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# Real time structural damage detection using recursive singular spectrum analysis

Basuraj Bhowmik
*PhD, Dept. of Civil Engineering, Indian Institute of Technology, Guwahati, India*

Budhaditya Hazra
*Assistant Professor, Dept. of Civil Engineering, Indian Institute of Technology, Guwahati, India*

Vikram Pakrashi
*Assistant Professor, Dynamical Systems and Risk Laboratory, School of Mechanical and Materials Engineering & Marine and Renewable Energy Ireland (MaREI), University College Dublin, Ireland*

ABSTRACT: A baseline independent approach towards identifying structural damage in real time for dynamically vibrating systems using recursive singular spectral analysis (RSSA) is proposed. The principal characteristic of the method lies in its inherent ability to identify events using the inputs from a *single sensor*. The algorithm takes the input of a single channel in real time and produces a time lagged Hankel matrix from the series. Streaming acceleration data is used to obtain recursive proper orthogonal components online using first order perturbation (FOP) method. This generates transformed responses at each time stamp, which provide information regarding the current state of the structure. Subsequently, the need for offline post processing is eliminated by facilitating a continuous real time monitoring, without resorting to baseline data. As time series models work extremely well in discerning the key patterns of such responses, time-varying autoregressive (TVAR) modeling is adopted. Numerical simulations performed on a nonlinear base isolated system excited using white noise provide successful detection results. The applicability of the proposed algorithm towards experimental studies is described in detail using a cantilever beam setup subjected to earthquake excitation. Successful detection results indicating a change of state in real time validate the robustness of the algorithm towards catering a wide range of application specific problems.

## 1. INTRODUCTION

Assessing the performance of existing infrastructure requires a thorough evaluation of all the components in a system. However, cost, accessibility, unavailability of sensors and other factors hinder a successful monitoring process. Literature [1,2,3] provides instances where structures instrumented at *multiple floor levels* have been monitored using signal processing tools. In this paper, a recently developed approach known as *recursive singular spectrum analysis (RSSA)* is considered, which aids in real time monitoring of structures using a *single sensor input*. The method functions by separating a multi-frequency signal into its simpler components and excluding the noise components, thereby generating transformed responses [6,8,9]. This paper addresses the damage detection capabilities of the algorithm through the use of damage sensitive features (DSFs) [9].

The process of structural health monitoring (SHM) entails the implementation of damage detection strategies to monitor the state of a structure or mechanical system over time using periodically spaced response measurements [10]. The extraction of DSFs from time histories and their subsequent statistical analysis aids in determining the current state of system health [1-5,7,9]. Damage detection strategies are well reported in literature [2,3,4,5]. However, *real-time damage detection* schemes capable of baseline-free damage identification in the presence of both sparse and dense sensor leads to challenges. Added to these issues are the complexities due to computational load [10]. The present study addresses some of these issues by online detection and localization of damage under a single framework, using data from a single channel.

The key motivation of the present work isin developing a damage detection framework using *single sensor characteristics* that can process online data and detect damage in a system in real time [9]. Conventional SSA requires gathered response data to process and identify the damage parameters. The implementation of RSSA through first order perturbation (FOP) techniques however, provides online updates [8]. RSSA provides real time processing of data based on rank-one eigenvector updates in a recursive framework for streaming data[9]. Once the eigen space updates are obtained, an approximation of the original time series is carried out by reconstruction, where the framework then utilizes time-varying autoregressive (TVAR) modeling in conjunction with DSFs for identifying the instant of damage.

The main contributions of this work are as follows: *First*, a damage detection framework using RSSA as a real time approach is provided. *Second*, the use of RSSA for a fairly representative class of nonlinear systems achieves a lower limit of detectability of the order of 15%, in real time. *Finally*, a detailed description of simultaneous spatio-temporal damage detection is presented. The proposed method is shown to perform through real time updates without demanding computational expense or memory allocations of a higher order.

