Determination of Optimum Post-earthquake Restoration Strategies for Highway Bridges by Markov Decision Process

Weifeng Tao
Post-doctoral Research Fellow, School of Civil Engineering and Environmental Science, University of Oklahoma, USA. Email: weifeng.tao-1@ou.edu

Naiyu Wang*
Professor, Zhejiang University, China. Email: naiyuwang@zju.edu.cn (Previously, School of Civil Engineering and Environmental Science, University of Oklahoma, USA.

ABSTRACT: In this paper, a life-cycle cost (LCC) based Markovian framework for determining optimum post-earthquake restoration strategies of highway bridges is proposed. LCC analysis is performed for a highway transportation network as a whole considering seismic hazards on an infinite time horizon, and the optimum restoration strategy for each bridge is determined by a continuous-time Markov decision process (CTMDP) model. The indirect economic losses are calculated according to earthquake-induced traffic congestion which is estimated by the Frank-Wolfe algorithm. The state transition probabilities used in the CTMDP are derived from probabilistic seismic hazard analysis as well as fragility analysis. A 7-node, 12-link transportation network with three bridges is studied to demonstrate the application of the proposed model.

A highway transportation network consists of various components, such as roads, bridges, and tunnels, which are spatially distributed but connected to each other to meet the needs of the community served. Severe damage to bridges after a major earthquake can impair traffic flow within the transportation network significantly. Apart from the direct costs required for repairing or rebuilding damaged bridges, significant indirect economic losses (e.g. delayed emergency response, increased travel time, business disruption, among others) will also be incurred, as have been observed in past destructive earthquakes, such as the San Fernando earthquake in 1971, the Loma Prieta earthquake in 1989, the Northridge earthquake in 1994, and the Kobe earthquake in 1995 (Priestley et al. (1996)). The state or local transportation authority must decide to either restore the damaged bridges to their original intact states or to upgrade their seismic capacities. This is a rare opportunity for enhancing the resilience of the transportation infrastructure system through well-supported restoration and rebuilt decisions.

The idea of upgrading infrastructure facilities in post-hazard recovery is encapsulated in the concept of Build Back Better (BBB), which was proposed during the multi-national recovery effort following the Indian Ocean Tsunami (Clinton (2006)). The principle of BBB provides a promising means for improving post-hazard resilience level of communities. Most of the current work in this field, however, has focused on definition and evaluation of resilience, short-term post-hazard recovery modeling, and restoration prioritization. For instance, Cimellaro et al. (2010) provided a framework for the quantitative definition of resilience using an analytical function that may fit both technical and organizational issues; Lin and Wang (2017a, b) proposed a simulation-based building portfolio recovery model to predict the functionality recovery time and recovery trajectory of both individual buildings and building portfolios following a natural hazard event, and applied the model to a midsize community. The common objective of these approaches is to restore the network functionality to their pre-hazard condition quickly through efficient resource allocation in the recovery process. Such analyses have been confined to a one-event timeframe and do not investigate the appropriate (or better) performance levels for rebuilt activities in order to mitigate a community's risk to future events.

Considering a future time horizon of hazard exposure and the need to amortize possible added restoration costs to enhance the resilience of the highway network as a whole, it is essential to adopt a life-cycle perspective as the basis for designing BBB strategies. Life-cycle cost (LCC) analysis was initially applied to the optimum design of structures subject to hazards more than four decades ago by Liu and Neghabat (1972) and Rosenblueth (1976a, b), and was gradually improved by other scholars (Kanda and Ellingwood (1991); Ang and De Leon (1997); Wen and Kang (2001a, b); Frangopol and Maute (2003)). In recent years, some LCC-based optimization frameworks for finding optimum maintenance schedules of infrastructures have been proposed (Bocchini and Frangopol (2011); Dong et al. (2014); Tapia and Padgett (2016)). In these studies, it is tacitly assumed that a bridge is restored to their original states after an earthquake, which is, however, not always the case in practice. On one hand, essential interventions (e.g. replacement of damaged structural components) are not necessarily required before the strength of the bridge degrades to a certain performance threshold; On the other hand, upgrading the bridge beyond its original intact strength is sometimes preferable from the viewpoint of network resilience and future risk.

