainty of the representation of variables. The results are also insensitive against model biases as long as it can be assumed that the bias affects all reliability elements that are subject to the calibration in the same way. However, the explicit assessment of the absolute safety level is not possible when only a relative comparison is done.

All listed interpretations of targets for reliability based calibration of semi-probabilistic design codes are lacking consistency. The design situations that a regulated by a semi-probabilistic design code are very in-homogeneous in regard to their representation by design equations and the corresponding accounted uncertainties and the inherent model biases. The introduction of an absolute value of a target reliability seems therefore not to be a feasible solution.

2.2. *Framework*

The objective of the assessment of the safety factors of existing design codes is to confirm their absolute magnitude and to evaluate whether a change of the safety factors would lead to a better correspondence with the reliability requirements stated in the code. The key points of the followed framework are:

- The assessment is formulated as a calibration problem, where the load partial factors $\boldsymbol{\gamma}_S$ are the calibration variables.
- The objective of the calibration is the minimisation of variability of reliability levels in the considered domain of design equations D. The corresponding objective function (to be minimised) is

$$M(\boldsymbol{\gamma}_S) = \sum_{k \in D} w_k \left(\beta_k(\boldsymbol{\gamma}_S) - \beta_{target}\right)^2 \qquad (1)$$

- The domain of design equations D is defined as all design equations for which the load partial factors apply.
- The domain of design equations is represented by a reduced set of design equations representing the main structural materials and loads induced by wind and snow, and permanent, sustained and intermittent actions. The importance of the different design equations is represented by the weighting factor w_k Assumptions are made for the formulation of this representative set.
- Design equations are represented in a partial factor design format as specified in the Eurocodes and with the corresponding limit states. All load and resistance related variables and model uncertainties are represented as random variables. Assumptions are made for the specification of the random variables, whereas the Probabilistic Model Code of the Joint committee on Structural Safety (JCSS, 2001) is used as the main reference.
- The target reliability level β_{target} is defined as the average reliability level of the considered set of design equations with the partial factors of the present Eurocode (EN1990:2002) (CEN, 2002).

3. Case Study Eurocodes

Published in 2002, the Eurocodes consist of 10 European Standards, EN 1990 - EN 1999, providing a common basis for the design of buildings and other civil engineering structures (CEN, 2002). In 2012 the European Commission issued a mandate (M/515 EN) for a revision of the Eurocodes in order to amend and extend the scope. The revision is currently ongoing and part of it is the assessment of the load related partial factors that are recommended by the Eurocodes by means of reliability based code calibration.

3.1. Method

3.1.1. Limit state function and design situations

A generic linear limit state function is formulated for assessing the load partial safety factors. With the limit state function different structural materials and different variables can be considered, i.e. in Eq. (2) the failure mode is dominated by a material property R_i and the loads are the effects of the self-weight (G_S), the permanent load (G_P) and one variable load (Q_j).

The design variable in Eq. (2) is determined by the design equations of CEN (2002) for the material property i and the variable load j in Eq.(3) for "6.10ab" and in Eq. (4) for "6.10". Six material properties listed in Table 2 are considered. Wind ($j = 1$), snow ($j = 2$) and imposed ($j = 3$) loads are considered.

The notation, the random variables and the probabilistic models are reported in Table 1 and Table 2. a_Q is a parameter representing different proportions between variable and permanent loads ($a_Q = 1$ for variable load only). Ten equally spaced and equally weighted values in the ranges reported in Table 2 are considered. a_G is a parameter representing different proportions between permanent load and self-weight ($a_G = 1$ for self-weight only). Three equally spaced and equally weighted values in the ranges reported in Table 2 are considered.