The paper is organized as follows. First, a brief description of RSSA is presented using a data driven approach. The use of TVAR models on transformed responses and the associated formulation is described next in detail. The utility of the proposed method is demonstrated with the aid of numerical examples on a 5 DOF Buoc-Wen (B-W) model undergoing damage in real time. To validate these numerically simulated findings, detection results using experimental setups devised in a lab environment are also provided. These aid in to demonstrating the efficiency and robustness of the proposed algorithm towards practical situations.

## 2. RECURSIVE SINGULAR SPECTRUM ANALYSIS (RSSA)

Recent research in signal processing and information technology has identified SSA as an efficient damage detection tool, leading to the development of SSA-based structural damage detection algorithms [8,9]. However, the major disadvantage associated with SSA is its conventional nature that allows the processing of data only in batch mode operations. One of the key challenges faced in tailoring basic SSA towards an online stature lies in the recursive update of the Hankel covariance matrix that is computationally expensive. In order to address this shortcoming, the FOP based approach provides recursive updates of eigen subspace from the previous eigen space of the data at a particular time instant. The basic RSSA expression can be structured as [9]:

$$\mathbf{C}_k = \frac{k-1}{k}\mathbf{C}_{k-1} + \frac{1}{k}\mathbf{X}_k\mathbf{X}_k^T \tag{1}$$

The covariance estimate at $k^{th}$ instant, $\boldsymbol{C}_k$ can be written in terms of eigenvalue ( $\boldsymbol{P}_k$ ) and eigenvector matrix ( $\boldsymbol{\omega}_k$ ) at a particular time instant as: $\mathbf{C}_k = \mathbf{P}_k\boldsymbol{\omega}_k\mathbf{P}_k^T$ with $\theta_k = \mathbf{P}_{k-1}^T\mathbf{X}_k$ ,denoting the projection of the sample vector ( $\boldsymbol{X}_k$ ) into the previous eigen subspace ($\boldsymbol{P}_{k-1}$). Substituting these expressions in Eq. (1), one can obtain:

$$\mathbf{P}_k k\boldsymbol{\omega}_k\mathbf{P}_k^T = \mathbf{P}_{k-1}\{(k-1)\omega_{k-1}+\theta_k\theta_k^T\}\mathbf{P}_{k-1}^T \quad (2)$$

Corresponding to a finitely large sample size ($k$) for systems having low to moderate damping estimates, the term $(k-1)\omega_{k-1}+\theta_k\theta_k^T$ shows diagonally dominant characteristics. This ensures the subsequent application of Gershgorin's theorem in order to recursively estimate the eigenspace at this point forward [9]. Considering the eigen decomposition of the term in the form $\mathbf{U}_k\boldsymbol{\gamma}_k\mathbf{U}_k^T$, where the eigenspace is represented by $\boldsymbol{U}_k$ and $\boldsymbol{\gamma}_k$, the following expression can be obtained:

$$\mathbf{P}_k k\boldsymbol{\omega}_k\mathbf{P}_k^T = \left(\mathbf{P}_{k-1}\mathbf{U}_k\right)\boldsymbol{\gamma}_k\left(\mathbf{U}_k^T\mathbf{P}_{k-1}^T\right) \quad (3)$$

The recursive update of the eigenspace can be finally estimated using the following set of equations:

$$\left.\begin{aligned}\mathbf{P}_k &= \mathbf{P}_{k-1}\mathbf{U}_k\\ \omega_k &= \frac{\gamma_k}{k}\end{aligned}\right\} \quad (4)$$

After obtaining the eigen space updates at a particular time instant from the previous eigen space and current sample vector, principal component (PC) values of the time series at a particular time stamp ($\xi_i(k)$) can be extracted as follows:

$$\xi_i(k) = P_i^T(k)X_k \quad (5)$$

Depending upon the relative contribution of the eigenvectors, the number of PCs required for reconstruction can be automated. The present study the PCs of eigenvectors that explain more than 90% of the system's kinetic energy. The $i^{th}$ PCs are projected back into its original subspace to obtain last column of the corresponding elementary matrix, can be updated as [9]:

$$\mathbf{R}_i(k) = \mathbf{P}_i(k)\xi_i(k) \quad (6)$$

The above expression provides the final form of reconstruction provided by the RSSA algorithm. The importance of this reconstructed signal is that it has simpler components, making it amenable towards a lower TVAR order model.