In this paper, we propose an LCC-based Markovian framework to determine the optimum post-earthquake restoration strategies for damaged bridges in a highway transportation network. Since the occurrence of earthquakes is modeled as a Poisson process, the problem is formulated as a continuous-time Markov decision process (CTMDP).

1. PROBABILISTIC SEISMIC HAZARD ANALYSIS

The goal of probabilistic seismic hazard analysis (PSHA) is to quantify the exceedance probabilities of different ground motion intensities by taking into account the randomness in the seismic source (occurrence and intensity), propagation path, and site condition. The framework for PSHA was proposed by Cornell (1968), which has since evolved into modern PSHA.

Different models have been proposed for earthquake occurrence and intensity (McGuire (2004)). The attenuation of seismic ground motion from the epicenter to the engineering site is described by a mathematical model that relates a ground motion intensity (e.g. peak ground acceleration, spectral acceleration) to several seismological parameters including earthquake magnitude, source-to-site distance, and local site condition (Campbell (2003); Bozorgnia et al. (2010)). For simplicity, we adopt the Poisson model for earthquake occurrence, an idealized point model for earthquake source, the truncated Gutenberg-Richter law for magnitude-frequency relationship (Gutenberg and Richter (1944)), and the Campbell empirical model (Campbell (2003)) for attenuation propagation.

The seismic source model and attenuation relationship suffice to calculate the probabilistic distribution of the ground motion intensity at an individual facility. However, in the analysis of a spatially distributed infrastructural network, the correlation relationship between the ground motions must be considered. Assuming that the network consists of N facilities, the correlation between two ground motions resulting from the common hazard can be described by an exponential function (Wang and Takada (2005)):

$$\rho_{ij} = \exp\left(-\|i-j\|/L_{\mathrm{c}}\right) \tag{1}$$

where $\|i-j\|$ denotes the distance between the ith site and the jth site; L_{c} denotes the correlation length, which is taken as the maximum distance between two sites in the network. In this case, for a given moment magnitude $M_W=m$, the conditional joint PDF of the ground motion is described by the multivariate lognormal distribution:

$$f_{\mathrm{Net}}\left(\boldsymbol{y}|m\right)=\frac{\exp\left(-\frac{1}{2}\left(\ln\boldsymbol{y}-\boldsymbol{\mu}(m)\right)^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(m)\left(\ln\boldsymbol{y}-\boldsymbol{\mu}(m)\right)\right)}{\sqrt{(2\pi)^{N}\left|\boldsymbol{\Sigma}(m)\right|}} \tag{2}$$

where

$$\ln\boldsymbol{y}=\begin{pmatrix}\vdots\\ \ln y_i\\ \vdots\end{pmatrix}_{N\times 1},\ \boldsymbol{\mu}(m)=\begin{pmatrix}\vdots\\ \mu_{\ln Y_i}(m)\\ \vdots\end{pmatrix}_{N\times 1}$$
$$\boldsymbol{\Sigma}(m)=\begin{pmatrix}\ddots & & \\ & \rho_{ij}\sigma_{\ln Y_i}(m)\sigma_{\ln Y_j}(m) & \\ & & \ddots\end{pmatrix}_{N\times N} \tag{3}$$

The mean $\mu_{\ln Y_i}(m)$ and standard deviation $\sigma_{\ln Y_i}(m)$ are given in Campbell (2003). The total joint PDF of the ground motions at facility sites is

$$f_{\mathrm{Net}}\left(\boldsymbol{y}\right)=\int_{M_0}^{M_u}f_{\mathrm{Net}}\left(\boldsymbol{y}|m\right)f_{M_{\mathrm{W}}}(m)\mathrm{d}m \tag{4}$$

where the conditional PDF of magnitude $f_{M_{\mathrm{W}}}(m)$ can be derived from the Gutenberg-Richter (G-R) Law.