3.2. Assessment Strategy

3.2.1. Safety level of the existing EN1990

The assessment of the reliability level of the present Eurocodes design equations, i.e. format and partial factors according to CEN (2002) is assessed first. The weighted average reliability index of a domain D of design equations representing 6 material resistances, 3 different dominant variable loads, 3 different permanent load proportions and 10 different relative proportions of variable load relative to the total load, i.e. the weighted average from 6 x 3 x 3 x 10 = 540 design equations is computed in Eq.

$$E\left[\beta_{EC}\right] = C \sum_{k \in D} w_k \beta_k \left(\boldsymbol{\gamma}_{EC}\right) \tag{5}$$

C is a normalisation constant equal to one divided of the sum of all weights w_k. Independent from the detailed assumptions taken, the pattern as shown in Figure 1 is observed when assessing the safety level of the existing Eurocodes. The following aspects

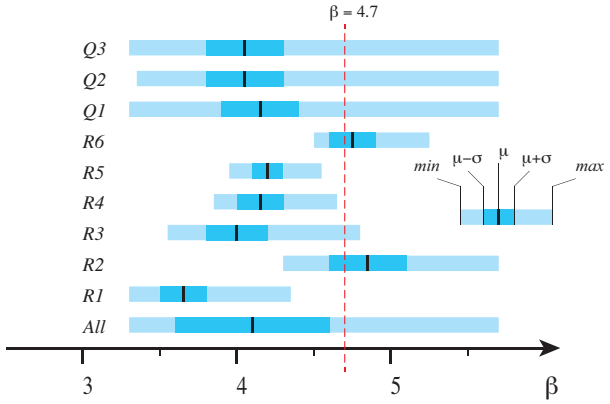

Figure 1: Magnitude and variability of the estimated yearly reliability indexes: situation with recommended safety factors given in EN1990 (CEN (2002))

are indicated by Figure 1 :

- The scatter of the reliability level considering all design equations is large.
- There is higher variability of the reliability level between materials than within a material for different loads.
- The computed average reliability levels are significantly lower than the reliability target in EN 1990:2002 that is $\beta_{target,EC} = 4.7$ for the yearly reference period.

3.2.2. Calibration of the load partial factors

The calibration is performed with the objective to reduce the variability of reliability indexes. As only the load factors are subject to calibration, it is suggested to consider the weighted average yearly reliability associated to the Eurocode recommended reliability elements as a reliability target, i.e. as determined with Eq. (5) as $\beta_{target} = E\left[\beta_{EC}\right]$.
The calibrated load partial safety factors are identified by solving the following minimisation problem over the domain of considered design equations D:

$$\boldsymbol{\gamma}_{S,opt} = \arg\min_{\boldsymbol{\gamma}_S} \sum_{k \in D} w_k \left(\beta_k(\boldsymbol{\gamma}_S, \boldsymbol{\gamma}_{R,EC}) - \beta_{target}\right)^2 \tag{6}$$

$$g(\mathbf{X}, p_{ij}) = p_{ij}\Theta_{R,i}R_i - (1-a_Q)(a_G G_S + (1-a_G)G_P) - a_Q\Theta_{Q,j}Q_j \tag{2}$$

$$p_{ij} = max\left\{\begin{array}{l} \frac{\gamma_{M,i}}{\theta_{Ri,k}r_{i,k}}\left\{(1-a_Q)\left[a_G g_{S,k}\gamma_{GS} + (1-a_G)g_{P,k}\gamma_{GP}\right] + a_Q\psi_{0,j}\gamma_Q\theta_{Qj,k}q_{j,k}\right\} \\ \frac{\gamma_{M,i}}{\theta_{Ri,k}r_{i,k}}\left\{(1-a_Q)\left[a_G g_{S,k}\zeta\gamma_{GS} + (1-a_G)g_{P,k}\zeta\gamma_{GP}\right] + a_Q\gamma_Q\theta_{Qj,k}q_{j,k}\right\} \end{array}\right\} \tag{3}$$

$$p_{ij} = \frac{\gamma_{M,i}}{\theta_{Ri,k}r_{i,k}}\left\{(1-a_Q)\left[a_G g_{S,k}\gamma_{GS} + (1-a_G)g_{P,k}\gamma_{GP}\right] + a_Q\gamma_Q\theta_{Qj,k}q_{j,k}\right\} \tag{4}$$