## 3. RECURSIVE DAMAGE SENSITIVE FEATURES (DSFs)

RSSA functions online by providing a set of transformed responses at each instant. The recursive updates of eigenvectors and eigenvalues, referred to as eigenspace, subsequently gets updated in real time. As the eigenspace cannot indicate deviations corresponding to damage inflicted to the system, it is processed by a set of DSFs [1,7,9]. The key DSF utilized in the paper are the TVAR coefficients. The estimation of AR coefficients frequently requires the use of windowing and baseline data to detect damage, making online implementation difficult. These drawbacks motivate the need of using a TVAR modelling based framework independent of baseline to detect damage in real time. The proposed method tracks the TVAR coefficients in real time, which are used to ascertain damage induced in the structure by a change in the mean level of the plot, at a time point very close to the instant of damage. The near mono-component nature of the transformed response ensures that low model order is sufficient to capture its dynamics [1,9]; therefore, in the proposed framework, a model order of 2 (two) is pre-selected for all the cases. The basic expression can be written as follows:

$$\psi_1(\mathrm{k}) = \mathrm{a}_1(k)\psi_1(\mathrm{k}-1)+\mathrm{a}_2(k)\psi_2(\mathrm{k}-2)+\nu(\mathrm{k}) \quad (7)$$

where, $a_1(k)$ and $a_2(k)$ are the associated TVAR coefficients. As these coefficients are expected to show alterations in their values at the instant of damage, tracking them in real time serves as a DSF.

## 3. PROPOSED ALGORITHM

The RSSA algorithm comprises of two detection modules that occur in succession. The primary module deals with identifying the temporal aspect of damage, while the spatial module localizes the damage in the system. At the onset of the streaming physical response, the RSSA algorithm provides transformed response at each time stamp. The eigenspace spanned by the Hankel covariance estimated gets updated online. At a possible damage event, distortions in the eigenspace need to be tracked using certain DSFs (such as the TVAR coefficients) that are recursively tracked in real time. Once the damage instant is identified, the spatial damage detection module is invoked that localizes the damage to the system. The basic steps of the algorithm are enumerated as follows:

- First, traditional SSA is employed on some initial data points (around 100 in number) in order to estimate the initial eigenspace. The number of data points chosen are pre-selected, only used for initial stabilizing and subsequent calculations for the recursive approach.
- The RSSA algorithm then operates on the real-time input of the streaming data. This generates a time lagged Hankel matrix out of the set of physical responses obtained from a single sensor.
- Using the memory shift parameter, the Hankel covariance estimate at the present time instant is derived using the covariance estimate at preceding time instant. From the recursive updates, the eigenspace is updated using FOP approach and the transformed responses are obtained using the RSSA algorithm at each time point. An approximate time series is reconstructed according to the relative order of significance of the corresponding eigenvalues.
- TVAR models are fit to the transformed responses that act as DSFs. These are tracked in real time to extract the changes inflicted due to damage in the system. Once the presence of damage is ascertained, the spatial damage detection module helps in localizing the damage to the system.

## 4. NUMERICAL CASE STUDIES

In order to illustrate an application of the proposed method, numerical simulations are performed on a 5-DOF system modeled with a B-W oscillator at the base [1,7,9]. Alterations in the nonlinear force term are contextually defined as damage. The equation of motion for the system can be summarized as:

$$\mathbf{M}\ddot{u}+\mathbf{C}\dot{u}+\mathbf{K}u=\Lambda\mathbf{f}-\mathbf{MI}\ddot{\mathbf{u}}_g \tag{8}$$