2. STATES AND ACTIONS

The state of a bridge network is the Cartesian product of the states of the bridges it contains. The same holds for the action. The state of an individual bridge is defined by its seismic capacity, assumed to be related to its design response spectrum. The design response spectrum for buildings in the United States is anchored to the pseudo-spectral accelerations (PSAs) S_S and S_1 associated with the risk-adjusted maximum considered earthquake (MCER) at periods of 0.2s and 1.0s, respectively (ASCE (2017)). The collapse probability of a code-compliant building should be approximately 10%. The AASHTO Guide Specifications for LRFD Seismic Bridge Design (Transportation Officials (2011)) does not have comparable requirements. Thus, to achieve a consistent measure of public safety for bridges and buildings, we assume that the collapse probability of an intact bridge under the MCER should also be approximately 10%. Obviously, the MCER that a damaged bridge can bear with the same collapse probability decreases as its aseismic capacity deteriorates. Thus, we define the aseismic capacity (C^E) of a bridge as the MCER PSA at its fundamental period corresponding to a 10% collapse probability.

Fragility analysis is used for estimating the collapse probabilities under different PSAs. A collapse fragility function takes the following form:

$$F(y)=\Phi\left(\frac{\ln(y/\alpha)}{\beta}\right) \tag{5}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function; y denotes the MCER PSA of ground motion; α and β denote the median and dispersion parameters, respectively. α is related to the aseismic capacity of the bridge, while β is set as a constant 0.6 (MRl (2003)). Given the α value, the aseismic capacity of the bridge can be easily calculated from Eq. (5):

$$C^E=\exp\left\{\Phi^{-1}(10\%)\cdot\beta\right\}\cdot\alpha \tag{6}$$

Conversely, the collapse fragility curve corresponding to a specified aseismic capacity C^E is

$$F(y)=\Phi\left(\frac{\ln\left(y\cdot\exp\left\{\Phi^{-1}(10\%)\cdot\beta\right\}/C^E\right)}{\beta}\right) \tag{7}$$

Further, the continuous range of aseismic capacity can be discretized into a series of values. Without loss of generality, we set ten bridge states from *State* 1 to *State* 10, corresponding the aseismic capacities $1.2C_0$, $1.1C_0$, …, $0.4C_0$, and $\leq 0.3C_0$. Note that $0.8C_0$, C_0, and $1.2C_0$ represent the aseismic capacities according to the low standard, the normal standard and the high standard, respectively.

Given the state of a bridge, the optional restoration actions are as follows: If it is above the prescribed threshold, which is taken as $0.5C_0$ in this paper, the bridge can continue to work or be elevated to a better state. Otherwise, the bridge has to be shut down for major repairs or complete reconstruction.

3. CONTINUOUS-TIME MARKOV DECISION PROCESSES

Markov decision process (MDP) theory can date back to as early as the 1950s (Bellman (1957)). An MDP can be discrete-time or continuous-time, depending on whether the decision epochs are uniform or random. CTMDP is briefly introduced in this section.

For the infinite-horizon decision-making problem for post-earthquake restoration, the governing stochastic optimality equation is as follows:

$$\min_{\pi} \mathrm{E}^{\pi}\left\{\sum_{n=0}^{\infty} e^{-\lambda t_n} C\left(S_n, A^{\pi}\left(S_n\right)\right)\right\} \tag{8}$$

where $\mathrm{E}\{\cdot\}$ denotes an expectation operator; π denotes a restoration policy; S and A denote a state and the action determined by the state and the policy, respectively; C is the cost incurred; λ denotes the discount rate; t_n denotes the time point of the nth earthquake occurrence.

The stochastic optimality equation can be decomposed by dynamic programming into a set of subproblems (Bradtke and Duff (1995))

$$\begin{aligned} V^*(s) &= \min_{\pi}\left(C\left(s, A^{\pi}(s)\right) + \sum_{s' \in S} P\left(s' \middle| s, A^{\pi}(s)\right)\left[\int_0^{\infty} e^{-\lambda t}\left(\nu e^{-\nu t}\right)\mathrm{d}t\right] V^*(s')\right) \\ &= \min_{\pi}\left(C\left(s, A^{\pi}(s)\right) + \frac{\nu}{\lambda+\nu}\sum_{s' \in S} P\left(s' \middle| s, A^{\pi}(s)\right) V^*(s')\right), \quad s \in S \end{aligned} \tag{9}$$

where ν denotes the mean occurrence rate of earthquakes; $P\left(s' \middle| s, A^{\pi}(s)\right)$ denotes the state transition probability from s to s', given that the restoration action $A^{\pi}(s)$ is implemented.