where $\boldsymbol{\gamma}_{S,opt} = [\gamma_{GS}, \gamma_{GP}, \gamma_{Q1}, \gamma_{Q2}, \gamma_{Q3}]$ are the load partial factors that are calibrated; $\boldsymbol{\gamma}_{R,EC}$ are the material partial factors that are not calibrated (they are fixed to the values reported in Table 2).
The results of the calibration for the design equations corresponding to Eq. (3) and Eq. (4) (equations 6.10a&b and 6.10 in the Eurocodes correspondingly) are listed in Table 3.

The results show that the safety factors for permanent actions are lower than the present value but relative similar for self-weight and permanent load and also similar for Eurocode equations 6.10 and 6.10 ab. For variable load it can be seen that all safety factors have similar values for different loads and also for Eurocode equations Eq. 6.10 and 6.10 ab. The resulting effect on the reliability indexes is illustrated in Figure 2.

The following observations can be made:

- By calibration of the load factors the variability of reliability indexes can be reduced.
- Also the variability within the different materials can be reduced considerably.
- The variability in between the materials is unaffected by the calibration of the load factors. This confirms that the resulting partial factors are insensitive to the simplistic representation of material resistance.
- This variability can only be reduced by calibration of the material factors.

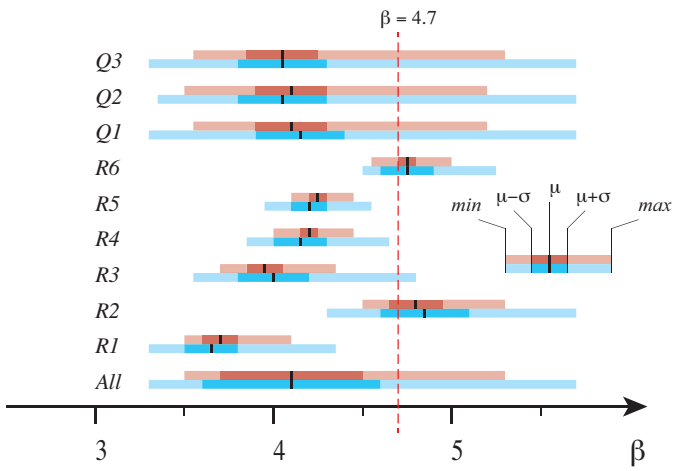

Figure 2: Magnitude and variability of the estimated yearly reliability indexes: comparison of the safety factors of EN1990 (blue) with the case where the load factors are calibrated (red). (Analysis based on Eq. 6.10 of EN 1990:2002 (CEN (2002)).

4. Conclusion

The presented framework for the assessment and calibration of existing semi-probabilistic design codes. A case study was introduced for the calibration of the load safety factors of the Eurocodes. Conditional on the assumptions, the study indicates that:

- it is not necessary to distinguish two types of permanent loads in the safety concept;
- it is reasonable to apply the same safety factor to all variable loads considered in this study;
- the existing partial factors seem too high for permanent loads;
- the existing partial factors seem too low for variable loads;