A simple shear building representation is assumed to arrive at the expressions for assembled mass ($\boldsymbol{M}$), damping ($\boldsymbol{C}$), and stiffness ($\boldsymbol{K}$) matrices respectively, which are skipped here for brevity. For each of the four floor levels above the base, the values for the respective parameters are $7461\,kg$, $23{:}71\,kNs/m$ and $11912kN/m$; while the values for the base are $6800\,kg$, $3{:}74\,kNs/m$ and $232\,kN/m$ respectively. In Eq. (8), $\ddot{\mathbf{u}}_g$ represents the ground acceleration and $\Lambda$ represents the location of the base at the point of application of the nonlinear force ($\boldsymbol{f}$) due to the LRB base isolator, given by:

$$f=\kappa z Q_{pb}-k_b x_b-c_b\dot{x}_b \tag{9}$$

Here, $Q_{pb}=\left(1-\dfrac{k_{yield}}{k_{nitial}}\right)Q_y$, $Q_y$ being the yield force, and $\boldsymbol{k_b}$ and $\boldsymbol{c_b}$ are the base stiffness and viscous damping respectively. The evolutionary variable $z$ is obtained by the solution of the following nonlinear differential equation:

$$z=-\gamma z\left|\dot{x}_b\right|\left|z^{n-1}\right|-\beta\dot{x}_b\left|z^n\right|+A\dot{x}_b \tag{10}$$

where, $\gamma, \beta, A\ and\ n$ are the shape parameters for the system.

### *5.1 Temporal damage detection results*

Case studies for 15% change in nonlinear force term for the B-W model is considered. The numerically simulated case study considers a damage event at 31s from the start of the excitation. Out of the available response measurements obtained from the B-W system, the acceleration of the top floor is considered as an input to the RSSA algorithm. As the transformed responses obtained using the RSSA algorithm consist of simpler components, a relatively low model order (which is 2 in this article) can be used. The transformed responses are modeled using TVAR coefficients that are tracked at each time stamp. As evident from Fig. 1, the distinct mean shift of the TVAR coefficients indicate the exact instant of damage at 31s.

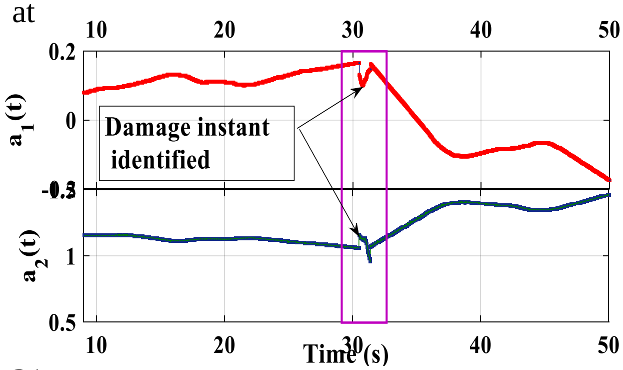


*Figure 1: Temporal damage detection using TVAR coefficients as DSFs.*

The efficacy of the DSFs for less than 15% damage is slightly questionable which indicates that the current online framework is less reliable when the extent of damage suffered is low (i.e. less than 15%). The results are therefore not reported here for brevity. However, hybrid approaches have shown real time damage detectability in the order of 10%, which has been reported in an independent work by the authors.

*5.2 Spatial damage detection results*

The local damage is induced to the B-W model through a change in the linear story stiffness at each floor level. Case studies for successive floor-wise local damage at different instants of time are provided that are validated through the use of TVAR plots. Such instances occur in practical scenarios such as an earthquake, where the stiffness of each floor level degrades with the passing time of excitation. The simulation case considered here involves reduction of the linear stiffness of individual story by 15% at each 10-s span that continues for a total duration of 50s. Based on this notion, damage was numerically induced to the fifth floor at 10 s from the start, to the fourth floor at 20 s, to the third floor at 30 s, to the second floor at 40 s, and to the first floor at 50 s. The acceleration response from individual floor levels is provided as input to the algorithm to be processed in real time.

The response from the fifth floor is first provided as input to the proposed method and subsequent TVAR models are fit. As the first damage occurrence takes place at 10s, Fig. 2 shows a peak corresponding to the 5$^{th}$ floor. The damage proceeds to the 4$^{th}$ floor where the input response is again processed using the RSSA approach. An interesting observation to make here is that as the damage occurs at 20s for this floor level, there is a peak corresponding to the 4$^{th}$ floor. However, an additional peak indicating a possible mean shift is also observed at 10s, clearly indicating that a damage had previously occurred at that instant, corresponding to the top floor. Similar observations can also be made for the 3$^{rd}$ floor, where the storey-level damage took place at 30s. But the damage indicative of the previous storey-event is clearly evident by a peak at 20s, indicating that the damage constitutes a steadily progressive degradation event. Following same lines of development, the mean shifts corresponding to the remaining floors are illustrated in Fig. 2. This clearly concludes that the proposed method is potentially robust in determining local damage cases using single sensor input.