Eq. (9) can be solved by policy iteration which has good convergence properties (Powell (2007)). Policy iteration randomly selects an initial policy and then performs the following two steps iteratively: (1) Given a policy, evaluate the corresponding values of the states; (2) Given the values, find a better policy. The iteration procedure is illustrated below.

1. Initialization:
   Select a policy π^0 and set n=1.
2. Policy evaluation:
   Given a policy π^{n-1}, compute the values $\left\{V^{\pi^{n-1}}(s),\ s \in S\right\}$ by solving a set of linear equations as follows:
   $$V^{\pi^{n-1}}(s) = C\left(s, A^{\pi^{n-1}}(s)\right) + \frac{\nu}{\lambda+\nu}\sum_{s' \in S} P\left(s' \middle| s, A^{\pi^{n-1}}(s)\right) V^{\pi^{n-1}}(s'),\ s \in S \tag{10}$$
3. Policy improvement:
   Based on the values $\left\{V^{\pi^{n-1}}(s),\ s \in S\right\}$, find a better policy π^n defined by
   $$A^{\pi^n}(s) = \arg\min_{a \in A}\left(C(s,a) + \frac{\nu}{\lambda+\nu}\sum_{s' \in S} P\left(s' \middle| s, a\right) V^{\pi^{n-1}}(s')\right),\ s \in S \tag{11}$$
4. If $\pi^n = \pi^{n-1}$, set $\pi^* = \pi^n$ and stop; otherwise, set n=n+1 and go back to step 1.

Figure 1: Procedure of Policy Iteration

4. STATE TRANSITION

The amount of decrease in aseismic capacity is measured by damage, the conditional probability distribution of which can be derived by fragility analysis. In order to simplify the derivation, we assume that damage is independent of the state.

We define structural collapse as a complete loss of aseismic capacity. Therefore, in combination with the definition of bridge states in Section 2, the collapse of a bridge during an earthquake means the damage caused is no less than its aseismic capacity. Therefore, according to Eq. (7), given a ground motion PSA y, the conditional cumulative distribution function of damage is

$$P\left(D \geq d \middle| y\right)=\Phi\left(\frac{1}{\beta}\cdot\ln\left(\frac{y\cdot\exp\left(\Phi^{-1}(10\%)\cdot\beta\right)}{d}\right)\right) \tag{12}$$

Further, we can obtain the conditional probability masses of discrete damage magnitudes through lumping, as follows:

$$\begin{cases} P\left(D=0|y\right)=F\left(0|y\right)-F\left(0.05C_0|y\right) \\ P\left(D=0.1C_0|y\right)=F\left(0.05C_0|y\right)-F\left(0.15C_0|y\right) \\ P\left(D=0.2C_0|y\right)=F\left(0.15C_0|y\right)-F\left(0.25C_0|y\right) \\ \vdots \end{cases} \tag{13}$$

The conditional state transition probabilities can be derived from the conditional probability masses of damage as follows:

$$P\left(s'|s,y\right)\equiv\begin{cases} P\left(D=s-s'|y\right) & \text{if } s'>0 \\ \sum_{d_i\geq s} P\left(D=d_i|y\right) & \text{if } s'=0 \end{cases} \tag{14}$$

For a network of N bridges, statistical independence of conditional state transition is further assumed. Thus, the network-level conditional state transition probabilities are products of the individual components:

$$P_{\text{Net}}\left(\boldsymbol{s}'|\boldsymbol{s},\boldsymbol{y}\right)=\prod_{j=1}^{N}P\left(s_j'|s_j,y_j\right) \tag{15}$$

Finally, the total state transition probabilities $TP_{\text{Net}}\left(\boldsymbol{s}'|\boldsymbol{s}\right)$ are the convolution of $P_{\text{Net}}\left(\boldsymbol{s}'|\boldsymbol{s},\boldsymbol{y}\right)$ and $f_{\text{Net}}\left(\boldsymbol{y}\right)$, which can be dealt with by Monte Carlo simulation.