*Table 1: Stochastic models based on JCSS (2001) (*yearly maxima).*

Random variable		Distr. type	Mean(μ)	COV	Ch. Fract.(value)
Resistance model unc. (steel)	$\Theta_{R,1}$	Logn.	1.00	0.05	(μ)
Resistance model unc. (concrete)	$\Theta_{R,2}$	Logn.	1.00	0.10	(μ)
Resistance model unc. (rebar)	$\Theta_{R,3}$	Logn.	1.00	0.10	(μ)
Resistance model unc. (glulam)	$\Theta_{R,4}$	Logn.	1.00	0.10	(μ)
Resistance model unc. (solid timber)	$\Theta_{R,5}$	Logn.	1.00	0.10	(μ)
Resistance model unc. (masonry)	$\Theta_{R,6}$	Logn.	1.16	0.175	(μ)
Mat. property (steel yielding strength)	R_1	Logn.	1.00	0.07	($\mu - 2\sigma$)
Mat. property (concrete compr. capacity)	R_2	Logn.	1.00	0.15	0.05
Mat. Property (rebar yielding strength)	R_3	Logn.	1.00	0.07	0.05
Mat. property (glulam bending strength)	R_4	Logn.	1.00	0.15	0.05
Mat. property (solid timber bending strength)	R_5	Logn.	1.00	0.20	0.05
Mat. property (masonry compr. strength)	R_6	Logn.	1.00	0.16	0.05
Self-weight (steel)	$G_{S,1}$	Norm.	1.00	0.04	0.50
Self-weight (concrete)	$G_{S,2}$	Norm.	1.00	0.05	0.50
Self-weight (rebar)	$G_{S,3}$	Norm.	1.00	0.05	0.50
Self-weight (glulam)	$G_{S,4}$	Norm.	1.00	0.10	0.50
Self-weight (solid timber)	$G_{S,5}$	Norm.	1.00	0.10	0.50
Self-weight (masonry)	$G_{S,6}$	Norm.	1.00	0.065	0.50
Permanent load	G_P	Norm.	1.00	0.10	0.50
Permanent load (large COV)	$G_{P,v}$	Norm.	1.00	0.20	0.95
Wind time-invariant part	$\Theta_{Q,1}$	Logn.	0.79	0.24	(1.095)
Snow time-invariant part	$\Theta_{Q,2}$	Logn.	1.00	0.30	($\mu + \sigma$)
Imposed load model uncertainty	$\Theta_{Q,3}$	Logn.	1.00	0.10	(1.00)
Wind mean reference velocity pressure*	Q_1	Gumb.	1.00	0.25	0.98
Snow load on roof*	Q_2	Gumb.	1.00	0.40	0.98
Imposed load*	Q_3	Gumb.	1.00	0.53	0.98

Table 2: Material properties, weights and ranges of variations of a_G and a_Q.

i	Mat. property	$w_{R,i}$ (weight)	a_G range	a_Q range	$\gamma_{M,i}$ recommended in current Eurocodes
1	Structural steel yield strength	40%	[0.6; 1.0]	[0.2; 0.8]	1.00
2	Concrete compressive strength	15%		[0.1; 0.7]	1.50
3	Re-bar yield strength	25%		[0.1; 0.7]	1.15
4	Glulam timber bending strength	7.5%		[0.2; 0.8]	1.25
5	solid timber bending strength	2.5%		[0.2; 0.8]	1.30
6	Masonry compression strength	10%		[0.1; 0.7]	1.50

- the reliability target in the existing Eurocodes is higher than the average reliability level implied by the current safety factors.

For thew continuation of the study, the load partial factors should be included in the calibration. This would be done by fixing the partial load factors to the ones obtained in the presented study and calibrate the partial resistance factors per relevant ma-

Table 3: Results of the calibration.

	Eq. (3) / (6.10a&b in Eurocodes)		Eq. (4) / (6.10 in Eurocodes)	
	Eurocode values	Calibrated values	Eurocode values	Calibrated values
$\boldsymbol{\gamma}_{R,EC}$	$\boldsymbol{\gamma}_{R,EC} = [1.00, 1.50, 1.15, 1.25, 1.30, 1.50]$			
γ_{GS}	1.35	1.18	1.35	1.15
γ_{GP}	1.35	1.23	1.35	1.22
ζ	0.85	0.85	/	/
γ_{Q1} (wind)	1.50	1.62	1.50	1.63
γ_{Q2} (snow)	1.50	1.59	1.50	1.64
γ_{Q3} (imposed)	1.50	1.62	1.50	1.65

terial. This implies a representation of load bearing capacity on a higher level of detail and with due consideration of the existing expertise and literature that is available for the corresponding materials.

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