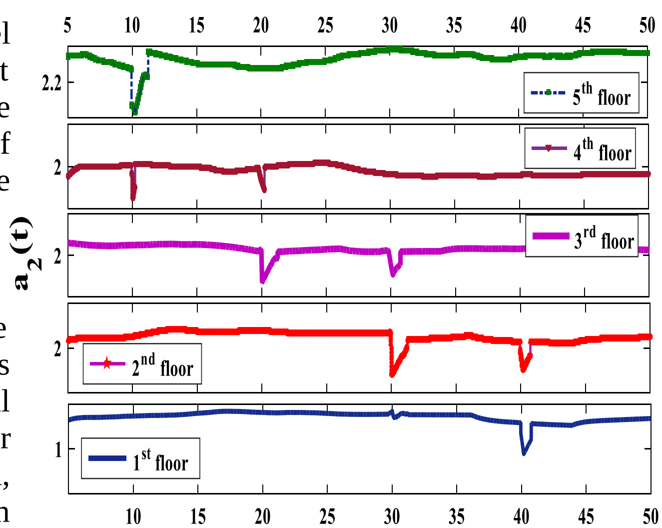


*Figure 2: Local damage detection for all floors*

## 5. EXPERIMENTAL CASE STUDIES

Towards identifying the change of state from a nonlinear to a linear regime and subsequently determining the presence of damage, an experimental setup has been devised under controlled laboratory environment. The setup consists of an aluminium beam of dimension $120\ cm x 3.5\ cm x 3.5\ cm$ fixed on a base plate, drilled on top of a shake table. For detailed specification of the setup, the readers are kindly referred to **[**1,7,9**]** The intent of this trial was also to assess the detection capability of the proposed method towards identifying damage scenarios for nonstationary excitations as well. The setup has a thin rubber strip attached to the free end of the cantilever beam that is snapped at an instant of time in order to emulate a damage condition. The experimental trial commences by subjecting the setup to earthquake excitation of scaled magnitude for a period of about 80s. The acceleration data are collected using QuantumX MX410 HBM Data Acquisition System (DAQ) at a sampling frequency of 75 Hz [9]. The aluminum beam model is instrumented using Honeywell accelerometers TEDS by HBM at four positions. The positions of the four accelerometers from the free end are $1, 30, 47$, and $81\ cm$, respectively. The rubber strip serves the purpose of providing a stiffening type of nonlinearity, the change of which can be construed as damage.

In order to simulate a real-time damage scenario, the rubber strip attached to the free end of the cantilever beam is snapped accurately at a fixed time instant, during the vibration of the beam [12]. As there remains uncertainties during experimentation, human errors and noisy environments, the trials were recorded in the form of video clips in order to cross-check the exact instant of damage, manually. Through repeated trials, it was observed that there is at least a possible error of 0.5-1s during the time of snap, that should be accounted for, while estimating the instant of damage. At 33s from the start of the excitation, the rubber strip was snapped. The snapping action closely emulates a real life damage case where the stiffness of a system suddenly deteriorates due to forced excitation. The representation of the damage event can be represented using Hilbert Huang spectrum (HHS) plot that identifies the changes in frequency with respect to time [11]. This provides a first-hand approximation of a possible event. As clearly evident from Fig. 3, the mean shift that takes place corresponding to 33s from the start, indicates a possible event that might correspond to damage. However, the shift in the mean plot of the HHS should not be straightaway adjudged as a damage event as the presence of noisy background compromises the reliability of such a visual indicator. Hence, an accurate estimation of the same can be done using the proposed RSSA based approach.