5. CASE STUDY

5.1. Description of a Hypothetical Highway Transportation Network

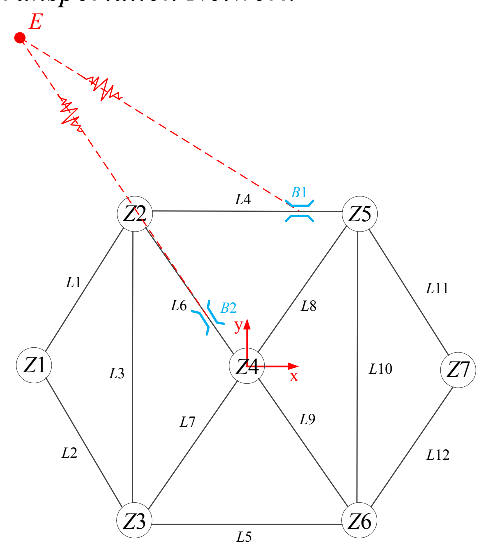

Figure 2: Schematic Diagram of Highway Transportation Network

The proposed method is applied to a hypothetical 7-node, 12-link highway transportation network, as illustrated in Figure 2. E denotes the epicenter. $Z1$ - $Z7$ and $L1$ - $L12$ are the nodes and two-way highways of the transportation network, respectively. The flow capacity of $L3$ and $L10$ is 30,000 passenger car units per day (pcu/d), while for other links the number is 60,000 pcu/d. The design traffic speed of all the links is assumed to be 60 km/h. $B1$ and $B2$ are two bridges. The origin-destination (O-D) demands are given in the following O-D matrix (pcu/d)

$$\begin{bmatrix} - & 11{,}250 & 11{,}250 & 15{,}000 & 7{,}500 & 7{,}500 & 7{,}500 \\ 11{,}250 & - & 10{,}500 & 15{,}000 & 11{,}250 & 7{,}500 & 7{,}500 \\ 11{,}250 & 10{,}500 & - & 15{,}000 & 7{,}500 & 11{,}250 & 7{,}500 \\ 15{,}000 & 15{,}000 & 15{,}000 & - & 15{,}000 & 15{,}000 & 15{,}000 \\ 7{,}500 & 11{,}250 & 7{,}500 & 15{,}000 & - & 10{,}500 & 11{,}250 \\ 7{,}500 & 7{,}500 & 11{,}250 & 15{,}000 & 10{,}500 & - & 11{,}250 \\ 7{,}500 & 7{,}500 & 7{,}500 & 15{,}000 & 11{,}250 & 11{,}250 & - \end{bmatrix}$$

And the coordinates of the nodes, bridges and the epicenter are given in Table 1.

Table 1: Two-dimensional Coordinates (km)

	Z1	Z2	Z3	Z4	Z5	Z6	Z7	E	B1	B2
X	-24	-12	-12	0	12	12	24	-24	7	-3
Y	0	16	-16	0	16	-16	0	32	16	4

The depth of the epicenter is assumed to be 20 km. The lower and upper limits of earthquake magnitude are taken as M_0=5.0 and M_u=8.5, and the mean return period of earthquakes in this range is taken as 20 yrs. The discount rate is assumed to be 4%. The *b*-value in the G-R model is assumed to be 0.8 (Petersen et al. (2008)). The attenuation coefficients in the Campbell empirical model corresponding to T=0.30s can refer to Campbell (2003). The site condition of the two bridges belongs to Class *B*. Since the Campbell empirical model is only valid for estimating ground motions on hard rocks, estimates of ground motion for a different site condition need to be modified using empirical or theoretical site factors. According to ASCE (2017), all the PSAs given by the Campbell empirical model are adjusted by an amplification factor of 1.25 (Table 11.4-1). The normal aseismic capacity is taken as C_0=0.8g.

The two bridges are assumed to be the same. The direct economic costs required for restoration or reconstruction are listed in Table 2. Note that although these data are, in fact, random, they are represented by their mean values for simplicity here.

Table 2: Direct Economic Cost (10^3 \$)

	Post-decision state		
	State 1 (High standard)	*State 3 (Normal standard)*	*State 5 (Low standard)*
State 1	*0*	-	-
State 2	*200*	-	-
State 3	*400*	*0*	-
State 4	*600*	*200*	-
State 5	*800*	*400*	*0*
State 6	*1,000*	*600*	*200*
State 7	*1,200*	*800*	*400*
State 8	*1,400*	*1,000*	*600*
State 9	*1,600*	*1,200*	*800*
State 10	*3,400*	*3,000*	*2,600*

Further, the durations required for major restoration are 45 days for *State* 8 and 60 days for *State* 9, and complete reconstruction of a bridge takes 210 days. The social influence caused by earthquake-induced traffic delay is converted into indirect economic losses, which is simply assumed to be proportional to the difference of total travel time (*TTT*) before and after an earthquake event. The empirical coefficient β is taken as 20 \$/pcu/hr. *TTT* is the summation of the daily travel time of all the passengers in the transportation network. The average daily travel flows and times on the links are estimated by the Frank-Wolfe algorithm.

5.2. Results and Discussion

Based on these parameters, the optimum post-earthquake restoration strategies for the two bridges in the highway transportation network is obtained by the proposed method, as shown in Figure 3.

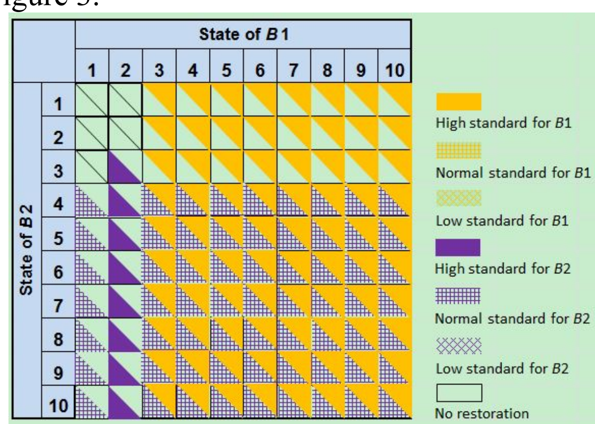

Figure 3: Optimum Post-earthquake Restoration Strategies

For *B*1, if it is in *State* 1 or *State* 2, no restoration is required; otherwise, it should be restored according to the high design standard. The optimum performance requirement for *B*2 is lower than for *B*1. When *B*1 is in *State* 2, *B*2 should be restored according to the high design standard; otherwise, it is sufficient to restore *B*2 according to the normal standard. We can tell that *B*1 is more important than *B*2 in this transportation network, which is mainly determined by the topology of the network alone since the other two critical influencing factors (source-to-site distance and normal traffic volume) are set to be equal. In addition, the restoration strategy of *B*2 depends not only on the state of its own but also on the state of *B*1.

6. CONCLUSIONS

A CTMDP-based framework for determining optimum post-earthquake restoration strategies of highway bridges is introduced in this paper and is applied to a simple highway transportation network with two bridges. Two primary conclusions are drawn from the results: (1) The importance ranking of bridges in a transportation network is determined by their locations, apart from the common influencing factors; (2) The optimum restoration strategy of a bridge not only depends on its own state, but may also depend on the states of other bridges.

7. ACKNOWLEDGEMENT

This research was supported by the National Key R&D Program of China (Grant No. 2016YFC0800200) and by the US National Institute of Standards and Technology (NIST) under Cooperative Agreement No. 70NANB15H044.

8. REFERENCES

Ang, A.-S., and De Leon, D. (1997). "Determination of optimal target reliabilities for design and upgrading of structures." *Structural Safety*, 19(1), 91-103.

ASCE (2017). "Minimum design loads for buildings and other structures (Standards ASCE/SEI 7-16)." Amer Society of Civil Engineers.

Bellman, R. (1957). "A Markovian decision process." *Journal of Mathematics and Mechanics*, 679-684.

Bocchini, P., and Frangopol, D. M. (2011). "A probabilistic computational framework for bridge network optimal maintenance scheduling." *Reliability Engineering & System Safety*, 96(2), 332-349.

Bozorgnia, Y., Hachem, M. M., and Campbell, K. W. (2010). "Ground motion prediction equation ("attenuation relationship") for inelastic response spectra." *Earthquake Spectra*, 26(1), 1-23.