In this approach, the acceleration response from the sensor placed closest to the damage location is considered for analysis. The proposed algorithm is applied on the raw acceleration data streaming in real time that generates transformed responses at each instant of time. Using time-series models of appropriate order, the TVAR coefficients can be found. It can be clearly observed from Fig. 4 that a mean shift of the plot at 33s indicates a damage event. Further, the

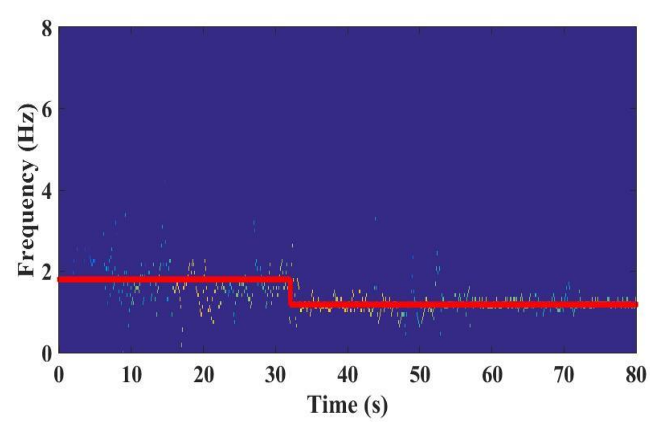


*Figure3: Use of HHS to represent damage event*

figure illustrates that the change in mean continues till the remainder of the excitation, thereby indicating that the damage has certainly not repaired over the course of time. Therefore, the applicability of the proposed method towards identifying damage events under nonstationary excitations can be justified.

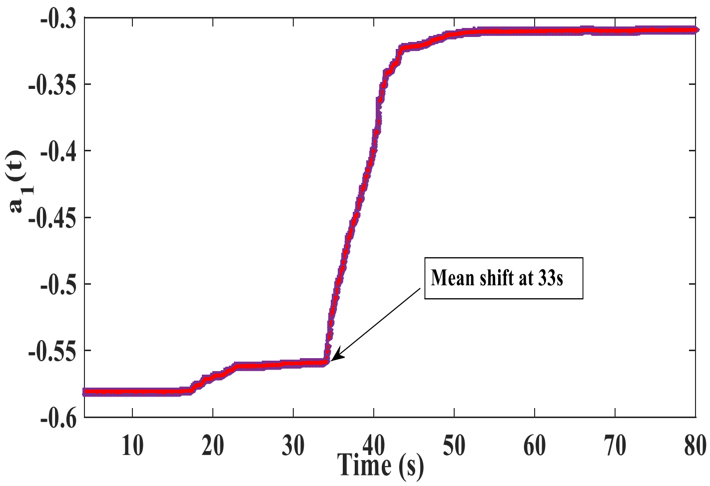


*Figure4: Identification of damage instant for the experimental case*

## 9. CONCLUSIONS

A real time damage detection algorithm for vibrating systems based on RSSA in conjunction with TVAR model is presented. Recursive updates of the eigen subspace using rank one perturbation facilitated real time evolution of the principal components. Subsequent modelling of principal component explaining maximum variance makes the transformed response amenable to a low order TVAR model which is a key step of the proposed framework. The use of TVAR models served as DSFs for estimating the presence of damage. Subsequent analysis through the spatial module revealed the location of damage for an application specific problem. . The proposed framework provided successful detection results for damages even up to 15% for both the global and local damage cases, using single sensor input data. This is of prime significance especially in situations where instrumentation at all DOF is not feasible considering cost and other factors. Numerically simulated case studies reveal that the proposed RSSA based approach is capable to identify the damage patterns to a system in real time. Additionally, the method also captures a real time degradation event of the floor levels for the B-W model. This is of significant importance considering real life cases where forced excitations induce a step wise deterioration to a vibrating system. Presented case studies show that the proposed approach results in successful damage detection and works well even when used with both experimentally acquired data under controlled laboratory conditions. The robustness of the method is validated through its detection for test beds subjected to nonstationary input excitation. Global damage detection problems for different categories of nonlinearities associated with a system is kept as an extension of the current work to be dealt with in the future.

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