Bradtke, S. J., and Duff, M. O. "Reinforcement learning methods for continuous-time Markov decision problems." *Proc., Advances in neural information processing systems*, 393-400.

Campbell, K. W. (2003). "Prediction of strong ground motion using the hybrid empirical method and its use in the development of ground-motion (attenuation) relations in eastern North America." *Bulletin of the Seismological Society of America*, 93(3), 1012-1033.

Cimellaro, G. P., Reinhorn, A. M., and Bruneau, M. (2010). "Framework for analytical quantification of disaster resilience." *Engineering Structures*, 32(11), 3639-3649.

Clinton, W. J. (2006). "Lessons learned from tsunami recovery: Key propositions for building back better." *New York: Office of the UN Secretary-General's Special Envoy for Tsunami Recovery*.

Cornell, C. A. (1968). "Engineering seismic risk analysis." *Bulletin of the seismological society of America*, 58(5), 1583-1606.

Dong, Y., Frangopol, D. M., and Saydam, D. (2014). "Pre-earthquake multi-objective probabilistic retrofit optimization of bridge networks based on sustainability." *Journal of Bridge Engineering*, 19(6), 04014018.

Frangopol, D. M., and Maute, K. (2003). "Life-cycle reliability-based optimization of civil and aerospace structures." *Computers & Structures*, 81(7), 397-410.

Gutenberg, B., and Richter, C. F. (1944). "Frequency of earthquakes in California." *Bulletin of the Seismological Society of America*, 34(4), 185-188.

Kanda, J., and Ellingwood, B. (1991). "Formulation of load factors based on optimum reliability." *Structural safety*, 9(3), 197-210.

Lin, P., and Wang, N. (2017a). "Stochastic post-disaster functionality recovery of community building portfolios I: Modeling." *Structural Safety*.

Lin, P., and Wang, N. (2017b). "Stochastic post-disaster functionality recovery of community building portfolios II: Application." *Structural Safety*.

Liu, S. C., and Neghabat, F. (1972). "A cost optimization model for seismic design of structures." *Bell Labs Technical Journal*, 51(10), 2209-2225.

McGuire, R. K. (2004). *Seismic hazard and risk analysis*, Earthquake Engineering Research Institute.

MRl, H. (2003). "Multi-hazard loss estimation methodology: Earthquake model." *Department of Homeland Security, FEMA, Washington, DC*.

Petersen, M. D., Frankel, A. D., Harmsen, S. C., Mueller, C. S., Haller, K. M., Wheeler, R. L., Wesson, R. L., Zeng, Y., Boyd, O. S., and Perkins, D. M. (2008). "Documentation for the 2008 update of the United States national seismic hazard maps." Geological Survey (US).

Powell, W. B. (2007). *Approximate Dynamic Programming: Solving the curses of dimensionality*, John Wiley & Sons.

Priestley, M. N., Seible, F., and Calvi, G. M. (1996). *Seismic design and retrofit of bridges*, John Wiley & Sons.

Rosenblueth, E. (1976a). "Optimum design for infrequent disturbances." *Journal of the Structural Division*, 102(9), 1807-1825.

Rosenblueth, E. (1976b). "Towards optimum design through building codes." *Journal of the Structural Division*, 102(3), 591-607.

Tapia, C., and Padgett, J. E. (2016). "Multi-objective optimisation of bridge retrofit and post-event repair selection to enhance sustainability." *Structure and Infrastructure Engineering*, 12(1), 93-107.

Transportation Officials, S. o. B. (2011). *AASHTO guide specifications for LRFD seismic bridge design*, AASHTO.

Wang, M., and Takada, T. (2005). "Macrospatial correlation model of seismic ground motions." *Earthquake Spectra*, 21(4), 1137-1156.

Wen, Y., and Kang, Y. (2001a). "Minimum building life-cycle cost design criteria. I: Methodology." *Journal of Structural Engineering*, 127(3), 330-337.

Wen, Y., and Kang, Y. (2001b). "Minimum building life-cycle cost design criteria. II: Applications." *Journal of Structural Engineering*, 127(3), 338-346.